Succeed with maths – Part 2

Contents

Introduction and Guidance

Introduction

Succeed with maths – Part 2 is a free badged course which lasts 8 weeks, with approximately 3 hours' study time each week. You can work through the course at your own pace, so if you have more time one week there is no problem with pushing on to complete another week's study.

You'll start with looking at different units of measurement for length and how to convert between these, before moving onto the same for volume and mass (known as weight in everyday language). In following weeks you'll move on to look at scientific notation, basic geometry and ways of representing and analysing data. You'll use plenty of real-life examples to help with this and get plenty of opportunities to practise your new understanding and skills.

Part of this practice will be the weekly interactive quizzes, of which Weeks 4 and 8 will provide you an opportunity to earn a badge to demonstrate your new skills. You can read more on how to study the course and about badges in the next sections.

After completing this course you will be able to:

- understand the SI and imperial systems of measurement
- understand and use scientific notation in different situations
- describe and calculate the basic properties of circles, four sided shapes and triangles
- understand some different ways of representing and analysing data.

Moving around the course

The easiest way to navigate around the course is through the 'My course progress' page. You can get back there at any time by clicking on 'Go to course progress' in the menu bar. From the quizzes click on 'Return to Succeed with maths − Part 2'.

It's also good practice, if you access a link from within a course page, including links to the quizzes, to open it in new window or tab. That way you can easily return to where you've come from without having to use the back button on your browser.

Viewing maths and equations

As you view the different text styles and symbols used to display maths throughout the course, you may notice that some of the text appears clickable or linked to something else. This is part of the equation processing software used by The Open University. You can ignore these clicks or links as they won't provide you with any useful information, but if you do click by accident you can just come back to where you were by using the back button.

What is a badged course?

While studying *Succeed with maths – Part 2* you have the option to work towards gaining a digital badge.

Badged courses are a key part of The Open University's mission *to promote the educational well-being of the community.* The courses also provide another way of helping you to progress from informal to formal learning.

To complete a course you need to be able to find about 24 hours of study time, over a period of about 8 weeks. However, it is possible to study them at any time, and at a pace to suit you.

Badged courses are all available on The Open University's OpenLearn website and do not cost anything to study. They differ from Open University courses because you do not receive support from a tutor. But you do get useful feedback from the interactive quizzes. What is a badge?

What is a badge?

Digital badges are a new way of demonstrating online that you have gained a skill. Schools, colleges and universities are working with employers and other organisations to develop open badges that help learners gain recognition for their skills, and support employers to identify the right candidate for a job.

Badges demonstrate your work and achievement on the course. You can share your achievement with friends, family and employers, and on social media. Badges are a great motivation, helping you to reach the end of the course. Gaining a badge often boosts confidence in the skills and abilities that underpin successful study. So, completing this course should encourage you to think about taking other courses.

How to get a badge

Getting a badge is straightforward! Here's what you have to do:

- read each week of the course
- score 50% or more in the two badge quizzes in Week 4 and Week 8.

For all the quizzes, you can have three attempts at most of the questions (for true or false type questions you usually only get one attempt). If you get the answer right first time you will get more marks than for a correct answer the second or third time. If one of your answers is incorrect you will often receive helpful feedback and suggestions about how to work out the correct answer.

For the badge quizzes, if you're not successful in getting 50% the first time, after 24 hours you can attempt the whole quiz, and come back as many times as you like.

We hope that as many people as possible will gain an Open University badge – so you should see getting a badge as an opportunity to reflect on what you have learned rather than as a test.

If you need more guidance on getting a badge and what you can do with it, take a look at the OpenLearn FAQs. When you gain your badge you will receive an email to notify you and you will be able to view and manage all your badges in My OpenLearn within 24 hours of completing the criteria to gain a badge..

Get started with Week 1.

Week 1: Measurement of length

Introduction

Welcome to the first week of *Succeed with maths – Part 2*. The first two weeks of this course will focus on measurement and the units that are used to communicate this.

People use numbers to solve a wide variety of different problems, including when designing buildings, navigating and working out the fuel consumption of an aircraft. These examples all use measurements that involve distance and it is these types of situations this week will be concentrating on. For this, it is important that you have a working knowledge of various units of measurement and the two different measurement systems used in the UK – The Système Internationale (SI) or International System, on which the metric system is based, and imperial. Both of these will be covered over the next two weeks, and this week will look in particular at measuring length.

Watch the course author Maria Townsend introduce Week 1:

Video content is not available in this format.

After this week's study, you should be able to:

- understand how the International System (SI) is formulated
- understand the more common SI units for length and convert between them
- recognise the more common imperial units for length and convert between these
- understand how the SI and imperial units of length relate to each other.

If you haven't seen *Succeed with maths – Part 1* yet and would like to study this first please follow the link: *Succeed with maths – Part 1*.

Before you start, The Open University would really appreciate a few minutes of your time to tell us about yourself and your expectations of the course. Your input will help to further improve the online learning experience. If you'd like to help, and if you haven't done so already, please fill in this optional survey.

1 Using units of measurement

Everyday problems where you want to determine how much there is of a quantity involve measurement. Things are measured for a variety of different reasons, but in everyday life this is usually to find out how big, how long or how heavy something is, or its volume, length or mass.

For these numbers to have any meaning, you need to specify what units the measurement is in. If you were told that it was 'three' to the nearest hospital that wouldn't be much good unless you knew the units as well. Is that three feet, three metres, three miles or three kilometres? Each is a very different distance from you. So, it is very important when dealing with measurements to always state the units being used.

The units will also tell you what system of measurement has been used. Most of the scientific world has adopted an internationally agreed system of measurement, using metric units such as metres and litres. The rules for their use form the Système Internationale (International System), simply known as the SI, whereas in everyday life you may still be using some imperial units, such miles and pints. The system you prefer will depend on what you are most comfortable with. This course looks at both systems of measurement because you probably encounter both in your daily life. This may well also be the case in any university level study. Although, most work will be carried out using the SI, it may be that work from other countries, such as the USA, and historical studies will employ imperial measures.

The next section starts with a look at how the SI is put together generally and then moves onto the SI and imperial units when working with length.

2 The International System (SI)

One major advantage to the SI is that everything you could ever want to measure can be measured using a few basic units, or combinations of them. As well as this, the different sizes of units to measure the same quantity, such as a distance, are all based upon the number ten. This makes calculations with SI units relatively straightforward compared to the imperial system, where there was no standard relationship between units, as you'll see later.

Most of the scientific and engineering world agreed to use the SI system to standardise their work. However, there are some important exceptions to this, and the USA is one of these. This means that it is very important to check the units being used if an international team is working on a project, as the following costly example demonstrates:

In 1999, a NASA Mars Orbiter spacecraft was destroyed on arrival in the Martian atmosphere at a loss of $125 million. An inquiry established that the flight system software on board the Orbiter was written to calculate thruster performance in metric newtons (N), but mission control on earth was inputting course corrections using the imperial measure, pound-force (lbf). One newton is about 0.22 pounds-force, so there was a considerable difference between the two. An expensive mistake to make!

The SI system works with a combination of base units and prefixes. An example of a base unit is the metre – this is the unit of measurement on which the other length units are based. You may well recognise some of prefixes, such as 'kilo-', 'centi-' and 'milli-'.

Combined with the metre, these give kilometre, centimetre and millimetre respectively. All these prefixes have specific numerical meanings, as shown below:

- 'kilo-' a thousand or 1000
- 'centi-' a hundredth or 0.01
- 'milli-' a thousandth or 0.001.

When they are added to a base unit, such as the metres in our example, they alter the size of the unit by an amount defined by the prefix.

So, kilo combined with a base unit means a thousand times the size of the base unit, centi means a hundredth of the size and milli means a thousandth of the size of the base unit.

This idea extends to cover all seven SI base units, which are shown in Table 1:

Table 1 The seven base SI units

Unit name	Unit abbreviation	Measurement
metre	m	distance
kilogram	kg	mass
second	s	time
ampere	A	electrical current
kelvin	K	temperature
mole	mol	number of particles
candela	cd	light intensity

In these two weeks of study you will be using the units for distance and mass from the SI and volume from the related metric system. The SI unit for volume is based upon the metre, but in everyday situations the litre is used, since that is a more appropriate size. If you move on to study science or engineering you will come across some, if not all, of the other SI base units.

For now though, it is time to look at measurements of length.

3 Length: SI units

Length is one of the most common measurements that is used every day. This can tell you how far away the nearest town is, the width of a fridge or your height. In science it can be used on very different scales to measure the size of the universe, or at the other extreme, the diameter of an atom.

The base unit for length in the SI is the metre, abbreviated with a lower case m. An upper case M has a very different meaning, it is the prefix for a million times larger, so care needs to be taken with this! For those of you who are more familiar with imperial measurements, a metre is very roughly the same size as a yard.

Putting the metre together with the prefixes covered in the previous section gives:

kilometre (km) – one thousand times bigger than a metre
centimetre (cm) – a hundredth the size of a metre
millimetre (mm) – a thousandth of the size of a metre.

This not only gives an idea of the size of these units but also how they relate to each other; that is, how many centimetres and millimetres there are in a metre and how many metres there are in a kilometre. This is important knowledge for when you want to change between units – usually known as converting.

So, if one centimetre is a hundredth of a metre, that means one metre must contain 100 centimetres. Similarly, one metre contains 1000 millimetres and one kilometre is the same as 1000 metres.

This can be summarised as follows:

1 km = 1000 m

1 m = 100 cm

1 m = 1000 mm.

or in a diagram that shows how to change between units, as in Figure 1 below:

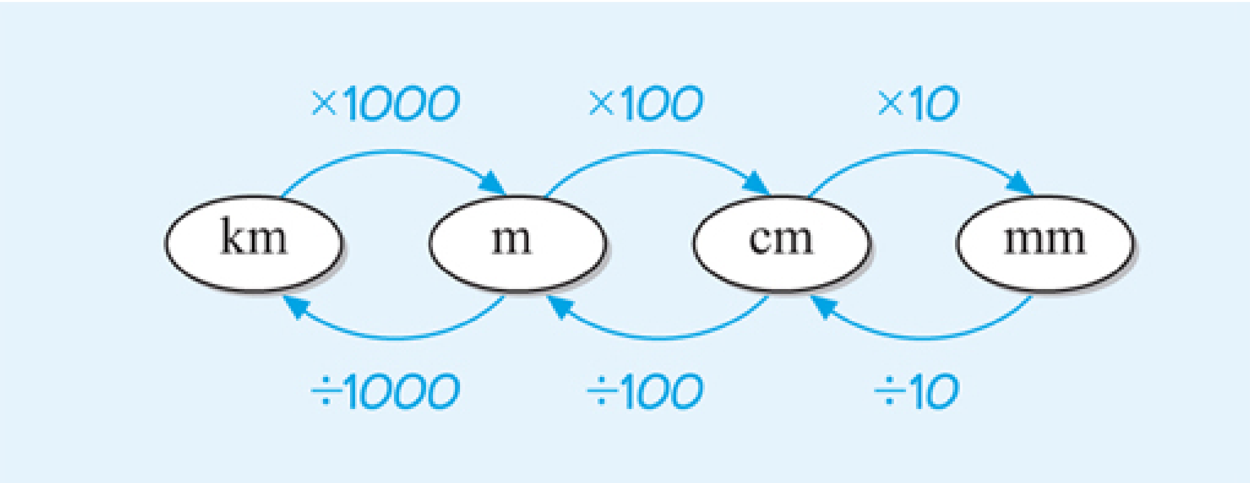


Figure 1 Relationships between SI units of length

This should make sense from what has been established so far about how the different units are connected.

For example, looking at kilometres, you know that there are 1000 m in 1 km, so in 2 km there will be 2000 m or 2 x 1000 m. So, to change from kilometres to metres you just need to multiply by 1000, as shown in Figure 1.

You'll look at how to convert between different SI units of length in more detail in the next section.

3.1 Converting between SI units and length

You started to consider converting between different SI units of length in the last section. Take another look at Figure 1 below, showing how these relate to each other:

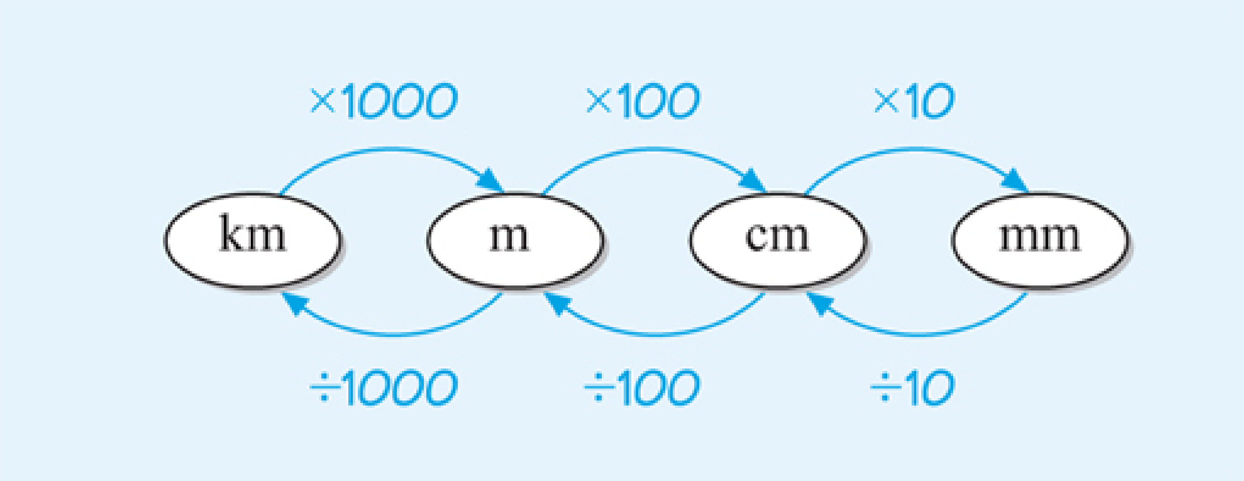


Figure 1 Relationships between SI units of length

Notice that to convert from a physically **larger** unit, such as kilometre (km), to a physically **smaller** unit, such as metre (m), you always **multiply**. This makes sense – you will need a lot more of the smaller units to make one of the larger units! On the other hand, to convert a measurement in a **smaller** unit to a **larger** unit, you always **divide**.

Another way to think about this is whether you expect your final answer, when converting between units, to be a bigger or smaller number than the original value you had. If it should be a larger number, then multiply, if smaller, then divide.

Let's see how this might work in an example:

A girl's height has been measured in metres, but for a school project this needs to be shown in centimetres. Her height was measured as 1.32 metres.

Looking at Figure 1, to convert from metres to centimetres you need to multiply by 100. That makes sense, as a centimetre is a physically smaller unit than a metre, so you would expect the final answer to be a bigger number than the original value that you started with. So the

$$\begin{aligned}\text{girl's height} &= 1.32 \times 100\,\text{cm} \\ &= 132\,\text{cm}\end{aligned}$$

Use these ideas to have a go at this first activity of the course.

Activity 1 SI units of length

Allow approximately 5 minutes

(a) Write 6.78 metres in centimetres.

Comment

Do you expect a bigger or smaller number for your answer?

Answer

(a) There are 100 cm in 1 m and you are converting from a larger unit to a smaller unit, so multiply by 100.

$$\begin{aligned}6.78\,\text{m} &= 6.78 \times 100\,\text{cm} \\ &= 678\,\text{cm}\end{aligned}$$

(b) Write 327 centimetres in metres.

Answer

(b) There are 100 cm in 1 m. This time you are converting from a smaller unit to a larger unit, so divide by 100 to change cm into m.
Hence,

$$\begin{aligned} 327\,\mathrm{cm} &= (327 \div 100)\,\mathrm{m} \\ &= 3.27\,\mathrm{m} \end{aligned}$$

(c) The distance to your friend's house is about 2324 m. How far is this in kilometres?

Answer

(c) There are 1000 m in 1 km. You are converting from a physically smaller unit to a physically larger unit, so divide by 1000 to change metres into kilometres. So,

$$\begin{aligned} \text{distance to the house} &= (2324 \div 1000)\,\mathrm{km} \\ &= 2.324\,\mathrm{km} \end{aligned}$$

Well done, you've just completed your first activity for the week and the course. Many people find converting between units a tricky skill to master, but using the ideas introduced here should help you to feel more confident with this. You'll be getting lots of practice over this week and the next to build on this as well.

The next section introduces imperial units of measurement before turning your attention back to length.

4 Length: imperial measurements

Before the 1970s, the UK used the imperial system of measurement, which had its basis in historical measurements and the need to have common weights and measures to enable the sale of goods and services to operate efficiently. For example, the foot, a measurement of length of around 30 cm (or the length of standard ruler), was first defined in law by Edward I in 1305, and is thought to be derived from the length of a man's foot with shoes.

For those not brought up using the imperial system for measurement it may seem to be a very difficult way to measure things. It does not operate on a system of base units and standard prefixes, like the SI, so this means that there are lots of different relationships to remember for each set of measurements. These are also not based upon the number ten (as the SI is), so calculations and conversions between units isn't quite so straightforward.

However, the same techniques can be used to help decide whether you need to divide or multiply when converting, as you'll see.

The common units used for measuring length in the imperial system are inches (in), feet (ft), yards (yd) and miles (mi). These units are listed in increasing order of size.

If you haven't worked in these units before you may not have a good idea of their actual sizes. Table 2 below shows approximate values for how the imperial units relate to the SI units to help with this.

Table 2 Size of imperial units of length

Imperial unit	SI unit
1 inch	2.5 cm
1 foot (singular of feet)	30 cm
1 yard	0.9 m
1 mile	1.6 km

Now back to how these imperial units of length relate to each other.

There are:

- 12 inches in one foot
- 3 feet in one yard
- 1760 yards in one mile.

So as you can see, no regular relationship based on tens here!

This means that when you convert between the different units of length it becomes more important to think carefully about the answer that you will be expecting. Should it be bigger or smaller than the number you started with?

Let's have a look at an example before you have a go yourself. If you have a photograph that measures eight inches by ten inches, as shown in Figure 2, what is the total distance around the photo in feet and inches?

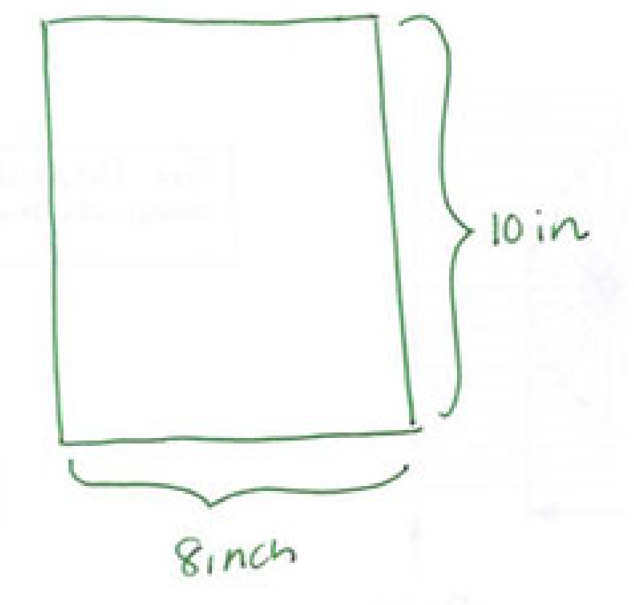


Figure 2 What is the perimeter?

The top and bottom of the photo have the same measurement, eight inches. Similarly, the left- and right-hand sides share a measurement of ten inches.

$$\begin{aligned}\text{The length of the border} &= 8\text{ inches} + 10\text{ inches} + 8\text{ inches} + 10\text{ inches}\\ &= 36\text{ inches}\end{aligned}$$

Or as there are two sides measuring 8 inches and two sides measuring 10 inches:

$$\begin{aligned}\text{The length of the border} &= 8\ \text{inches}\ \times 2\ +\ 10\ \text{inches}\ \times\ 2\\ &= 36\,\text{inches}\end{aligned}$$

Now looking at how many feet there are in 36 inches:

You already know that: $1\ \text{foot} = 12\ \text{inches}$

So if a measurement is in feet instead of inches, the converted number should be smaller than the original. This means the number of inches has to be divided by 12. So,

$$\begin{aligned}\text{length in feet} &= 36 \div 12\\ &= 3\,\text{feet}\end{aligned}$$

This means that 36 inches is the same as 3 feet. Thus, the length of the border of the picture is 3 feet.

Now it's your turn. Remember to think about the size of the final answer you are expecting.

Activity 2 Length in imperial units

Allow approximately 10 minutes

For each of the following scenarios, use the appropriate operation, multiplication or division, and unit to determine the answer. Click on 'reveal comment' if you would like a hint to get going.

(a) The height of a three-year-old girl is measured as 34 inches. Over her lifetime, she grows another 34 inches. When fully grown, how tall is she in feet and inches?

Discussion

If you have studied *Succeed with Maths Part 1* you may remember that drawing pictures can help to visualise a situation.

Answer

(a) The girl is 34 inches when she is three and then grows another 34 inches before reaching her full height.

There are two ways to calculate the final height, and either is perfectly fine!

$$\begin{aligned}\text{Girl's height when fully grown} &= 34\,\text{inches}\ \times\ 2\\ &= 68\,\text{inches}\end{aligned}$$

Or

$$\begin{aligned}\text{Girl's height when fully grown} &= 34\,\text{inches}\ +\ 34\,\text{inches}\\ &= 68\,\text{inches}\end{aligned}$$

$1\,\text{feet} = 12\,\text{inches}$

So for a measurement using feet instead of only inches, the number should be smaller than the original. This means the number of inches has to be divided by 12.

$$\begin{aligned}\text{Girl's height in feet} &= 68\,\text{inches} \div 12\\ &= 5.67\,\text{feet}\end{aligned}$$

This is the same as 5 feet and 0.67 feet.

Now convert 0.67 feet into inches.

This time you are converting from a larger to a smaller unit, so you need to multiply.

$$\begin{aligned} 0.67\,\text{feet} &= 0.67 \times 12\,\text{inches} \\ &= 8\,\text{inches} \end{aligned}$$

This means 68 inches is the same as 5 feet and 8 inches.

(b) A rectangular garden measures 12 feet by 18 feet. What is the length of the border of the garden in yards?

Answer

(b) Here's a quick sketch of the garden to help visualise the problem:

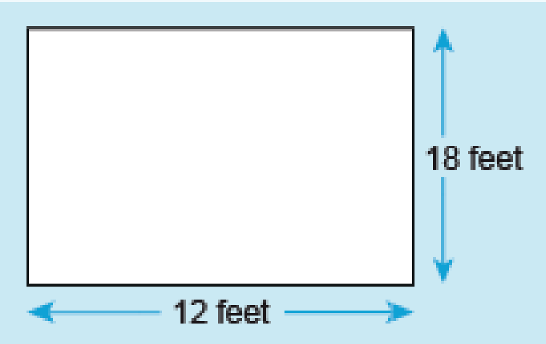


Figure 3 A rectangular garden measuring 18 feet by 12 feet

$$\begin{aligned} \text{Length of border} &= 18\,\text{feet} + 12\,\text{feet} + 18\,\text{feet} + 12\,\text{feet} \\ &= 60\,\text{feet} \end{aligned}$$

$$1\,\text{yard} = 3\,\text{feet}$$

As you are converting from a smaller to a larger unit, you would therefore expect the converted answer to be smaller than the original value. Hence, you need to divide in this case.

So the

$$\begin{aligned} \text{length of the border in yards} &= 60 \div 3\,\text{yards} \\ &= 20\,\text{yards} \end{aligned}$$

Thus, the length of the garden's border is 20 yards

Now you've looked at units of length in both the SI and the imperial system, it's time to look at converting between the systems of measurement in the next section.

5 Relating the SI and imperial systems of measurement

This section of the week has two aims: to give a better idea of the size of unfamiliar units and to provide more practice at converting between units. You can't get too much practice at this as it is one of those skills that just needs a bit of extra work to perfect.

In order to be able to do this, you need to know in a bit more detail about how units in the SI relate to units in the imperial system. So, listed below are some of the more common conversion factors:

- 1 metre = 39.4 inches
- 1 metre = 1.1 yards
- 1 yard = 0.9 metres
- 1 mile = 1.6 kilometres
- 1 inch = 2.54 cm.

A practical way that you can help yourself become more familiar with units that you don't usually use is to use them in your everyday life. So, for example, if you measure materials for DIY projects in millimetres, see what this would be if you used inches instead. This should help you to get a much better feel for the units than just looking at a lot of numbers!

Now it is time to use this information.

5.1 Converting between different unit systems

Many people find converting between units quite challenging at first – but the more you practise, the easier it will become. Hopefully, you are already finding this yourself. The process that you have been following has three steps:

1. Find out how the two units are related to each other – the 'conversion factor'.
2. Ask yourself if the final answer should be bigger or smaller than the original value.
3. Divide or multiply by the conversion factor to get your answer.

If you are lucky, and with practice, you may be able to miss step two out altogether – but that may depend on what you are trying to convert.

The process to use when converting between metric and imperial, or imperial and metric, is just the same as outlined above.

Let's look at an example to see this approach in action.

A man's height

A man measures 1.8 m and needs to know his height in feet and inches.

The first step is to find out how the units relate to each other. From the previous section, 1 m is the same as 39.4 inches.

This means that the answer in inches must be bigger than the original value, so multiply by 39.4. So, the

$$\begin{aligned}\text{man's height in inches} &= (1.8 \times 39.4)\,\text{inches}\\ &= 70.92\,\text{inches}\end{aligned}$$

Now, this needs to be converted to feet and inches. There are 12 inches in 1 foot, so this time the conversion is from a physically smaller unit to a physically larger unit and the answer will be smaller. So now divide by 12 to convert from inches to feet.

$$\begin{aligned}\text{Man's height in feet} &= (70.92 \div 12)\,\text{feet}\\ &= 5.91\,\text{feet}\end{aligned}$$

Now you know that 1.8 m is the same as 5.91 feet, which is 5 feet and 0.91 of a foot. However, the final answer should be in feet and inches, so now find out what 0.91 of a foot is in inches.

This is a conversion from a larger to a smaller unit, so multiply by the conversion factor in this case.

$$\begin{aligned}\text{Number of inches} &= (0.91 \times 12)\,\text{inches}\\ &= 10.92\,\text{inches}\\ &= 11\,\text{inches (to the nearest inch)}\end{aligned}$$

Putting this altogether, a man who is 1.8 m is about 5 feet 11 inches.

You can apply the same technique for any units, so see how you get on with these next activities. Take your time and think carefully about whether you need to divide or multiply.

Activity 3 Converting length measurements between systems

Allow approximately 10 minutes

(a) The distance between Fort William and Glasgow is about 108 miles. What is this distance in kilometres, to the nearest km? Click on 'reveal comment' if you feel you need more guidance.

Discussion

1 mile = 1.6 km

Answer

You know that 1 mile = 1.6 km

This means converting from a physically larger unit to a smaller one, so the expected converted answer will be larger. Hence, you need to multiply by the conversion factor.

$$\begin{aligned}\text{Distance from Fort William to Glasgow} &= 108 \times 1.6\,\text{km}\\ &= 173\,\text{km (to the nearest kilometre)}\end{aligned}$$

So, the distance from Fort William to Glasgow is 173 km.

(b) The North Downs Way is a footpath in the South of England. The total length of the route is about 250 km. What is this distance in miles?

Discussion

1 mile = 1.6 km

Answer

(b) You know that 1 mile = 1.6 km

This means converting from a physically smaller unit to a bigger one, so the expected converted answer will be smaller. Hence, you need to divide by the conversion factor.

$$\begin{aligned}\text{Length of route} &= 250 \div 1.6\,\text{km}\\ &= 156\,\text{miles}\end{aligned}$$

So, the North Downs Way is 156 miles long.

(c) A reel of thread contains 182 yards of cotton. How long is the thread in metres, to the nearest metre?

Discussion

1 yard = 0.9 metres

Answer

(c) You know that 1 yard = 0.9 metres

This means converting from a physically smaller unit to a larger one. However, the conversion factor is less than one so the expected converted answer will be larger. Hence, you need to multiply by the conversion factor.

$$\begin{aligned}\text{Length of thread} &= 182 \times 0.9\,\text{metres}\\ &= 164\,\text{metres (to the nearest m)}\end{aligned}$$

So, the reel has 164 metres of cotton on it.

You've got one final activity to look at before finishing this week. This brings together your new conversion skills, as well as giving you a refresher on problem solving from *Succeed with maths – Part 1*.

6 Home improvement project

This section looks at a real-world problem that uses measurement. The following instructions were given on a DIY store's website for calculating how many rolls of wallpaper are needed to decorate a room:

- **Step 1:** A standard roll of wallpaper is approximately 33 ft long and 1 ft 9 in wide. If you measure the height of the walls from the skirting board to the ceiling, you can determine how many strips of paper you can cut from a standard roll – four strips are about average.
- **Step 2:** Measure around the room (ignoring doors and windows) to work out how many roll widths you need to cover the walls. Divide this figure by the number of strips you can cut from one roll to calculate how many rolls you need to buy. Make a small allowance for waste.

Now try this next activity.

Activity 4 Following the instructions

Allow approximately 5 minutes

Read through the directions above. Write down the important information you need to know to work out the number of rolls of wallpaper required. Remember, you can click on 'reveal comment' for additional help.

Discussion

Try drawing a sketch, and think through the steps you would take to apply new wallpaper in your kitchen.

Answer

This is how Rebecca, a student, tackled the problem. Her notes are given below.

There are **three** calculations:

- working out how many strips of paper you can cut from a standard roll
- working out how many roll widths there are round the room
- calculating how many rolls are needed.

A roll of wallpaper measures 33 feet in length and 1 ft 9 in, in width.

You may have a slightly different answer here, but as long as the main points convey the same ideas then that is fine!

Now use your notes from this activity to complete this next one.

Activity 5 How many rolls?

Allow approximately 10 minutes

The room that you want to wallpaper measures 3.2 m by 4 m and the height of the walls is 2.34 m. Work out how many rolls of wallpaper you will need.

Click on 'reveal comment' if you would like a quick hint.

Discussion

As well as using your notes, drawing a diagram of the room from above and one wall, may help you to visualise what you need to do.

Answer

The first thing you need to do is to make all the units the same. The room has been measured in metric and the wallpaper is imperial. It doesn't matter if you changed from imperial to metric or metric to imperial!

Measurements for wallpaper in metric units:

$$\begin{aligned}\text{Length of roll} &= 33\,\text{feet}\\ &= (33\times 12)\,\text{inches}\\ &= 396\,\text{inches}\end{aligned}$$

$1\,\text{inch} = 2.54\,\text{cm}$

$$\begin{aligned}\text{Length of roll in cm} &= (396\times 2.54)\,\text{cm}\\ &= 1005.84\,\text{cm}\end{aligned}$$

$1\,\text{m} = 100\,\text{cm}$

$$\begin{aligned}\text{Length of roll in m} &= (1005.84\div 100)\,\text{m}\\ &= 10.06\ \text{m}\ (\text{to 2 decimal places})\end{aligned}$$

For these purposes this can be called 10 m. As the assumption is that each roll is slightly shorter rather than longer, buying too few rolls would not therefore cause a problem.

$$\begin{aligned}\text{Width of wallpaper} &= 1\,\text{ft}\ 9\,\text{in}\\ &= 12\,\text{inches} + 9\,\text{inches}\\ &= 21\,\text{inches}\end{aligned}$$

$$\begin{aligned}\text{Width of wallpaper in cm} &= (21\times 2.54)\,\text{cm}\\ &= 53.34\,\text{cm}\end{aligned}$$

$1\,\text{m} = 100\,\text{cm}$

$$\text{Width of wallpaper in m} = (53.34 \div 100)\,\text{m}$$
$$= 0.53\,\text{m (to 2 decimal places)}$$

Number of strips from one roll?

The first calculation is to work out how many lots of the height (2.34 m) you can get from a roll of wallpaper 10 m long.

$$\text{Number of strips of wallpaper} = 10\,\text{m} \div 2.34\,\text{m}$$
$$= 4.27\,\text{strips}$$

As 0.27 of a strip is not very useful, so it is best to say that you can get 4 strips from each roll.

How many strips are required to cover the room?

$$\text{The total length around the room} = 3.2\,\text{m} + 3.2\,\text{m} + 4\,\text{m} + 4\,\text{m}$$
$$= 14.4\,\text{m}$$

It is helpful to draw a quick sketch to show what the walls would look like if they were flattened out into one big wall:

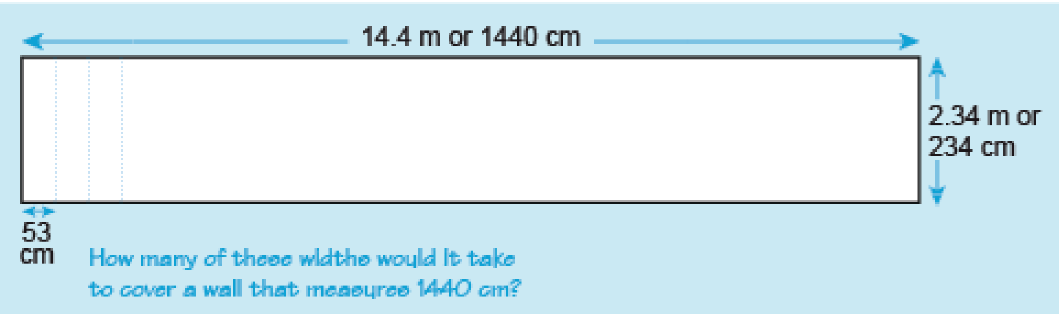


Figure 4 A rough sketch representing how the walls would look as one big wall

Here again you could either calculate this using centimetres or metres.

$$\text{Number of strips required} = 14.4\,\text{m} \div 0.53\,\text{m}$$
$$= 27.1\,\text{m}$$

To make sure there is enough paper, this has to be rounded-up, giving 28 strips altogether.

How many rolls of wallpaper?

These calculations show:

1 That 28 strips of wallpaper are needed
2 Each roll will give 4 strips.

So, the

$$\text{number of rolls needed} = 28 \div 4$$
$$= 7$$

This should be plenty, as doors and windows have not been taken into account.

Well done for completing this activity. It had lots of steps to get to the answer and some unit conversions as well. It was therefore much more involved than any other activity this week and required you to make use of your problem-solving skills as well as your measurement knowledge.

That completes the study for this week, except for this week's quiz.

7 This week's quiz

Go to:

Week 1 practice quiz.

Open the quiz in a new tab or window (by holding ctrl [or cmd on a Mac] when you click the link).

8 Summary

Congratulations for making it to the end of the first week in *Succeed with maths – Part 2*. You will use the skills that you have developed this week as you work through Week 2, which will introduce the measurement of volume and mass. So try and think back to what you have learnt as you continue your studies, particularly when converting between units.

You should now be able to:

- understand how the International System (SI) is formulated
- understand the more common SI units for length and convert between them
- recognise the more common imperial units for length and convert between these
- understand how the SI and imperial units of length relate to each other.

You can now go to Week 2.

Week 2: Measurement of mass and volume

Introduction

This week follows on from Week 1, which looked at measurement of length in both the International System (SI) and imperial, to consider two other common measurements: mass and volume. As well as learning about the units for these measurements in both systems, you'll get much more practice in converting between units. Conversions can often be a challenge, so the more practice you get with this the better!

Watch Maria introduce Week 2:

Video content is not available in this format.

After this week's study, you should be able to:

- understand the more common SI units for mass and volume, and convert between them
- recognise the more common imperial units for mass and volume, and convert between these
- understand how the SI and imperial units of mass and volume relate to each other.

1 Using units of measurement

Before looking at mass, here is a quick reminder of how the SI works from last week.

The SI system uses a combination of base units and prefixes. The base unit that you used in Week 1 was the metre. This is then combined with a prefix, if relevant, to complete the unit. What unit you use will depend upon the size of the measurement that you are taking. For example, it wouldn't make much sense to measure the length of a finger in kilometres! The most common everyday prefixes are:

- 'kilo-' a thousand
- 'centi-' a hundredth
- 'milli-' a thousandth.

This gives the following units of length:

kilometre = 1000 metres

centimetre = 0.01 metres

millimetre = 0.001 metres

This idea is extended to all the SI units, as you'll see in the next sections on mass.

2 Mass

You'll start by quickly looking at why this section is called mass rather than weight. Weight is the word that most people use in everyday language to describe how much 'stuff' is in an object. However, in science weight does not measure this quantity. In fact, the correct term for the amount of 'stuff' in an object is 'mass'. The two quantities are related, but have different meanings.

Weight, on the other hand, is the force due to gravity on an object (or a mass). As mass is a measure of the amount of 'stuff' in an object, this doesn't change unless you physically alter that object. However, the same object would not have the same weight on the moon as it does on the earth. Its weight would reduce because the moon has a lower gravity than the earth. They are also measured in different units! So, to be strictly accurate, the course uses the correct term: 'mass'.

Now it's time to look at the SI units for mass.

2.1 Mass: SI units

Unlike all the other SI base units, the base unit for mass has a prefix. You may have noticed that in Week 1, when you looked at the summary of the seven SI base units.

The base unit for mass in the SI is the kilogram (abbreviated to 'kg'), not the gram as you may have logically thought. However, the gram can be seen as the base unit from the point of view of adding prefixes to produce the other units.

Adding the prefixes gives the milligram (mg), as well as the kilogram (kg). Note that you could have a centigram as well, but this is not a unit that is commonly used.

Use your knowledge from the previous section on length in this next short activity.

Activity 1 SI units of mass

Allow approximately 5 minutes

(a) What would you need to do to convert from kilograms (kg) to grams (g)?

Answer

Kilo means a thousand, so kilogram is the same as saying 1000 grams.

Hence to convert from kilograms to grams you should multiply by 1000.

(b) What would you need to do to convert from grams (g) to milligrams (mg)?

Answer

Milli means a thousandth, so milligram is the same as saying a thousandth of a gram. This means that 1 gram = 1000 milligrams.

Hence, to convert from grams to milligrams you should multiply by 1000.

(c) What would you need to do to convert from grams (g) to kilograms (kg)?

Answer

From part (a) to convert from kilograms to grams you needed to multiply by 1000, so the opposite must be true to convert from grams to kilograms.

Hence, to convert from grams to kilograms you should divide by 1000.

The findings from this activity can be summarised as:

- kilograms (kg) to grams (g) – multiply by 1000
- grams (g) to kilograms (kg) – divide by 1000
- grams (g) to milligrams (mg) – multiply by 1000
- milligrams (mg) to grams (g) – divide by 1000.

