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Exploring distance time graphs

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Introduction

Graphs are a common way of presenting information. However, like any other type of representation, graphs rely on shared understandings of symbols and styles to convey meaning. Also, graphs are normally drawn specifically with the intention of presenting information in a particularly favourable or unfavourable light, to convince you of an argument or to influence your decisions.

This OpenLearn course provides a sample of level 1 study in Mathematics.

Learning Outcomes

After studying this course, you should be able to:

- explain in English and by using examples, the conventions and language used in graph drawing to someone not studying the course
- use the following terms accurately, and be able to explain them to someone else: ‘time-series graph’, ‘conversion graph’, ‘directly proportional relationship’, ‘“straight-line” relationship’, ‘gradient’, ‘intercept’, ‘x-coordinate’, ‘y-coordinate’, ‘coordinate pair’, ‘variable’, ‘independent variable’, ‘dependent variable’, ‘average speed’, ‘velocity’, ‘distance-time graph’
- draw a graph on a sheet of graph paper, from a table of data, correctly plotting the points, labelling the graph and scaling and labelling the axes
- draw and use a graph to convert between a quantity measured in one system of units to the same quantity measured in a different system
- write down the formula of a straight-line graph, and be able to explain, using sketches, the meaning of the terms ‘gradient’ and ‘intercept’.

1 Introduction

1.1 A shared understanding

All representations (including graphical ones) rely on shared understandings of symbols and styles to convey meaning. Like maps, graphical representations stress some features and ignore others. As you work through this course, bear in mind that graphs are selective representations of information. When you come across different graphs ask yourself what is being stressed and what is being ignored.

In the newspapers, you are likely to find graphs used to present all sorts of information: how the number of people who are jobless or homeless is changing; how interest rates and share prices are varying; how support for different political parties has changed.

It is easy to be seduced by graphs. They have an air of authority which defies you to challenge them. After all, many graphs used by the media have been drawn from data collected by reputable survey organisations. The graphs apparently simply show trends, variations, peaks and troughs as they are. However, no graph is simply a neutral representation of facts. All graphs are drawn from *some* point of view, drawn this way rather than that for a particular purpose. That is to say, someone has made some choices about how the graph should look. When you are faced with just a single graph, however, the choices may not be evident. One reason for knowing about a variety of types of graph, therefore, is to be more aware of what *could* have been drawn, when faced with what *was* drawn.

So an important message is that no graph is a value-free representation. Even if the intention is to present information in as unbiased a way as possible, that is itself a point of view. But some graphs will have been drawn specifically with the intention of presenting information in a particularly favourable or unfavourable light, to convince you of an argument or to influence your decisions. So take care when you are reading graphs that you are not misled or, perhaps more to the point, so that you do not mislead yourself. And when you are drawing your own graphs, you need to be clear about how you present your information to others so that your intentions are not misinterpreted or misunderstood.

As the American writer James Thurber put it: ‘Get it right or let it alone. The conclusion you jump to may be your own.’ (James Thurber (1956) *Further Fables for Our Time*, New York)

1.1.1 About the course

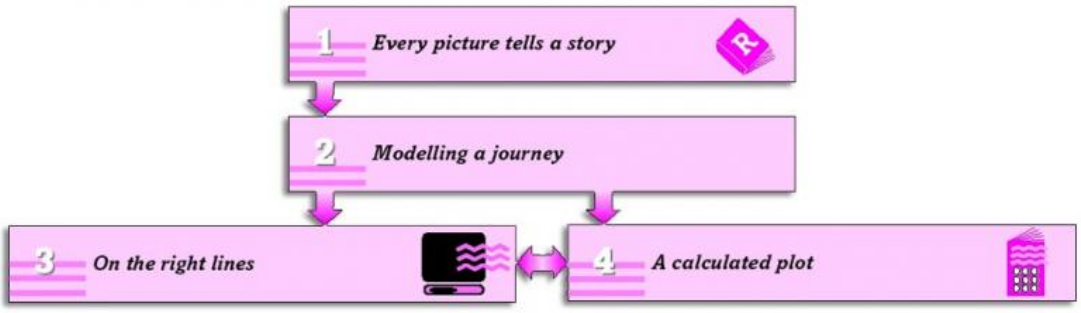
This course teaches on drawing and interpreting graphs. It has eight sections, each dealing between them with 4 different types of graph. You will need to draw some graphs, so make sure you have a flat surface to work on. You will also need to have centimetre graph paper and your calculator to hand as you study the material.

Section 3 is based on a video band, ‘Single-track minders’. This is split into four separate viewing sessions, each followed by an activity. You should expect to spend up to about thirty minutes on some of the activities. You may want to watch the video all the way through first, and then go back and watch the appropriate section again before trying the associated activity. These activities require you to draw graphs and make some

calculations, so make sure you have some graph paper and your calculator with you at this point.