Use this information to help you in the next activity, as well as the steps from Week 1 to convert between different units. These are summarised again here for you:

1. Find out how the two units are related to each other – the conversion factor.
2. Ask yourself if the final answer should be bigger or smaller than the original value.
3. Divide or multiply by the conversion factor to get your answer.

You can click on 'reveal comment' if you would like some additional guidance.

Activity 2 Everyday mass

Allow approximately 10 minutes

(a) During a shopping trip you buy 1.5 kg of apples, 450 g of bananas and 0.75 kg of pears. How much fruit in grams have you bought altogether?

Discussion

Remember that 1 kg = 1000 g.

Answer

(a) As there is a mixture of different units, start by converting all the units into the same one. The final answer should be in grams, so it makes sense to convert all the masses into grams to start with.

$$1\,\mathrm{kg} = 1000\,\mathrm{g}$$

$$\begin{aligned}\text{Mass of apples} &= (1.5 \times 1000)\,\mathrm{g} \\ &= 1500\,\mathrm{g}\end{aligned}$$

$$\begin{aligned}\text{Mass of pears} &= (0.75 \times 1000)\,\mathrm{g} \\ &= 750\,\mathrm{g}\end{aligned}$$

$$(\text{Mass of bananas} = 450\,\mathrm{g})$$

$$\begin{aligned}\text{So, total weight of fruit bought} &= 1500\,\mathrm{g} + 750\,\mathrm{g} + 450\,\mathrm{g} \\ &= 2700\,\mathrm{g}\end{aligned}$$

(b) One batch of mortar mix required for building a brick wall requires 250 g of cement and 750 g of sand. If you need 5 batches to finish the wall, how much sand and cement will you need and what is the total mass in kilograms?

Answer

(b) $\text{Cement required} = 5 \times 250\,\text{g} = 1250\,\text{g}$

$\text{Sand required} = 5 \times 750\,\text{g} = 3750\,\text{g}$

$\text{Total mass of sand and cement} = 1250\,\text{g} + 3750\,\text{g} = 5000\,\text{g}$

$1\,\text{kg} = 1000\,\text{g}$

This time you need to divide by 1000 to convert from g to kg

So, $\text{total mass} = (5000 \div 1000)\,\text{kg} = 5\,\text{kg}$

(c) The daily recommended intake of vitamin C in the UK for adults is 0.04 g. Convert this to milligrams, the more usual way to quote this value.

Answer

(c) 1 milligram (mg) is a thousandth of a gram.

This tells us that:

$1\,\text{g} = 1000\,\text{mg}$

So, the $$\begin{aligned}\text{daily recommended intake} &= (0.04 \times 1000)\,\text{mg}\\ &= 40\,\text{mg}\end{aligned}$$

As with length, imperial units for mass are not quite as straightforward as the SI units. So, you may need to take your time over this next section.

2.2 Mass: imperial units

To measure mass in the imperial system, ounces (abbreviated to ‘oz’), pounds (lb) and stones (st) are used. These are listed in increasing order of size and you can see the relationship between these units below:

16 ounces in 1 pound

14 pounds in 1 stone

It's easy to confuse the two!

These relationships can in part be explained by the fact that an ounce was originally based on the mass of that very useful item, the barleycorn, with 480 barleycorns to the ounce (although it couldn't have been much fun counting 480 of them!). Having 16 ounces to the pound was useful because 16 can be divided into halves (8, 4, 2, 1) easily, which was very useful for shop keepers measuring out and selling their wares.

The same logic also applies to changing, or converting, between these different mass units as with any others. So, if you convert a value from ounces to pounds, you would expect the final number to be smaller, and hence need to divide by 16 (16 ounces in 1 pound). Using the knowledge you have already gained this week you may be able to work out this next example without looking at our workings. See how you get on!

Let's suppose you were moving boxes of electronic equipment. The first box has a mass of 14 pounds 11 ounces and the second, 17 pounds 8 ounces. You want to find the total mass of both boxes in stones, pounds and ounces to ensure they can be moved safely.

First, add the pounds together:

$$14 \text{ pounds} + 17 \text{ pounds} = 31 \text{ pounds}$$

Next add up the ounces:

$$11 \text{ ounces} + 8 \text{ ounces} = 19 \text{ ounces}$$

Since there are 16 ounces in 1 pound, this means:

$$19 \text{ ounces} = 16 \text{ ounces} + 3 \text{ ounces} = 1 \text{ pound} + 3 \text{ ounces}$$

$$\begin{aligned} \text{Thus, the total mass} &= 31 \text{ pounds} + 1 \text{ pound} + 3 \text{ ounces} \\ &= 32 \text{ pounds} + 3 \text{ ounces} \end{aligned}$$

There are 14 pounds in 1 stone.

So, $32 \text{ pounds} = 32 \div 14 = 2 \text{ stones and } 4 \text{ pounds.}$

So the total mass is 2 stones, 4 pounds and 3 ounces.

Now, have a go at this next activity yourself.

Activity 3 Mass in imperial

Allow approximately 10 minutes

For each of the following scenarios, use the appropriate operation and unit conversion factors to determine the answer.

(a) At birth, Samuel's mass is 7 pounds 4 ounces. After one week, he has gained 13 ounces. What is Samuel's mass at the end of the week in pounds and ounces?

Answer

$$\text{Samuel's mass} = 7 \text{ pounds} + 4 \text{ ounces} + 13 \text{ ounces} = 7 \text{ pounds } 17 \text{ ounces}$$

There are 16 ounces in 1 pound, so you can convert 17 ounces in pounds and ounces.

$$17 \text{ ounces} = 16 \text{ ounces} + 1 \text{ ounce} = 1 \text{ pound} + 1 \text{ ounce}$$

So, $\text{Samuel's mass} = 7 \text{ pounds} + 1 \text{ pound} + 1 \text{ ounce}$

The correct way to say this is that Samuel's mass is now 8 pounds 1 ounce.

(b) When he started dieting, Derek's mass was 203 pounds. He lost 37 pounds. What is Derek's mass now in stones and pounds?

Answer

(b) $\text{Derek's new mass} = 203 \text{ pounds} - 37 \text{ pounds} = 166 \text{ pounds}$

There are 14 pounds in a stone. So, this means that our final answer needs to be smaller, and that it is necessary to divide by 14 to achieve this.

$$166 \text{ pounds} = (166 \div 14) \text{ stone} = 11 \text{ stone and } 12 \text{ pounds.}$$

So, now Derek's mass is 11 stone and 12 pounds.

As with length, you need to be able to relate the two different systems of measurement to each other for mass – this is the subject of the next section and will help give you an idea of the size of the different units.

2.3 Relating SI and imperial: mass

For the units covered in the course, the two systems are related as shown below:

Mass

- 1 kg = 2.2 lb
- 1 ounce = 28.3 g
- 1 stone = 6.4 kg

If you are more familiar with imperial measurements, this will hopefully show how small a gram actually is. To get a feel for this, a paperclip has a mass of around a gram – so not very much in everyday terms!

To give yourself a good feel for both systems of measurement, try weighing a few everyday objects around you. Note down the mass in both systems of measurement.

Hopefully, you will now feel confident enough to convert between the SI and imperial system for mass without an example. So, it's straight into our next activity. Remember to click on 'reveal comment' if you need a hint.

Activity 4 Relating SI and imperial units of mass

Allow approximately 5 minutes

(a) A baby weighs 3.7 kg when it is born. How much is this in pounds and ounces?

Discussion

$1\,\mathrm{kg} = 2.2\,\mathrm{lb}$

This means converting from a physically larger unit to a smaller one. The final answer should therefore be larger.

Answer

(a) $1\,\mathrm{kg} = 2.2\,\mathrm{lb}$ $\text{Mass of baby in lb} = 3.7 \times 2.2\,\mathrm{lb}$

$$= 8.14\,\mathrm{lb}$$

So the mass is 8 lb and 0.14 lb

$1\,\mathrm{lb} = 16\,\mathrm{oz}$

$$\begin{aligned} 0.14\,\mathrm{lb\ in\ ounces} &= 0.14 \times 16\,\mathrm{oz} \\ &= 2.24\,\mathrm{oz} \end{aligned}$$

So, the baby weighed 8 lb and 2 oz (to the nearest ounce) when it was born.

(b) One litre of water has a mass of 1 kg. If a rainwater butt contains 120 litres of water, what mass of water does it contain?

Answer

b. $1\,\mathrm{kg} = 2.2\,\mathrm{lb}$

$$\text{Mass of water in kg} = 120 \times 1\ \text{kg} = 120\ \text{kg}$$

$$\text{Mass of water in lb} = 120 \times 2.2\ \text{lb} = 264\ \text{lb}$$

Now, onto our next section, which is about how the capacity of something is measured – that is, its volume. This is another of those measurements that is very familiar in everyday life, whether it is the quantity of milk in a container or how much fuel is needed to fill a car.

3 Volume

Here the units that are used in everyday life differ from the SI units for volume. In the SI, units of volume are based upon the metre, with a cubic metre being standard. However, this is a very large volume and not of much use to us when measuring everyday objects. To get an idea of how large 1 cubic metre is, imagine a box that is 1 metre high by 1 metre wide by 1 metre long – that is 1 cubic metre and would be a lot of milk!

The base unit for volume that is used in everyday situations is the litre (abbreviated as 'l' – lower case 'L', not upper case 'i'). This is from the metric system of measurement on which the SI is based. Adding the prefixes as before gives us the related units of millilitre (ml) and centilitre (cl). Note this could also be continued to include a kilolitre, but larger volumes are usually measured in cubic metres, where 1 cubic metre is the same as 1000 litres (or a kilolitre).

Again, using the knowledge of prefixes it can be deduced that a millilitre is a thousandth of a litre, and a centilitre is a hundredth of a litre. From this the following can be stated:

- 1 litre = 1000 ml
- 1 litre = 100 cl.

These relationships can also be displayed in a diagram, showing how to convert between the different units, as shown below:

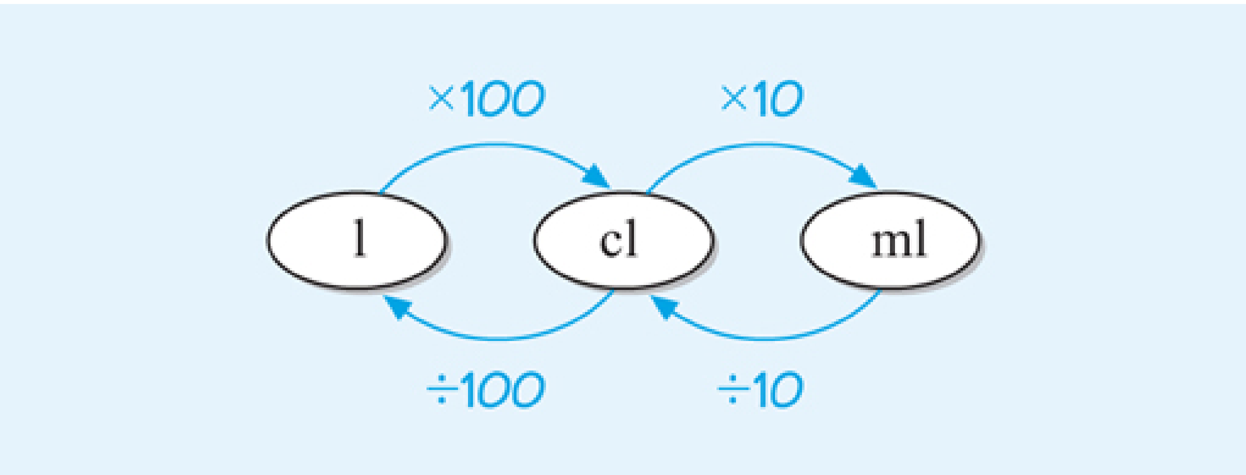


Figure 1 Converting between units of volume

Now you've had lots of practice with converting between units, see how you get on with this next activity. Think back to the start of Week 1 – do you feel that the activities are now becoming more straightforward?

Activity 5 Volume

Allow approximately 5 minutes

(a) You fill your car with 35.6 litres of fuel one day and four days later, another 15.2 litres. How much fuel in total have you put in your car this week in centilitres? Remember, clicking on 'reveal comment' will give you additional hints and tips.

Answer

$$\begin{aligned}\text{Volume of fuel} &= 3.6\text{ litres} + 15.2\text{ litres}\\ &= 50.8\text{ litres}\end{aligned}$$

$1\,\text{litre} = 100\,\text{cl}$

Converting from a physically larger to a physically smaller unit, means you need to multiply.

$$\begin{aligned}\text{So volume of fuel in cl} &= (50.8 \times 100)\text{ cl}\\ &= 5080\,\text{cl}\end{aligned}$$

(b) A bottle contains 14 cl of medicine. The dose is 5 ml. How many doses can be given from this bottle?

Click on reveal comment if you need a hint.

Discussion

You are dealing with different units here, so you need to start by converting 14 cl to ml or 5 ml to cl.

You may find it easier to convert 14 cl to ml.

Answer

To find the number of doses, the volume of the bottle and the dose need to be in the same units.

There are 10 ml in 1 cl.

Therefore, $14\,\text{cl} = (14 \times 10)\,\text{ml} = 140\,\text{ml}$

$$\begin{aligned}\text{So, number of doses in the bottle} &= 140\,\text{ml} \div 5\,\text{ml}\\ &= 28\end{aligned}$$

You have probably guessed by now that the next topic will be the imperial units for volume. Some of these will no doubt prove more familiar than others, as they are still used in some everyday situations, such as buying drinks.

3.1 Volume: the imperial units

To measure volume, the imperial system uses, amongst other units, gallons and pints. You may also have heard of a gill and a quart. As with length and mass in the imperial system there are no constant relationships between all these units, although there is more similarity with these than with some!

There are:

- 4 gills in 1 pint
- 2 pints in 1 quart
- 4 quarts in 1 gallon.

For our purposes it is most useful to know that:

- 8 pints = 1 gallon.

As the methods used to convert between different units of volume are just the same as those already covered for both length and mass, you can move straight on to an activity to complete. Bear in mind what you have already learned over this week and the last. You can always look back at the previous sections or click on 'reveal comment' for some hints if you need them!

Activity 6 Imperial units of volume

Allow approximately 5 minutes

For each of the following scenarios, use the appropriate operation and unit to determine the answer.

(a) At the beginning of the month a garage purchases 240 pints of motor oil. Each week it uses 44 pints. Assuming that there are four weeks in this month, how many pints of oil will be left over at the end of the month? What is this value in gallons?

Discussion

Write down all the information from the question in a list and think about the size of the answer that you expect.

Answer

(a) There are 44 pints used each week. $\text{So the total volume used in a month } = 44\,\text{pints } \times 4 = 176\,\text{pints}$

The garage started with 240 pints.

$\text{Remaining volume of oil } = 240\,\text{pints } - 176\,\text{pints } = 64\,\text{pints}$

There are 8 pints in a gallon.

So, $64\text{ pints in gallons } = 64 \div 8 = 8\,\text{gallons}$

Therefore there are 8 gallons of oil remaining at the end of the month.

(b) A large barrel holds 50 gallons of beer. If in one week 14.5 gallons of beer are sold from the barrel and the next week 15.5 gallons are sold, how many pints of beer will have been sold in total and what will be left in the barrel (in pints)?

Answer

$\text{Number of gallons sold } = 14.5\,\text{gallons } + 15.5\,\text{gallons } = 30\,\text{gallons}$

There are 8 pints in a gallon.

So, $\text{number of pints sold } = (30 \times 8)\text{ pints } = 240\text{ pints}$

$\text{Gallons left in the barrel } = 50\,\text{gallons } - 30\,\text{gallons } = 20\,\text{gallons}$

$\text{So pints left in the barrel } = 20 \times 8 = 160\,\text{pints}$

To double-check the answer add the pints sold to the pints left in the barrel and then convert back to gallons. This should be the same as the original capacity given:

$\text{Total number of pints } = 240\,\text{pints} + 160\,\text{pints} = 400\,\text{pints}$

$1\,\text{gallon} = 8\,\text{pints}$

$\text{So total volume in gallons } = 400 \div 8 = 50\,\text{gallons}$

You can now confidently state that 240 pints were sold and there were 160 pints remaining in the barrel.

Finally, you need to know how these two measurement systems relate to each other for volume. This is the subject of the next brief section.

3.2 Relating the two systems: volume

For volume you're going to look at how just two units compare, that is pints and litres. This is because there is no real equivalent of the gallon in the metric system.

To see how the two are related, it is helpful to use an example.

Milk is still sold in containers that are exact sizes in pints, although the quantity in litres is displayed with equal prominence. On a 4-pint container of milk the label states that this is also 2.272 litres.

You can use this information to determine how pints and litres are related to each other. Try now in this next activity.

Activity 7 How do pints relate to litres?

Allow approximately 3 minutes

A milk bottle label states that it contains 4 pints or 2.272 litres. Use this information to work out how pints and litres are related by stating how many pints are in 1 litre. Remember you can click on 'reveal comment' for hints and tips.

Discussion

The information on the label means that $4\,\text{pints} = 2.272\,\text{litres}$

Answer

You know that: $4\,\text{pints} = 2.272\,\text{litres}$

So 1 litre is 2.272 times smaller than shown here. This means to determine how many pints are in 1 litre you must divide by 2.272.

$1\,\text{litre} = 4 \div 2.272\,\text{pints}$

$1\,\text{litre} = 1.76\,\text{pints}$

---

4 Less familiar units

You've nearly reached the end of this week but there is one final activity to check your newly honed skills with converting between units. This involves some units that you may not be familiar with. However, all the techniques that you have been building over the last two weeks are still relevant, so bear these in mind as you work your way through Activity 8. Don't forget to click on 'reveal comment' for an additional hint.

Activity 8 Converting between less familiar units

Allow approximately 5 minutes

(a) Given the fact that:

$1\,\mathrm{kWh} = 3\,600\,000\,\mathrm{J}.$

$(\mathrm{kWh} = \text{kilowatt-hour}, \mathrm{J} = \text{joules and both are units of energy})$

Now convert 322 kWh to J.

Discussion

Remember to consider whether the final answer should be bigger or smaller than the original value given. This will tell you whether to multiply or divide by the conversion factor.

Answer

$1\,\mathrm{kWh} = 3\,600\,000\,\mathrm{J}$

This means that for every 1 kWh there are 3 600 000 J. That is a lot of joules for each kilowatt-hour! So the final answer will be much larger than the original value. That means multiplying to convert between the two.

$322\,\mathrm{kWh} = (322 \times 3\,600\,000)\,\mathrm{J} = 1\,159\,200\,000\,\mathrm{J}$

(b) Given that 1 chain = 66 feet, convert 4345 feet to chains.

Answer

(b) This time the final answer will be smaller than the original value, so you need to divide.

$4345\,\text{feet} = (4345 \div 66)\ \text{chains} = 65.83\,\text{chains (to 2 decimal places)}$

(c) Given that: $1\,\text{carat} = 20\,\mathrm{mg}\ (\mathrm{mg} = \text{milligrams})$

convert 3.5 carats to mg.

Answer

(c) The final answer needs to be larger than the original value. This means you must multiply. $3.5\,\text{carats} = (3.5 \times 20)\,\mathrm{mg} = 70\,\mathrm{mg}$

5 This week's quiz

Hopefully, the last activity has given you a feel for how the basic ideas that you have been building up and using in the last two weeks can be applied in any situation where you need to convert between different units. Always keep in mind the three steps that are needed:

1. Find out how the two units are related to each other – the ‘conversion factor’.
2. Ask yourself if the final answer should be bigger or smaller than the original value.
3. Divide or multiply by the conversion factor to get your answer.

Go to:

Week 2 practice quiz.

Open the quiz in a new tab or window (by holding ctrl [or cmd on a Mac] when you click the link).

6 Summary

Congratulations for making it to the end of another week! Many people find converting between different units a challenge, even at higher levels of study, but you now have a set of tools that will help in the future. This means that if you go on to study, in many different subject areas, you will already be another step on your way to achieving your goals. And even if this is not your ultimate aim, you will still have gained some very valuable skills for your everyday and work life.

You should now be able to:

- understand the more common SI units for mass and volume, and convert between them
- recognise the more common imperial units for mass and volume, and convert between these
- understand how the SI and imperial units of mass and volume relate to each other.

You can now go to Week 3.

Week 3: Everyday patterns and formulas

Introduction

Patterns occur everywhere in art, nature, science and especially mathematics. Being able to recognise, describe and use these patterns is an important skill to have when tackling a variety of different problems and in understanding the world, for example, predicting the effect on fish numbers for a given amount harvested from the sea. This course explores some of these patterns, ranging from ancient number patterns to the latest mathematical research. It also looks at useful practical applications that are relevant to everyday life. Your study of this subject is spread over two weeks, and includes plenty of activities for you to do to gain confidence with these ideas.

In the following video Maria introduces you to Week 3:

Video content is not available in this format.

After studying this unit, you should be able to:

- recognise patterns in a variety of different situations
- use word formulas to help solve a problem
- write your own word formulas to help solve a problem.

1 Exploring patterns and processes

Patterns occur in many different ways in everyday life. They include how petals are arranged on a flower and the repeating notes of bird song. These patterns give us a powerful tool in understanding the world and universe. Many centuries ago ancient civilisations recognised the yearly pattern of changing sunset and sunrise and used this to build places of worship, such as Stonehenge in the UK. Today, patterns in the progress of the sun and moon are used to work out accurate heights and times of tides. This course does not look at anything so complicated, but this example does show that patterns are important in many different ways.

The first section starts by looking at a do it yourself (DIY) problem using tiles. See how you get on with describing the pattern in this first activity.

Activity 1 Tile pattern

Allow approximately 5 minutes

Suppose that somebody is tiling a bathroom, and the last row of square tiles is going to be a decorative border made up of blank tiles and patterned tiles, as shown in Figure 1 below:

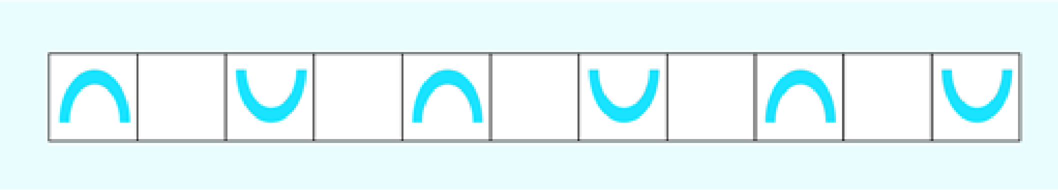


Figure 1 Tiles

A friend has offered to help with the job. How would you describe the pattern so that they laid the tiles correctly? Click on ‘reveal comment’ if you would like a hint.

Discussion

There is no one correct answer to how to tackle this description, so just get stuck in! Think about what might be the best way to describe this to somebody who didn’t have the pattern in front of them.

Answer

There are lots of ways you could have tackled this. For example, you might say that you will need some blank tiles and some patterned tiles with the ‘bridges’ on. Start with a bridge tile, and then put a blank tile next to it. Take another bridge tile, but turn it around so that the bridge is upside down, like a smile, and put it next to the blank tile in the same line. Then put another blank tile next to the smile tile. This is the pattern: bridge, blank, smile, blank, bridge, blank, smile …

Or maybe you numbered the tiles, something like this:

1 bridge
2 blank
3 smile

The pattern would then be:

1,2,3,2,1,2,3,2 etc.

The decorative border is an example of a type of geometric pattern that has many applications in art, crafts and design. You may have something similar in your own home. If you have, see if you can describe this as some extra practice.

The next section looks at a number pattern that has fascinated people for many centuries.

1.1 Pascal’s triangle

Numbering the tiles in Activity 1 gave a number pattern that described a geometric pattern. This next example is a number pattern that appeared in China and Persia more than 700 years ago, but is still used by students in mathematical and statistical problems today – it even appears in chemistry. It is known as ‘Pascal’s triangle’ after the French mathematician Blaise Pascal who studied the properties of this triangle. The first part of Pascal’s triangle is shown in Figure 2:

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

Figure 2 Pascal's triangle

You can continue the triangle indefinitely by following the pattern. So how is it created?

If you look at the triangle you can see that each row of numbers starts and ends with the number 1. Now look at the numbers in the second to last line (1 4 6 4 1) and the last line of the triangle. Look carefully at them both - can you see a way that you can make the numbers in the last line from the line above? If you add each pair of numbers in the second to last line, this gives you numbers in the last line.

Starting from the left-hand side the pairs of numbers are 1 and 4, 4 and 6, 6 and 4, and 4 and 1. So:

$$
\begin{aligned}
1+4 &= 5\\
4+6 &= 10\\
6+4 &= 10\\
4+1 &= 5
\end{aligned}
$$

This process, plus adding a 1 at end of the line, generates the next row of the triangle. Have a go at creating the next line for yourself in this next activity.

Activity 2 Pascal’s triangle – next line

Allow approximately 5 minutes

Create the next line of Pascal’s triangle by adding the pairs of numbers in the last line. Remember to add the 1s to the end of the rows when you are done.

Answer

Starting from the left-hand side the pairs of numbers are 1 and 5, 5 and 10, 10 and 10, 10 and 5, and lastly 5 and 1.

Adding these pairs gives us:

$$
\begin{aligned}
1+5 &= 6\\
5+10 &= 15\\
10+10 &= 20\\
10+5 &= 15\\
5+1 &= 6
\end{aligned}
$$

So the next line in the triangle should look like the bottom one in Figure 3:

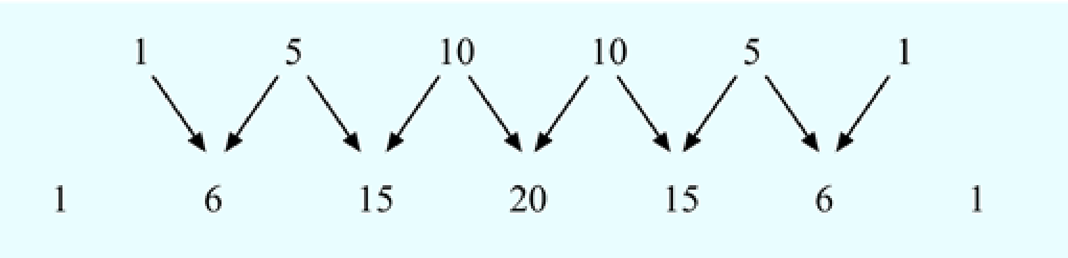


Figure 3 How to generate row 7 of Pascal's triangle from row 6

You can watch a larger version of Pascal's triangle being built in this video:

Now you've seen how to build Pascal's triangle by adding pairs of numbers it's time to see if there are more patterns hiding in this number triangle in the next section.

1.2 Pascal's triangle – a closer look

As well as the patterns within the triangle being interesting in their own right, they are used in maths and science. If you continue your study at university level in these areas you may well come across Pascal's triangle and will already be prepared for its delights. Also, having a go at spotting the patterns now will give you a chance to practise your pattern-spotting skills and you may well be surprised by how many there are!

Activity 3 Identifying patterns

Allow approximately 10 minutes

Study this six-row version of Pascal's triangle and note down any patterns that you can spot.

Click on 'reveal comment' for tips on how to get started.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Figure 4 A six-row version of Pascal's triangle

Discussion

Play around with the numbers to find patterns. Try looking down the diagonals and adding the numbers in each row.

Answer

- Each row starts and ends with a 1.
- The counting numbers, 1, 2, 3, 4, 5, are in the diagonal rows next to the sides.
- In the next diagonal row, there is the sequence 1, 3, 6, 10. These numbers are known as the 'triangular numbers' because they create triangular patterns as shown in Figure 5:

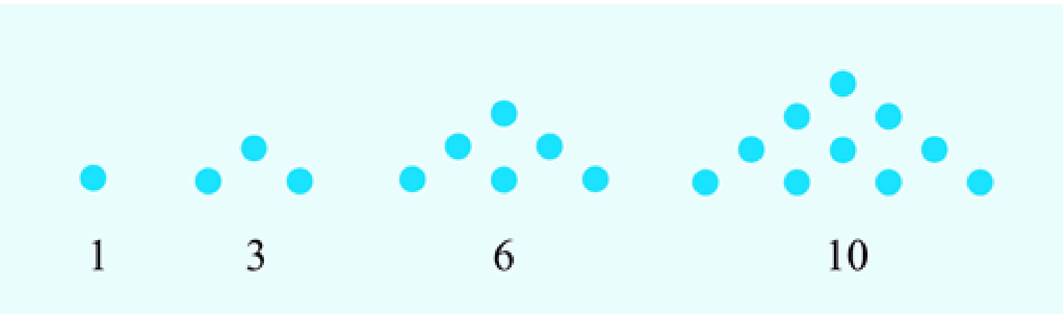


Figure 5 Triangular numbers

- Pascal's triangle is symmetrical, too. If you draw a vertical line down through 1, 2, 6, one side of the triangle is a mirror image of the other.
- The sums of the first six rows are 1, 2, 4, 8, 16, 32. The total for the next row is double the total for the current row.

These numbers can also be written as 1, 2^1, 2^2, 2^3, 2^4 and 2^5.

The pattern in the powers of 2 shown in Figure 6 suggests that the first total, 1, might be written as 2^0. Check on your calculator to see if this is correct. So it can also be said that the sums of each row are square numbers and these can be generated by raising

2 to the row number minus 1. Figure 6 may help you to see this more clearly than a written description:

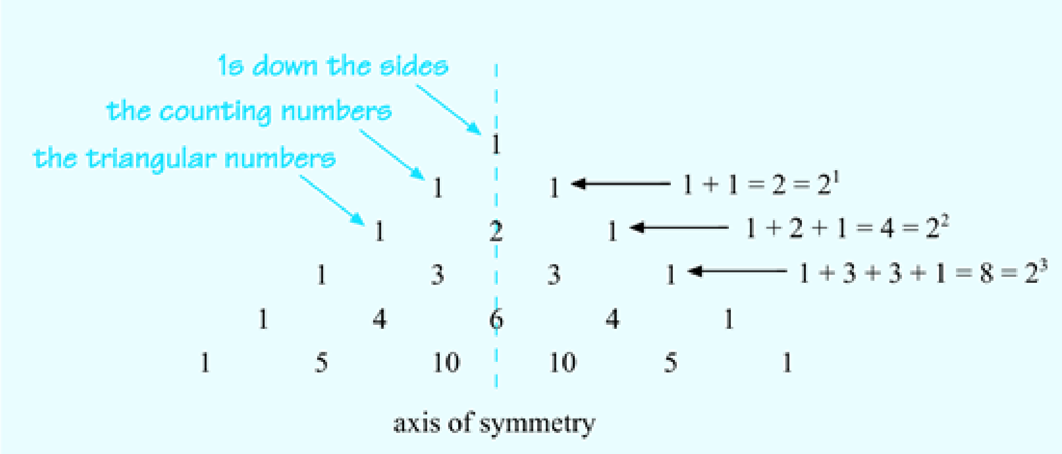


Figure 6 Powers in Pascal's triangle

Well done if you spotted some or all of those patterns. There are many more than those looked at here, so this is just a flavour of what can be found in Pascal's triangle.

What do the two examples have to do with mathematics? Well, recognising patterns in shapes, sets of numbers, processes or problems, and noticing what is the same and what is different about situations, often makes a task easier to solve. You saw how recognising the tiling pattern in Activity 1 made it easier to remember and describe, and by using the number patterns in Pascal's triangle, you could work out the sum of each row without adding the individual numbers.

If you can spot a pattern and then describe what happens in general, this can lead to a rule or formula. If you can prove that this rule will always work, it can be used elsewhere. For example, if you can work out the general process for calculating a quarterly electricity bill and then give these instructions to a computer, many electricity bills can be generated, printed and sent out in just a few minutes. It may also help you to understand your bill!

In the next section you will start to look at how this process of looking for relationships can be used to write rules.

2 Looking for relationships

This section considers the relationship between quantities in two practical situations and shows you how to describe these relationships by writing down a general rule or a **word formula**. You can think of these as a set of mathematical instructions. A good way to start is to use an everyday example.

Suppose you are planning a visit abroad. Your map marks all the distances between places in kilometres rather than miles, which you are used to dealing with. How can you work out what these distances are in miles? The first type of information you will need is how long one kilometre is, measured in miles. If you want to, you could use the internet to verify that 1 km is equivalent to approximately 0.6214 miles.

The next step is to work out how, using maths operations, you would convert kilometres into miles. You may be able to see how to convert from kilometres into miles immediately from your study of weeks 1 and 2 of this course – but, if not, try to visualise a few simple examples. The diagram in Figure 7 should help with this, as it shows how kilometres relate to miles:

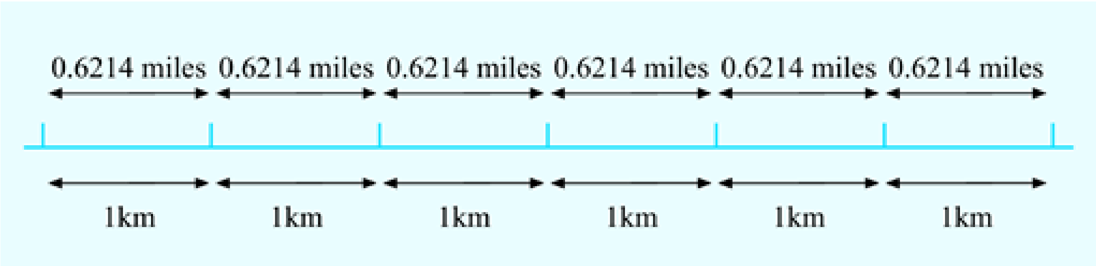


Figure 7 Converting miles to kilometres

So if the distance is 3 km, you will have 0.6214 miles three times; if the distance is 10 km, you will have 0.6214 miles 10 times; if the distance is 250 km, you will have 0.6214 miles 250 times, and so on.

You can write this last example down mathematically as:

$250\text{ km} = 250 \times 0.6214\text{ miles} = 155\text{ miles}$ (to the nearest whole number)

Notice that in each of the examples above, the process for calculating the number of miles was the same: multiply the number of kilometres by 0.6214 (the conversion factor). This technique will work for any distance, and so the following word formula can be written:

$\text{distance in miles} = \text{distance in kilometres} \times 0.6214$

Note that what the word formula works out is stated first – this is the standard way of presenting any formula.

You can use this formula to convert any distance in kilometres into miles. For example, suppose you wanted to convert 500 km, using the formula and replacing ‘distance in kilometres’ with 500 gives:

$\text{distance in miles} = 500 \times 0.6214 = 310.7\text{ miles}$

So 500 km, rounded to the nearest whole number, is equivalent to 311 miles.

In this example, you used the formula by replacing the phrase ‘distance in kilometres’ on the right side of the equation by the corresponding value, 500. This is known mathematically as **substituting** the value into the formula.

Have a go at this yourself in our next activity.

Activity 4 How many miles?

Allow approximately 5 minutes

Using the word formula just worked out, calculate how far 350 km is in miles. Show your answer to the nearest mile.

Answer

Substituting 350 for ‘distance in km’ gives:

$\text{Distance in miles} = 350 \times 0.6214\text{ miles} = 217.49\text{ miles}$

So, 350 km is approximately 217 miles to the nearest mile.

Now that you have used a word formula, you can move on to finding your own word formula in an activity with a similar idea behind it. Remember that drawing diagrams can help you with spotting relationships and patterns!

2.1 Finding a word formula

Instead of looking for a way to show the relationship between miles and kilometres in a word formula, the next activity asks you to find a word formula to help work out the value of one currency in another. This is very similar to the previous example, as the focus is again on how much of one quantity is represented by another. The activity takes you step-by-step through the process of writing your own word formula.

(Again, remember that drawing diagrams can help you to spot relationships and patterns)

Activity 5 Exchanging currencies

Allow approximately 10 minutes

Suppose somebody is visiting Europe, and they want to exchange some money from pounds (£) into euros (€). One agency offered an exchange rate of £1.00 to €1.18 and did not make any additional charges.

(a) How many euros would you get for £5? How many euros would you get for £10?

Click on ‘reveal comment’ if you would like a hint to get going.

Discussion

Try drawing yourself a diagram to help visualise what you have been asked to find.

Answer

(a) For each pound, you would get €1.18.

So for two pounds, you get two lots of €1.18, and for three pounds, three lots of €1.18 and so on.

$$\text{£5 in euros} = 5 \times \text{€}1.18 = \text{€}5.90$$

Similarly, if £10 were exchanged, the person would get 10 lots of €1.18.

$$\text{£10 in euros} = 10 \times \text{€}1.18 = \text{€}11.80$$

(b) Now, write down a word formula that can be used to convert pounds into euros.

Answer

(b) To change pounds into euros, multiply the number of pounds by the exchange rate of €1.18. The word formula to represent this is:

$$\text{Number of euros} = \text{number of pounds} \times \text{€}1.18$$

You may also have thought of the more general word formula that will work with any exchange rate of:

$$\text{Number of euros} = \text{number of pounds} \times \text{current exchange rate in euros}$$

(c) Check that your formula works by using it to convert £5 into euros.

Answer

(c) Substituting 5 for ‘number of pounds’ gives:

$$\text{number of euros} = 5 \times €1.18 = €5.90$$

This agrees with the answer in part (a) as expected, giving confidence that the word formula is correct.

These examples illustrate how a word formula can be used to summarise a mathematical process such as converting units of length or currencies. Once a formula has been derived, it can then be used in other situations, both for calculations by hand or by computer – for example, for currency transactions in a bank. You will be able to practise writing your own formulas next week.

Now let's look at using formulas in some more real-life examples.

3 Real-life examples of using formulas

The previous section considered how a formula could be built and then how it could be used. This section considers some more complicated formulas that have already been developed and are used in a variety of different situations – such as cooking, health care, business and archaeology. These examples illustrate some of the broad applications of maths and how mathematical relationships can be used in making decisions. As you work through these examples, you might consider where else maths could be used in each of these topics.

When you are trying to solve a real-life problem mathematically, you often use formulas that have already been developed, so it is important to understand how to apply them. This section will help you do that.

3.1 Formulas in cooking

The time to cook a fresh chicken depends on its mass (commonly called weight in everyday language), as given by the following formula:

$$\text{cooking time in minutes} = 15 + \frac{\text{mass in grams}}{500} \times 25$$

Roughly how long will a chicken with a mass of 2.2 kg take to cook?

To use the formula, you need to substitute the mass of the chicken into the right side of the formula and then work out the resulting calculation. However, the formula asks for the mass in grams, so the first step is to convert 2.2 kilograms (kg) into grams (g).

Since there are 1000 g in 1 kg, $\text{mass in grams} = 2.2\ \text{kg} = 2.2 \times 1000\ \text{g} = 2200\ \text{g}$

Substituting 2200 for the ‘mass in grams’ in the formula gives:

$$\text{cooking time in minutes} = 15 + \frac{2200}{500} \times 25$$

Following the BEDMAS rules, the multiplication and division must be carried out first, which gives:

$\text{cooking time in minutes} = 15 + 110 = 125$

(If you are unsure about what BEDMAS is, you can find out more in *Succeed with maths – part 1*.)

So the cooking time would be about 125 minutes, or 2 hours and 5 minutes.

Now use the same formula for a different sized chicken in the next activity. Remember to check carefully what units of mass are required.

Activity 6 A smaller bird

Allow approximately 5 minutes

How long would it take to cook a chicken with a mass of 1.6 kg?

Answer

First, convert 1.6 kg into grams, since the formula requires the mass in grams.

$1\ \text{kg} = 1000\ \text{g}$

So, $\text{mass in grams} = 1.6\ \text{kg} \times 1000 = 1600\ \text{g}$

Substituting this value into the formula gives:

$\text{cooking time in minutes} = 15 + \frac{1600}{500} \times 25 = 95\ \text{minutes}$

The chicken would take about 95 minutes, or 1 hour and 35 minutes to cook.

The next example is an application from health care.

3.2 Formulas in health care

The body mass index (BMI) is sometimes used to help determine whether an adult is underweight or overweight. It is calculated as follows:

$$\text{Body mass index} = \frac{\text{mass in kilograms}}{(\text{height in metres})^2}$$

Although care needs to be taken in interpreting the results – for example, the formula isn't appropriate for children, old people or those with a very muscular physique – a BMI of less than 20 suggests the person is underweight and a BMI of over 25 suggests the person is overweight.

In this formula, the units have been included in the expression on the right-hand side of the equals sign. It is important to change any measurements into these units before you substitute the values into the formula. (No units have been included for the BMI on the left-hand side, in line with current health care practice.) This is true for any formula that you will come across, so it is very important to remember.

This time, instead of giving you an example first it is straight into an activity to apply the formula.

Activity 7 Body mass index

Allow approximately 5 minutes

If an adult man is 178 cm tall and weighs 84 kg, calculate his BMI and decide whether he is overweight.

Answer

The formula needs the mass in kg and the height in metres. You have the man's height in centimetres, so this must first be converted into metres.

$1\text{m} \ = 100\,\text{cm}$

So, his $\text{height in metres} = (178 \div 100)\text{m} \ = 1.78\text{m}$

Substituting the mass and height into the formula for the BMI gives:

$\text{BMI} = 84 \div 1.78^2 = 26.5 = 27$ (to the nearest whole number)

Since his body mass index is over 25, the man is probably overweight.

Hopefully, you can see that this relatively simple formula gives any health care professional a way to check the health status of any patient.

The next section example introduces a bit more complexity in a formula, but all the same principles apply that you have already used. These are, converting to required units before substituting into a formula and following BEDMAS as necessary.

3.3 Formulas in business

One of the advantages of identifying the general features of a calculation and then describing it mathematically is that the formula can then be used in a computer to work out different calculations quickly and efficiently. Many utility suppliers (gas, water, electricity, telephone) have rates based on a fixed daily charge and a further charge based on how much you have used during the billing period.

For example, a mobile phone network charged £20 per month for 300 minutes (or less) of phone calls. Extra calls above the 300 minutes were charged at 40p per minute.

Assuming that more than 300 minutes were used per month, the formula for the total monthly cost can be expressed by the following word formula:

$$\text{Total monthly cost in £} = 20 + \Big(\text{total number of minutes used} - 300\Big) \times 0.4$$

For simplicity data usage or texting are not included.

In this next activity you'll unpick how this formula relates to the information given about the way in which bill is worked out. This insight will help when moving onto building your own formulas.

Activity 8 Understanding the formula

Allow approximately 10 minutes

Suppose that in one month, 375 minutes of phone calls were made. Explain how you would calculate the cost for the extra minutes (above the 300 minute allowance) and then how to calculate the total cost for that month. Can you then explain how the formula has been put together?

Answer

Because 300 minutes are included in the £20 charge, the number of minutes that are charged separately is:

$375 - 300 = 75\,\text{minutes}$

Each extra minute costs 40p. This is the same as saying £0.40 per minute, as $\pounds 1 = 100\text{p}$. So 75 extra minutes will cost $75 \times 0.4 = \pounds 30$

So the total charge for that month is $\pounds 20 + \pounds 30 = \pounds 50$

The formula says that the fixed charge is £20, and then each minute in excess of the 300 minutes allowed costs 40p. How this information relates to our formula can be shown by including some extra notes with it, as shown in Figure 8:

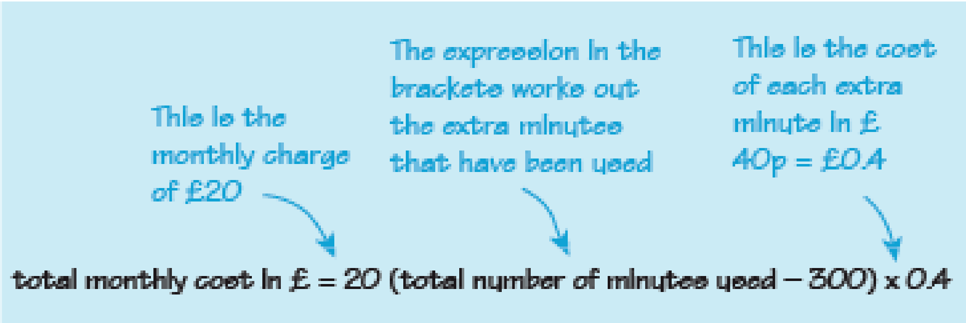


Figure 8 Annotated word formula

Now you know how the formula works, and has been put together (or derived), try applying it in this next activity.

Activity 9 Mobile Phone bills

Allow approximately 5 minutes

Use the formula to work out the total monthly cost for the following total number of minutes used:

(a) 348 minutes

Answer

(a) Substituting 348 for the total number of minutes gives the total cost in pounds as:

$$\begin{aligned}\text{Total monthly cost in } \pounds &= 20 + (348 - 300) \times 0.4\\ &= 20 + 48 \times 0.4\\ &= 20 + 19.2\\ &= 39.2\end{aligned}$$

So, the bill for the month is £39.20.

Note how this calculation is set out with all the working and a concluding sentence that answers the question precisely. This follows the ideas for correct mathematical communication that were covered at the end of Week 8 in *Succeed with Maths Part 1*.

(b) 250 minutes

Answer

(b) The formula only applies if more than 300 minutes of calls are made, so it cannot be used in this case. This is another important step in using formulas – check that they apply in your situation before using them. Here, the charge is £20 for up to 300 minutes of calls, so the charge for this month is £20.

If you have a mobile phone with a similar deal, you could work out a formula that fits yours and use it to check your bills – and get some more practice at the same time. It might also help you to decide if a different deal may suit you better.

Moving on now from everyday formula to those used in academic study, the next section looks at how one set of formulas are used in archaeology to work out the approximate height of some of our ancestors.

3.4 Formulas in archaeology

Footprints from prehistoric human civilisations around the world have been found preserved in either sand or volcanic ash. From these tracks, it is possible to measure the foot length and the length of the stride. These measurements can be used to estimate both the height of the person who made the footprint and also whether the person was walking or running. This can be done by using three formulas:

$$\text{height} = 7 \times \text{length of foot}$$

$$\text{relative stride length} = \frac{\text{stride length}}{\text{hip height}}$$

$$\text{hip height} = 4 \times \text{length of foot}$$

Note that no units have been included in these formulas – so it is important to make sure that you use the same units, such as centimetres, throughout the calculation.

The formula for relative stride length is used to establish if the person was walking or running. If the value of the relative stride length is less than 2, the person was probably walking; if the value is greater than 2.9, the person was probably running. Between these two values, it is difficult to be sure if the person was running or walking. The expected value for relative stride length is usually between 0 and 5.

Notice that the relative stride lengths quoted have no units. The stride length and the hip height are both measured in cm, and when one is divided by the other, these units cancel each other out, just like cancelling numbers when dealing with fractions. If you go on to study physics or other sciences, you will find many examples like this.

Before trying the next activity, if you would like to see one of these formulas being used in practice, have a look at this short video.

Activity 10 Footprints in the sand

Allow approximately 10 minutes

From one set of footprints, the length of the foot is measured as 21.8 cm and the stride length as 104.6 cm. What does the data suggest about the height and the motion of the person who made these footprints? Remember you can click on ‘reveal comment’ if you get stuck.

Discussion

Break the problem down into simpler parts. Start by asking yourself what you need to know to tell if the person was running or walking.

Answer

$\text{height} = 7 \times \text{length of foot}$

So,

$$\begin{aligned}\text{estimated height of the person} &= 7 \times 21.8\ \text{cm} \quad \text{(to the nearest cm).}\\ &= 153\ \text{cm}\end{aligned}$$

To work out if the person was running or walking, it is important to know the relative stride length. This is given by:

$$\text{relative stride length} = \frac{\text{stride length}}{\text{hip height}}$$

The question gives the information about the stride length but not the hip height. To calculate the hip height use this formula.

$\text{hip height} = 4 \times \text{length of foot}$

Fortunately, this can be done, as the foot length is also given in the question.

So, $\text{hip height} = 4 \times 21.8\,\text{cm} = 87.2\,\text{cm}$

This has given all the information needed to calculate the relative stride length. So, substituting into the formula:

$$\text{relative stride} = \frac{104.6}{87.2} = 1.2 \quad \text{(rounded to 1 decimal place).}$$

As the relative stride length is less than 2, the person was probably walking.

This was a slightly more complicated problem than our previous examples as it required a step in the middle to calculate some extra information needed. So, well done for having a go!

3.5 Combining formulas

There is a quicker way to the answer which requires two of the formula to be combined to give a new formula for relative stride length.

The formula for relative stride length was:

$$\text{relative stride length} = \frac{\text{stride length}}{\text{hip height}}$$

It is known that:

$\text{hip height} = 4 \times \text{length of foot}$

This means that it is possible to substitute the formula for hip height in the formula for relative stride length. This then gives:

$$\text{relative stride length} = \frac{\text{stride length}}{4\ \times\ \text{foot length}}$$

This new formula could be used directly with the information that was given in the question. This version would also make more sense in the field, as it contains the two pieces of information that would be known on site.

In the final activity for this week you'll consider why it is important to always think about whether the answer you have obtained looks sensible. This is something that you should always ask yourself. It can act as a prompt to check over your working or logic in any problem.

Activity 11 Relative stride length: what's wrong?

A student used the formula $\text{relative stride length} = \dfrac{\text{stride length}}{4 \times \text{foot length}}$ to calculate the relative stride length in the footprints activity.

The student entered the key sequence on their calculator for the calculation as: 104.6 ÷ 4 × 21.8. This gave an answer of approximately 570, so the student concluded that the person was probably running very fast. Can you explain where the student made the mistake and why they should have been suspicious of their answer?

Answer

The calculator will perform this calculation from left to right, using the BEDMAS rules (order of operations), which treat multiplication and division as equally important. So, it will first divide 104.6 by 4 to get 26.15, and then multiply by 21.8 to get approximately 570. However, this is not the correct calculation from the formula.

The stride length (104.6) should be divided by (4 × foot length), so the calculation should be 104.6 ÷ (4 × 21.8). When explaining the formula at the start of this section you were told that the expected value for relative stride length lies between 0 and 5. So, an answer of 570 should have immediately set alarm bells ringing for this student that something had gone wrong somewhere. As well as checking your answer using known information, carrying out a quick estimate of what size of answer you are expecting can also be very useful. So, if you are expecting a value in the order of hundreds, say around 200, and your answer is in the millions, then those alarm bells should be ringing again!

Well done, you've just completed the last activity for Week 3! You've just got one final section to look at before moving on to the Week 3 quiz. This summarises the top tips for using any formula that you've covered here.

4 Tips for using a formula

The following tips for using any formula you come across will be useful, both in next week's study and any further study at university level you may undertake, particularly if you are interested in maths, science or technology. You might find it useful to make a note of these now, or bookmark them to refer to later:

- Check that the formula can be applied to your particular problem.
- Check what values you need to substitute and also what units these should be measured in. Convert the measurements if necessary before substituting.
- Make a rough estimate for the answer.Substitute the values into the formula carefully.

- Use BEDMAS to work out the resulting calculation, step by step. Explain your steps carefully using words like 'substituting' and 'converting'.
- Check that your answer seems reasonable, both practically and from the estimate. Round your answer appropriately.

Remember your concluding sentence should both answer the question and include units.

5 This week's quiz

Use the tips as you are completing the questions in this week's quiz.

Go to:

Week 3 practice quiz.

Open the quiz in a new tab or window (by holding ctrl [or cmd on a Mac] when you click the link).

6 Summary

Congratulations for making it to the end of another week. You may not have realised it, but you have now started on groundwork that will lead you to taking your first steps with algebra. This is a fundamental tool of maths and is used in many different subject areas and other, possibly unexpected subjects. In fact anywhere you want to work out an unknown value from numbers that you already know will probably need some algebra. This is just the start of your journey though, and this course takes you through it step-by-step to help you build your confidence. What you should now feel more confident in doing is:

- recognising patterns in a variety of different situations
- using word formulas to help solve a problem
- writing your own word formulas to help solve a problem.

You can now go to Week 4.

Week 4: Working with patterns and formulas

Introduction

You finished last week by using some word formulas from different aspects of everyday life. Some of these examples will not have been relevant for some people (not everyone is an archaeologist!) but they should still have all given you some great practice at using different formulas. This week the course turns to writing your own word formula. You'll start by looking at spreadsheets and then do some number tricks to amaze your friends and family with. Finally, you'll use doing and undoing diagrams to change what is being calculated by a formula. This is another aspect of mathematical study that is directly related to building skills for algebra. The final section concentrates on inequalities. These are used in the place of the usual equals sign, to show that something is greater or less than something else.

Watch Maria introduce Week 4:

Video content is not available in this format.

After this week's study, you should be able to:

- understand how to read a spreadsheet
- understand how a spreadsheet is created
- use doing and undoing diagrams to change formulas
- interpret and use notation for inequalities.

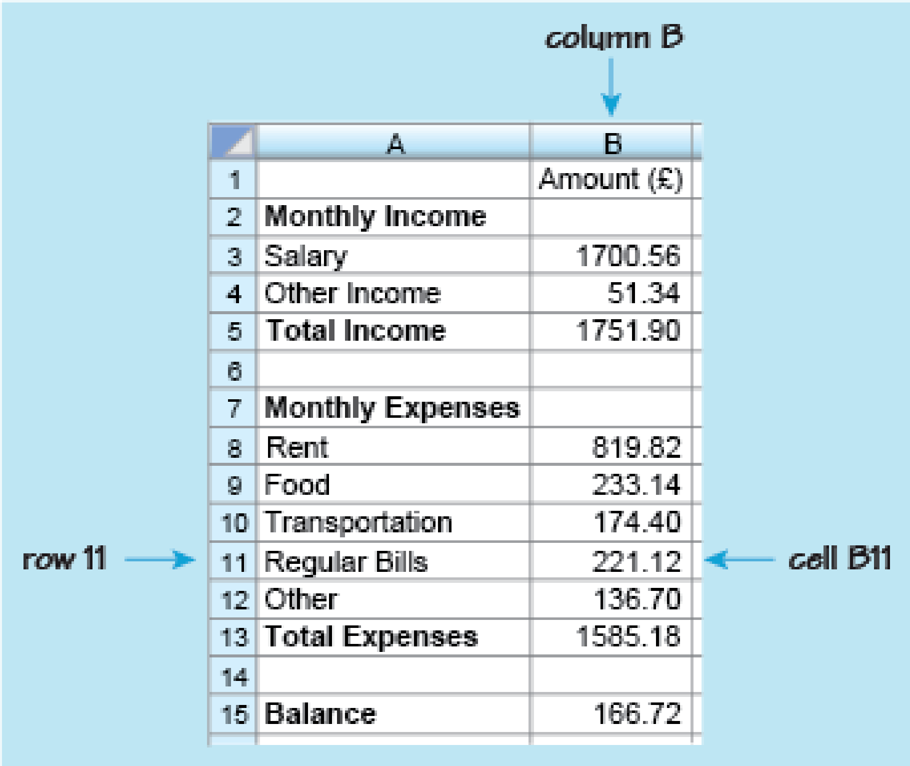
1 Your formulas: using a spreadsheet

Last week, the course looked briefly at how to construct a word formula but concentrated on how to use formulas you were given successfully. This week moves onto building your own formulas, starting with a practical application using spreadsheets. As well as having relevance to future university level studies, if you need to work with data that you or somebody else has collected, being able to work with spreadsheets may well come in useful at home or at work, now and in the future. It is also a useful way to learn more about constructing your own formulas.

Before getting going though, it is worth pausing to think about what a spreadsheet is, how they may be useful and the way they are set up.

At the basic level, spreadsheets can be used to display data, such as fuel prices over a number of years, in tables. Once, the data are in a spreadsheet then the real strengths of spreadsheets can be used. You can perform calculations on the data to work out other information from the data and even use a spreadsheet to create charts for you to display the data or results. Spreadsheets come into their own when you have a mountain of data and calculations to perform on it.

Figure 1 shows part of a spreadsheet that has been constructed to record monthly income and outgoings, or expenditure. It is similar to a bank balance sheet that you might draw up by hand to check your monthly income and expenses, the totals, and the overall balance. However, the spreadsheet has been created on a computer and has formulas inserted into it to enable automatic calculations without using a calculator.



	A	B
1		Amount (£)
2	**Monthly Income**	
3	Salary	1700.56
4	Other Income	51.34
5	**Total Income**	1751.90
6		
7	**Monthly Expenses**	
8	Rent	819.82
9	Food	233.14
10	Transportation	174.40
11	Regular Bills	221.12
12	Other	136.70
13	**Total Expenses**	1585.18
14		
15	**Balance**	166.72

Figure 1 A spreadsheet used for a monthly budget

As with any other spreadsheet, this example is made up of rows and columns containing boxes. These boxes are called ‘cells’. The columns are identified by letters and the rows by numbers. This enables you to identify each cell in the spreadsheet. For example in this spreadsheet, the number 221.12 is in column B, row 11, so is in cell B11.

Cells can thus be found, from their reference, by looking down the column and across the row. So cell A3 can be found by looking down column A and across row 3. This cell contains the word ‘Salary’. Notice that cells can contain either text or numbers.

Use this knowledge when completing your first activity of the week.

Activity 1 Spreadsheet cells

Allow approximately 5 minutes

Use the spreadsheet in Figure 1 to answer the following questions.

(a) What word or number is contained in the following cells?

(i) A5
(ii) A12
(iii) B15
(iv) B1

Answer

(a)

(i) The cell that is in both column A and row 5 contains the words ‘Total Income’.
(ii) Down column A and across row 12, cell A12 contains the word ‘Other’.
(iii) The cell that is in column B, row 15 contains the number 166.72.
(iv) The cell that is in column B, row 1 contains the heading ‘Amount (£)’.

(b) What is the reference for the cells that contain the following?
(ii) The number 1585.18.
(iii) The word ‘Food’.

Answer

(b)

(ii) 1585.18 is in column B and row 13, so its reference is B13.
(iii) ‘Food’ is in column A and row 9, so its reference is A9.

So, remember when you are referring to a particular cell, you always state the column letter, followed by the row number.

Looking again at Figure 1 you can see what other information it shows. For example, if you look at row 3, this shows that the monthly salary is £1700.56. Although cell B3 only contains the number 1700.56, you know that this is measured in pounds from the heading in cell B1 ‘Amount (£)’. Overall, the spreadsheet shows the items that make up the monthly income and where money has been spent over the month. If you were keeping these records by hand, you would then need to calculate the total income, the total expenses and the balance.

For example, to find the total income for the month, you would need to add the salary of £1700.56 to the other income of £51.34. This gives the total income of £1751.90. In other words, to calculate the value in cell B5, you need to add the values in cells B3 and B4. This can be written as the following formula:

$$\text{Value in B5} = \text{value in B3} + \text{value in B4}$$

1.1 Finding the balance

A similar approach can be used for working out other values in the spreadsheet. Have a go yourself in this next activity.

Activity 2 Working out the balance

Allow approximately 5 minutes

Formulas have also been used to calculate the total monthly expenses and the balance. If you were working these calculations out by hand, what would you do? Write this using the appropriate cell numbers and maths operators as shown in the previous example. Finally, check these comparing your answers using the formulas with the values in cell B13 and B15.

Answer

To calculate the total monthly expenses, you need to add the individual expenses of ‘Rent’, ‘Food’, ‘Transportation’, ‘Regular Bills’, and ‘Other’. The formula will therefore be:

$$\text{value in B13} = \text{value in B8} + \text{value in B9} + \text{value in B10} + \text{value in B11} + \text{value in B12}$$

To find the balance, you need to take the expenses away from the total income. So, the formula to calculate the balance will be:

$$\text{value in B15} = \text{value in B5 - value in B13}$$

Checking these now:

$$\begin{aligned} \text{Value in B13} &= 819.82 + 233.14 + 174.40 + 221.12 + 136.70 \\ &= 1585.18 \\ \text{Value in B15} &= 1751.90 - 1585.18 \\ &= 166.72 \end{aligned}$$

Comparing these with the values in the spreadsheet, it is clear that they agree. This should give us confidence that the formulas built are correct!

How are these formulas actually put into the spreadsheet though? To do this you type the formula directly into the relevant cell, starting with an equals sign to tell the software that you are entering a formula rather than a word. This is shown in Figure 2:

	A	B
1		Amount (£)
2	**Monthly Income**	
3	Salary	1700.56
4	Other Income	51.34
5	**Total Income**	=B3+B4
6		
7	**Monthly Expenses**	
8	Rent	819.82
9	Food	233.14
10	Transportation	174.4
11	Regular Bills	221.12
12	Other	136.7
13	**Total Expenses**	=SUM(B8:B12)
14		
15	**Balance**	=B5-B13

Figure 2 Finding the balance

Notice that in cell B13, a shorthand form for the sum (total) has been entered. The formula '= B8 + B9 + B10 + B11 + B12' could have also been entered, but it is much quicker to use the shorthand form '= SUM(B8:B12)', which is shown here. This instruction tells the computer to add the values in all the cells from B8 to B12.

If you have access to spreadsheet software, try and recreate this example now and see how you get on with entering formulas yourself.

Another advantage of using a spreadsheet is that if you change some of the numbers, all the calculations that use that particular number will be automatically updated to reflect the change. For example, in this budget, the amounts for the salary, rent and the regular bills are likely to remain the same from one month to the next, and may only be updated once or twice a year. However, the amounts for food, transportation, other bills and other income will probably change from month to month. These values can be changed easily on the spreadsheet and the revised balance produced immediately, taking some of the work from you and reducing the risk of calculation errors.

1.2 Spreadsheet formulas

There is one final activity in this section on spreadsheets, bringing all these ideas together.

Activity 3 Spreadsheet formulas

Allow approximately 10 minutes

The formulas used in a spreadsheet are displayed as shown in Figure 3. Note that the asterisk (*) is the notation for multiplication used in spreadsheets and that all entries have been rounded to the closest penny. This example shows UK VAT, which was 20 per cent in 2014 (20 per cent as a decimal is 0.2).

	A	B	C	D
1	Item	Cost (£)	VAT (£)	
2	Radio	64.99	=B2*0.2	=B2+C2
3	Kettle	16.49	=B3*0.2	=B3+C3
4	Fan	12.99	=B4*0.2	=B4+C4
5				

Figure 3 Spreadsheet formulas

(a) The values in columns C and D will be displayed to two decimal places because they represent an amount of money. What values will be displayed in cells C3 and D3?

Answer

(a) The formula in C3 is: $\text{value in C3} = \text{value in B3} \times 0.2$ (to 2 decimal places)

So, $\text{value in C3} = 16.49 \times 0.2 \approx 3.30$

The value in D3 is obtained by adding together the values in B3 and C3.

So, $\text{value in D3} = \text{value in B3} + \text{value in C3} = 16.49 + 3.30 = 19.79$

(b) What do you think is being calculated in the cells in column D? Can you suggest a suitable heading for this column to be entered into D1?

Answer

(b) Column D represents the total cost of the item plus VAT. A suitable heading might be ‘Total (£)’. You can think of other correct titles such as ‘Final Price (£)’.

(c) Cell D5 calculates the sum of the values in D2, D3 and D4. Write down a formula that could be entered in cell D5. What does this value represent?

Answer

(c) The sum can be found by adding together the values in cells D2, D3 and D4. The formula ‘=D2+D3+D4’ could therefore be entered into cell D5. (Note that you do not use the formulas that are present in each of the cells you are adding. Just the cell title is sufficient.) Alternatively, you could use ‘SUM(D2:D4)’, which also adds together the cells from D2 to D4. The resulting entry represents the total cost (including VAT) of the radio, kettle and fan together.

With these changes, and the title ‘Total (£)’ typed into cell C5, the spreadsheet will look like the example shown here:

	A	B	C	D
1	Item	Cost (£)	VAT (£)	Total (£)
2	Radio	64.99	13.00	77.99
3	Kettle	16.49	3.30	19.79
4	Fan	12.99	2.60	15.59
5			Total	£113.36

Figure 4 Spreadsheet

This section has served a few purposes. It has introduced spreadsheets and shown you how you can use these to carry out calculations as well as helping you make a start on writing your own formula. The next section leaves spreadsheets behind to continue with this latter skill, to build your confidence with formulas further.

2 Your own formulas

Let’s start this section by looking at a number trick and seeing if a formula can derived, or built, for it. The aim of the trick is to end up with the name of an animal.

To do this follow the instructions listed below. The final number represents a letter of the alphabet, numbered in ascending order from A to Z. So, A is 1, B is 2, C is 3 etc. Then write down the name of an animal beginning with that letter.

The instructions for the trick are:

- Think of a number.
- Add 5.
- Double this.
- Subtract 8.
- Divide by 2.
- Take away the number you first thought of.
- Add 4.
- The final number represents a letter of the alphabet, numbered in ascending order from A to Z. So, A is 1, B is 2, C is 3 etc.
- Now write down the name of an animal beginning with that letter.

Activity 4 What animal?

Allow approximately 5 minutes

Carry out the number trick, starting with the number 3 and then any other number you would like to start with. What do you notice about the result in both cases?

Answer

Starting with 3, the instructions work out as follows

Instruction	**Expression**
Add 5	$3 + 5 = 8$
Double it	$8 \times 2 = 16$
Subtract 8	$16 - 8 = 8$
Divide by 2	$8 \div 2 = 4$
Take away the number you first thought of	$4 - 3 = 1$
Add 4	$1 + 4 = 5$

You will find that you always get 5, whatever number you start with! This gives the letter E. Most people will then think of elephant, although you cannot always guarantee that. The odd eel or emperor penguin may sneak in!

What is really interesting here though, to a mathematician, is how the trick works. The activity in the next section will help you to work this out.

2.1 Understanding the number trick

One way to understand how the previous number trick works is to use a visual representation of each stage of the instructions, and that is how you are asked to approach this next activity. Take your time, and read over the instructions and hints carefully to start you off.

Activity 5 How does this trick work?

Activity 5 How does this trick work?

To see how this trick works, read through the instructions from before. Because you could have thought of any number, replace this unknown number with a thought bubble like this:

Keep the thought bubble as part of the new number that you get at each stage. So after the first step you will have something like this:

+ IIIII (to represent 5).

Discussion

When doubling, draw the cloud and associated number twice.

Answer

This is what our visualised representation of the number trick looks like:

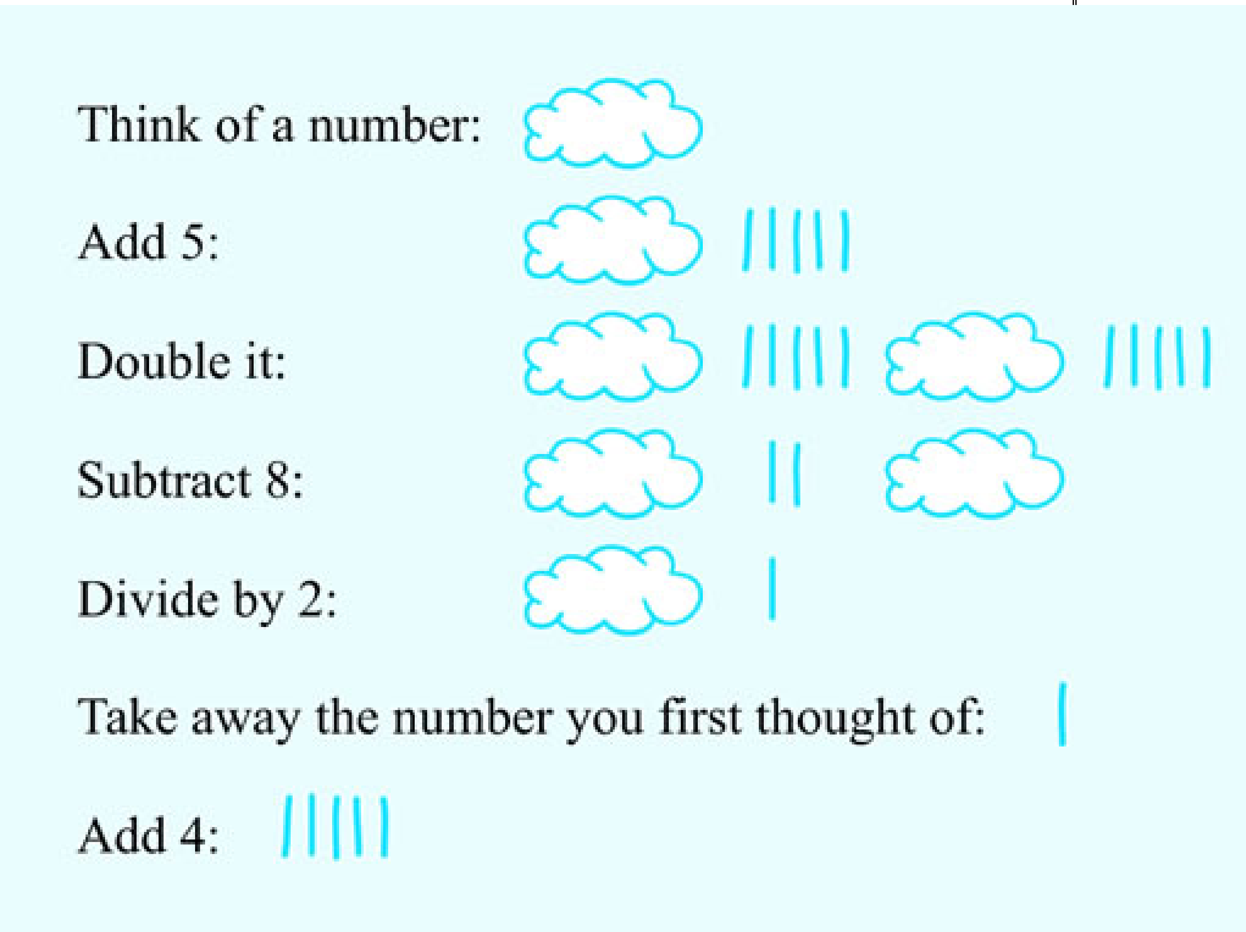


Figure 5 The number trick explained

This shows that the numerical answer is always going to be 5. It does not depend on which number was chosen first since there are no clouds, representing the initial number, involved in the final answer. So the letter of the alphabet chosen is always E. There are not many animals with names beginning with E and most people do think of an elephant first, but we've already thought about the pitfalls of that!

Rather than using a cloud to represent the number and explaining the trick visually, you could write this in a more mathematical way by using either a word or letter to represent the initial number. In maths unknown numbers are usually represented using a letter. Using 'n' to stand for this number the trick can be written out again without the clouds like this:

Table 1 Showing an algebraic representation of the number trick

Instruction	Expression
Think of a number	n
Add 5	$n + 5$

Double it	$n+5+n+5$ This can be re-written as $n+n+5+5$ $n+n$ is the same as 2 lots of n, so can be written in shorthand as $2\times n$ So $n+5+n+5=2\times n+10$
Subtract 8	$2\times n+10-8=2\times n+2$
Divide by 2	$(2\times n\div 2)+(2\div 2)$ Dividing each part by 2 separately is the same as dividing the whole of $2\times n+2$, by 2. Taking this former approach helps us show the next step more clearly. 2 lots of n divided by 2 gives 1 lot of n, and clearly 2 divided by 2 gives 1. So, finally giving: $n+1$
Take away the number you first thought of	$n+1-n=1$
Add 4	$1+4=5$

This gives the same result, where n is not involved in the final answer. This time a more conventional mathematical representation was used rather than a cloud!

At first this may have appeared to be quite a daunting example, using both numbers and letters but hopefully you found that by working your way carefully through each stage you were able to understand how the final answer was obtained. Working step by step carefully through problems will really pay dividends as problems and concepts start to look more complicated. Remember this when completing the next activity with another number trick.

2.2 Another number trick

Here is another number trick which uses the same techniques you learned in the last section to see how it works.

Activity 6 Another number trick

Activity 6 Another number trick

Try the following trick several times; make a note of the number you started with and your final answer each time:

- Think of a number between 1 and 10 (this will work with numbers greater than 10, but the restriction is to keep the arithmetic manageable).
- Multiply by 4.
- Add 6.
- Divide by 2.
- Subtract 3.
- Divide by 2.

What do you notice about the answer? See if you can explain why this happens, either by using a diagram or by writing down the expressions for the answer at each stage, replacing your initial number with n again.

Answer

You should find that this time the answer is always the number you chose at the start. Both ways of showing why the number trick works are shown below.

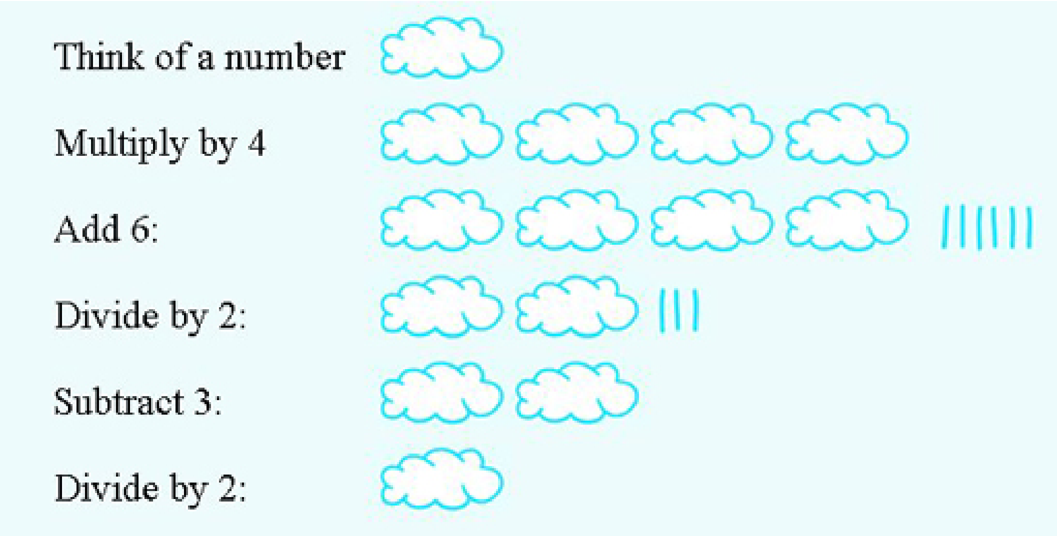


Figure 6 Graphic representation of how the number trick works

Table 2 Table format showing how the number trick works

Instruction	Expression
Think of a number between 1 and 10	n
Multiply by 4	$4 \times n$
Add 6	$4 \times n + 6$
Divide by 2	$(4 \times n \div 2) + (6 \div 2)$ $= 2 \times n + 3$
Subtract 3	$2 \times n + 3 - 3$ $= 2 \times n$
Divide by 2	$2 \times n \div 2$ $= n$

These two number tricks are sets of instructions that can be used to build, or derive, a formula. They are in essence not much different from those you looked at for spreadsheets. They just involved a few more steps to reach the answer.

Sometimes the answer and the set of instructions are available, while, for some reason, the starting value is not known. For example, imagine something had happened to your bank balance spreadsheet and the starting balance was no longer displayed but you still knew the final balance and all the formulas. Could you then rebuild the spreadsheet to work out initial balance? The next section shows you how.

3 Doing and undoing

The next example helps you to start to think about how to work out an initial number if you know the answer and instructions using a more straightforward example than our spreadsheet. This time there are only two instructions:

- Think of a number.
- Add 4.

If my answer is 11, can you work out what number I was thinking of? You might have said 'What number do I have to add to 4 to get 11?' or perhaps 'If I take away 4 from 11, what number do I get?'. In both cases, you should have arrived at the answer 7.

In the second method, 'subtracting 4' undoes the 'adding 4' in the original instructions, and this can be illustrated with a 'doing-undoing diagram' (see Figure 7).

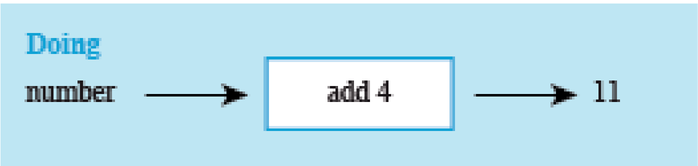


Figure 7 A doing-undoing diagram

In the 'doing' part of the diagram, start with the number and write down the operations applied in turn until you get the answer. Here there is just one operation: 'add 4'.

For the 'undoing' part of the diagram, start on the right with the answer, in this case 11. Then work back towards the left, undoing each operation in turn until you find the starting number. In this case, 'subtract 4' undoes 'add 4' and $11 - 4 = 7$

You can then construct an undoing diagram that starts from the left, as shown in Figure 8. Notice how the arrows indicate the direction to read the diagram.

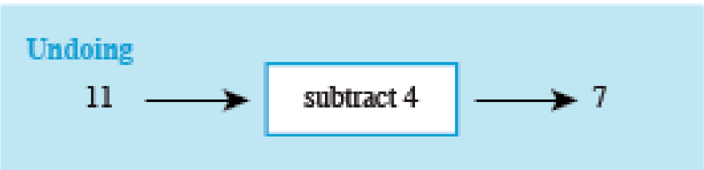


Figure 8 Doing-undoing diagram

So this also shows that the number first thought of was 7.

3.1 Doing some doing and undoing

Now you've had a look at an example, see how you get on with the next activity.

Activity 7 Working out the number

Allow approximately 13 minutes

Try to work out what number was thought of in the following problems. You may find it helpful to use some doing-undoing diagrams to visualise, and keep track of these.

(a) Think of a number, subtract 3, and the answer is 2.

Answer

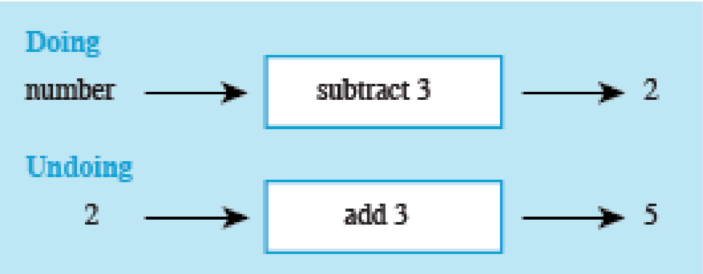


Figure 9 Doing-undoing diagram for part (a)

So, the initial number was 5.

Remember, you can check your answer by using it with the instruction to check you obtain the correct answer given. So, 5 − 3 = 2!

(b) Think of a number, multiply by 5, and the answer is 35.

Answer

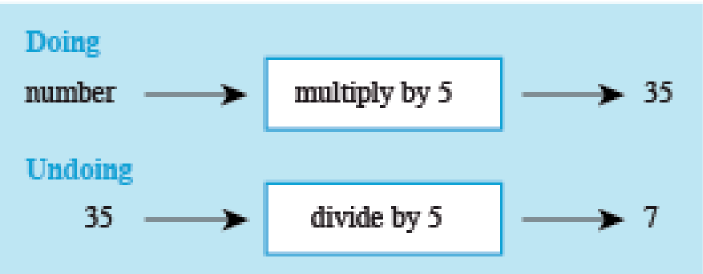


Figure 10 Doing-undoing diagram for part (b)

So, the initial number was 7. (Check 7 × 5 = 35).

(c) Think of a number, add 4, double it, and the answer is 14.

Click on 'reveal comment' if you would like a hint

Discussion

When you have to deal with more than one operation, just take each step in turn. Write down the doing diagram and draw the undoing one underneath, working backwards to undo each operation.

Answer

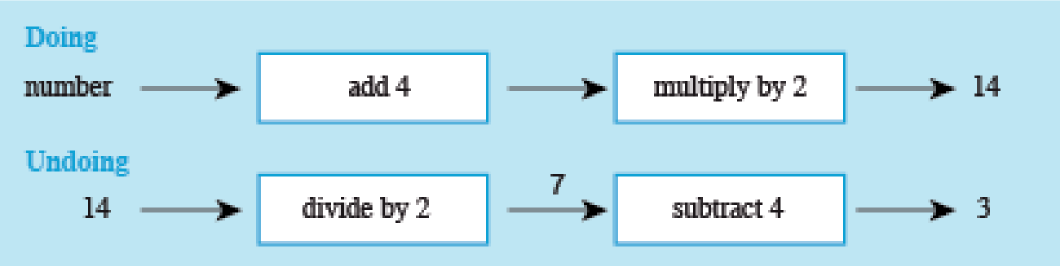


Figure 11 Doing-undoing diagram for part (c)

So, working from the left in the undoing diagram, 14 ÷ 2 = 7 and 7 − 4 = 3.

If you like, you can write the output from each box on the arrow as shown in Figure 13 above.

So, the initial number was 3. (Check: 3 + 4 = 7, then 7 × 2 = 14).

(d) Think of a number, add 5, multiply by 3 and subtract 4. The answer is 17.

Answer

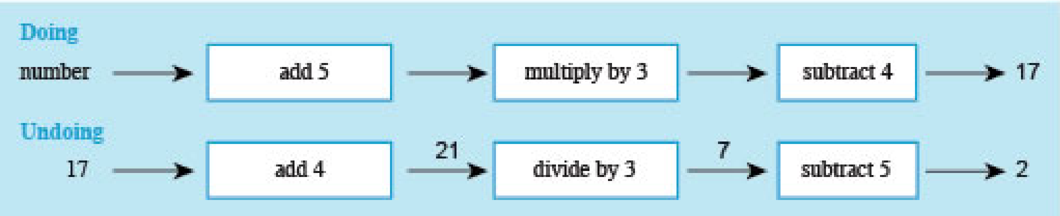


Figure 12 Doing and undoing diagrams to solve a multistep problem

So, if the answer is 17, undoing the steps by working from right to left gives:

Instruction	**Expression**
Add 4	$17 + 4 = 21$
Divide by 3	$21 \div 3 = 7$
Subtract 5	$7 - 5 = 2$

So, the initial number was 2 this time. (Check: 2 + 5 = 7, then 7 × 3 = 21 and 21 − 4 = 17).

You may be wondering what is the point of all this doing/undoing business! Sometimes there is a need to change a formula so that you can work out something different from it. This is called rearranging the formula and does occur quite frequently in any subject area that uses formulas. The same technique that you have been looking at in this section on 'undoing' instructions can be used when rearranging formulas, as you'll see in the next section.

4 Rearranging formulas

Rearranging a formula makes it possible to show the formula in a different way, whilst making sure the relationships between the various elements that it describes are not changed. If these relationships are changed, the formula will no longer work as expected. Thinking about rearranging using the 'doing' and 'undoing' technique from the last section will help to ensure that the relationships are maintained.

Instead of a using a set of instructions, the next example is from Week 3. Hopefully, you recognise this word formula:

$$\text{Number of euros} = \text{number of pounds} \times \text{€}1.18$$

How could a visitor from Europe use this word formula to convert euros into pounds, say, while shopping on holiday? They would need a formula for the number of pounds based on the number of euros. This can tackled by drawing the 'doing and undoing' diagrams for this situation.

In this case, the starting point for the doing diagram will be number of pounds on the left, the operation in the box will be multiply by 1.18 and output on the right will be number of euros (our answer). Therefore, to undo the operation in the box divide by 1.18. So, the diagrams will look like those shown in Figure 13.

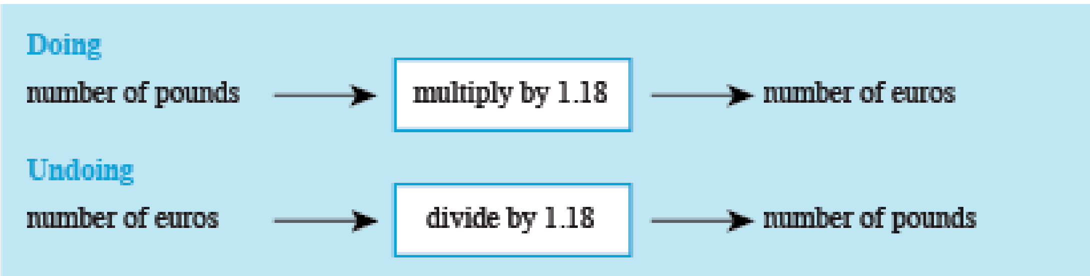


Figure 13 Doing and undoing diagrams for converting pounds to euros and euros to pounds

From the undoing diagram the new word formula can be built. This gives:

$$\text{Number of euros} \div 1.18 = \text{number of pounds}$$

This just needs to be swapped around, so that the formula is shown in the conventional way with what you are calculating stated first. This results in:

$$\text{Number of pounds} = \text{number of euros} \div 1.18$$

Now use what you have learned here in this next activity.

Activity 8 Miles and kilometres

Allow approximately 5 minutes

To change kilometres into miles, you can use the formula:

$$\text{Distance in miles} = \text{distance in kilometres} \times 0.6214$$

Starting with ‘distance in kilometres’, draw a doing diagram to show how to calculate the distance in miles. Then draw the undoing diagram and write down the formula for changing miles into kilometres.

Your formula should start ‘distance in kilometres = ...’.

Answer

The doing and undoing diagrams are shown in Figure 14.

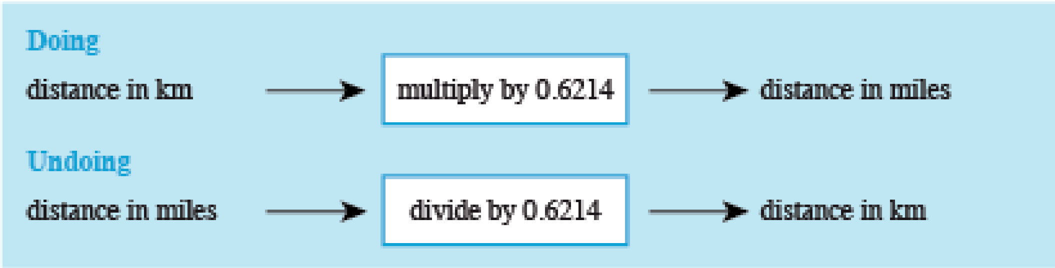


Figure 14 Doing and undoing diagrams for converting kilometres to miles and miles to kilometres

So the formula for converting miles into kilometres is:

$$\text{distance in kilometres} = \text{distance in miles} \div 0.6214$$

4.1 Time formula

There is just one last activity for you to complete now before moving on to a different subject area. This will give you another opportunity to rearrange a word formula but with a few more elements to consider.

Activity 9 How many minutes for chatting?

Allow approximately 5 minutes

This was the formula for the monthly cost in £ of a mobile phone that was used in week 3:

$$\text{Total monthly cost in £} = 20 + \left(\text{total number of minutes used} - 300\right) \times 0.4$$

The owner wishes to stick to a monthly budget of £45. To do this they need to know the maximum number of minutes they can use, so that they don’t exceed the budget. Use doing and undoing diagrams to work the maximum number of minutes. Substitute ‘Total monthly cost in £’ for £45.

See if you can then write a word formula that will allow the number of minutes to be calculated for any budget.

Answer

Here is the doing diagram:

Figure 15 Doing diagram for £45 monthly budget

Undoing each step in turn gives the following undoing diagram.

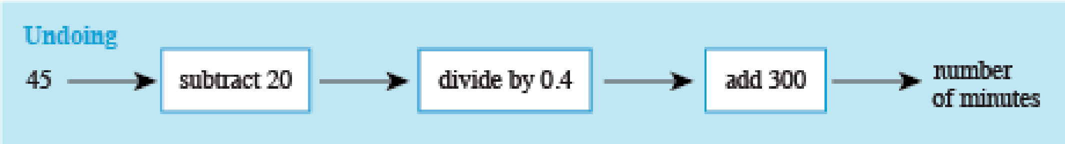


Figure 16 Undoing diagram for number of minutes

Applying these operations in turn gives:

- Subtracting 20 gives: $45 - 20 = 25$
- Dividing by 0.4 gives: $25 \div 0.4 = 62.5$
- Adding 300 gives: $62.5 + 300 = 362.5$

So, 362 minutes can be used for a monthly budget of £45.

Now all you need to do to provide a word formula for any monthly budget is to change from £45 back to 'Total monthly cost in £'.

So the word formula will be:

$$\text{Total number of minutes} = \left(\text{Total monthly cost in £} - 20\right) \div 0.4 + 300$$

or

$$\text{Total number of minutes} = \frac{\text{Total monthly cost in £} - 20}{0.4} + 300$$

Mathematicians prefer the second way of showing this formula just because it looks much neater – otherwise there is no difference between them.

The last part of the activity was more challenging than our other examples, so well done for having a go at it. What you may not realise is that during these last few sections you have been taking your first steps into the wonderful world of algebra. This is one of the fundamental tools of most maths, science and technology, so if you continue into university level study in any of these areas you will certainly find these skills useful. You won't be using doing and undoing diagrams but techniques used for rearranging formulas (and equations) will be just the same – looking for what 'undoes' each operation.

In the next section you are going to look at inequalities. This is the name mathematicians use for expressions that use notation meaning, for example 'greater than' and 'less than'. You have actually come across three examples of these in this week's and last week's study, as you will discover when you move on.

5 Inequalities: greater than or less than?

There have been three occasions in these last two weeks when checks have been made to see whether a result is greater than or less than some other value. These were when:

- calculating the BMI and determining whether the person was overweight or underweight
- determining from footprints whether a person was walking or running
- checking whether a phone had been used for more than 300 minutes.

In all these situations the statements could have also been written mathematically using inequality symbols.

This checking of whether values are greater than, or less than, some limit happens frequently – occasions when you may see this could be safety issues or age limits. For example, medicines may have to be stored at a temperature of 25 °C or less; child train tickets can be bought for children who are over 5 but under 16 years old.

Rather than writing out ‘greater than’ or ‘less than’, shorthand notation is often used – as shown below.

- **>** greater than
- **≥** greater than or equal to
- **<** less than
- **≤** less than or equal to

If you have difficulty remembering these symbols, you can think of them as arrows that point to the smaller number or note that ‘<’ looks like an ‘L’, which stands for ‘less than’.

The symbols are read from left to right. For example, ‘11 > 9’ is read as ‘11 is greater than 9’; the cost of a holiday in pounds < 1000 is read as ‘the cost of a holiday is less than £1000’.

To use the symbols in your own writing, decide what you want to say first, then use the symbol. For example, since 10 is greater than 5, this would be written as ‘10 > 5’. On the number line, because –4 lies to the left of –3, –4 is less than –3; this would be written as ‘–4 < –3’.

Similarly, the instructions for the medicine that has to be stored at a temperature of 25 °C or less could be written as ‘Medicine storage temperature in °C ≤ 25’.

Sometimes, it can be helpful to draw a number line to visualise this kind of information. For example, the ages that children are eligible for the child train fare are from their fifth birthday up to, but not including, their sixteenth birthday. This means that the age has to be greater than or equal to 5 and less than 16. This range is shown on the number line below. Does this help you to visualise the information?

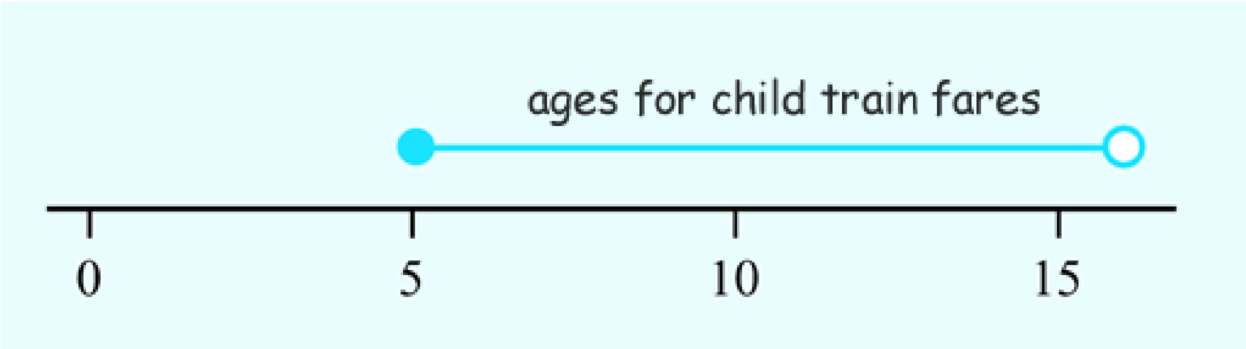


Figure 17 Ages that children are eligible for the child train fare

The empty circle means that this number (16) is not included and the filled-in circle means that this number (5) is included in the interval. This is then written as ‘5 ≤ age for child train fare < 16’.

Note carefully the format in which this last inequality is written. The variable that is being described – that is, the age for a child train fare – is always in the centre when defining a range of values with an upper and lower limit. This is the maths convention that is followed with ranges of values. This can be a little confusing, as you have to turn your logic around for the lower limit. You are in effect saying that 5 is less than, or equal to, a child train fare, rather than a child train fare is greater than, or equal to, 5. This amounts to exactly the same thing, but how you may usually think about the lower limit is turned on its head!

Now put these ideas into practice in this final activity for the week.

Activity 10 Inequalities

Allow approximately 10 minutes

(a) Which symbol (< or >) should go in the blank spaces below? Click on ‘reveal comment’ if you would like a hint to get going.

- (i) 4__7
- (ii) 18__10
- (iii) 3__–2

Discussion

Write the inequality using words first and then convert this to the correct symbol.

Answer

(a)

- (i) 4 < 7
- (ii) 18 > 10
- (iii) 3 > –2

(b) Work out what the following mathematical statements mean. Write your answers as full sentences and be as precise as you can.

- (ii) Balance in account > 0.
- (iii) Speed (in mph) on motorway ≤ 70.
- (iv) 18 ≤ age (in years) ≤ 50.

Answer

(b) Your answers may be worded slightly differently, but they should still have the same meaning:

(ii) The balance in the account is greater than zero.

(iii) The speed on the motorway is less than or equal to 70 mph.

(iv) The age in years is between 18 and 50, inclusively.

(c) Rewrite the following sentences as statements using the inequality symbols:

(iii) The refrigerator temperature should be less than 4 °C.

(iv) There must be at least five people on the committee.

(v) For an ideal weight, a person’s BMI should be greater than 20 and less than 25.

Answer

(c) Your answers may be worded slightly differently again, but they should still have the same meaning:

(iii) Refrigerator temperature (in ºC) < 4.

(iv) Number of people on committee ≥ 5.

(v) $20 < \text{BMI} < 25$

As well as in the examples given here, inequalities are also an important part of many computer programs. So you can see that maths pops up in all sorts of areas of study and everyday lives.

This is the end of this section on inequalities and the week as a whole. You may have found some ideas that were new to you here and others that were more familiar. Whatever the case for you, hopefully you feel even more confident with your maths skills now.

6 This week's quiz

Well done, you've just completed the last of the activities in this week's study before the weekly quiz.

Go to:

Week 4 compulsory badge quiz.

Remember, this quiz counts towards your badge. If you're not successful the first time you can attempt the quiz again in 24 hours.

Open the quiz in a new tab or window (by holding ctrl [or cmd on a Mac] when you click the link).

7 Summary

This week completes your study of patterns and formulas. Starting in Week 3 with describing a visual pattern and looking for number patterns in the famous Pascal's triangle, you moved on to using word formulas and writing your own. Finally, you learned how to rearrange a word formula so that you could find out something new from it before taking a quick look at inequalities. These have been a busy two weeks as well as an important two weeks, as you've been developing skills that will stand you in good stead for any future studies that involve algebra. So, congratulations on making it to the end of another week. Next week will be all about using powers, and in particular how they can help to write very large and very small numbers concisely. See you there!

You should now be able to:

- understand how to read a spreadsheet
- understand how a spreadsheet is created
- use doing and undoing diagrams to change formulas
- interpret and use notation for inequalities.

You can now go to Week 5.

Week 5: More power

Introduction

This week extends your study of exponents, or powers (*Succeed with maths – Part 1* provides an introduction to this). You'll consider how to use these to write large and small numbers to make them easier to understand, write and work with. To do this, a system called scientific notation is used, based on different powers of ten, which allows small and large numbers to be written using fewer digits. Once such numbers have been rewritten this system enables calculations to be carried out with the numbers efficiently. Finally, this week will cover the operation that 'undoes' taking the power of a number – roots. This will all give you a good basis to continue your study in maths and many other subject areas, including technology and science.

Watch Maria introduce what this week involves:

Video content is not available in this format.

After this week's study, you should be able to:

- understand and use scientific notation
- perform calculations with numbers written in scientific notation
- understand and calculate square roots of a number.

1 Big and small

The numbers that you encounter in everyday situations are usually of a reasonably manageable size, that is, they are generally not very big or very small. A person's height may be 1.8 metres or the balance in a current account around a few hundred pounds. Both of these are numbers that are easy to understand and read. But what about the UK's national debt? This is often quoted in billions of pounds, a number with at least ten digits! That is not quite so easy to write or understand when written down. Large numbers like this are often encountered in science and technology, as well as very small numbers. This is where scientific notation comes into its own.

One example of a very large distance is the width of the observable universe. This is about 92 billion light years, where a light year is about 9.5 trillion kilometres. A trillion is one thousand billion or a million million – which means a number followed by 12 zeroes. So to find the width of the universe in kilometres, you would need to multiply 92 billion by 9.5 trillion, since each light year is 9.5 trillion kilometres. How would you do that?

Well, you may want to use a calculator, but there's a problem: 92 billion is 92 000 000 000, and that is too big a number for many calculators.

So how can you work this out? You could have worked the calculation out on paper, or multiplied 92 by 9.5 to get 874 and deduced that the distance must therefore be 874 billion trillion kilometres – or you may just have been bewildered by the enormity of the numbers! This is an example where scientific notation, which is based upon different powers of the number ten, could usefully be employed.

The next section will be a quick refresher on powers of ten, before moving onto how to use these in scientific notation. If you would like to refresh your knowledge of powers, take a look at *Succeed with maths – Part 1*.

1.1 Powers of 10

100 can be calculated by multiplying 10 by itself, that is 100 = 10 × 10 and, any number multiplied by itself can also be written using power notation. The value of the power being the number of times the number is multiplied by itself. Written in power notation, 10 × 10 is 10^2, said as ten to the power of two or ten squared.

Similarly, 1000 is the same as 10 × 10 × 10, or 10^3. This can be extended indefinitely, to give larger and larger numbers and their corresponding powers of 10.

Our first activity will give you the chance to practise writing numbers using powers of 10, before moving on to how to use these in scientific notation. If you need a hint to get going, click on Reveal comment.

Activity 1 Powers of 10

Allow approximately 5 minutes

Write each of the following numbers as a power of ten.

(a) 10 000

Discussion

(a) Start by working out how many tens you need to multiply together, to give you 10 000, then use the knowledge that $10 \times 10 = 10^2$.

(a) $10\ 000 = 10 \times 10 \times 10 \times 10 = 10^4$

(b) 1 000 000

Answer

(b) $1\ 000\ 000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$

(c) 1 000 000 000

Answer

(c) $1\ 000\ 000\ 000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

(d) What do you notice about the number of zeroes in the original number and the power of 10 in parts (a) to (c)?

Answer

(d) The number of zeroes in the original number is equal to the power of 10.

Use the answer to part d) to write each of the following as a number using zeroes and then as a power of 10.

(e) There are about one hundred thousand hairs on an average human head.

Answer

(e) One hundred thousand is 100 000, or 10^5.

(f) By 2050, the population of earth may be about 10 billion people.

Answer

(f) Ten billion is 10 000 000 000, or 10^{10}.

(g) In 1961, the French poet Raymond Queneau wrote a book called *A Hundred Thousand Billion Poems*.

Answer

(g) One hundred thousand billion is 100 000 000 000 000, or 10^{14}.

If you would like to know more about Queneau's book, click on 'reveal comment'.

Discussion

Queneau's book contained ten sonnets, each with 14 lines. Each page, containing one sonnet, was cut into 14 strips with one line on each strip, so it was possible to combine lines from different sonnets to form a new sonnet. There are 10^{14} different ways of making a sonnet in this way.

A digital version of A Hundred Thousand Billion Poems allows you to change lines in one sonnet. The number of sonnets created by visitors to website already is displayed at the bottom of the page. When I first visited the site, fewer than one million had been created.

Now let's look at how to use powers of ten to write large numbers using scientific notation. You'll learn about its use with small numbers later in the week.

2 Scientific notation and large numbers

So how can you use powers of ten to write numbers in scientific notation? Let's look at the example of six million to start with. Written in full this number is six followed by six zeroes: 6 000 000. This can also be written as: $6 \times 1\ 000\ 000$. Now 6 million is shown like this it can be written using a power of ten by noting that:

$1\ 000\ 000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$

10 multiplied by itself 6 times is the same as 10^6

So, $1\ 000\ 000 = 10^6$

So, this now means that 6 million can be written as:

$6 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 6 \times 10^6$

You would therefore write 6 million using scientific notation as: 6×10^6 .

Similarly, if the example had been six and a half million (6 500 000), this can be written in scientific notation as:

$6.5 \times 1\ 000\ 000$ or 6.5×10^6 in scientific notation.

So, a number written in scientific notation takes the form of: a number between **1 and 10** multiplied by a whole number power of 10. This can be shown mathematically as:

$\left(1 \leq \text{number} < 10\right) \times 10^{\text{whole number}}$

Thus, there are two steps to writing a number using scientific notation, as follows:

1. Work out what the number between 1 and 10 will be.
2. From this, decide on the power of 10 required.

So, taking the example of 130 000, the number between 1 and 10 must be 1.3, as it cannot be 0.13 or 13. 0.13 is less than 1, and 13 is greater than 1.

$130\ 000 = 1.3 \times 10 \times 10 \times 10 \times 10 \times 10 = 1.3 \times 10^5$

So, 130 000 written in scientific notation is 1.3×10^5 . Now, it's your turn to try some examples.

Activity 2 Understanding and writing numbers in scientific notation

Allow approximately 10 minutes

Write the following numbers without using powers of 10.

(a) 2×10^4

Answer

(a) $2 \times 10^4 = 2 \times 10 \times 10 \times 10 \times 10 = 20000$

(b) 3.82×10^8

Answer

(b) $3.82 \times 10^8 = 3.82 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 382\ 000\ 000$

(c) 9.3567×10^2

Answer

(c) $9.3567 \times 10^2 = 9.3567 \times 10 \times 10 = 935.67$

Write the following numbers in scientific notation.

(d) 92 billion

Discussion

1 billion is 1 followed by 9 zeroes.

Answer

(d) $92\,\text{billion} = 92\;000\;000\;000 = 9.2 \times 10^{10}$

(e) 400 trillion

Discussion

1 trillion is a million million.

Answer

(e) $400\,\text{trillion} = 400\;000\;000\;000\;000 = 4{\times}10^{14}$

(f) 9 500 000 000 000

Answer

(f)

$$9\;500\;000\;000\;000 = 9.5 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 9.5 \times 10^{12}$$

(g) Which of these numbers is the biggest?

Discussion

Compare the powers of ten.

Answer

(g) The highest power of the three numbers in this activity is 14, so 400 trillion is the biggest number here.

Now you've found out how to write large numbers using scientific notation, in the next section you'll turn your attention back to the problem posed at the beginning of this week: how to work out the width of the observable universe in kilometres.

2.1 Multiplying powers with the same base number

At the beginning of the week the first example used was the width of the observable universe, which is about 92 billion light years, where each light year is about 9.5 trillion kilometres.

So, to work this out in kilometres, multiply 92 billion by 9.5 trillion. Both these numbers can be shown in scientific notation as follows:

$92\,\text{billion} = 9.2 \times 10^{10}$

$9.5\,\text{trillion} = 9.5 \times 10^{12}$

So the calculation becomes:

$9.2 \times 10^{10} \times 9.5 \times 10^{12}$

This still leaves what looks like a complicated calculation. However, there are some handy short cuts that can be used when dealing with powers with the same base number (in this case 10 is the base number). To explore this, here is a simpler example: $10^2 \times 10^4$.

Writing these numbers out in full, the calculation is:

$$\begin{aligned} 10^2 \times 10^4 &= (10 \times 10) \times (10 \times 10 \times 10 \times 10) \\ &= 100 \times 10\ 000 = 1\ 00\ 000 \end{aligned}$$

1 000 000 written in powers of 10 is 10^6.

So, $10^2 \times 10^4 = 10^6$.

Can you spot anything that relates the powers of ten in the answer to those in the original numbers?

The powers of 10 in the original numbers were 2 and 4, and in the answer 6. It would seem that if you add the powers of 10 in the original numbers, you arrive at the power of 10 in the answer. So: $10^2 \times 10^4 = 10^{(2+4)} = 10^6$

This rule can be used whenever you are dealing with multiplication of numbers with the same base number and gives a quick way to calculate the width of the observable universe. Note that the base number remains the same! You can do this in the next activity after you've had a go at some other examples.

Activity 3 Multiplying powers with the same base number

Allow approximately 10 minutes

Work out the following, giving your answer first in power form before calculating the answer to the sum. You can use a calculator to work these out for parts b) to d).

(a) $10^4 \times 10^3$

Discussion

These numbers both have the same base number (10), so add the powers for the final answer.

Answer

(a) $10^4 \times 10^3 = 10^{(4+3)} = 10^7 = 10\ 000\ 000$

$10^7 = 10\ 000\ 000$

(b) $2^2 \times 2^4$

Discussion

The base number in this case is 2.

Answer

(b) $2^2 \times 2^4 = 2^{(2+4)} = 2^6$

$2^6 = 64$

(c) $(-3)^2 \times (-3)^3$

Answer

(c) $(-3)^2 \times (-3)^3 = (-3)^{(2+3)} = (-3)^5$

$(-3)^5 = -243$

(d) Now work out the width of the universe. Remember that the universe is 92 billion light years across and a light year is about 9.5 trillion kilometres.

Answer

(d) $\text{Width of the universe} = 9.2\,\text{billion} \times 9.5\,\text{trillion km}$ First express both numbers in scientific notation:

$$92\,\text{billion} = 9.2 \times 10^{10}$$

$$9.5\,\text{trillion} = 9.5 \times 10^{12}$$

$$\begin{aligned}\text{So the width} &= 9.2 \times 10^{10} \times 9.5 \times 10^{12}\,\text{km} = 9.2 \times 9.5 \times 10^{10} \times 10^{12}\,\text{km} \\ &= 87.4 \times 10^{22}\ \text{km}\end{aligned}$$

Our first number is not between 1 and 10, so this is not yet shown in correct scientific notation.

$$87.4 = 8.74 \times 10^{1}$$

$$\begin{aligned}\text{So}, 87.4 \times 10^{22} &= 8.74 \times 10^{1} \times 10^{22} \\ &= 8.74 \times 10^{23}\end{aligned}$$

That means the width of the observable universe is about, 8.74 x 10^{23} km and you've just worked out the answer to the original problem – well done!

You might reasonably be asking yourself if there is a similar rule for dividing numbers with the same base as there is for multiplying them. Continue to the next section to find out now.

2.2 Dividing powers with the same base number

The rule for multiplying powers with the same base number is to add together the powers of these numbers. Let's now consider what happens with division using the example of $10^6 \div 10^2$.

As you may already know, any division can be rewritten as a fraction and then simplified if appropriate. (Again *Succeed with maths – Part 1.* looks at this topic.)

. This means that $10^6 \div 10^2$ can be shown as:

$$\frac{10^6}{10^2} = \frac{10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10}$$

To simplify this both the top and bottom of the fraction can be divided by 10 and then by 10 again, giving:

$$\frac{10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10} = 10 \times 10 \times 10 \times 10 = 10^4$$

Therefore, $10^6 \div 10^2 = 10^4$

If you look at the powers in this sum and the answer, you may notice that you can get the same result by subtracting the powers, giving:

$$10^6 \div 10^2 = 10^{(6-2)} = 10^4$$

This rule works with any power where the base numbers (in this case 10) are the same.

As with any new idea in maths the best way to cement these is to have a go at some examples. So see how you get on with the next activity on dividing numbers with the same base number.

Activity 4 Dividing powers with the same base number

Allow approximately 5 minutes

Without using your calculator, work out each of the following:

(a) $4^7 \div 4^5$

Answer

(a) $4^7 \div 4^5 = 4^{(7-5)} = 4^2$

$4^2 = 16$

(b) $3^5 \div 3^4$

Answer

(b) $3^5 \div 3^4 = 3^{(5-4)} = 3^1$

$3^1 = 3$

(c) $2^4 \div 2^{-2}$

Answer

(c) $2^4 \div 2^{\text{-}2} = 2^{(4-(\text{-}2))} = 2^6$

$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Now you have two rules giving you a short cut when faced with calculations involving powers of the same base number. These can be summarised as follows:

- When multiplying powers with the same base number, add the powers.
- When dividing powers with the same base number, subtract the powers.

All the powers that have been dealt with so far were positive whole numbers and shortly you'll look at negative powers. This leaves the number separating the positive and negative numbers, zero. Can a number actually be raised to the power of zero and if so, what does that mean? Let's see in the next short section.

2.3 Zero as a power?

The easiest way to investigate whether a number raised to the power of zero means anything mathematically, is to look at an example.

Here, start with $10^3 \div 10^3$. Using the rules from the last section about dividing numbers with the same base number this gives us:

$10^3 \div 10^3 = 10^{(3-3)} = 10^0$.

So, $10^3 \div 10^3$ is the same as 10^0

However, another way of looking at $10^3 \div 10^3$ is by writing this as a fraction and then simplifying it.

This gives:

$$10^3 \div 10^3 = \frac{10^3}{10^3} = 1$$

But $10^3 \div 10^3 = 10^0$, this means that:

$$10^0 = 1$$

So, although 10^0, that is '10 multiplied by itself zero times' does not make any practical sense, mathematically this has the value of 1. You can use a similar argument to show that any number to the power zero has a value of 1. For example,

$2^0 = 1, 3.25^0 = 1$, and $(-6)^0 = 1$.

You could check these on your calculator now if you would like to confirm this is correct.

Remembering this rule will be very useful if you continue your study of maths, science or technology based subjects, so it is not just an interesting diversion!

Now let's get back to scientific notation and how to show small numbers in this format. The first thing to do is though is to look at negative powers, or exponents, in the next section.

2.4 Negative exponents

The same approach used to understand what zero as a power means can also be used for negative exponents or powers.

This time showing $10^2 \div 10^6$ as a fraction, simplifying and using the rules for powers with the same base. This gives:

$$10^2 \div 10^6 = \frac{10^2}{10^6} = \frac{10 \times 10}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = \frac{1}{10 \times 10 \times 10 \times 10} = \frac{1}{10^4}$$

So,

$$10^2 \div 10^6 = \frac{1}{10^4}$$

Now, using the rule for dividing numbers with the same base:

$$10^2 \div 10^6 = 10^{(2-6)} = 10^{-4}$$

Putting these two results together gives:

$$10^{-4} = \frac{1}{10^4}$$

In the same way, 5^{-2} means $\frac{1}{5^2}$, or $\frac{1}{5 \times 5}$, which is equivalent to $\frac{1}{25}$.

Since 5^2 is 25 and 5^{-2} is $\frac{1}{25}$, in maths 5^{-2} is said to be the **reciprocal** of 5^2. So, negative powers represent the reciprocal of the same positive power.

Negative powers of ten then enable small numbers to be shown using scientific notation. Before that though you will practise understanding and writing negative powers in the next activity.

Activity 5 Understanding and writing negative powers

Allow approximately 10 minutes

Without using your calculator, write the following numbers as fractions or whole numbers, as appropriate.

Remember you can always click on ‘reveal comment’ for a hint if you get stuck.

(a) 2^{-3}

Answer

(a) $2^{-3} = \frac{1}{2^3} = \frac{1}{2 \times 2 \times 2} = \frac{1}{8}$

(b) 3^{-4}

Answer

(b) $3^{-4} = \frac{1}{3^4} = \frac{1}{3 \times 3 \times 3 \times 3} = \frac{1}{81}$

(c) 10^{-6}

Answer

(c) $10^{-6} = \frac{1}{10^6} = \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = \frac{1}{1\,000\,000}$

Write the following as powers of ten.

(d) 0.01

Discussion

Convert the number to a fraction first, think about place value if you need to.

Answer

(d) $0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$

(e) 0.00001

Answer

(e) $0.00001 = \frac{1}{100000} = \frac{1}{10^5} = 10^{-5}$

2.5 Small numbers and scientific notation

By using negative powers of ten it is possible to write numbers less than one in scientific notation. Let’s take the example of 0.03.

0.03 is three one hundredths, so this can be shown as a fraction, with 100 as the denominator. This is important, as 100 is a multiple of 10, and can therefore be shown using powers of 10 as follows:

$0.03 = \frac{3}{100} = \frac{3}{10 \times 10} = \frac{3}{10^2}$

From the previous section you know that $\frac{1}{10^2} = 10^{-2}$.

Hence, $0.03 = 3 \times 10^{-2}$

Now 0.03 is shown in scientific notation, since there is a number between 1 and 10, that is then multiplied by a power 10.

This shows you why the process works, but it may not necessarily be the most straightforward way of writing numbers that are less than one in scientific notation. Another way of thinking about this is in two steps, as with large numbers:

- Write the number as one between 1 and 10.
- From this, decide on the power of 10 required.

To establish the power of 10 required, work out how many times you would need to divide the number from step 1 by ten to reach the original number. Each time you divide by 10, the negative power of 10 reduces by 1, starting from -1.

Following these steps for 0.03 again:

1 Write the number as one between 1 and 10.

This gives us 3.

2 From this, decide on the power of 10 required.

To do this divide by 10 and then by 10 again to return to the original number. Hence, the negative power will be $-2\ (-1-1)$.

To convert a number in scientific notation back into decimal form, write down the negative power of 10 as a fraction and then divide the numerator by the denominator. For example:

$5.72 \times 10^{-4} = \frac{5.72}{10^4} = \frac{5.72}{10\ 000} = 5.72 \div 10\ 000 = 0.00057$

Now use what you have learned in this section, as well as your previous knowledge from the week, to complete the following activity and hone your skills.

Activity 6 Understanding and writing small numbers in scientific notation

Allow approximately 10 minutes

Write down the following numbers as decimals.

(a) 3×10^{-4}

Discussion

Start by converting the negative power into a positive power, hence showing the number as a fraction.

Answer

(a) $3 \times 10^{-4} = \frac{3}{10^4} = \frac{3}{10 \times 10 \times 10 \times 10} = \frac{3}{10\ 000} = 0.0003$

(b) 7.5×10^{-5}

Answer

(b) $$7.5 \times 10^{-5} = \frac{7.5}{10^5} = \frac{7.5}{10 \times \times 10 \times 10 \times 10 \times 10} = \frac{7.5}{100\ 000} = 0.000075$$

(c) 1.46×10^{-3}

Answer

(c) $$1.46 \times 10^{-3} = \frac{1.46}{10^3} = \frac{1.46}{10 \times 10 \times 10} = \frac{1.46}{1000} = 0.00146$$

Write down the following numbers in scientific notation:

(d) 0.000007

Answer

(d) $$0.000007 = \frac{7}{1\ 000\ 000} = \frac{7}{10^6} = 7 \times 10^{-6}$$

(e) 0.0742

Answer

(e) $$0.0742 = \frac{7.42}{100} = \frac{7.42}{10^2} = 7.42 \times 10^{-2}$$

(f) 0.0000000098

Answer

(f) $$0.0000000098 = \frac{9.8}{1\ 000\ 000\ 000} = \frac{9.8}{10^9} = 9.8 \times 10^{-9}$$

If you want some more practice with scientific notation before you move on to look at how to use it on a calculator, have a go at a scientific notation game.

2.6 Calculators and scientific notation

So far you have carried out calculations involving scientific notation by hand. To make life a little easier, you can input numbers in scientific notation into most calculators.

How you do this will vary between different calculators, but the keys to look for are 10^x, EXP or EE. Generally you just enter the first number, followed by the 10^x, EXP or EE key, and then the power. There is no need to press the multiplication button because this is done automatically by the calculator.

So to enter 1×10^3, press ‘1’, then 10^x, EXP or EE, and finally 3.

Have a go now and see if you can work out how to enter 4.5×10^3 and 4×10^{-4}.

You may need to be careful with some calculators when performing calculations with scientific notation in terms of what they are including as part of the power. For example, if you input 4.5 × 10^3 × 2, some calculators may include the 2 as part of the power – so you may need to include the power in brackets to differentiate this clearly.

Here's an activity to practise working with numbers in scientific notation in general and particularly when using a calculator.

Activity 7 Working with numbers in scientific notation

Allow approximately 5 minutes

Work out the following using your calculator. Show all your final answers in correctly formatted scientific notation.

(a) $9.2 \times 10^{10} + 9.5 \times 10^{9}$

Answer

(a) $9.2 \times 10^{10} + 9.5 \times 10^{9} = 1.015 \times 10^{11}$

(b) $3.3 \times 10^{-6} \times 4.2 \times 10^{-5}$

Answer

(b) $3.3 \times 10^{-6} \times 4.2 \times 10^{-5} = 1.386 \times 10^{-10}$

(c) In a country, there are 7 532 000 beehives, and each hive contains about 50 000 bees at the peak of the bee season. Write each of these numbers in scientific notation, then use your calculator to find the total number of bees in the country.

Answer

(c) $7\ 532\ 000 = 7.532 \times 10^{6}$ and $50\ 000 = 5 \times 10^{4}$

$\text{Total number of bees} = 7.532 \times 10^{6} \times 5 \times 10^{4} = 3.766 \times 10^{11}$.

That's a lot of bees – about 380 billion!

This completes your study of scientific notation, but not the work associated with powers. Before finishing for this week, there is one final area to look at with regards to powers. The operation that reverses or undoes these – roots.

3 Roots

Taking the root of a number reverses the operation of raising a base number to a certain power so the answer will be the original base number. Since a number can be multiplied by itself any number of times, there are also any number of different roots. Here though, you are going to concentrate on square roots – the reverse of squaring or raising a base number to the power of two.

If you take the square root of 25, written as $\sqrt{25}$, the answer is 5, the original base number. You can check this by reversing the root taken and squaring the result. So, squaring $5\left(5^2\right)$ gives 25 – the original number.

However, with any square root of a positive number there is not just one possible answer. Looking at the original example of the square root of 25, the answer was 5 because $5 \times 5 = 25$. But there is another square root because $(-5) \times (-5)$ is also 25. So, any positive number has two square roots, a positive one and a negative one.

To avoid confusion the following convention is used to distinguish between these two outcomes. When both roots are relevant $\pm\sqrt{}$ is used to distinguish between these two outcomes (where the symbol $\pm$ is read as ‘plus or minus’). Hence, for both the negative and positive root of 25 you would write:

$\pm\sqrt{25} = \pm 5$

When only the positive root is relevant $\sqrt{}$ is used without the $\pm$. Hence for the positive square root of 25 you would write:

$\sqrt{25} = 5$

But how do you actually work out square roots? This usually relies on one of two methods. The first is just memory, and the more you use roots the more you will remember what they are, but this only helps in a few situations. The other method is to use a calculator to do the work for you. If you continue your study of maths, you may well come across methods to work out square roots from scratch as well, but this is definitely something for another time.

To use a calculator to find square roots, use the $\sqrt{}$ key.

Try using your calculator to find $\sqrt{625}$. Depending on what calculator you are using, you will either have to press the square root button before you enter the number, or after.

Hopefully you should have obtained 25 as the answer.

It is worth noting that a calculator will only ever give you the positive square root, so it can be easy to forget there is another answer as well!

As well as the form of notation covered here, roots can also be shown using power notation. The next section will look at how this is done.

3.1 Square roots in power notation

Using power notation to show a root is very useful for further maths, when you may need to show roots using this method to help solve a problem. By using the rules for multiplication of powers from 2.1 Multiplying powers with the same base number this week it is possible to work out the appropriate value of the power. Remember that when multiplying powers with the same base, the power in the answer will the sum of the original powers.

So, let’s consider the following example of $4^{1/2}$ (4 raised to the power of ½) and see what happens when this is squared:

To square $4^{1/2}$, $4^{1/2}$ must be multiplied by itself giving:

$4^{1/2} \times 4^{1/2}$

To work out the answer to this, add the powers together:

$4^{1/2} \times 4^{1/2} = 4^{(1/2+1/2)} = 4^1 = 4$

This shows that when you multiply $4^{1/2}$ by itself (squaring $4^{1/2}$), the result is 4. In other words, $4^{1/2}$ is another way of writing the square root of 4. This is defined as the positive square root and may be expressed as:

$$4^{1/2} = \sqrt{4} = 2$$

To satisfy yourself that this is correct, try using your calculator to work out $4^{1/2}$.

There is one final point to consider about square roots before finishing, and that is the square root of a negative number.

3.2 Square roots of negative numbers

So, far you have only been looking at the square roots of positive numbers, but what about negative numbers? Let's see how a calculator handles these in this next activity.

Activity 8 Square roots of negative numbers

Allow approximately 5 minutes

Before you find out the answer that your calculator gives to the square root of −16, think about what the answer could be.

Discussion

Remember that a positive number multiplied by a negative number (or vice versa) gives a negative result.

Answer

What answer you get from your calculator will depend upon the calculator you are using.

Most will simply give you an error message!

This is because to obtain −16 you have to multiply 4 and −4, so the root of −16 is both of these different numbers at the same time. Hence, most calculators will show an error message.

More advanced graphics calculators will give the answer for the square root of −16 as 4*i*.

In this context, the *i* is shorthand for $\sqrt{-1}$ and is known as an 'imaginary number'. This is the concept that mathematicians have used to get around the quandary of having two answers simultaneously to the same question. This is not the same as the issue of a square root of a positive number having two possible answers. That is either a positive or a negative number, not both a positive and a negative number.

You may well be thinking that this just more maths for maths sake, but many fields within technology and science require the use of imaginary numbers to provide solutions to real problems.

For now though, it is enough to know that there is no 'real' answer to the square root of a negative number. This is another area that you will learn more of if you continue studying mathematics.

This section also wraps up the extended study of exponents, or powers. In everyday life there is not much need for the particular skills and ideas that you have been studying here, but they do form part of the basic concepts that are used in more complex maths and therefore also other subject areas that use maths. This makes scientific notation, roots and how to perform calculations efficiently with powers of the same base number some of the fundamental areas to study if you want to continue with any maths related area in the future.

4 This week's quiz

Well done, you've just completed the last of the activities in this week's study before the weekly quiz.

Go to:

Week 5 practice quiz.

Open the quiz in a new tab or window (by holding ctrl [or cmd on a Mac] when you click the link).

5 Summary

Congratulations for making it to the end of another week! You have again covered a lot of ground but hopefully you will feel more confident after completing this week’s quiz that you can now tackle scientific notation and related calculations, as well as understand some of the subtleties of roots. There was a lot to take in, but remember that you can always look back at your notes to remind you of any rules or ideas if you need to. Next week will cover shapes and how to calculate basic quantities for these, so this is back to maths that is easier to relate to the world around us. See you there!

You should now be able to:

- understand and use scientific notation
- perform calculations with numbers written in scientific notation
- understand and calculate square roots of a number.

You can now go to Week 6.

Week 6: Shapes around us

Introduction

The study of shapes is fundamental to understanding the world that people see around them. Being able to define the properties of both two dimensional shapes, such as triangles, and solid shapes, such as cylinders, enables engineers and designers to build bridges and design new products. Here you'll begin this area of study by concentrating on basic four-sided shapes, triangles and circles, as well as some solid shapes and how to calculate some of the basic properties of these, such as area and volume.

Maria introduces Week 6 in the video:

Video content is not available in this format.

After this week's study, you should be able to:

- understand some terms used to describe shapes
- understand the notation used when drawing shapes
- work out the perimeter and area of simple shapes
- calculate the volume of simple solids.

1 Geometry

The word geometry comes from two Greek words: 'geo' meaning earth and 'metros' meaning measurement. So, geometry literally means earth-measurement and in the sense of the earth that is around us, this is what geometry does. It is a branch of maths that, at its basic level, is concerned with describing shapes and space, such as triangles and circles. In order to do this effectively, and to be able to communicate with others exactly what is being measured or described, there is a set of vocabulary and basic definitions that is used to describe angles, lines and shapes. These will be the focus of the first part of this week.

1.1 Angles and lines

Angles measure the amount of turning from one position to another, so for example, you can describe how far around a circle you have moved. Imagine looking straight ahead and then turning around until you return to your starting position. The angle you have turned through is a full turn. If you turn so that you are facing in the opposite direction, you will have made half of a full turn. If you turn from looking straight ahead to facing either directly to your right or directly to your left, you will have made a quarter turn.

A full turn, or a circle, is defined as having 360º (said as 360 degrees). A circle is split into 360 equal parts, each part being 1º. Week 3 of *Succeed with maths – Part 1* looks at fractions by using pizzas, and this is very similar. This system was inherited from the Babylonians, whose counting system was based on 60 and who were the first people to use degrees in astronomy.

If one full turn is 360º, this means that a half turn is 180º and a quarter turn is 90º, as shown in Figure 1. This helps us to define some basic ideas in geometry.

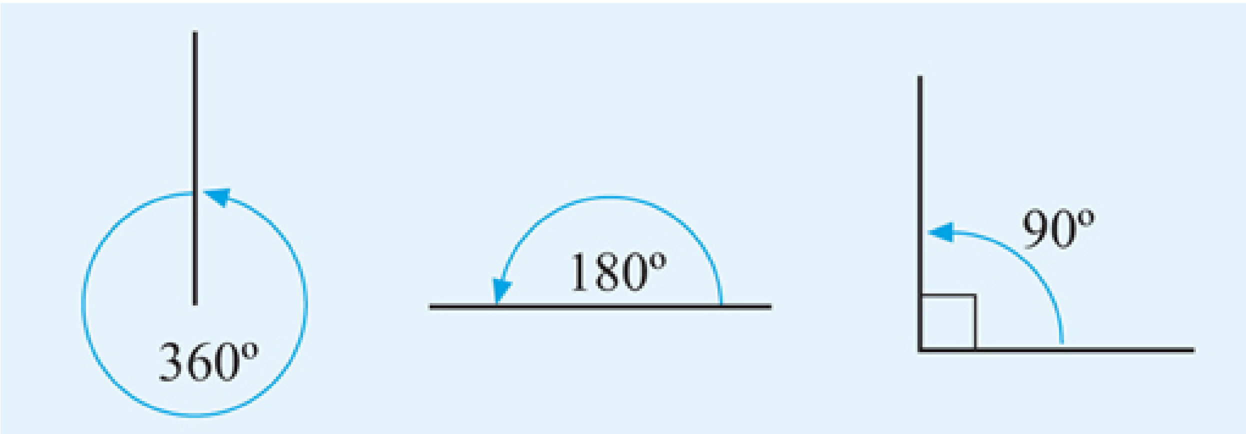


Figure 1 Angles in different anticlockwise turns

Figure 1 helps us to define some of the basic ideas of geometry.

These are:

- A circle is 360º.
- A straight line is 180º.
- A right angle is 90º.

Now, you've looked at one common way of measuring the amount of rotation, let's think briefly about how to describe and show lines precisely.

Figure 1 shows a right angle, which is denoted by a small square drawn at the angle. This tells us that the angle is exactly 90º and not 89º or 91º. When two lines are at right angles to each other, they are also said to be perpendicular.

Lines that will never meet and are always the same distance apart, however far you extend them in either direction are called parallel lines. A railway track is an example of a set of two parallel lines and these are shown on diagrams by using an arrow (or double arrow) drawn on each line, as shown in Figure 2.

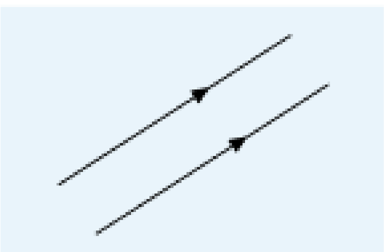


Figure 2 Parallel lines

This use of precise language and definitions removes any confusion when you assess angles and lines on a diagram. These also allow us to move onto looking at shapes and the angles and lines that these are constructed from, in the next section.

1.2 Circles and triangles

There are many shapes in everyday life: rectangular windows, triangular roof sections and circular ponds. It is these types of more familiar shapes that will be the focus of this section.

Let's start with circles, since there is only one type of circle! A circle possesses the property that any point on the circle is always the same distance from the centre point. This distance is known as the **radius** of the circle. The **diameter** of a circle is the distance from one edge of the circle to the other, passing through the centre. Finally, the edge of the circle (or the length of this edge) is known as the **circumference**. You can see all these properties of a circle in Figure 3.

As shown in Figure 3:

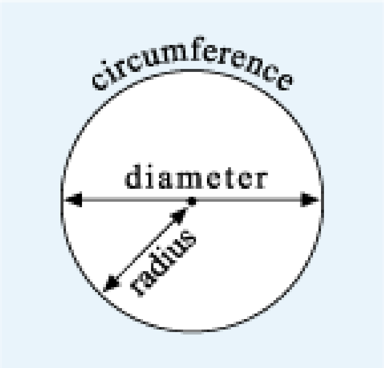


Figure 3 A circle showing the radius, diameter and circumference

Of course, you also know that a circle has 360º.

Moving on now to think about triangles. These present us with more to consider, as there are several different standard forms that a triangle can take. Some of these, the **right-angled triangle**, **isosceles triangle** and **equilateral triangle**, are shown in Figure 4. Note that sides that are the same length are marked with the same symbol, usually a short line, perpendicular, to the side.

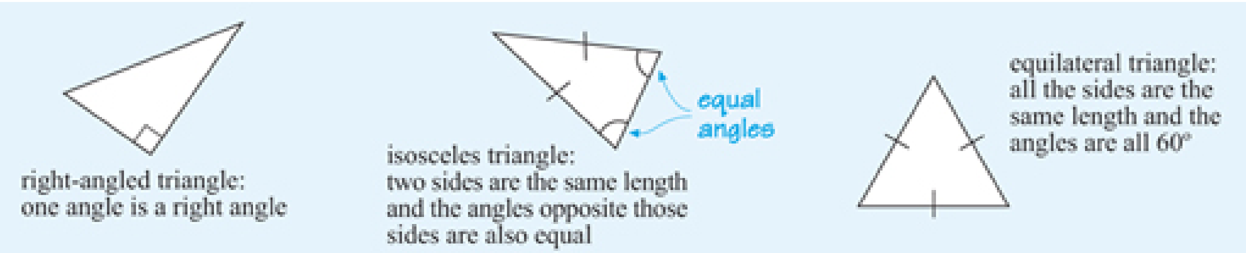


Figure 4 Right-angled, isosceles and equilateral triangles

A triangle in which all the sides have different lengths is known as a **scalene** triangle. Some examples are shown below. You will see that a right-angled triangle can also be described as a scalene triangle, as all the sides are different lengths.

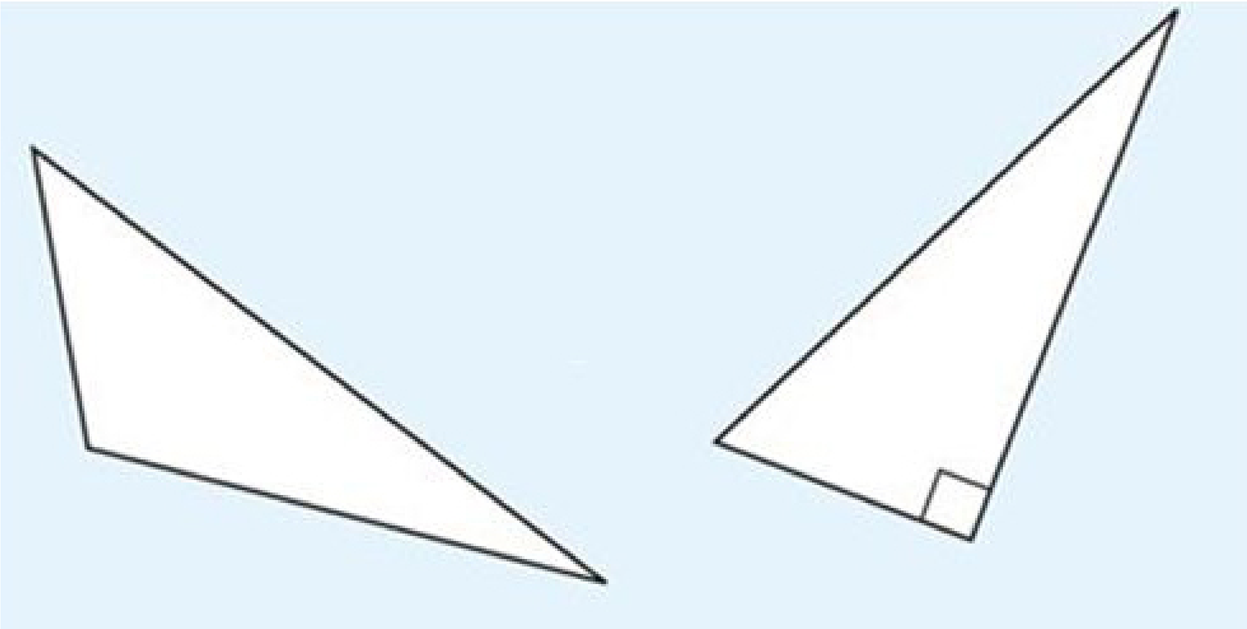


Figure 5 Scalene triangles

Another important fact is that the angles of a triangle always add up to 180º. To illustrate this, if you cut out any triangle and then tear off the angles, you will be able to arrange them to form a straight line as shown in Figure 6. You might like to try this yourself.

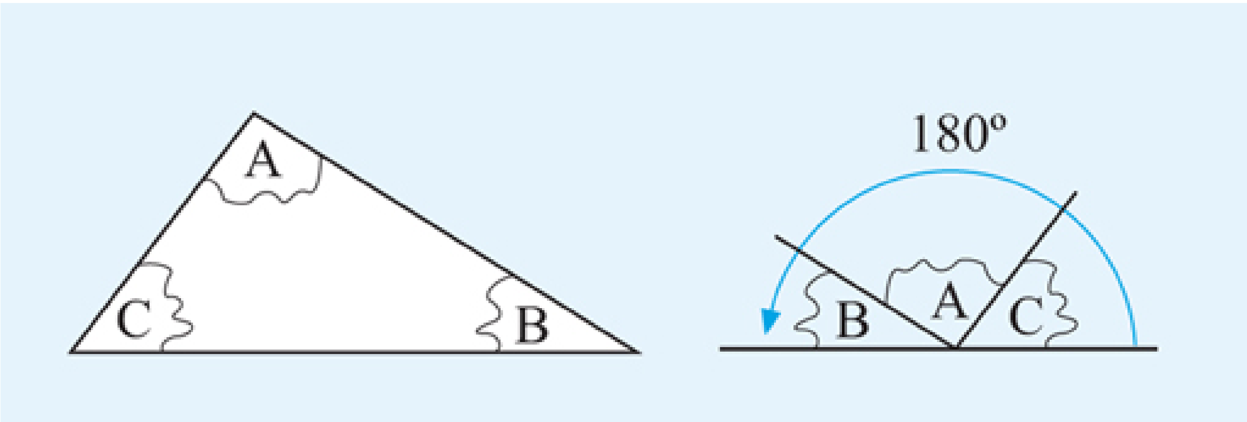


Figure 6 The angles of a triangle add up to 180º

The next section moves to four-sided shapes.

1.3 Four-sided shapes

Shapes that have four straight edges are known collectively as **quadrilaterals**. The most familiar of these are squares and rectangles. It might seem very obvious how these are defined but a rectangle (it's not called an oblong in maths!) has four straight sides that are all at right-angles to each other, with opposite pairs of lines being the same length. A square is a special kind of rectangle, where all the sides are the same length. This means that in both shapes, opposite pairs of lines are also parallel. These are both shown in Figure 7 – lines of the same length have been marked with a single or double short perpendicular line.

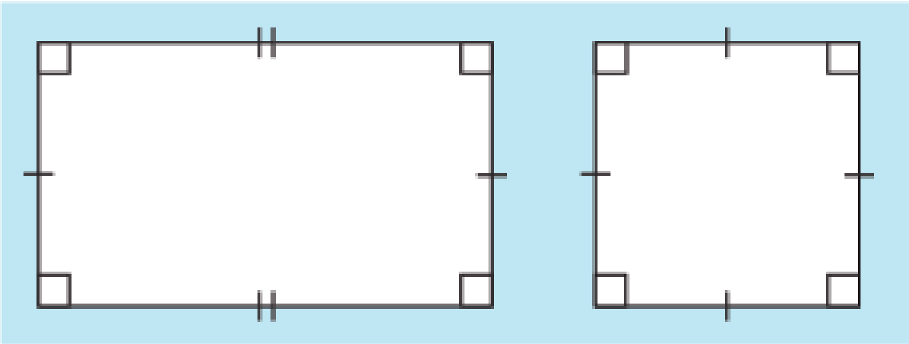


Figure 7 A rectangle and a square

What if you have a four-sided shape where only one pair of lines is parallel? This is known as **trapezium** and an example is shown below in Figure 8. The parallel lines are marked with arrows.

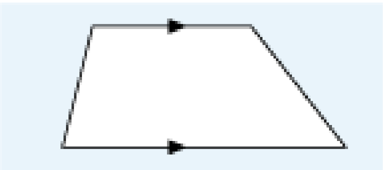


Figure 8 A trapezium

If the quadrilateral, a four-sided shape, has two sets of parallel sides, it is called a **parallelogram**:

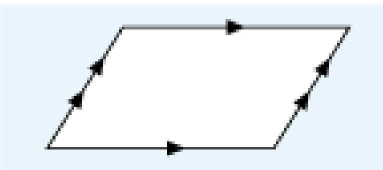


Figure 9 A parallelogram

Of course, this definition also means that squares and rectangles are parallelograms!

Now these more familiar shapes have been defined, it is useful to look at how to refer to specific sides or angles in a shape so that these can be clearly communicated to others. This is the subject of the next brief section.

1.4 Describing shapes

When you are describing a geometrical figure or shape, you often need to refer to a particular line or angle on the diagram, so others know what you are referring to. This can be done by labelling the diagram with letters. For example, Figure 10 shows a triangle labelled clockwise at the corners with A,B,C. This is then known as triangle *ABC*, in which the longest side is *AB* and the angle $A\widehat{C}B$ is a right angle. $A\widehat{C}B$ is the angle formed by the lines *AC* and *CB*. The point where two lines meet is known as a **vertex** (the plural is vertices). So A, B and C are **vertices** of the triangle.

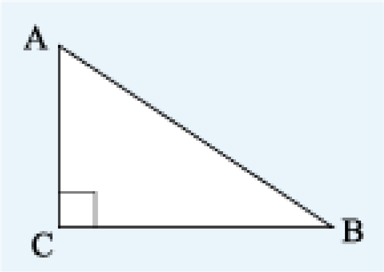


Figure 10 A triangle labelled *ABC*

Note that you can use the shorthand notation '$\triangle_{ABC}$' for 'the triangle *ABC*' if you wish. There is a lot of new maths vocabulary in these last few sections, so you might find it useful to make a note of these to refer back to when completing this next activity, or for this week's quiz and the badged quiz in Week 8.

Activity 1 What can you see?

Allow approximately 10 minutes

Look at the image below and then answer the following questions using the letters shown.

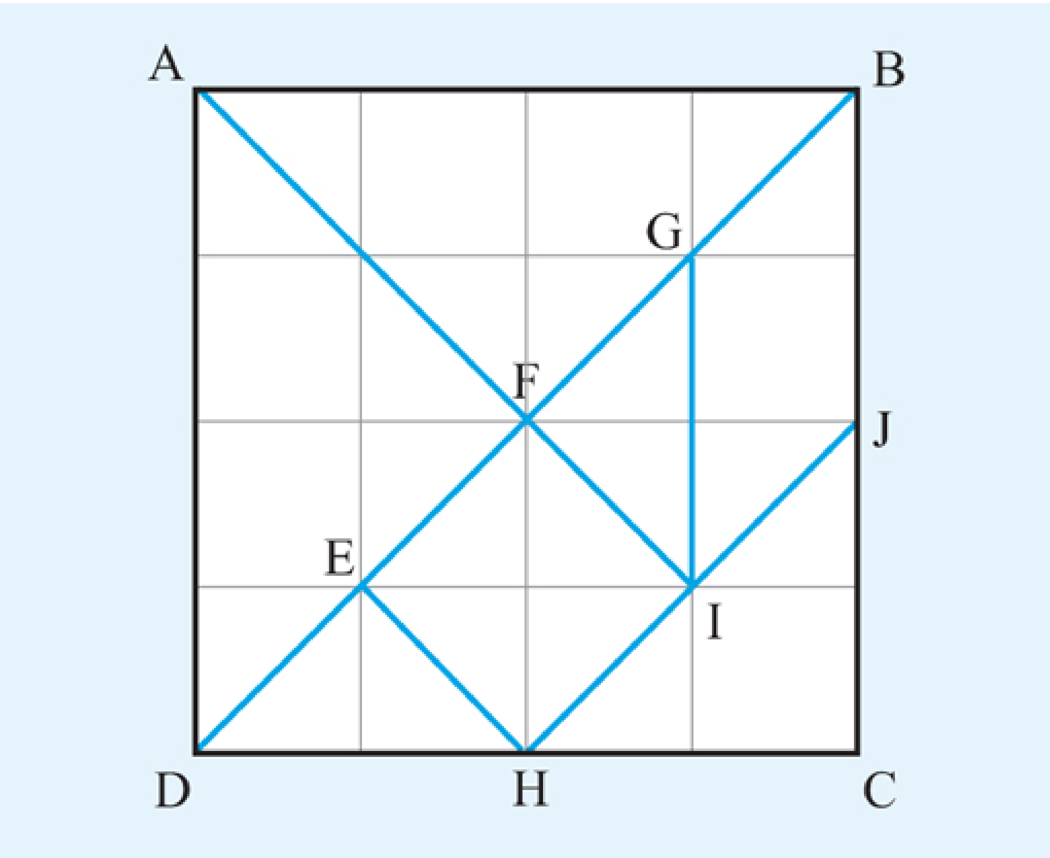


Figure 11

(a) Which sets of lines appear to be parellel?

Answer

(a)

(b) *AB* is parallel to *DC*.
AD is parallel to *GI* and *BC*.
AI is parallel to *EH*.
BD is parallel to *HJ*

(b) Which lines are perpendicular?

Answer

(b) *AB* is perpendicular to *AD* and to *BC*.
DC is perpendicular to *AD* and *BC*.
EH is perpendicular to *DB* and *HJ*.
AI is perpendicular to *HJ* and *DB*.

(c) How many triangles can you see?

Answer

(c)

$\triangle\ ABD,\ \triangle\ DBC,\ \triangle\ ABF,\ \triangle\ ADF,\ \triangle\ DEH,\ \triangle\ FGI\ \triangle$ and $\triangle\ HJC.$

(d) What other shapes can you see?

Answer

(d) The parallelogram, *GBJI*.
The squares, *ABCD* and *EFIH*.
The trapeziums, *DHIF*, *EHIG*, *DBJH*, *BJHE* and *DGIH*.

This completes your work on defining shapes and how to label them in order to describe them clearly to others.

The next section looks at the different ways for measuring shapes.

2 Perimeters

One of the properties of a shape that is measured is its perimeter. This is the distance around the edge of a shape. For a shape with straight edges, you can work out the perimeter by measuring the length of each edge and then adding these together.

For example, one practical application of this would be someone wanting to decorate the room shown in Figure 12, using a border along the top of all the walls. To determine the length of the border they would need to work out the perimeter. So the border would measure:

$5\,\text{m} + 4\,\text{m} + 5\,\text{m} + 4\,\text{m} = 18\,\text{m}$

An alternative way to calculate this would be to note that the pairs of sides are the same length, giving us:

$5\,\text{m} \times 2 + 4\,\text{m} \times 2 = 18\,\text{m}$

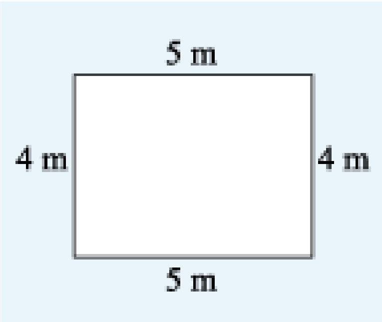


Figure 12 What is the perimeter of this rectangle?

Have a go yourself in this next activity.

Activity 2 Around the edge

Allow approximately 5 minutes

(a) This figure is formed from three of the shapes. What is its perimeter?

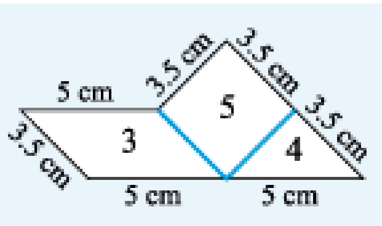


Figure 13 Figure formed from three of the shapes

Answer

(a) Starting on the left at the top of shape 3 and going clockwise around the figure, the perimeter is:

$5\,\text{cm} + 3.5\,\text{cm} + 3.5\,\text{cm} + 3.5\,\text{cm} + 5\,\text{cm} + 5\,\text{cm} + 3.5\,\text{cm} = 29\,\text{cm}$

You could have also shown the working as:

$3 \times 5\,\text{cm} + 4 \times 3.5\,\text{cm} = 29\,\text{cm}$

When a shape has straight sides, you can measure the perimeter fairly easily by considering each side in turn. However, measuring lines that are not straight can be more difficult. If you need the measurement for some practical purpose, then you can use a piece of string to wrap around an object and then measure the string. However, as you'll discover in the next section there more precise methods to calculate these in maths.

2.1 Perimeters of circles

As you saw earlier in this week, the perimeter of the circle has its own name: the circumference. But is there an easy way to calculate the circumference of a circle as there is with a rectangle?

Let's investigate this in the following practical activity. For this you will need a tape measure (or a piece of string and a ruler) and five objects such as mugs, cans, bowls or buckets that have a circular top or bottom.

Activity 3 Circles

Allow approximately 15 minutes

Measure the circumference and diameter of each object in centimetres to the nearest 0.1 cm (or 1 mm). Use the tape measure (or string and ruler) to find the circumference and the ruler to find the diameter.

Write down the diameter and the circumference of each object in a table and calculate the ratio $\frac{\text{circumference}}{\text{diameter}}$. What do you notice about your answers?

Answer

Here are some results that were obtained when carrying out the activity. The important data to consider are the ratios between the circumferences and the diameters.

Some sample results:

Table 1 Sample measurements of circular-shaped kitchen items

Item	Diameter (in cm)	Circumference (in cm)	$\frac{\text{circumference}}{\text{diameter}}$ (rounded to 1 decimal place)
Spice jar	4.4	14.1	3.2
Drinking glass	6.6	20.9	3.2
Tin container	8.6	27	3.1
Mug	10.3	32.3	3.1
Bowl	23.0	72.8	3.2

It is difficult to measure these quantities accurately. However, it is noticeable that in each case, the ratio of the circumference to the diameter seems to be about 3.1 or 3.2. In other words, the circumference is just over three times the length of the diameter.

2.2 Using π (pi)

If you could measure these objects more accurately, you would find that the circumference divided by the diameter always gives the same answer. This value is known as 'pi' (pronounced 'pie') and it is denoted by the Greek letter π . The value of π is approximately 3.142, although for most calculations you will be using the π button on your calculator, which shows pi to many more decimal places.

In fact, pi is an **irrational** number. This means that it can't be represented exactly as a fraction and has an infinite number of decimal places, with no repeating pattern of numbers.

This knowledge that the circumference divided by the diameter for any circle gives a constant value of around 3.142 enables a formula to calculate the circumference from the diameter or the radius of a circle to be formed. Therefore:

$$\text{circumference} = \pi \times \text{diameter} \;\text{ or } c = \pi \times d$$

Because the diameter is twice the radius, this can also be written as:

$$\text{circumference} = \pi \times \text{diameter} \;\text{ or } c = 2 \times \pi \times r$$

You will now be asked to select the relevant formula to use in the next activity.

Activity 4 Circumference of a circle

Allow approximately 5 minutes

Suppose a circular table has a diameter of 1.5 metres. How many people can sit down comfortably for a meal at the table, assuming that each person requires a space of about 0.75 m? Use the button marked with π on your calculator. If you need a hint or tip simply click on 'reveal comment'.

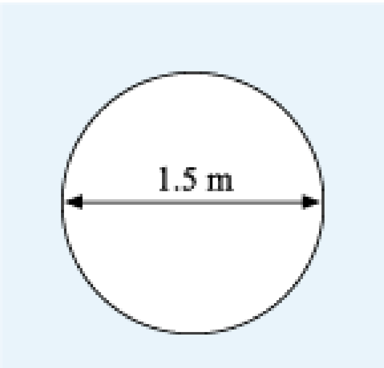


Figure 14 Diagram of a circular table with a 1.5 m diameter

Discussion

If you want some help to visualise the problem you could draw a quick sketch of the table and gaps for each person.

Answer

To see how many people will fit comfortably around the table you need to first work out the circumference of the table.

The diameter is given, so the circumference can be found with the formula:

$$\begin{aligned}\text{circumference} &= \pi \times \text{diameter}\\ &= \pi \times 1.5\,\text{m}\\ &= 4.7\,\text{m (to one decimal place)}\end{aligned}$$

Each person needs a space of about 0.75 m in width.

$$\begin{aligned}\text{Number of people who can fit around the table} &= 4.7\,\text{m} \div 0.75\,\text{m}\\ &\approx 6 \qquad (\approx \text{ means approximately equal to})\end{aligned}$$

Therefore, six people should be able to fit around the table.

More often than not, the shape of which you need to know the perimeter will not be a simple circle, or rectangle. So, what do you do then? You'll find out in the next short section.

2.3 Perimeters of mixed shapes

Suppose the shape that you wanted to determine the perimeter of is as shown in Figure 15.

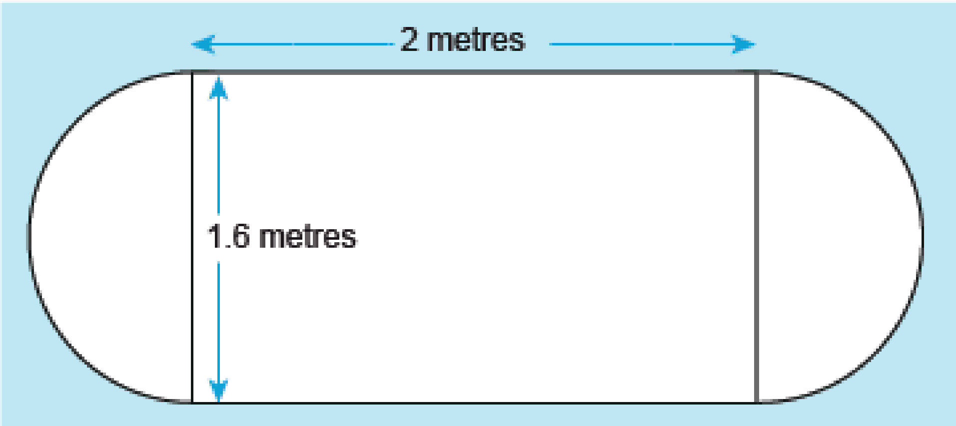


Figure 15 A shape formed of a rectangle and two semicircles

Start by noticing that the shape is formed of one rectangle and two semicircles. The distance around the two semicircles combined is the same as around a circle with a diameter of 1.6 m. So, calculating the circumference of a circle with this diameter will give us part of the answer. To find the total distance, or perimeter, you then just need to add in the lengths of the longest edges of the rectangle. This gives us the following calculation:

$$\begin{aligned}\text{Circumference of circle} &= \pi \times \text{diameter}\\ &= \pi \times 1.6\,\text{m}\\ &= 5.03\,\text{m (to 2 decimal places)}\end{aligned}$$

$$\begin{aligned}\text{Total perimeter} &= 5.03\,\text{m} + 2 \times 2\,\text{m}\\ &= 9.03\,\text{m (to 2 decimal places)}\end{aligned}$$

So, if you are faced with a complex shape, look for ways to split these up to help you calculate the perimeter.

Another property of a shape that can be useful to know is the area, or space that it occupies. This is slightly more complicated than the perimeter, but for regular shapes there are a set of formulas to help with this, as you will see in the next section.

3 Areas

If someone is planning to paint a wall, one of the first questions to ask will be how much paint they need. This will obviously depend on the size of the wall and how many coats needed. Fortunately, paint cans usually include information about the area that the paint will cover. For example, on the back of a 2.5-litre can of emulsion paint, it says that the paint will cover 'up to 35 square metres.' A square metre is the area that is covered by a square whose sides measure 1 m, as shown in Figure 16. This can also be written as 1 m^2. So 35 m^2 will be the same area as the area of 35 of these 1-metre squares.

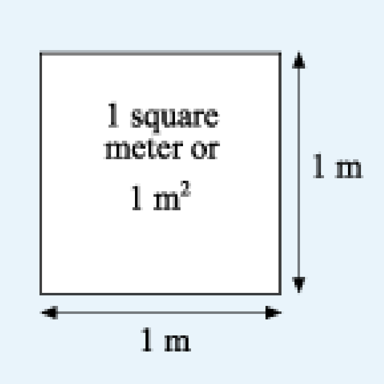


Figure 16 A square with an area of 1 m^2

Areas can also be measured in other square units, such as square centimetres (cm^2) or square kilometres (km^2), depending on what is appropriate to the situation. For example, if you wanted the area of a country, it would not make sense to show this in cm^2 or m^2 as the value would be very large! Similarly to a square metre, a square centimetre is a square whose sides are 1 cm long, and a square kilometre has sides that are 1 km long. If the diagram below (Figure 17) is drawn on a grid in which the lines are 1 cm apart, then each grid square will have an area of 1 cm^2. One way to work out the total area is counting the number of these squares. This gives us 16, so the grid shown is 16 cm^2 altogether.

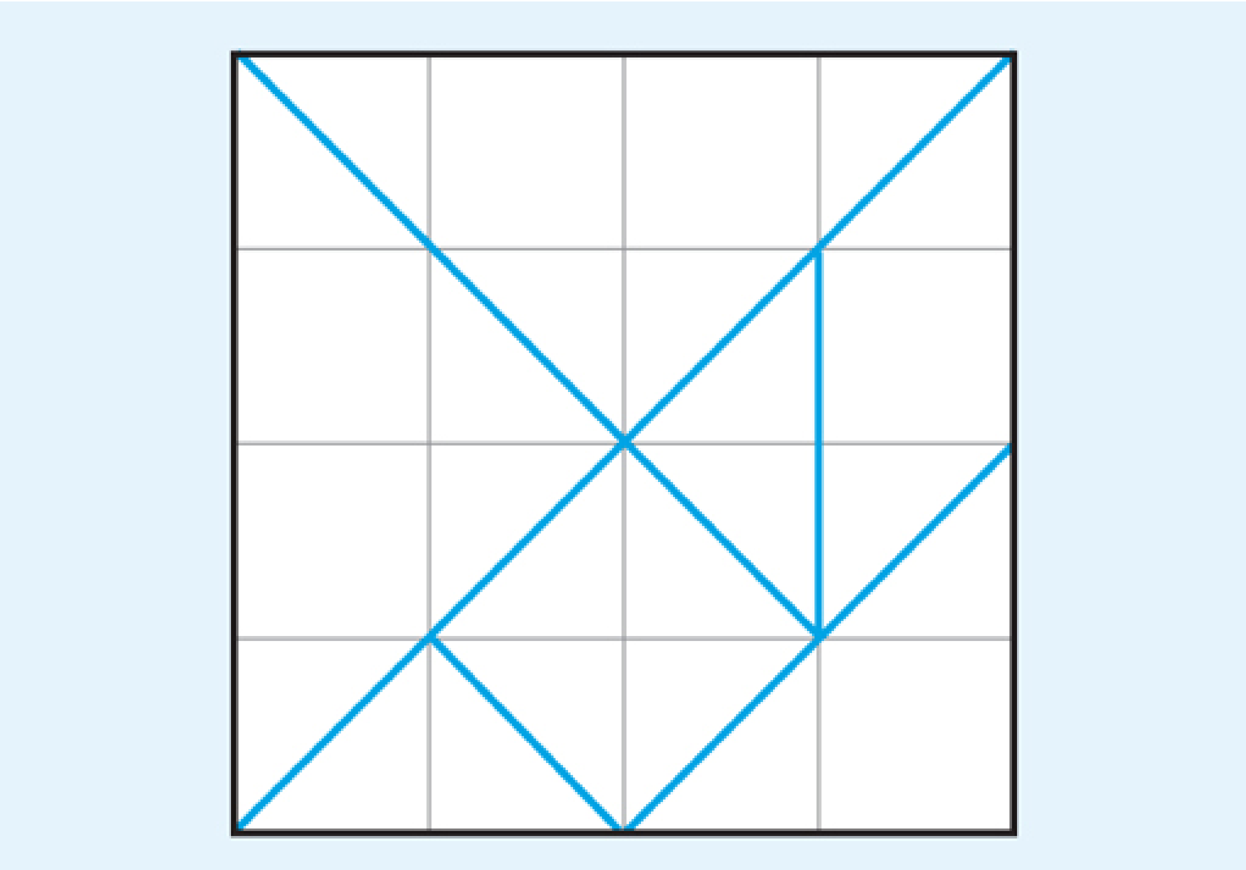


Figure 17 Grid containing different shapes

You can use the same technique to determine the areas of the shapes shown on the diagram. Try this now in the next activity.

Activity 5 Counting squares

Allow approximately 5 minutes

All the shapes on the diagram in Figure 17 are made up out of whole squares or half squares. Use this information to work out the areas of each of the shapes.

Answer

The areas in square centimetres are shown in Figure 18:

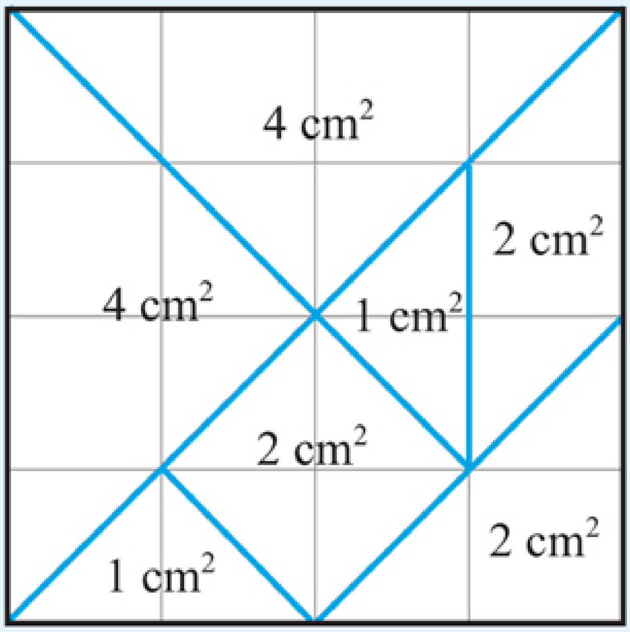


Figure 18 Grid showing square centimetres

Let's take the top triangle as an example to show how this was worked out.

This triangle is made up of two full squares and four half squares. Four half squares are the same as 2 full squares, so the total area of the top triangle is 4 cm^2.

To check the answers, you can add the individual areas and see if they total 16 cm^2.

The sum of the individual areas in cm^2 = 4 + 4 + 1 + 2 + 1 + 2 + 2 = 16.

This gives confidence in the individual answers and acts as a very useful check.

Counting squares is one way to work out the area of shapes, particularly if they are irregular. However, it is not very convenient in a lot of situations so this is where the power of formulas comes into play once again.

3.1 Formulas for areas

Many areas can be built out of basic shapes such as rectangles, triangles or circles, and these areas can be calculated using formulas. For example, if a rectangular room measures 4 m by 3 m and you are covering it in carpet tiles that are each 1 m square, the tiles can be arranged in three rows, each with four tiles. So the total number of tiles will be 3 x 4 = 12 and the area covered is written as 12 m^2.

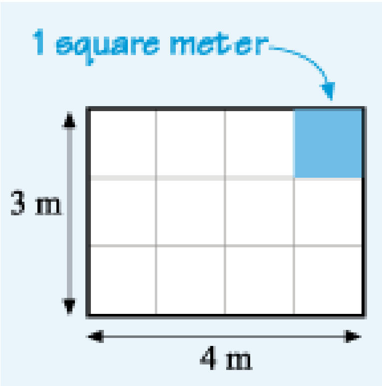


Figure 19 A rectangle

This area could have been calculated directly, without the need for squares, by multiplying the length of the room by its width. **Provided both measurements are in the same units**, the following formula holds for any rectangle (including a square):

$\text{Area of a rectangle} = \text{length} \times \text{width}$

From the formula for the area of a rectangle, the formulas for the area of a parallelogram and a triangle can be derived. Starting with a parallelogram, cut off the left edge and place it next to the right edge to make a rectangle, as in Figure 20. This must have the same area as the parallelogram, as nothing has been added or taken away.

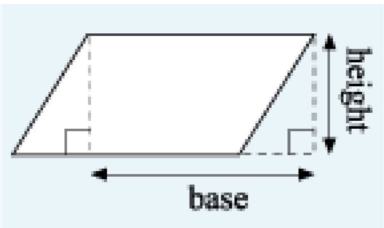


Figure 20 A parallelogram becoming a rectangle

The area of the rectangle can be found by multiplying its length by its height. In the case of a parallelogram, the length is called the base and the width the height. This gives the formula:

$\text{Area of a parallelogram} = \text{base} \times \text{height}$

Now if this parallelogram is cut in half along a diagonal, this gives two possibilities, as illustrated in Figure 21:

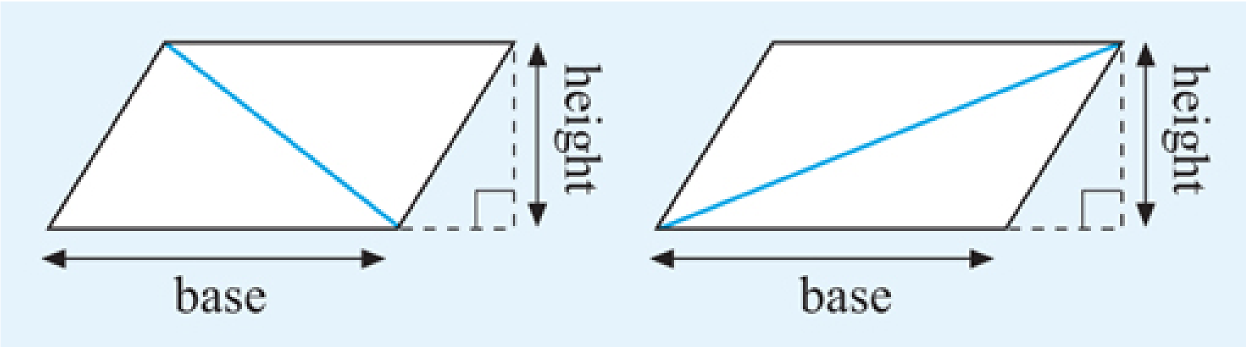


Figure 21 Two parallelograms with the diagonals marked

In each case, the parallelogram has been split into two triangles that are each half the area of the parallelogram.

This then gives a general formula for the area of a triangle:

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

The height goes through the vertex (the point where two lines meet) that is opposite, the base and is always perpendicular, to the base, as shown in Figure 22:

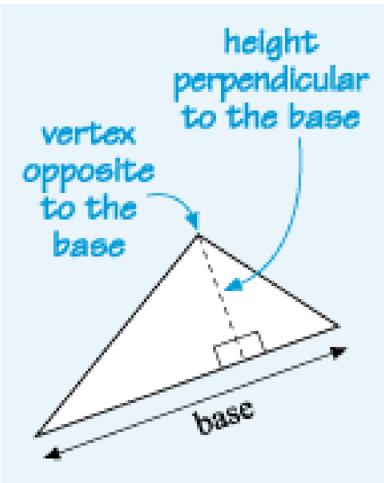


Figure 22 A triangle with the base marked

You'll look at an example in the next section before having a go yourself.

3.2 Finding an area using formulas

Figure 23 is a rough sketch of the gable end of a house that needs weatherproofing.

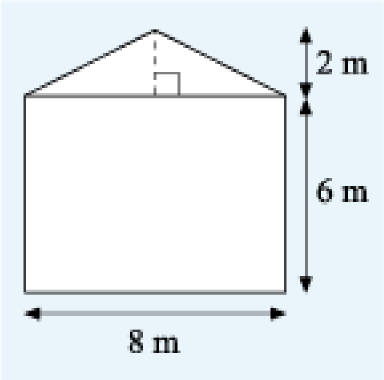


Figure 23 A sketch of the gable end of a house

To work out the quantity of materials required, the area of the wall is needed. This problem can be broken down by splitting the area into a rectangle and a triangle, then working out these areas and finally adding the two areas together to get the total.

$$\begin{aligned}\text{Area of rectangluar part} &= \text{length} \times \text{width}\\ &= 8\,\text{m} \times 6\,\text{m}\\ &= 48\,\text{m}^2\end{aligned}$$

The triangle has a base of length 8 m and a perpendicular height of 2 m. So, the area can be calculated as follows:

$$\begin{aligned}\text{Area of triangular part} &= \tfrac{1}{2} \times \text{base} \times \text{height}\\ &= \tfrac{1}{2} \times 8\,\text{m} \times 2\,\text{m}\\ &= 8\,\text{m}^2\end{aligned}$$

That means the total area of the gable end = 48 m^2 + 8 m^2.

Now, it’s your turn to put what you’ve learned in this section to use.

Activity 6 Using formulas to calculate areas

Allow approximately 5 minutes

Figure 24 shows the same diagram as you have already seen in previous activities. This time though, instead of counting the squares to determine the areas, use the relevant formulas from the previous section. Remember that the whole grid measures 4 cm by 4 cm, and each small square is 1 cm by 1 cm.

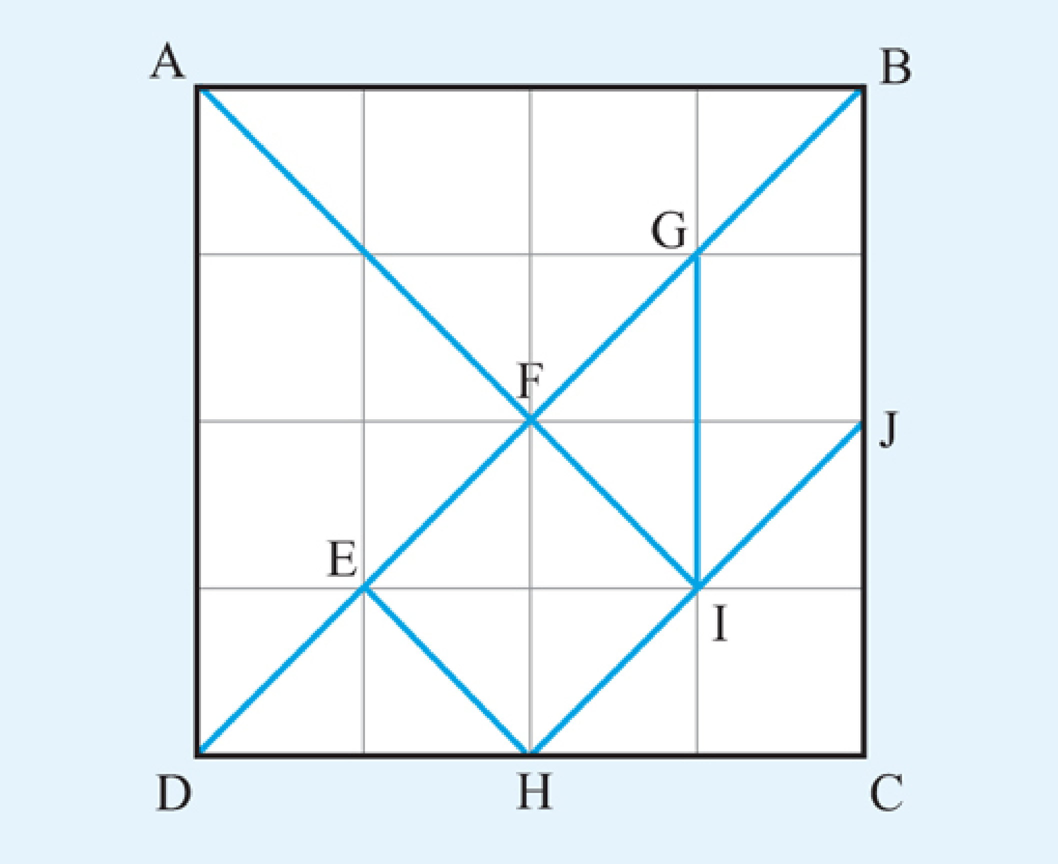


Figure 24 Grid

Calculate the areas of the following shapes.

(a) Triangle *ADF* with *AD* as the base

Answer

(a) The base of the triangle is *AD*. *AD* = 4 cm.
The perpendicular height from *F* onto *AD* = 2 cm.

$$\text{area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\begin{aligned}\text{So the area of triangle ADF} &= \frac{1}{2} \times 4\,\text{cm} \times 2\,\text{cm} \\ &= 4\,\text{cm}^2\end{aligned}$$

(b) Parallelogram *GBJI* with *BJ* as the base

Answer

(b) *BJ* = 2 cm. The perpendicular height from *G* onto *BJ* = 1 cm.

$$\text{Area of a parallelogram} = \text{base} \times \text{height}$$

$$\begin{aligned}\text{That means the area of GBJI} &= 2\,\text{cm} \times 1\,\text{cm} \\ &= 2\,\text{cm}^2\end{aligned}$$

Now that you know how to calculate the areas of basic shapes, you can calculate more complicated areas by breaking each shape into basic shapes and adding the individual areas together.

Many area problems can be calculated by using combinations of squares, rectangles and triangles. However, you often need to find circular areas, too. The next section will cover this aspect of areas.

3.3 Areas of circles

You can show that the area of a circle is calculated by multiplying the square of the radius by π , by splitting the circle into equal sized segments and then arranging these to form a rectangle as shown in Figure 25:

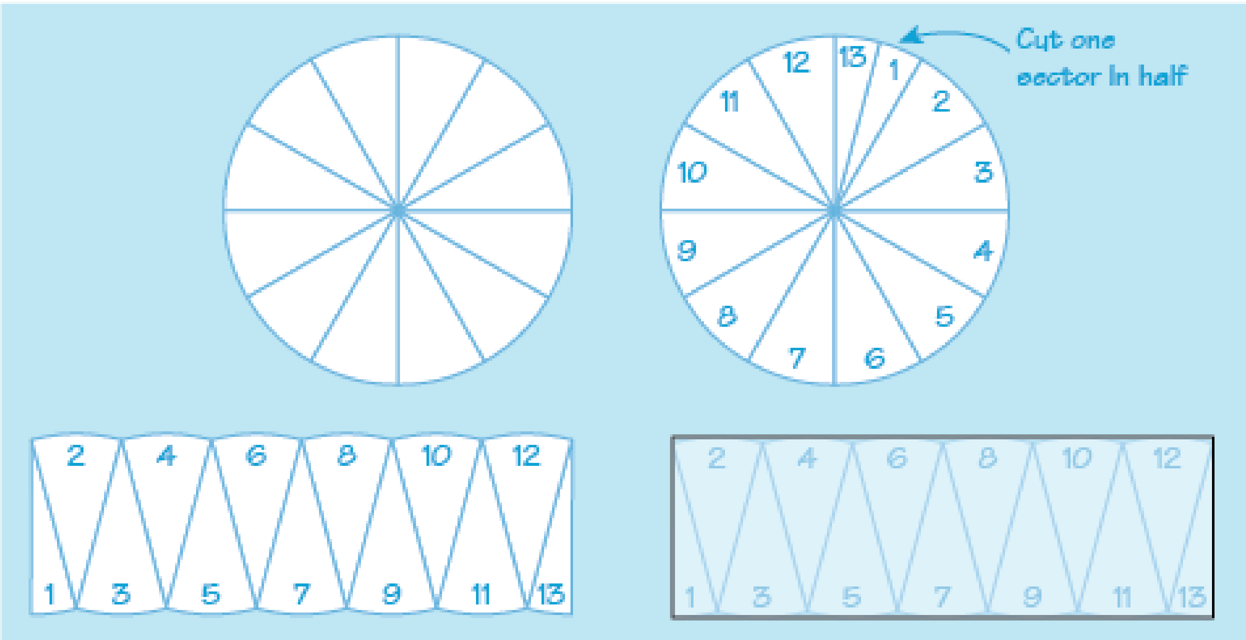


Figure 25 Circle area by sectors

If each segment is made gradually smaller, any 'bumps' along the top and bottom edges will be smoothed out to form a line that is closer and closer to being straight. The height of the rectangle will then be the same as the radius of the circle and the length half of the circumference. The area of this rectangle will therefore be equivalent to that of the circle.

The $\text{area of a rectangle (circle)} = \text{radius} \times \frac{1}{2} \times \text{circumference}$

The circumference and radius are related by the following formula:

$\text{Circumference} = 2 \times \pi \times \text{radius}$

So $\frac{1}{2} \times \text{circumference} = \pi \times \text{radius}$

So putting this together gives:

$$\begin{aligned} \text{Area of a circle} &= \text{radius} \times \pi \times \text{radius} \\ &= \pi \times \text{radius}^2 \end{aligned}$$

The formula for the area of a circle is therefore:

$\text{Area of a circle} = \pi \times \text{radius}^2$

If you are given the diameter of a circle instead of the radius, the first step to take when working out the area would be to halve the diameter.

See how you get on with applying this new formula in the next activity.

Activity 7 Everyday structures

Allow approximately 10 minutes

Calculate the areas of the following road and town developments, assuming that the measurements are sufficiently accurate to allow answers to the nearest whole number. Remember to click on 'reveal comment' for a hint or tip.

(a) A roundabout of radius 15.6 m.

Answer

(a) $\text{Area of a circle} = \pi \times \text{radius}^2$

$$\begin{aligned}\text{So the area of the roundabout} &= \pi \times (15.6\,\text{m})^2 \\ &= 765\,\text{m}^2\end{aligned}$$

(b) A circular building with a diameter of 54 metres.

Discussion

Remember that you need the radius, not the diameter for the formula for area of a circle.

Answer

(b) Here you are given the diameter. The radius is half the diameter.

$$\begin{aligned}\text{So radius} &= 54\,\text{m} \div 2 \\ &= 27\,\text{m}\end{aligned}$$

$$\begin{aligned}\text{So, the area of the circular building} &= \pi \times (27\text{m})^2 \\ &= 2290\,\text{m}^2 \text{ (to the nearest whole number)}\end{aligned}$$

(c) A semicircular lecture hall with a diameter of 46 m.

Answer

(c) The area of a semicircle is half the area of a circle.

So the $\text{area of a semicircle} = \frac{1}{2} \times \pi \times \text{radius}^2$

The lecture hall has a diameter of 46 m so the radius is 23 m.

$$\begin{aligned}\text{The area of the lecture hall} &= \tfrac{1}{2} \times \pi \times \left(23\,\text{m}\right)^2 \\ &= 831\,\text{m}^2 \text{ (to the nearest whole number).}\end{aligned}$$

The last few sections have included a number of different formulas, so you might appreciate a quick summary to bring them altogether. You might also like to take a note of them alongside any new vocabulary you've come across

- area of a rectangle $=$ length $\times$ width
- area of a parallelogram $=$ base $\times$ height
- area of a triangle $= \frac{1}{2} \times$ base $\times$ height
- area of a circle $= \pi \times \text{radius}^2$.

From your study of measurement in Week's 1 and 2 of the course you will know that there is one final property of shapes that hasn't been covered, that is capacity or volume. So, you'll move onto that now in the final part of this week's study.

4 Volumes

So far you have considered measuring perimeters and areas. But most things in life are not flat; that is, two-dimensional.

Questions like 'How much does that hold?' need you to be able to specify the volume of an object. Extending the ideas you learned earlier you can count how many cubes of a certain size will fit into the space. All the sides of a cube are the same length and its six faces are all square.

Useful cubic measurements to use are the cubic millimetre (written as mm^3), the cubic centimetre (written as cm^3) and the cubic metre (written as m^3).

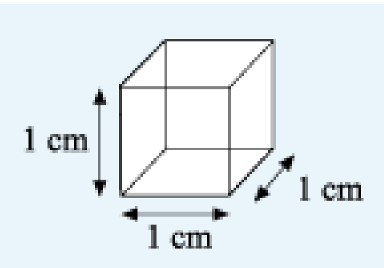


Figure 26 A cube with side lengths of 1 centimetre

In each case, the length of the side of the cube is 1 unit. So a cubic centimetre has all its sides of length 1 cm, as shown.

Now imagine filling a cubic centimetre with cubic millimetres. Since there are 10 mm in 1 cm, a cubic centimetre will contain 10 layers with each layer made up of 10 rows, each of 10 cubic millimetres.

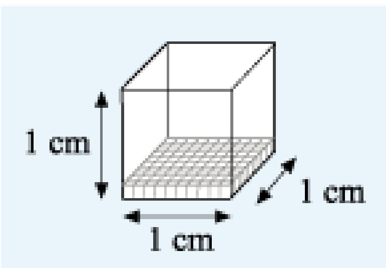


Figure 27 A cube of side 1 centimetre with one layer of 100 cubic centimetres shown

So, $1\text{cm}^3 = 10\,\text{mm} \times 10\,\text{mm} \times 10\,\text{mm} \times 10\,\text{mm} = 1000\,\text{mm}^3$

Or a 1 cm by 1 cm by 1 cm cube will hold 1000 cubic millimetres.

Now, suppose you have a box that measures 6 cm by 5 cm by 4 cm. What is its volume? Since all the dimensions are given in centimetres, you can measure the volume in cubic centimetres. Imagine filling the box with 1 cm^3 cubes: six rows with five cubes in each row would cover the bottom of the box, and the box would be filled by four of these layers.

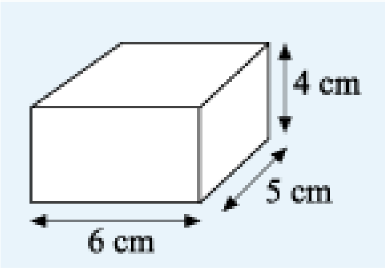


Figure 28 What is the volume of this box?

So, the total number of cubes used = 6 x 5 x 4 = 120. The volume of the box is therefore 120 cm^3. Before you move on to the next section, can you think of a suitable formula for working out the volume of a rectangular box, known as a cuboid?

4.1 Volume formulas

You may have already worked out that the formula for the volume of a cuboid can be calculated using the formula shown below.

$$\text{Volume of a cuboid} = \text{length} \times \text{width} \times \text{height}$$

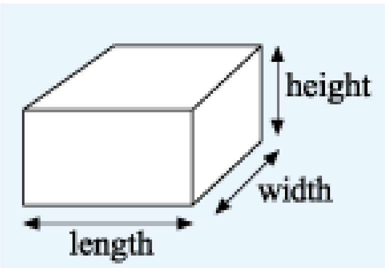


Figure 29 A cuboid with the length, width and height marked

Here are the formulas of various other shapes that you may encounter as well. Remember to make a note of them somewhere, as you may need one or another for the quizzes!

Volume of cylinder

$$\text{Area of base} \times \text{height} = \pi \times \text{radius}^2 \times \text{height}$$

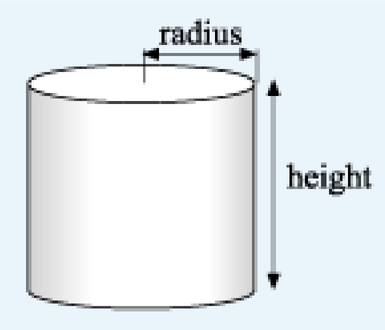


Figure 30 A cylinder with the radius of the base and the height marked

Volume of a sphere

$\frac{4}{3} \times \pi \times \text{radius}^3$

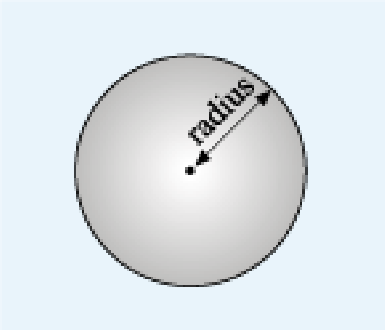


Figure 31 A sphere with the radius marked

Use the relevant formulas to complete this next activity and apply what you have learned in this section.

Activity 8 A fuel tanker

Allow approximately 5 minutes

(a) The container on a fuel tanker is approximately cylindrical, with an internal diameter of about 2.25 m and an internal length of 10 m. Roughly how many cubic metres of fuel will it hold?

Answer

(a) $\text{Volume of a cylinder} = \pi \times \text{radius}^2 \times \text{height}$

$\text{Radius} = 2.25\,\text{m} \div 2 = 1.25\,\text{m}$

The length is 10 m, which is in this case equivalent to the height.

Substituting these values into the formula for the volume of a cylinder:

$$\begin{aligned}\text{Volume of a cylinder} &= \pi \times (1.125\,\text{m})^2 \times 10\,\text{m} \\ &= 39.76\,\text{m}^3\end{aligned}$$

Rounding to the nearest cubic metre, the volume is 40 m^3.

(b) The petrol station has an underground tank that measures 2.5 m by 4 m by 3 m. If the tank is empty, will it hold all the fuel in the tanker?

Answer

(b) The tank must be a cuboid, as there are 3 dimensions given.

$$\begin{aligned}\text{So the volume of the underground tank} &= 2.5\text{m} \times 4\text{m} \times 3\,\text{m}\\ &= 30\,\text{m}^3\end{aligned}$$

Since the fuel tanker will hold about 40 m^3 the petrol station’s tank is not large enough to hold all the fuel.

This activity completes your study of shapes this week.

5 This week's quiz

Well done, you've just completed the last of the activities in this week's study before the weekly quiz.

Go to:

Week 6 practice quiz.

Open the quiz in a new tab or window (by holding ctrl [or cmd on a Mac] when you click the link).

6 Summary

This week you may well have encountered some new mathematical language and different ways of drawing shapes to precisely communicate their properties. As well as this new knowledge, there were also quite a few formulas introduced for perimeters, areas and volumes. That’s a lot to take in, so congratulations for working your way through the week!

You should now be able to:

- understand some terms used to describe shapes
- understand the notation used when drawing shapes
- work out the perimeter and area of simple shapes
- calculate the volume of simple solids.

You can now go to Week 7.

Week 7: Working with data

Introduction

More and more in every aspect of our lives, data – information, numbers, words or images – are collected, recorded, analysed, interpreted and used. For example, every time you go shopping, if you use a loyalty card, the store will be collecting details of what you purchased to help them to provide you with a better service and keep your custom. Information is also encountered in the form of statistics: everything from graphs of the latest house sales figures to census results, the current rate of inflation or the unemployment rate.

This week you'll consider how to summarise a set of data using averages to provide a typical value for that data. You'll also think about the importance of full information about the average that has been used and how to read and construct tables of data.

Watch Maria introduce you to the penultimate week of the course, Week 7:

Video content is not available in this format.

After this week's study you should be able to:

- calculate the average for a set of data using the mean, median and mode
- calculate the range of a set of data
- understand that different averages may give different results for the same set of data
- read and construct tables.

1 Averages

If somebody makes the same trip frequently, the length of that trip may be similar each time but there would be exceptions to this if there were delays along the way. So what then would be the typical time for the journey? Should you pick the time that occurs most often, or the time that is somewhere in the middle?

A value that is typical of the values in a set of data is known as an average, and just as with our example, there are different ways of calculating this: the mean, the median and the mode. In the next sections you will look at how each represents the typical value of a set of data.

Knowing a typical value, and also how the data is spread out around this typical value, is important in all sorts of situations. A clothes manufacturer, for example, would use this to decide the size of clothes to make, as well as how many of each size.

The first average to consider is the mean.

1.1 Calculating the mean of a data set

You may well have come across the mean before, as it is the most commonly used type of average. However, it may not be clear that this is being used as the average, as many uses of averages (in the media for example), do not say which average is being employed. You'll look at the difference to the average reported that this can make later in the week.

The important point to remember about the mean is that it takes all the data into account. It does this because to calculate the mean, all the values are added together and then divided by the number of values.

A word formula for this is:

$$\text{Mean} = \frac{\text{Sum of all the values}}{\text{Number of values}}$$

Let's look at an example now to explore this type of average:

Suppose eight students took an exam, with the following scores: 9, 7, 6, 7, 8, 4, 3 and 9.

Sum of all the values = 9 + 7 + 6 + 7 + 8 + 4 + 3 + 9 = 53

There are 8 values, so:

$$\begin{aligned}\text{Mean score} &= 53 \div 8\\ &= 6.6 \text{ (to one decimal place)}\end{aligned}$$

This feels right, as it lies between the smallest and the largest value, and is around the middle of the values. However, where the mean lies within the values will depend on the actual values in any data set. It may be close to the middle but equally, it could be closer to the smallest or largest value.

Here's an example for you to try:

Activity 1 Finding the mean time for a trip

Allow approximately 5 minutes

Allow approximately 5 minutes

The times for my trip to work during one week last month are shown in the table below:

Table 1 Time for trip to work over one week

Day of week	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Time in minutes	42	58	45	47	52

First, look at the data. What would you say is a typical length of time for the trip from this set of data? Write down your estimate.

Now calculate the mean commuting time.

Answer

The smallest time is 42 minutes and the largest is 58 minutes, so a typical time would lie between these, perhaps 50 minutes. Your estimate may be different from this, of course, because it is just a sensible guess at a typical value.

$$\text{Mean} = \frac{\text{Sum of all the values}}{\text{Number of values}}$$

$$\begin{aligned}\text{The sum of the values } &= 42 + 58 + 45 + 47 + 52 \\ &= 244\,\text{minutes}\end{aligned}$$

There are five data values.

$$\begin{aligned}\text{So the mean } &= 244\,\text{minutes } \div 5 \\ &= 48.8\,\text{minutes}\end{aligned}$$

The mean commute time over that week was about 49 minutes. (Remember to include the units with your answer!)

The mean is fairly close to the estimated typical value of 50 minutes, so it looks as if the calculated value for the mean is correct.

You probably used a calculator to help you arrive at the answer in the last activity and hopefully you got the same answer the first time. It is easy, however, to forget that your calculator probably knows the rules for the order of operations – **B**rackets, **E**xponents, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction, (this is covered in Week 2 of *Succeed with maths – Part 1*). So, if I had tried to calculate the mean in one step, without including any brackets, my calculator would have given me 202.4 as the answer. It would have calculated:

$$42 + 58 + 45 + 47 + 52 \div 5$$

only dividing 52 by 5, rather than the total.

Fortunately, a quick comparison with the smallest and largest values in our data set would have immediately told me that something was not right!

So, always check you have a sensible answer when compared to the data you have, and work out the mean in two steps.

The mean, however, may not always give the best idea of what a truly typical value is, and in these situations it is best to turn to one of the other options available. Let's explore this further in the next section.

1.2 Is the mean the best average to use?

As you learned earlier one of the advantages of using the mean as an average value is that it takes account of all the values in the data set. However, this also means that if one of the values is much higher or lower than the other values it can greatly affect the mean. The following activities will look at this.

Suppose somebody is planning a visit to Vermont, USA in October and they want an idea of how much rainfall there will be. To do this they consult records from the National Climate Data Center in the USA, and find the following:

Table 2 Precipitation (rainfall) in Vermont for October 2007–2011

Year	2007	2008	2009	2010	2011
Rainfall, in inches (to 1 d.p.)	5.8	5.2	4.6	9.3	4.5

(Enloe, 2013)

What the person really wants to know though is, is what the typical value for October would be. Let's start with calculating the mean to find this.

Activity 2 How wet is it in Vermont in October?

Allow approximately 5 minutes

Find the mean rainfall in Vermont in October for these five years.

Answer

$$\text{The sum of the five data values } = 5.8 + 5.2 + 4.6 + 9.3 + 4.5 = 29.4 \text{ inches}$$

$$\text{The mean rainfall } = 29.4 \div 5 = 5.88 \text{ inches}$$

Over these five years the mean rainfall was 5.9 inches (to 1 decimal place).

Now look at the five data values again. Does 5.9 inches give you a good idea of how much rainfall there has been? One way to look at this is using a number line, with all the data values plotted on it, as shown below in Figure 1:

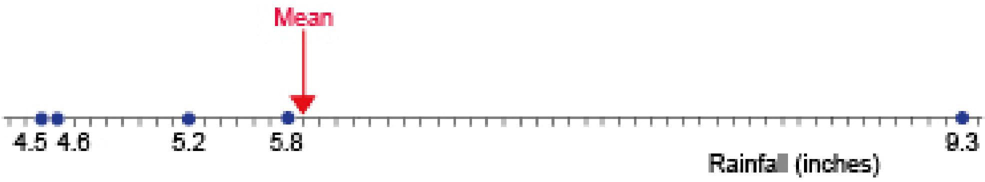


Figure 1 Rainfall number line with mean value indicated

This shows that, in four out of the five years, rainfall was below the mean, while it was above the mean in only one year. In that year, the rainfall was particularly high and this has pulled the mean up. So the mean is perhaps not a particularly good choice for a typical value in this case. So let's see if a different average would be better – this time the median.

1.3 Calculating the median value of a data set

The median is defined as the middle value of a data set when all the data is arranged in numerical order. If there is an even number of values, and therefore no middle value, then the median is calculated as the mean value of the two middle values.

So, let's now see how using the median, rather than the mean, affects the value for the average rainfall in Vermont. Here's a reminder of the data:

Table 3 Precipitation (rainfall) in Vermont for October 2007–2011

Year	2007	2008	2009	2010	2011
Rainfall, in inches (to 1 d.p.)	5.8	5.2	4.6	9.3	4.5

(Enloe, 2013)

To find the median rainfall, first arrange the data values in numerical order, to give:

4.5 4.6 **5.2** 5.8 9.3

There are five data values, an odd number, so the median is the middle value in this case. Therefore the median rainfall is 5.2 inches.

If you look at the number line again in Figure 2, with the median added this time, it appears that the median may be a better choice for an average or typical value in this case:

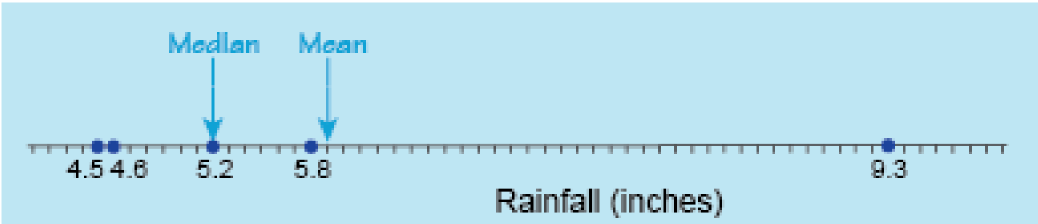


Figure 2 Rainfall number line with mean and median values indicated

Now try out the next activity.

Activity 3 A computer game for Charlie

Allow approximately 7 minutes

Over the course of one week Charlie plays her computer game every day except Friday, and achieves the following scores:

Table 4 Charlie's computer game scores over one week

Monday	Tuesday	Wednesday	Thursday	Saturday	Sunday
250	270	300	290	290	270

(a) Find Charlie’s mean score over these seven days.

Answer

(a) To find the mean, find the sum of the scores and divide by the number of scores, six.

Charlie’s mean score = 250 + 270 + 300 + 290 + 290 + 270 = 278 (to the nearest whole number).

(b) Find Charlie’s median score over these six days. (Click on reveal comment if you would like a hint)

Discussion

There is an even number of data values this time, so to find the median calculate the mean of the middle two values.

Answer

(b) To find the median score, arrange the data values in order:

250 270 **270** **290** 290 300

There is an even number of values (six), so Charlie's median score is the mean of the two middle values, 270 and 290.$$\text{Median score} = \frac{270 + 290}{2} = 280$$

(c) How do the median score and the mean score compare in this case?

Answer

(c) This time, the mean and median scores are very close. This is because the data values are fairly evenly clustered around the mean value without any extremely high or low values.

Having covered the mean and median, this leaves just the mode to consider in the next section.

1.4 Finding the mode of a data set

The final method of finding an average is simply to pick the value that occurs most often. This value is known as the mode or the modal value.

In some data sets, more than one value occurs most frequently, so there is then actually more than one mode. The mode is fairly easy to find, but it does not consider all the values in a data set. However, it can be useful in some situations, such as when making decisions about which sizes of a particular piece of clothing should be stocked in a shop.

Finding the mode of a data set can be summarised as follows:

- Arrange all the values in numerical order.
- Choose the value (or values) that occur most frequently.

Charlie's first set of scores from the previous example is shown below:

Table 5 Charlie's first set of scores

Monday	Tuesday	Wednesday	Thursday	Saturday	Sunday
250	270	300	290	290	270

Arranging Charlie's scores in numerical order gives:

250, 270, 270, 290, 290, 300

Two of the scores occur twice in this set, so there are two modal values: 270 and 290.

Try finding the mode yourself in this next activity.

Activity 4 Finding the mode of a data set

Allow approximately 4 minutes

Charlie's set of scores for her next week of play is shown below:

Table 6 Charlie's scores for her next week of play

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
270	260	280	270	310	300	290

Find the mode for this week's scores.

Answer

Arranging Charlie's scores in order gives: 260, 270, 270, 280, 290, 300, 310.

Each score occurs once, except 270 which occurs twice. The mode is therefore again 270.

This is perhaps not as representative this time of Charlie's typical score though, as 270 is at the lower end of the range. The median value of 280, or mean of 283 are probably better this time.

It's not really that important if you had one or other of these values for Charlie's typical score but didn't know which average had been used. However, in other situations this may not be the case. A specific average may be picked to support a particular point of view in the media or when arguing a case. You'll have a look at this in the next brief section.

1.5 Using different averages

Suppose a group of five employees in a small company each receive a wage of £250 per week and the director receives £2000 a week.

The mean wage per week $= \dfrac{5 \times £250 + £2000}{6} = £542$ (to the nearest pound)

However, if the data are sorted into numerical order as below, it is clear that the median (and also the modal) wage is £250:

250, 250, 250, 250, 250, 2000.

So, if there were a dispute about the employees' pay in the company, their representative could say 'The average pay in the company is only £250 per week,' whereas the director might say 'The average pay is £542 per week.'

Both statements are technically correct, because neither party has stated the type of average they have used. However, both parties have chosen the average that best suits their case!

So, watch out when you see averages used in future – are you being given all the information you need to know about them?

Knowing what average has been used is very important when you are trying to understand data that you encounter. Knowing something about how the data are spread will provide you with additional information that will shed more light on the average value that you are given. Think about the difference to the mean value, if one value is very high for example.

You are now going to look at the spread, or as it is more usually known, the range, of data sets.

2 Measuring the spread of a data set

As well as finding a typical value or average for a set of data, it is also useful to see how the data values are spread about the typical value. Here's an example to start exploring why this is.

Last year, Sam grew two varieties of sunflowers and measured the heights of the flowers in inches. Here are Sam's results:

Table 7 The heights of Sam's sunflowers shown in inches

Variety A	60	59	66	55	55	62	58	65
Variety B	52	69	61	60	54	67	50	67

Sam wants to compare how each sunflower variety performed based on height, so wants to look at the data in more detail.

Activity 5 How tall are the sunflowers on average?

Allow approximately 5 minutes

(a) Find the mean height of the sunflowers of variety A.

Answer

(a) $$\text{Mean height variety A} = \frac{60+59+66+55+55+62+58+65}{8} = 60 \text{ inches}$$

(b) Find the mean height of the sunflowers of variety B.

Answer

(b) On average, the sunflowers of the two varieties are the same height: 60 inches.

When Sam looked at the sunflowers, they did look about the same height on average, but there was still a difference. To investigate this difference have a look at the heights plotted on number lines in Figure 3.

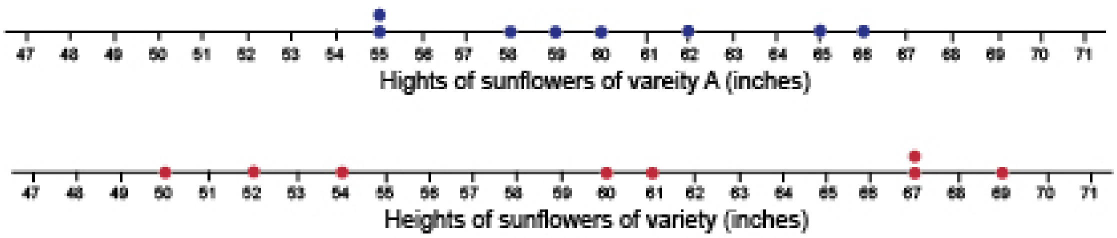


Figure 3 Number lines, showing sunflower heights

The blue dots represent the heights in inches of the sunflowers of variety A, and the red dots represent the heights in inches of the sunflowers of variety B. You should be able to see that the red dots are more spread out than the blue dots. This means that heights of sunflowers of variety A are more consistent (closer together), than the heights of the sunflowers of variety B, which are more variable (more spread out). If you had only looked at the mean value this would not have been evident. This shows how useful it is to look at the spread of data as well as the average.

One way of measuring the spread of a data set is called the range. The range is calculated as follows:

Range of data set = largest value – smallest value

For the sunflowers of variety A, the smallest value is 55 inches and the largest value is 66 inches.

So the range for variety A = 66 – 55 = 11 inches

The smallest value in the data set for variety B is 50 inches, and the largest value is 69 inches.

So the range for variety B = 69 – 50 = 19 inches

So even though both sunflower varieties had the same mean height, there was much more variation – that is, a wider range – of heights seen in variety B compared to variety A. If you grew sunflowers commercially, knowing this about a particular variety would be very useful at harvest time!

The range gives an idea of the spread of the data, although it can be affected by either very high or very low values. There are other ways of measuring the spread of a data set, and you'll meet these if you continue studying mathematics or statistics.

The next section gets you thinking about what questions to ask when you are presented with averages

3 More about data

Averages are used frequently in reports and in the media to summarise data. When you come across an average in this way, it's worth asking yourself some questions before drawing any conclusions.

- Do you know what kind of average was used?
- How many values were used to calculate the average?
- How were the data collected?

Then you'll be prepared to consider the data, and the type of average used, in a critical manner. The more data that has been used to calculate the average, the more likely it is to be a reliable result.

When averages are calculated, this may be based upon a sample of the total data available rather than all the data, particularly when dealing with large populations.

For example, if you wanted to know the average height of women in a country, it would be impractical to measure and record this value for all women – so a sample of the population would be taken. The more measurements you had from across the whole country, the better idea you would get of an average value for the whole population. As well as this, the more data that you have, the less influence extremes can have on the average if the mean is used, which it generally is.

Similarly, if the data were only collected from schools for example, that also would not be a true representation of the average height. So, how the data were collected is also important.

So, next time you see or hear an average value quoted, see if you can find out what lies behind it!

Now you have looked at a few ways to analyse data, let's move on to how to present data using tables.

4 Tables

Tables are often used to display data clearly, such as on a food label, or when a lot of data needs to be displayed in a concise form, such as on a bus or train timetable.

To read data from a table you need to ensure that you look at all the available information.

Using the table below taken from an Irish Tourism Employment Survey will help to consider this.

Table 8 Employment in the Irish tourist industry (numbers employed by sector)

Sector/year	2009	2010	% ± (2009–2010)
Hotels	52 308	46 373	–11
Guest houses	1931	1812	–6
Bed and breakfasts	n/a	3937	–
Self-catering accommodation	3058	2426	–21
Restaurants	41 049	38 657	–6
Non-licensed restaurants	16 134	14 336	–11
Licensed premises	59 983	51 693	–4
Tourism services and attractions	31 449	18 702	n/a*
Total	n/a	177 935	

Note: percentages have been rounded.

*Not all of the sectors included in 2009 are included in the 2010 survey

(Fáilte Ireland, 2011)

There is a lot of information here, so how can you start to understand it?

The first thing to do is to look at the title. This should explain clearly what the table contains. This table contains data on the number of people employed in the Irish tourist industry, divided into sectors. The next step is to examine the column and row headings. What information is being given here? Raw numbers, or percentages? What units are being used? Do you understand all abbreviations used and how the table is constructed?

The column headings are the years 2009 and 2010, that seems straightforward but the final column may be confusing. This tells us the percentage change between 2009 and 2010, given as either a positive or a negative number. Between 2009 and 2010, for instance, the number of people employed in hotels in Ireland fell by 11 per cent.

The rows give the types of tourist facilities, such as hotels and restaurants, with the total for all types given in the bottom row.

Finally, you can look at the main body of the table to find the information you need. For example, to find the number of people employed in restaurants in 2009, move along the row labelled 'Restaurants', and down the column labelled '2009'. Where this row and column meet, the value is 41 049. So in 2009, 41 049 people were employed in restaurants.

4.1 Reading a table

Before you go on to use the information in the table, here is a summary of the steps in reading a table:

- Read the title.
- Examine the column and row headings, particularly the units used.
- Check that you understand the meaning of any symbols or abbreviations.
- Extract the information you need from the main body of the table.

Now you've looked at how to go about understanding a table, try this activity for yourself, bearing in mind what you learned in the last section.

Activity 6 Employment in the Irish tourist industry

Allow approximately 5 minutes

Table 9 Employment in the Irish tourist industry (numbers employed by sector)

Sector/year	2009	2010	% ± (2009–2010)
Hotels	52 308	46 373	–11
Guest houses	1931	1812	–6
Bed and breakfasts	n/a	3937	–
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Total	n/a	177 935	

Note: percentages have been rounded.

*Not all of the sectors included in 2009 are included in the 2010 survey

(Fáilte Ireland, 2011)

The table above is the same table you saw on the previous page. Use it to answer the following questions:

(a) How many people were employed in Guest houses in Ireland in 2009?

(Click on reveal comment if you would like a hint to get going)

Discussion

Look down the first column for the sector and along this row until you reach the correct year.

Answer

(a) Going across the row marked ‘Guest houses’ and down the column headed ‘2009’ shows that 1931 people were employed in guesthouses in 2009.

(b) Why do you think that there is no total for 2009?Look carefully at all the data in this column.

Answer

(b) In the column for 2009, the number of people employed in bed and breakfast accommodation was not available (‘n/a’), so the overall total cannot be found.

(c) Which type of premises employed the highest number of people in 2010?

Answer

(c) Going down the column for 2010, the highest number (apart from the total) is 51 693. So, licensed premises employed the highest number of people in 2010.

(d) What type of premises has shown the largest percentage decrease between 2009 and 2010?When calculating the percentage, think carefully about what you have been asked to work out – which value is the original value that you are comparing the decrease to?

Answer

(d) The largest percentage decrease is 21 per cent, for self-catering accommodation.

The values in this table were neither very large nor very small. If this was the case it can make a table hard to read. You could, of course, use scientific notation instead, as you learned about in Week 5, but not everybody will understand this way of representing numbers. So you need another way to present these types of numbers clearly for the reader. This is the subject of the next section.

4.2 Representing data in tables clearly

Some tables contain very large numbers or very small numbers. To make these easier to read, the way they are presented may be changed. For example, suppose the number of tourists (rounded to the nearest thousand) visiting south-west Ireland in four quarterly periods were 556 000, 320 000, 284 000 and 355 000. These numbers can be expressed as 556 x 1000, 320 x 1000 and so on. Using units of 1000, the numbers in the table can be written simply as 556, 320, 284 and 355, and the column heading changed to ‘Numbers

(000s)' or 'Numbers (thousands)'. The values then become much easier to read. This is very similar to scientific notation but presented in a different way!

Table 10 indicates the number of visitors to the different regions of Ireland in 2011 and the revenue generated.

Table 10 Where did the tourists go and how much did they spend (2011)?

Numbers (000s) ***Revenue (€m)***	**Overseas tourists**	**Northern Ireland**	**Domestic**	**Total**
Dublin	3805 *1125.2*	311 *76.1*	1683 *314.6*	5799 *1515.9*
Midlands/east	760 *253.1*	185 *38.0*	1183 *198.5*	2128 *489.6*
South-east	720 *171.5*	73 *19.6*	1293 *271.3*	2086 *462.4*
South-west	1678 *595.9*	70 *19.6*	1906 *442.7*	3654 *1058.2*
Shannon	859 *256.8*	32 *8.1*	887 *169.1*	1778 *434.0*
West	1180 *423.6*	103 *27.7*	1303 *290.8*	2586 *742.1*
North-west	505 *144.7*	530 *80.7*	739 *135.2*	1774 *360.6*
Total revenue	*2970.8*	*269.8*	*1822.2*	*5062.8*

'Euro m' (or '€m') is an abbreviation for 'millions of euros', the currency used in Ireland.

(Fáilte Ireland, 2012)

Activity 7 Where did the tourists go?

Allow approximately 5 minutes

Use the table above to answer the following questions. Remember, 'Euro m' (or '€m') is an abbreviation for 'million of euros'. Don't forget to click on reveal comment if you would like a hint.

(a) How many domestic tourists visited Shannon in 2011?

Discussion

Remember to take note of the units used for numbers of visitors.

Answer

(a) Take care with the figures when you read them from the table. The number of people is given in thousands, so the number of domestic tourists who visited Shannon in 2011 was 887 000. The number in the table is 887, but the units are in thousands of visitors, making the final answer 887 000.

(b) Roughly how much did all the tourists spend in Dublin in 2011?

Answer

(b) Tourists visiting Dublin spent €1515.9 million, or about €1.52 billion.

1515.9 million = 1 515 900 000

1 billion = 1 000 000 000

So 1515.9 million = 1.52 billion (to 2 d.p.)

(c) In which regions were there more domestic tourists than overseas tourists?

Answer

(c) Comparing the columns for the overseas tourists and the domestic tourists shows that there were more domestic tourists than overseas tourists in all the regions except Dublin.

So, now you've had some practice at taking information from a data, it is time to think about constructing your own tables.

4.3 Constructing your own tables

As tables are a good way to show data, you may well come across situations where you will need to construct your own. This will be useful, not only in many areas of study but also at work or home. You might, for example, decide you want to show your monthly energy use over a year.

The first step is to decide how to sort the data into categories to stress the points you wish to make. For example, you may have collected data from a group of people that included their age.

Should you divide this into 'under 25', '25–40' and 'over 40'? Your choice of category would depend on the kind of data that you have and what you want to show.

The next would be to work out from this categorisation how many rows and columns you need and how to label these clearly. You may also have to consider what the best way to show the data is – do you have any very small or very large numbers, for example?

In this next activity the categories have already been decided for you, what you need to do is try and present the collected data in a much easier way to understand.

Activity 8 More about tourists

Allow approximately 10 minutes

The manager of a small Irish hotel has guests of different ages and nationalities. They would like to know what kinds of guests visit their hotel, so they decide to summarise this information in a table.

The categories they used for the data were: child (under 16), adult (16–60) and senior (over 60); and the nationalities Irish (Ir), British (B), mainland European (E) and the rest of the world (W).

The following data were collected. The number and letter combinations represent the age and nationality of the visitors.

Table 11 Hotel guest data

16E	7B	24Ir	26Ir	46Ir	43Ir	5Ir
50W	55W	13Ir	13B	15Ir	61B	8Ir
37W	48E	8Ir	62B	49E	6W	55Ir
11Ir	9Ir	12B	62W	65B	65B	67Ir
13W	12W	54B	72Ir	61Ir	48B	61W
15B	62Ir	67E	10W	27B	12Ir	31B
35W	8W	42B	43B	15W		

Construct a blank table with a title, the source of the data and clear column and row headings corresponding to the categories above. Also include totals for each column and row.

The data probably looks quite confusing – take your time and work methodically through the data to make sure you don't miss any!

The title, column and row headings should make it clear to the reader what information is contained in the table – so think carefully about these.

Discussion

To count the data in each category you could use a tally system. You may be familiar with tallying as representing sets of 5, with 4 slashes and a diagonal line across these, making what looks like a gate. This is known as a tally.

Answer

The final table should look something like this. It is fine if you have put the rows and columns the other way round.

Table 12 Origin and age categories of hotel guests

Nationality	Child	Adult	Senior	Total
Irish	8	5	4	17
British	4	6	4	14
Mainland European	0	3	1	4
Rest of the world	6	4	2	12
Total	18	18	11	47

Note that the two totals for nationality and age should agree, so it is useful to work out both as a check.

Hopefully, you agree that the table you constructed was much easier to understand and to find the information from than the raw, unordered data from the hotel manager.

5 This week's quiz

Well done, you've just completed the last of the activities in this week's study before the weekly quiz.

Go to:

Week 7 practice quiz.

Open the quiz in a new tab or window (by holding ctrl [or cmd on a Mac] when you click the link).

6 Summary

Analysing data using averages is one method of finding out more about data, in this case a typical value. Averages and ranges are the bases for the study of statistics, which provides many powerful tools to look more deeply into data sets. Statistics are used by many different people and in many different professions, but they are particularly important to anybody who collects data – if you don't have any tools to analyse information, the data is not much good on its own! Presenting data clearly in an ordered table is also a very useful skill to have. This allows you to communicate with others clearly and is used in a range of other academic study areas.

Congratulations for making it to the end of this week! You should now be able to:

- calculate the average for a set of data using the mean, median and mode
- calculate the range of a set of data
- understand that different averages may give different results for the same set of data
- read and construct tables.

You can now go to Week 8.

Week 8: Communicating with data

Introduction

At the end of last week you looked at one way of communicating with data, the use of tables. This week this theme will be continued by taking a look at graphs and charts. These give us a means of displaying data visually, and as the saying goes, ‘a picture paints a thousand words’! So, graphs and charts can be a very useful way of presenting data clearly and at the same time showing any possible patterns. You’ll be considering how to present and interpret graphs and charts, as well as the care that needs to be taken when doing this to ensure that you are really being presented with the full picture.

In the following video Maria introduces Week 8, the final week of the course, and the final quiz for your badge:

Video content is not available in this format.

After this week’s study, you should be able to:

- read and construct line graphs to display information
- read and construct bar charts to display information
- read and construct pie charts to display information
- critically interpret graphs and charts.

1 Graphs and charts

At the end of last week you looked at how useful tables are for summarising a lot of data concisely, clearly and accurately. However, sometimes you want to get an overall message across quickly and this may well stay hidden in a table of data. In this situation you can use a graph or a chart instead. Graphs can also be used to explore relationships between sets of data, such as the way in which a sunflower grows over a season.

You have probably seen different types of graphs and charts in your day-to-day life, such as when watching the TV, reading a newspaper or on the internet. Here is a flavour of some you may have encountered.

Bar charts

These allow a visual comparison between different categories. For instance, in Figure 1 you can see very quickly that the hiking/hill walking category was the most popular activity, and cruising the least popular.

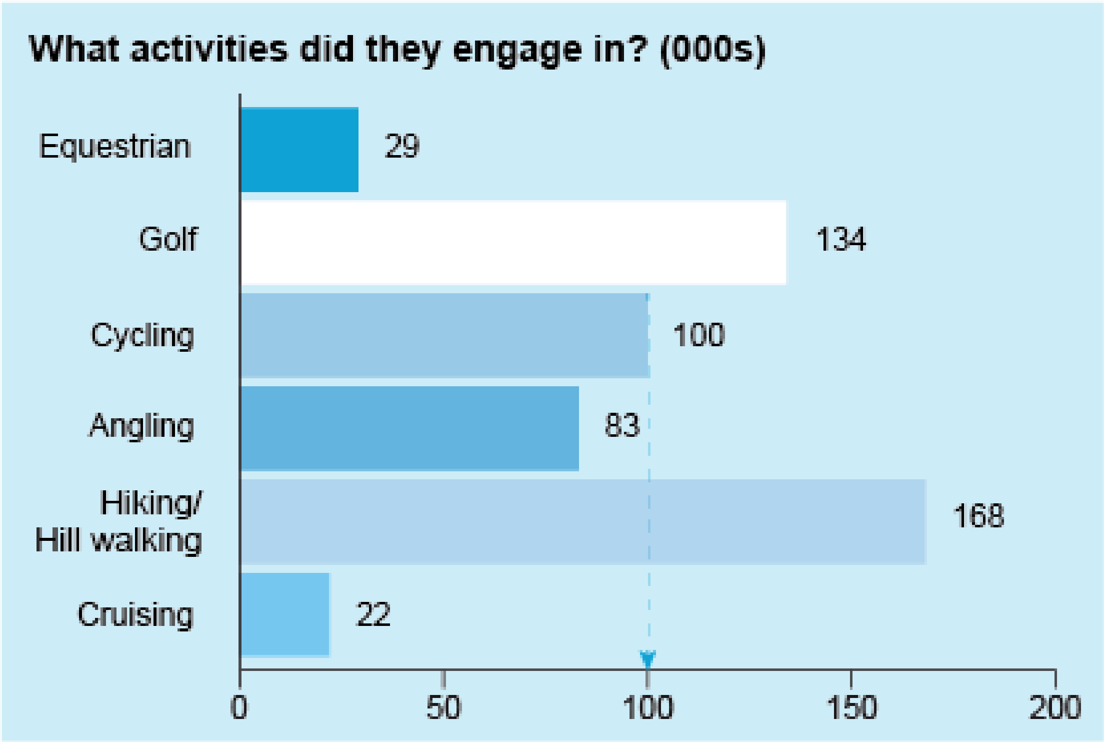

Figure 1 Example bar chart

Pie charts

Again, these allow a quick visual comparison between categories but this time using percentages rather than the actual data. This pie chart shows very clearly that walking is the favourite activity.

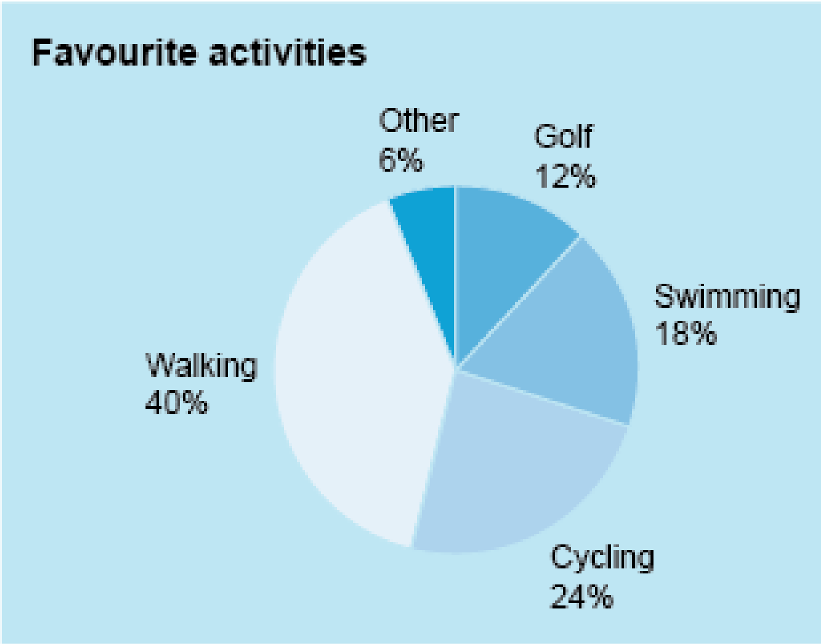

Figure 2 Example of a pie chart

Line graphs

Finally, a line graph, where all the plotted points are joined, can show clear relationships between the plotted data. In this case, the curve shows that the weight (or as it's properly known, mass!) of a woman increases over the last six weeks of her pregnancy, but that this weight gain slows towards the end. This is shown by the flattening out of the curve in Figure 3.

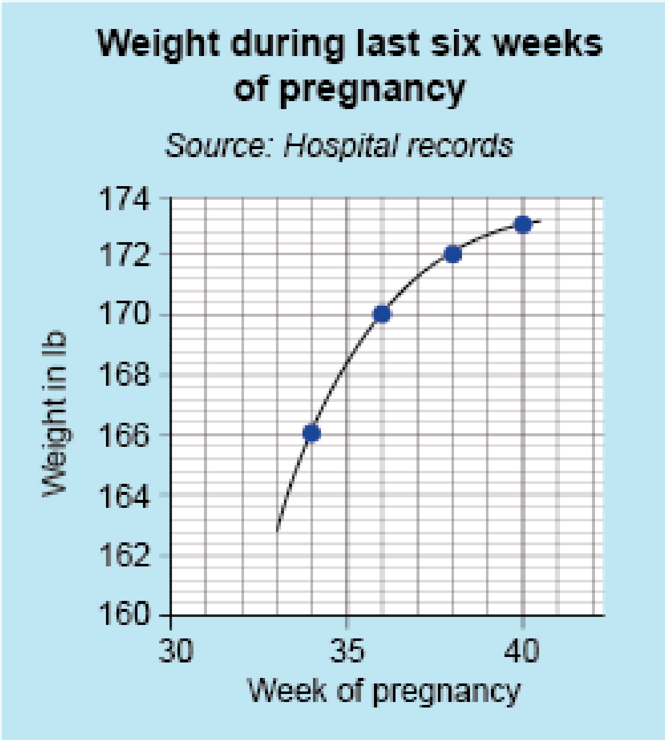

Figure 3 Example line graph

You may well have come across other charts or graphs, but as these are three of the most common types, bar and pie charts and line graphs will be the focus of this week. To get you started, in the next section you will look at plotting points on a line graph.

2 Line graphs

Line graphs are constructed by plotting points on a grid. Figure 4 shows two lines at right angles to each other that set up a grid. These are called the **horizontal axis** and the **vertical axis**. Each axis is marked with a scale, in this case from 0 to 3. The point where the two axes meet and where the value on both scales is zero is known as the **origin**. (Note: ‘axes’ is the plural form of ‘axis’.) This is not only the basis for all line graphs, but also for bar charts.

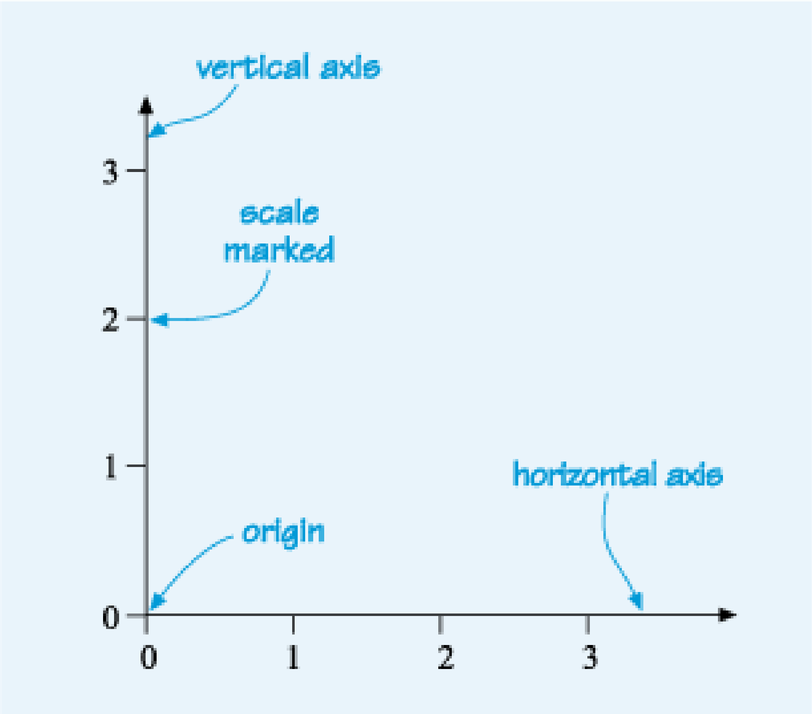

Figure 4 Example of a graph's axes

In Figure 5, the horizontal axis has been labelled with an 'x' and the vertical axis with a 'y'. This is because the horizontal axis is usually known as the x-axis and the vertical axis as the y-axis.

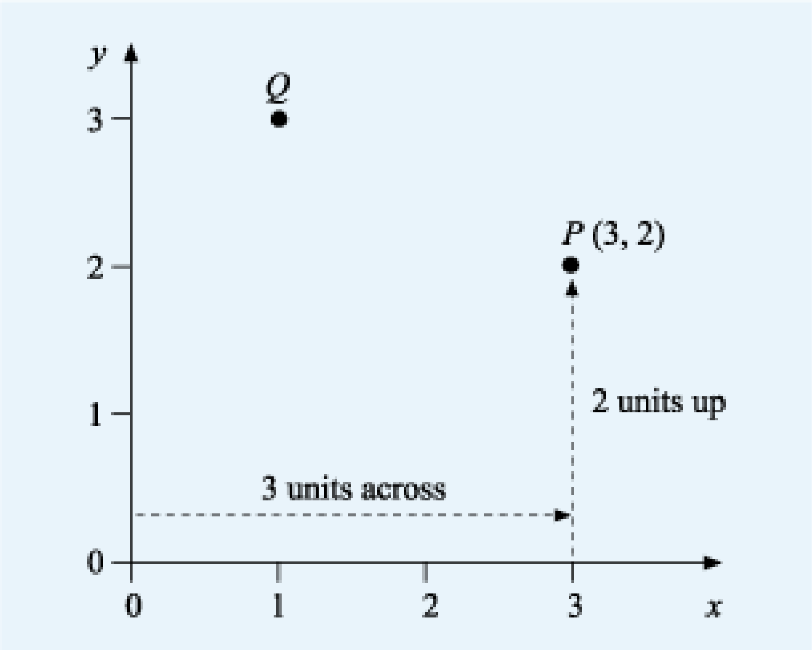

Figure 5 Two points labelled *P* and *Q* plotted on a graph

The location of any point on the grid by its horizontal and vertical distance from the origin. These are always shown with the horizontal distance first and in the form **(x,y)**, where x and y are the respective distances from the origin to the point. This is known as the **coordinates** of the point. The horizontal distance being the **x-coordinate** and the vertical distance the **y-coordinate**.

Look at point *P* in Figure 5. To get to point *P* from the origin, you move three units across and two units up. Alternatively you can say that point *P* is opposite 3 on the x-axis and 2 on the y-axis. This gives us the coordinates for *P* of as (3,2), with an x-coordinate of 3 and a y-coordinate of 2.

In the same way, point *Q* is one unit across and three units up, so *Q* has coordinates (1, 3).

coordinates do not need to be positive numbers though. You'll see in the next section how both the x and y axes can be extended to include negative numbers, just as a number line can.

2.1 Negative coordinates

Figure 6 shows the axes extended downward and to the left, to include negative values. The coordinates of the plotted points still have the same configuration, that is, *x* first, followed by *y*, and are read in the same way as before. For example, point *B* is three units to the left of the origin, so its x-coordinate is –3. It is one unit up, so its y-coordinate is 1. So *B* has coordinates (–3, 1).

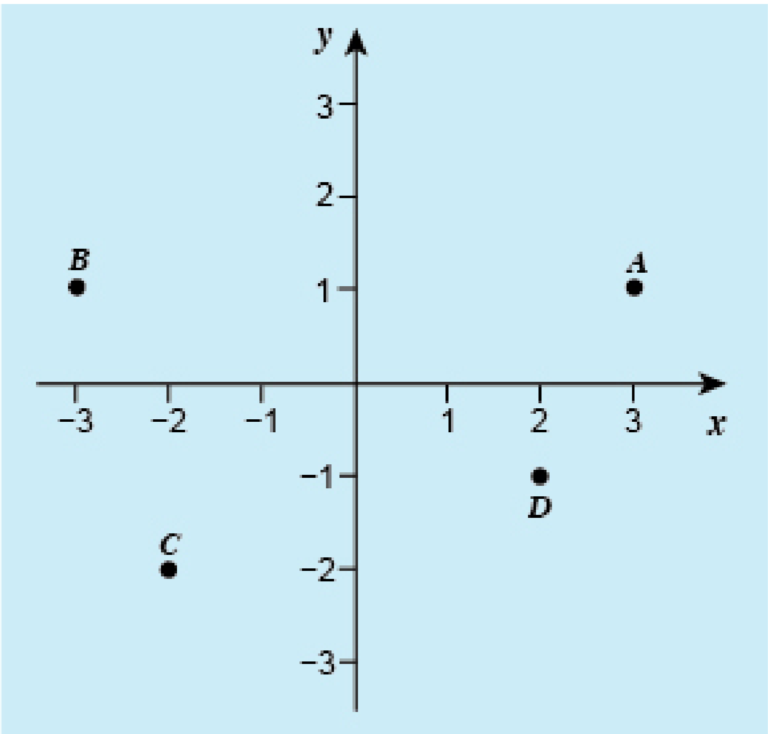

Figure 6 Graph showing negative coordinates

Now it is your turn to try your hand at reading the remaining coordinates from Figure 6 in this first activity of the week.

Activity 1 Reading coordinates

Allow approximately 5 minutes

Write down the coordinates of points *A*, *C*, and *D*, shown in Figure 6. What are the coordinates of the origin?

Answer

A is opposite 3 on the x-axis, so its x-coordinate is 3. It is opposite 1 on the y-axis, so its y-coordinate is 1. The coordinates of *A* are (3, 1).

C is opposite -2 on the x-axis, so its x-coordinate is -2. It is also opposite -2 on the y-axis, so its y-coordinate is also -2. The coordinates of *C* are therefore (-2, -2).

The coordinates of *D* are (2, -1).

The origin has coordinates (0, 0).

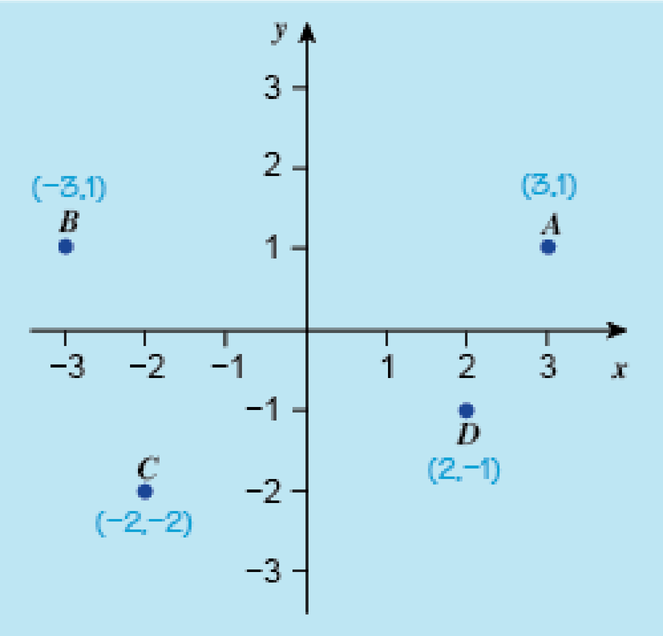

Figure 7 Graph showing negative coordinates

As well as being able to read coordinates of points on a graph it is also important to be able to plot them yourself, for when you need to draw your own line graph. There is probably not much need for this in your day-to-day life, but many different study areas make use of graphs to show data and relationships. So a good working knowledge of how to construct a graph will stand you in good stead if you continue with your studies in the future.

You'll look at plotting points in the next section.

2.2 Plotting points

Plotting points is essentially the same as reading the coordinates of a point. This means going along the x-axis to the given coordinate, and then vertically upwards until you are opposite the correct place on the y-axis. The point can then be marked with one of three symbols: a dot, as in the examples here, a cross, or a dot with a circle around it. This last option just makes the plotted point clearer if the points are joined with a line.

The next activity will give you some practice at plotting points. It would be useful to do this exercise on graph paper because it is much easier to plot the points accurately if you have a grid to work with. Fortunately, you don't have to rush out and buy lots of graph paper because you can print out as much as you need, if you have a printer.

Activity 2 Plotting points

Allow approximately 10 minutes

Draw x- and y-axes, with a scale on these from -3 to 3. Mark the points *E* at (1, 1), *F* at (–2, –1), *G* at (1, –3) and *H* at (–2, 3).

Answer

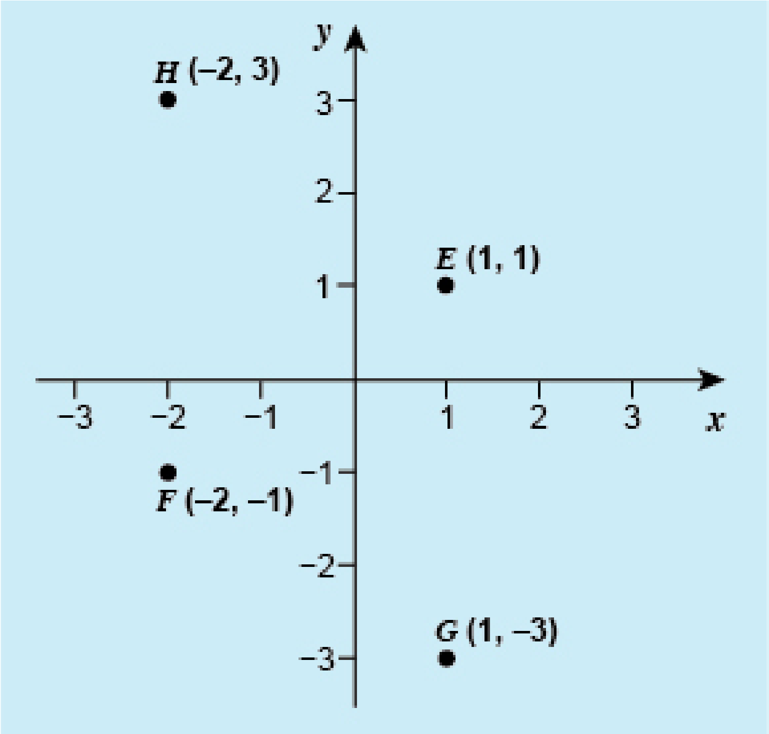

Figure 8 Graph with points *E*, *F*, *G* and *H* marked

The plot you drew should have points *E*, *F*, *G* and *H* marked in the same position they are shown on the plot above.

In the examples so far, all the coordinates were whole numbers and the scales were also marked at whole unit intervals. As with a number line though, there is nothing to stop us also showing fractions of whole units. Let’s take a look at this now.

2.3 Decimal coordinates

Marking a scale with fractions of whole units allows us to plot points using decimal coordinates such as (1.8, –2.1).

In Figure 9 below, each unit along the axes has been divided into ten, so each grey line represents a tenth of a whole unit, or 0.1. Point *A* has the coordinates (1.3, 0.8), as it is plotted 1.3 units across to the right of the origin and 0.8 units up.

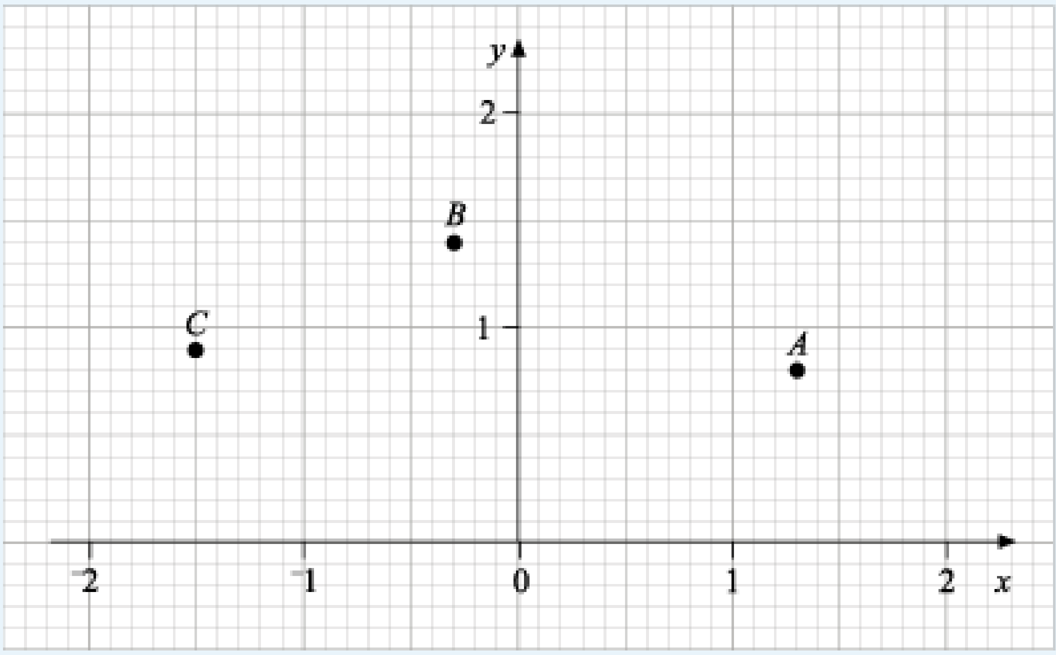

Figure 9 Decimal coordinates

Now, we've done the first point for you, complete the coordinates for the other points shown in the next activity.

Activity 3 Reading decimal coordinates

Allow approximately 5 minutes

Write down the coordinates for the points *B* and *C* on the graph above.

Answer

B is plotted at the point which is 3 intervals to the left of the origin on the horizontal scale and 4 intervals past the 1 mark on the vertical scale, i.e. opposite -0.3 on the x-axis and opposite 1.4 on the y-axis. So the coordinates of *B* are (-0.3, 1.4)

Similarly, the coordinates of *C* are (-1.5, 0.9).

2.4 Displaying data on a graph or chart

You have seen that, provided you have both an x- and a y-coordinate, you can plot a point on a chart. So if a set of data consists of pairs of data values, then that data can be plotted on a graph. This enables patterns between the data to be assessed visually, as well as mathematically. This is for future study though!

There are several steps to take when drawing a graph, which can be summarised as follows:

- Choose an appropriate scale for both axes.
- Label both axes, including a brief description of the data and the units.

- Give your graph a suitable title.
- State the source of data.
- Plot the points accurately.
- Join the plotted points with a line or smooth curve if appropriate.

Following these steps will ensure that any graph is clear, easy to read and well presented.

Let's look at one of these steps in more detail now – choosing the scales.

2.5 Choosing the scales for a graph or chart

Choosing appropriate scales can take more time than other elements of constructing a graph. They can need careful consideration, as they affect the whole look of the graph. Scales are usually chosen to illustrate the data clearly, making good use of the graph paper and using a scale that is easy to interpret.

In the examples considered so far, the scales on both axes were the same and both started at zero. However, this is not always appropriate. You're going to investigate this using the example of a nurse monitoring the weight of a woman during the last few weeks of her pregnancy. The woman's weight each week has been recorded as shown in Table 1 below.

Table 1 Weight of a woman during pregnancy

Week of pregnancy	34	36	38	40
Weight in kg	75.6	77.4	78.1	78.5

You could start both scales at 0, but then all the points would be plotted in the top right-hand corner, making the graph difficult to read, as can be seen in Figure 10 below.

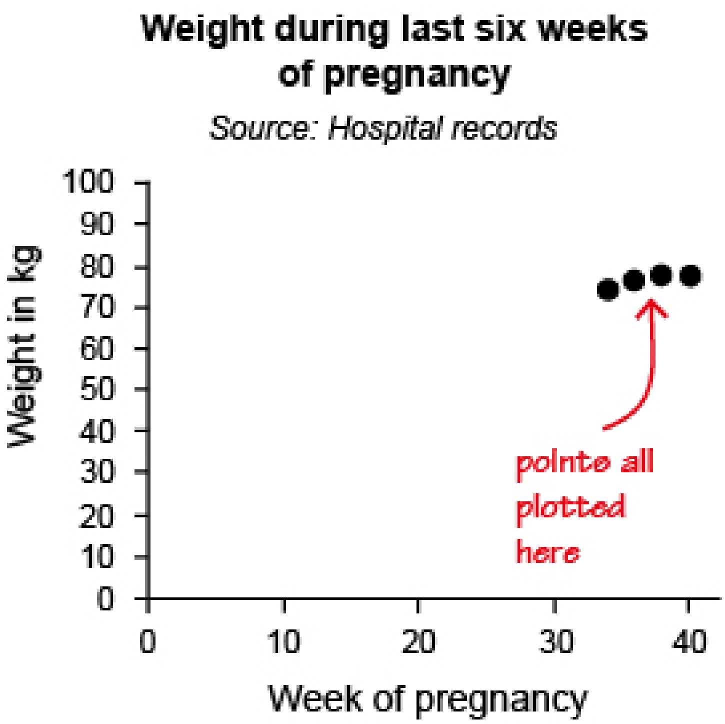

Figure 10 Weight during pregnancy data on a graph

Since the data on the x-axis start at 34 and finish at 40, a better scale on the x-axis would be one starting at 30 and ending at 42. Similarly, since the data for the y-axis are from about 75 to around 79, a scale on the y-axis that starts at 72 and ends at 79 would seem sensible. Using these scales, the points would be plotted as shown in Figure 11.

Hopefully you can see by comparing the two graphs that the second attempt, as well as being much clearer, just looks better! That is always a good test of how well a graph has been constructed.

Figure 11 Weight during pregnancy data on a graph with modified axes

Note that as well as the scales being marked on the axes, these are labelled to show what is being measured, and in what units. The graph also has a title and source of data.

The points have been joined with a smooth curve as, in this case, you would expect the woman's weight to change steadily between hospital visits over the period. So, it is reasonable to join the points to show the overall pattern or trend.

So, the graph has all the required elements.

If you have access to some graph paper, have a go at drawing your own graph in this next activity.

Activity 4 Drawing a graph

Allow approximately 10 minutes

The data shown in Table 2 are the circumferences and diameters of various circular objects. All measurements are in centimetres.

Table 2 Diameters and circumferences

Diameter/cm	Circumference/cm
4.4	14.1
6.6	20.9
8.6	27.0
10.3	32.3
23.0	72.8

(a) Construct a graph to show this data, with the diameter on the horizontal (x) axis and the circumference on the vertical (y) axis. Draw a straight line through the points.

Answer

(a) Your graph should look like this:

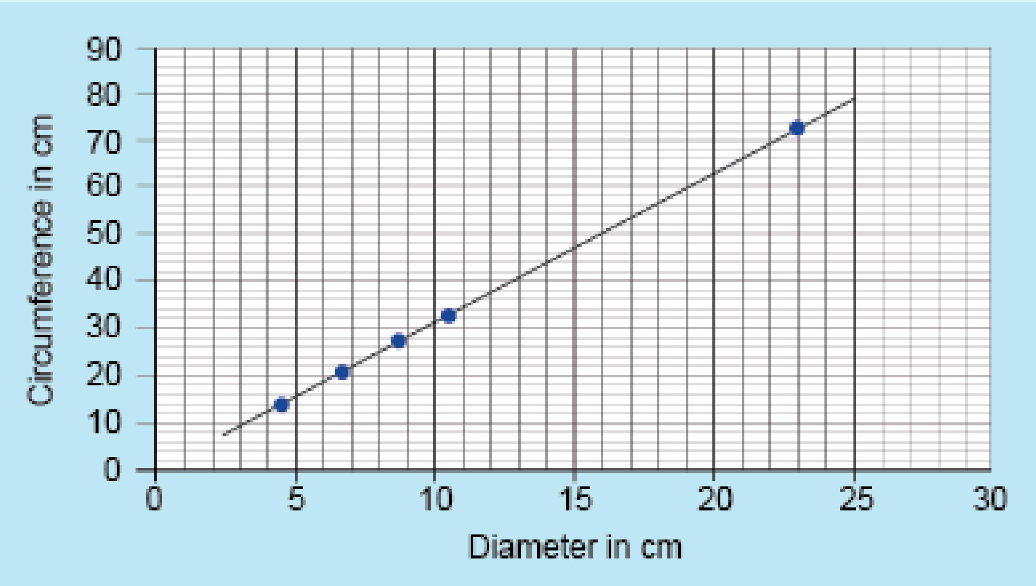

Figure 12 Graph showing the circumference and diameter of household objects

Note the title and labelling of the axes. You may have used a different scale from us if you had different graph paper. This is fine, as long as it makes best use of the paper and is easy to read from.

Finally in this section on line graphs you're going to take a look at how to read from, and use, a graph.

2.6 Using a graph

As well as providing a visual representation of data, a graph can be used to infer more information than the data that you started with.

Look again at the change in weight of a woman during the last six weeks of pregnancy. Since you would expect the woman's weight to change steadily between hospital visits, it is possible to use the graph to estimate her weight at other times during the six weeks.

For example, to estimate her weight at 35 weeks, first find 35 on the horizontal axis. Then from this point, draw a line parallel to the vertical axis, until the line meets the curve at point *P*. From *P*, draw a line horizontally, to meet the vertical axis. Read off this value: in Figure 13, it is about 76.2 kg.

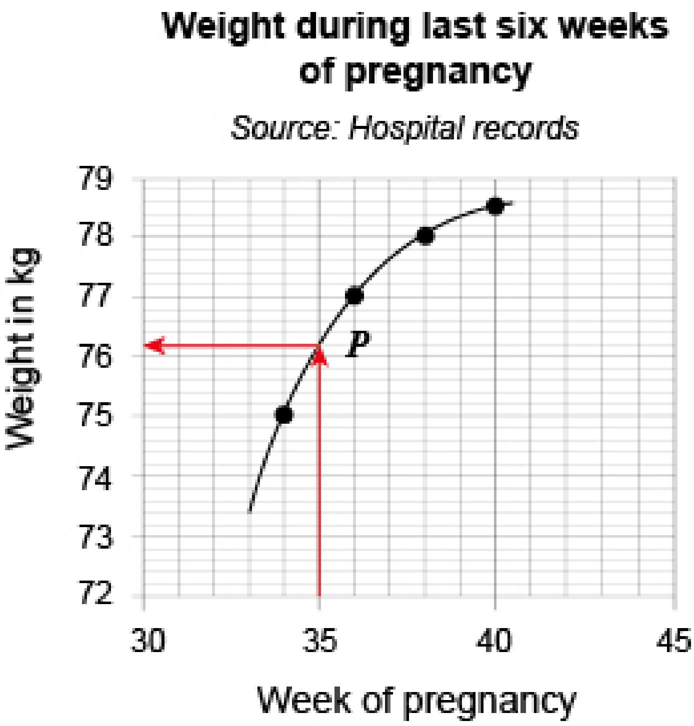

Figure 13 Weight during pregnancy data on a graph with modified axes and arrows

Determining values between the plotted points is known as **interpolation**.

Activity 5 Reading a value from a graph

Allow approximately 5 minutes

Use the graph above to answer the following.

It may be helpful to print this page with the graph, so that you can draw lines on the graphs to identify values. You may also use an object with a straight edge, such as a ruler or piece of paper, and hold it up to your monitor to help you visualise where certain points appear in relation to the axis.

(a) What is the woman's weight at 37 weeks?

Answer

(a) Find 37 on the horizontal axis and draw a line vertically up to the curve. Then draw a line horizontally from this point to intersect the vertical axis. This value is approximately 77.6 kg.

The woman's weight at 37 weeks is estimated to be about 77.6 kg.

(b) When do you think the woman's weight was 76 kg?

Answer

(b) Find 76 kg on the vertical axis. Draw a horizontal line to meet the curve. Then draw a line vertically down to meet the horizontal axis. Read off the value: it is just below 35 weeks.

So the woman's weight is estimated to have been 76 kg in week 34.

As well as inferring other data from the graph in some cases it is possible to estimate the coordinates of points that lie outside those plotted. This is known as **extrapolation**. However, you must be confident that the graph continues in a similar manner so you do need to be cautious: you cannot be certain that patterns shown in graphs will continue.

For instance, in our example it would not be sensible to extrapolate beyond 40 weeks, as this is the usual length of a normal pregnancy.

The graph can also be used to determine the overall trend in the data — that is, how one value is changing with the other. In this case, the graph shows that the woman gained weight quite rapidly between weeks 34 and 36, but from week 36 to week 40 she gained less.

So, careful use of a line graph can prove a very powerful tool when investigating collected data.

The next section looks at bar charts and the different ways these can be drawn, depending on the data that you have and what you want to show.

3 Bar charts

Last week, you saw data from Fáilte Ireland presented in tables. On their website they also illustrate other data in the form of bar charts, as shown in Figure 14 below. This represents the numbers of visitors participating in different activities.

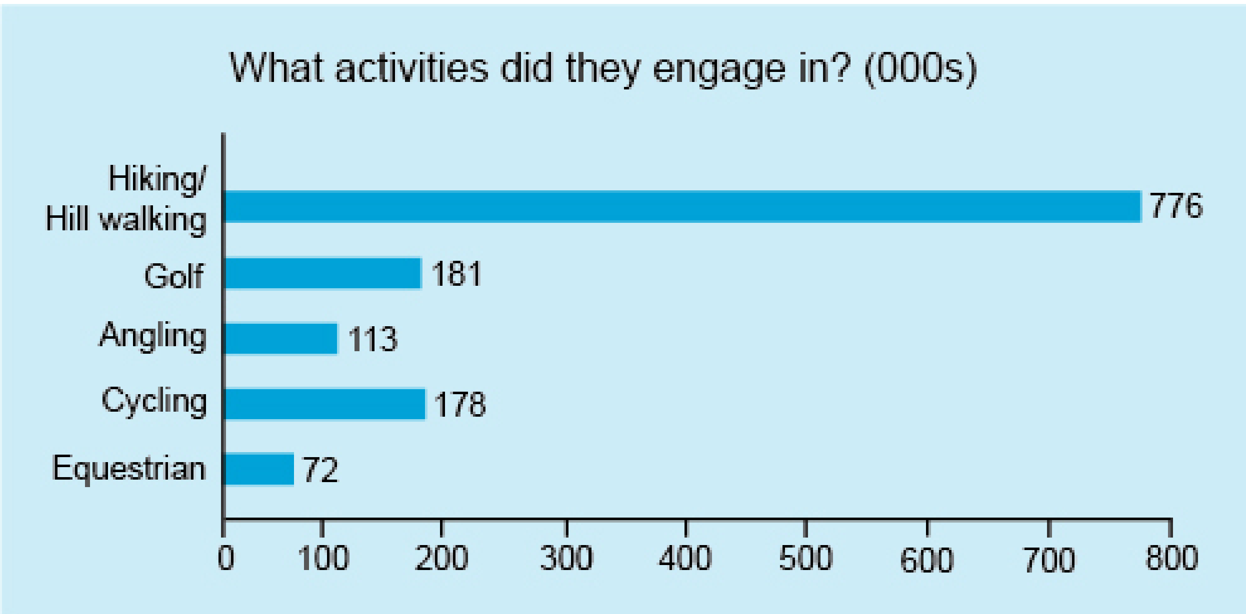

Figure 14 Bar chart from Fáilte Ireland showing the numbers of visitors participating in different activities.

Fáilte Ireland (2004)

In a bar chart, the length of each bar represents the number in that category. Because the bars on this chart are horizontal, the chart is known as a **horizontal bar chart**. However, bar charts can also be drawn with vertical bars. Note that each bar has the same width, and since the bars represent different and unrelated categories, the bars do not touch each other and are separated by gaps.

The authors of this chart have made it easy to read as they've marked the values represented by each bar directly on the chart. If the values had not been marked on the bars, they could have been estimated by drawing a line from the end of the bar and seeing where it intersected the horizontal axis, just as you did with the line graph.

So, rather than spending time on reading from a bar chart let's move on to constructing one.

3.1 Drawing a bar chart

The same basic guidelines apply to drawing a bar chart as they do to drawing a line graph. There are a few differences though. So, the steps to follow are:

- Choose an appropriate scale for both axes.
- Label both axes, including a brief description of the data and any units.
- Give your graph a suitable title.
- State the source of data.
- Draw the bars for each category accurately.

Use the skills that you have already gained when drawing a line graph to complete this activity.

Activity 6 Drawing a bar chart

Allow approximately 10 minutes

Last week you created the following table showing information about hotel guests:

Table 3 Origin and age categories of hotel guests

Nationality	Child	Adult	Senior	Total
Irish	8	5	4	17
British	4	6	4	14
Mainland European	0	3	1	4
Rest of the world	6	4	2	12
Total	18	18	11	47

Source: hotel accommodation records

Use the table to draw a vertical bar chart that shows the total number of guests in each of the nationality categories. Mark the nationality categories on the horizontal axis and the number of guests on the vertical axis.

Answer

Your bar chart should look like this:

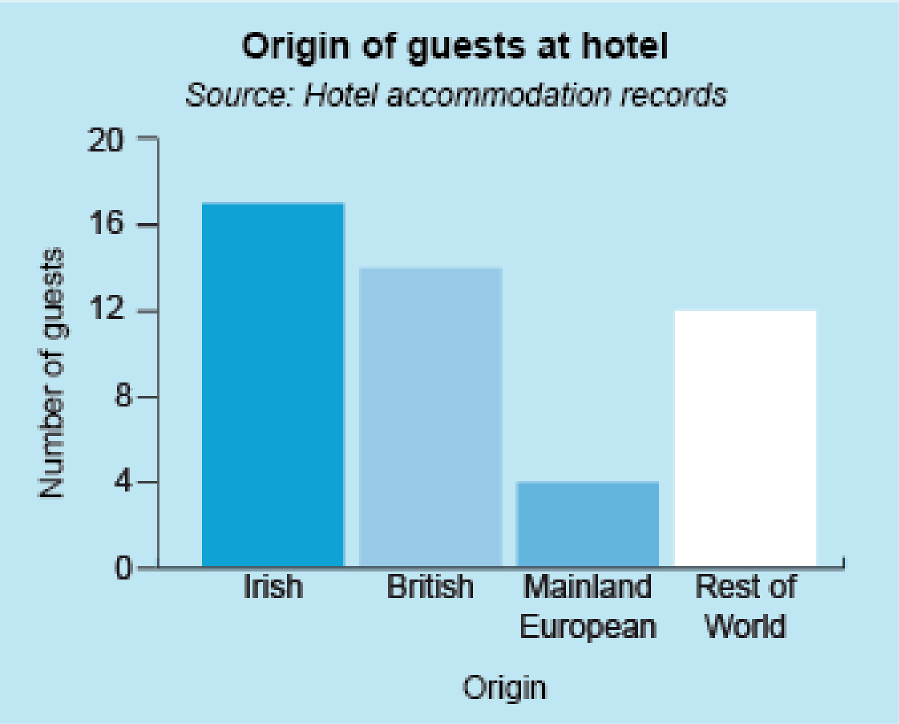

Figure 15 Origin of guests at hotel

Notice how this bar chart stresses the total number of tourists in each category. You can see easily that there were more tourists from Ireland than any of the other groups.

Another way of showing data on a bar chart is using a component bar chart, which can give more detail than the basic examples that have already been looked at. This will be the subject of the next section.

3.2 Component bar charts

In our last example of the quantity of guests from different nationalities in a hotel, you could add extra detail by showing how these totals were broken down into the different age groups. To do this you can split each bar into three different sections, where the length of each section represents the number of guests in that age group. The resulting bar chart would then be a **component bar chart**, as shown in Figure 16. Notice that a key (or ‘legend’) has been added to the graph to explain the shading for the different age categories.

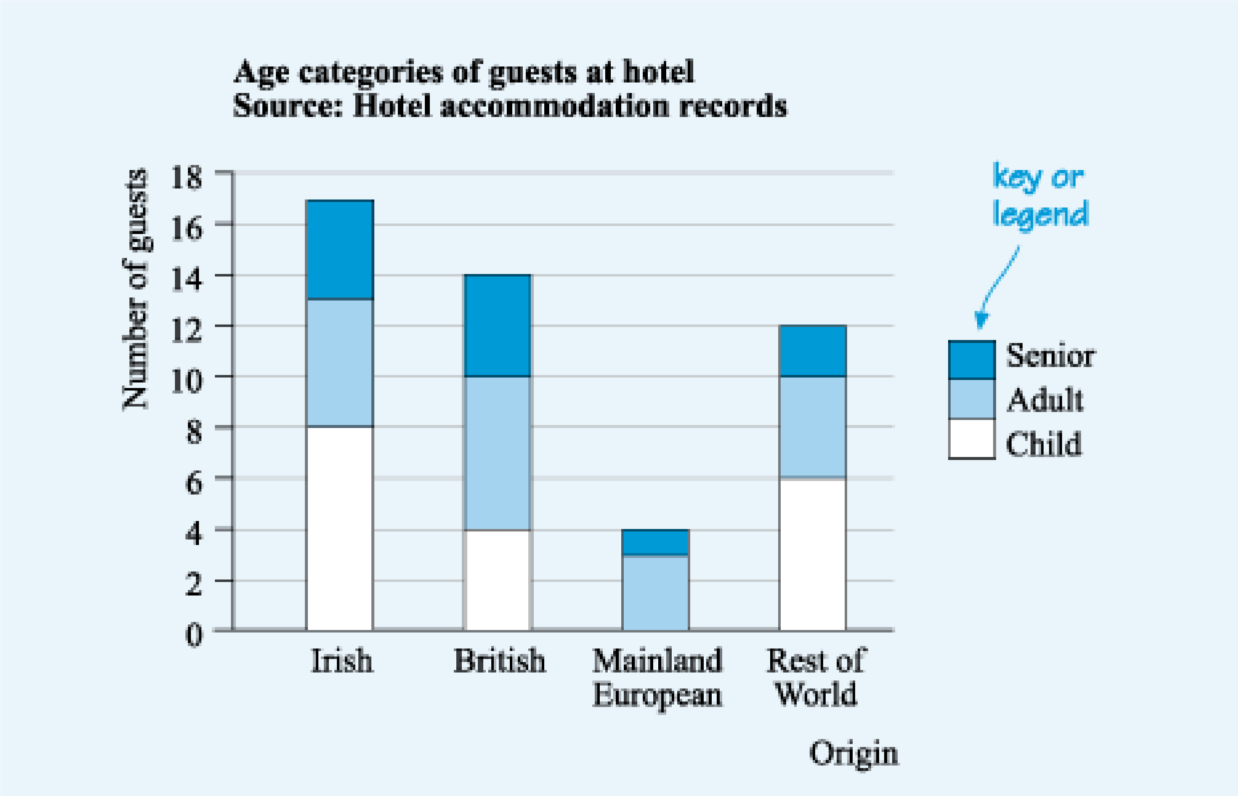

Figure 16 Component bar chart showing the age categories of hotel guests

When you read a component bar chart, you need to find the height of the relevant section to determine the quantity that it represents. For instance, the top of the section representing British adults is opposite the 10 on the vertical scale. The bottom of that section is opposite the 4. Thus, the number of British adults is the difference between the top and the bottom of the section, which is 6.

Another type chart is called a comparative bar chart. This again shows more detail than a standard bar chart, but in a different way to a component one, as you'll see now.

3.3 Comparative bar charts

A comparative bar chart places bars representing sections from the same category adjacent to each other. This allows for a quick visual comparison of the data.

For example, another way of categorising the data that you have been working with so far would be to split each nationality into the number of females and males. The resulting comparative bar chart would be as shown in Figure 17.

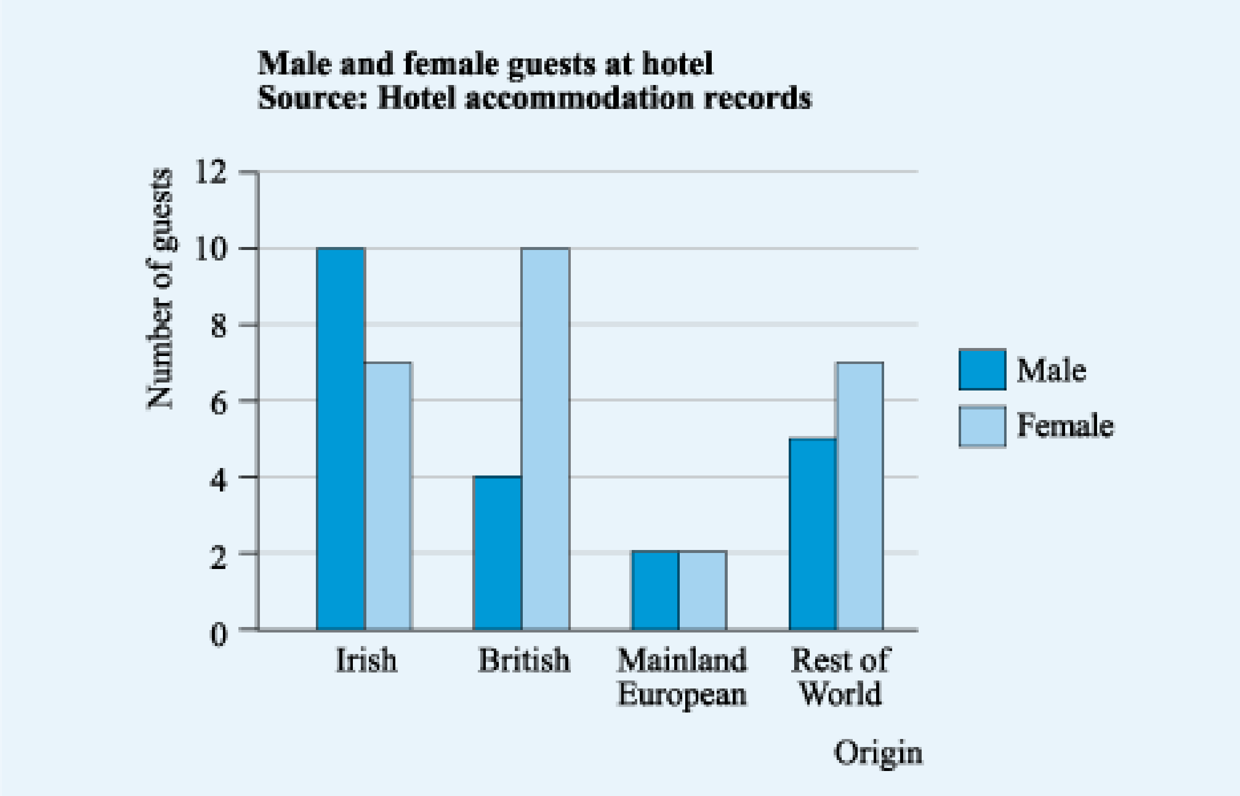

Figure 17 Comparative bar chart showing the gender of hotel guests

It is easy to see why this is called a comparative bar chart, as it is straightforward to compare the data – for example it is easy tell that many more visitors from Britain were female.

Finally in this section on bar charts, notice how all these charts followed the same format, with the title and source clearly labelled and the axes and scales clearly marked, but they emphasised different aspects of the data. The first showed the totals from each nation, whilst the latter two gave more detail on this data. So when you are displaying data in a graphical form, it is important to choose a chart or graph that stresses the main points as simply as possible so that your reader can understand your chart quickly and easily.

Now it’s time to look at our final type of chart, the pie chart.

4 Pie charts

Although bar charts are useful to display numbers and percentages, sometimes you might want to stress how different components contribute to the whole. Pie charts are often used when you want to compare different proportions in the data set. The area of each slice (or **sector**) of the pie chart represents the proportion in that particular category.

For example, the pie chart in Figure 18 illustrates the favourite type of exercise for a group of people:

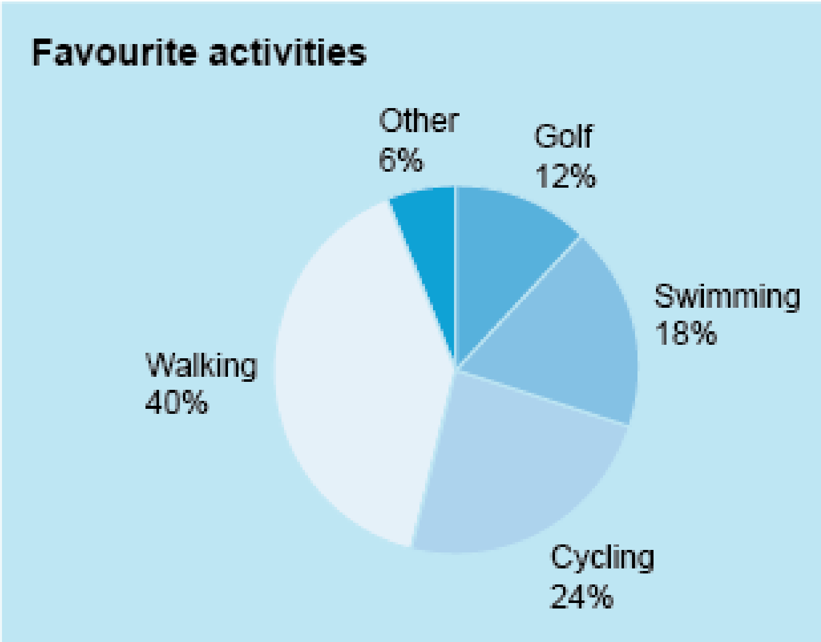

Figure 18 Example of a pie chart: favourite type of exercise for a group of people

This pie chart shows the percentages for each category, so you can read these off directly. Even without the percentages marked, it would be clear that walking was the most popular activity overall as a proportion of the whole group.

Sometimes, however, the percentages are not marked on the chart, so the proportions then have to be roughly estimated by eye. This, of course, is not ideal for all situations when accurate data is required!

The next activity gives you a chance to practise reading a pie chart.

Activity 7 How employees travel to work

Allow approximately 5 minutes

The pie chart in Figure 19 illustrates how employees in a particular company travel to work:

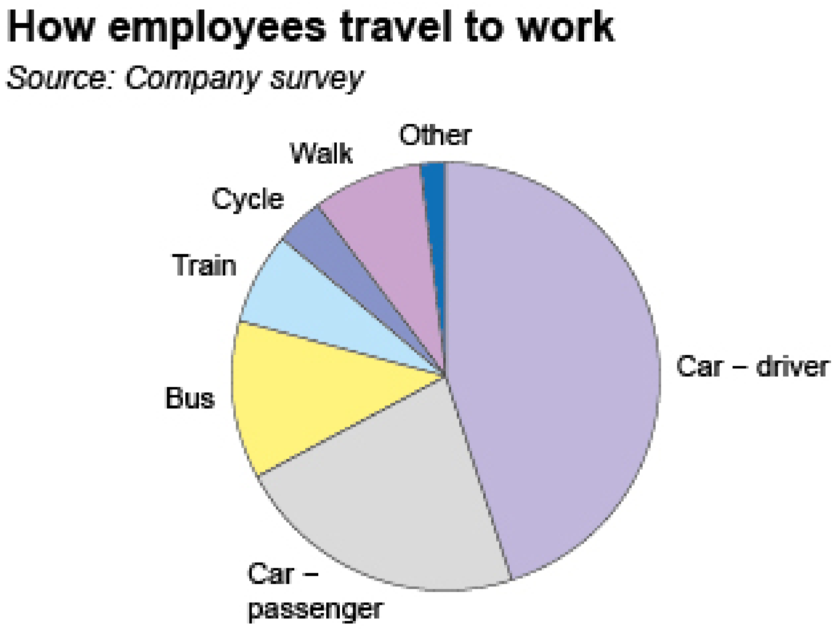

Figure 19 How employees travel to work

Use Figure 19 to answer the following questions:

(a) Do more people travel to work by bus or by train?

Answer

(a) The slice of the pie representing bus travel is larger than the slice representing train travel, so yes, more people travel by bus than by train.

(b) Would it be correct to say that the number of employees who travel to work as car drivers is about twice the number who travel as car passengers?

Answer

(b) The slice of the pie chart representing car drivers is about twice the size of the slice representing car passengers, so it is correct to say that the number of employees who travel to work as car drivers is about twice the number who travel as car passengers.

(c) Estimate the proportion of employees who travel to work by car, either as the driver or as a passenger.

Answer

(c) The two slices for car drivers and passengers cover about two-thirds of the circle, so about two-thirds of employees travel to work by car, either as the driver or as a passenger.

If necessary – and providing the pie chart has been drawn accurately – you can also work out the percentages by measuring the angle at the centre of the pie for each sector using a protractor. For example, the angle at the centre of the circle for the ‘Car passenger’ category in Activity 7 is about 80º.

Because a complete circle measures 360º, the fraction that this sector represents is $\frac{80}{360} = 0.22$, or 22%. So approximately 22% of the employees travel to work as car passengers.

However, as pie charts are often used to give an overall impression rather than detailed information, on many occasions a rough estimate will suffice.

This concludes our look at three different ways of displaying data visually. Before moving on to the last badged quiz for this course though, there are some words of caution for you on reading graphs.

5 Reading a graph with caution

The graph in Figure 20 below shows the monthly heating bills of a house before and after loft insulation was installed.

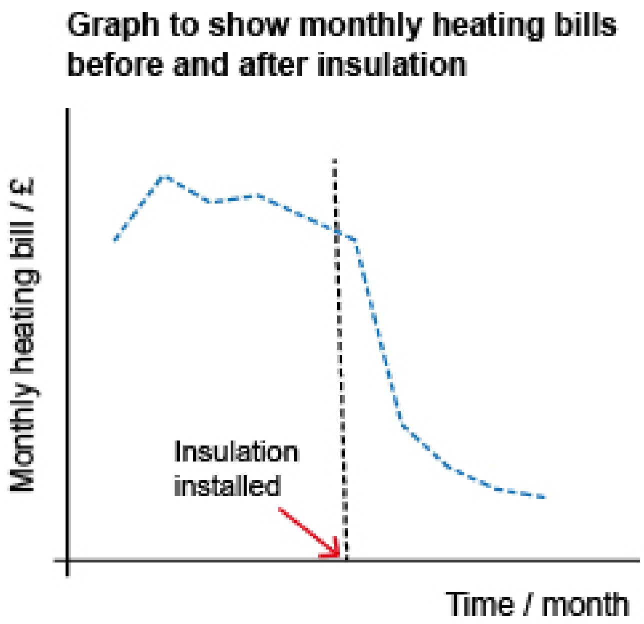

Figure 20 Graph to show monthly heating bills before and after insulation

You should have noticed that there are important pieces of information missing from this graph that you now know should always be included.

The first is the source of the data. Without this you have no idea how the data were collected or how reliable the information may be. Was just one house used in the survey, or were many houses used?

The next, and this makes the graph almost meaningless, is that neither axes have any scales marked.

Suppose the scale on the vertical axis was from £50 to £51; if it was, the apparent drop in the bill after insulation would be negligible. However, if the scale went from £0 to £50, the drop might be of more interest.

What about the horizontal axis? You don't know what time of year the data were collected or over what period. In fact, the data was collected from November to September, so the horizontal scale should have indicated this.

The graph appears to show a large drop in the heating bills after insulation was installed. However, without the scale on the vertical axis, it is impossible to say what kind of drop this is.

From the data you know that the drop in the monthly heating cost occurred in the May bill, just as the weather was warming up for the summer. So that means the reduced bills could simply be due to less heating being used in the summer.

So, overall, no conclusions can be drawn from this graph about the effectiveness of the insulation.

This illustrates an important point: when you are comparing two sets of data: you need to compare like with like. You would expect the bills for the summer to be less than those in the winter anyway, regardless of the presence or absence of loft insulation. It would be more appropriate to consider the amount of energy used for heating over two periods with similar weather.

So, although graphs and charts are very useful, it is important to read them critically, checking that all the information you need to interpret them is provided.

Another important point to note is that if the graph appears to show an association between two quantities, it does not prove that one has caused the other. For example, suppose the number of students in a town registering in a maths course rose from year to year and the number of burglaries in the town also rose – does this mean that the maths students have committed the burglaries? No, of course not! Both rises may be linked to some other factor, such as the number of people who have recently moved into the town.

6 This week's quiz

Well done, you've not only come to the end of this week's study but you have also completed the final week in *Succeed with maths – Part 2*!

Go to:

Week 8 compulsory badge quiz

Remember, this quiz counts towards your badge. If you're not successful the first time you can attempt the quiz again in 24 hours.

Open the quiz in a new tab or window (by holding ctrl [or cmd on a Mac] when you click the link).

7 Summary

After studying this week you should now be able to:

- read and construct line graphs to display information

- read and construct bar charts to display information
- read and construct pie charts to display information
- critically interpret graphs and charts.

So … congratulations on making it to the end of this week and *Succeed with maths – Part 2*! The subjects that you have studied, including measurement, patterns and formulas and working with data, have all laid the foundations of important areas of not only mathematical study but also many other areas. Maths is a fundamental tool for any scientist, economist, engineer, nurse or teacher, to name just a few. But, importantly, having a good grasp of these fundamentals will help you in your everyday life no matter what you do. So, as well as the personal achievement of completing the course, remember your new skills can help you every day as well as maybe opening new doors for you.

Every success with your onward journey, wherever this might lead.

If you've gained your badge you'll receive an email to notify you. You can view and manage your badges in My OpenLearn within 24 hours of completing all the criteria to gain a badge.

Now you've completed the course we would again appreciate a few minutes of your time to tell us a bit about your experience of studying it and what you plan to do next. We will use this information to provide better online experiences for all our learners and to share our findings with others. If you'd like to help, please fill in this optional survey.

You can now return to the course progress page.

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Course progress

Acknowledgements

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