Section 4 is the calculator book section. At the end of this section, you will be asked to complete a Learning File and Handbook activity. You will find it useful to have completed your response to earlier activities to tackle this activity successfully. The activity may also be useful for part of the assessment of this course.

Graph paper is required for Sections 1, 2 and 3.



Summary of sections and other course components

Activity 1: Organising your study

Before you start, take a few minutes to plan and organise your study of this course. If it is helpful, use the printed planning sheet.

Remember to consider the assessment and your overall study plans. What aspects of your study technique do you want to concentrate on as a result of your own review?

At this stage, you may spend little time on planning your study, or you may do it routinely. What is important is for you to adopt a system that works for you and that you find useful. Studying and learning at a distance is not easy and many students find that a few minutes spent thinking about how and when they work is time well spent. People plan in different ways, but it is generally the case that planning ahead helps you to be more effective in what you do.

Part of planning is also thinking about your own progress: self-monitoring. How are you getting on? Do you feel you need to spend more time on particular aspects of your studies? Are you going to complete the assignment questions for a course in the time you have allocated to study the course? Monitoring your own performance in particular areas helps to give you an insight into your progress generally. This is an important skill to acquire for independent learning. It helps you to focus on those areas you want to improve, and so enables you to communicate more effectively.

1.2 Every picture tells a story

The main aim of this section is to give you practice in reading, interpreting and drawing a variety of graphs created for many different purposes.You will need graph paper for this section.

Graphs occur in all sorts of different contexts and applications. Graphical representations can be used to show profiles of height plotted against distance for sections of the Peak

District walk, for example. This section looks at three sorts of graphs: time-series graphs, conversion graphs and mathematical graphs.

A time-series graph shows how a measured quantity changes with time. This is one of the most common graphical forms. Time-series graphs can be used to look for trends in the way things change over time in order to predict what might happen in the future. Or they may indicate changes in trends which invite investigation and explanation.

Conversion graphs, as the name suggests, are drawn to give an easy way of converting between a quantity measured in one system of units, and the same quantity measured in another. So you can use conversion graphs to change unfamiliar units into ones you know, such as converting perhaps from degrees Celsius to degrees Fahrenheit, changing millilitres into fluid ounces, or converting miles into kilometres.

One of the contexts in which you will meet graphs is within mathematics itself. Graphs are used not only to show how physical quantities change with respect to each other-how the height of a hillside changes with distance, for example, or how a person's temperature changes with time-but are also used to represent mathematical relationships. And as you get used to the idea, you can use graphs to explore the mathematical relationships themselves.

You will find, however, that the language and conventions of graphs remain pretty much the same whatever the graph is used to represent.

The final subsection looks more critically at graphs and encourages you to begin to move from looking at a particular graph to looking *through* the graph: looking beyond the actual lines to raise questions about the choices that have been made in drawing the graph, to ask why a particular graph looks the way it does.

Activity 2: Learning about graphs

You have just read a brief description of what this course is about. Imagine you have been asked to explain to a small group of people involved in market research work how to draw a graph, and how to extract information from a graph. They need to have some knowledge of graphs and graph drawing for their work, but some of them feel unsure about anything to do with graphs and numbers.

You have a wide variety of methods available to help you in your task. For example, you could use written instructions, activities, demonstrations, audio work and calculator exercises.

Plan which methods you might use to help this group and say why you would use those particular techniques.

Although you should make a start, you do not need to complete the activity at this point. You can come back to it again later. Towards the end of this course you will be asked to review your choices.

You may well have experiences of using graphs. You may be used to drawing them, or reading them. This activity asks you to focus on what you consider might be helpful for people who feel they have little understanding of graphs, and who feel they need to be more confident about using them in their work. It may be helpful to think about methods you have found useful, and those that have not been particularly useful in your own learning about graphs. As you work through the course, make some notes about your approach to working with graphs.

1.3 Time-series graphs

1.3.1 Introduction

> The time-series plot is the most frequently used form of graphic design. With one dimension marching along to the regular rhythm of seconds, minute, hours, days, weeks, months, years, centuries, or millennia, the natural ordering of the time scale gives this design a strength and efficiency of interpretation found in no other graphic arrangement.
>
> (Tufte, E. (1983) The Visual Display of Quantitative Information, Graphics Press, Connecticut, p. 28)

Graphs which show how a measured quantity varies as time changes are called time-series graphs. In a time-series graph, the horizontal axis represents the 'regular rhythm' of time in appropriate Units and the vertical axis represents the quantity that varies over time.

Time-series graphs are favourites of the media, particularly newspapers. They are useful tools of comparative analysis, and are frequently used in support of particular arguments. Figure 1 comes from an article in a national newspaper relating changing levels of crime and drug abuse; Figure 2 appeared in an article critical of the government and shows the changing support (in 1995) for the three largest political parties in the UK; Figure 3 was found on the financial pages of a newspaper – it shows how exchange rates and stock market prices varied over three months.

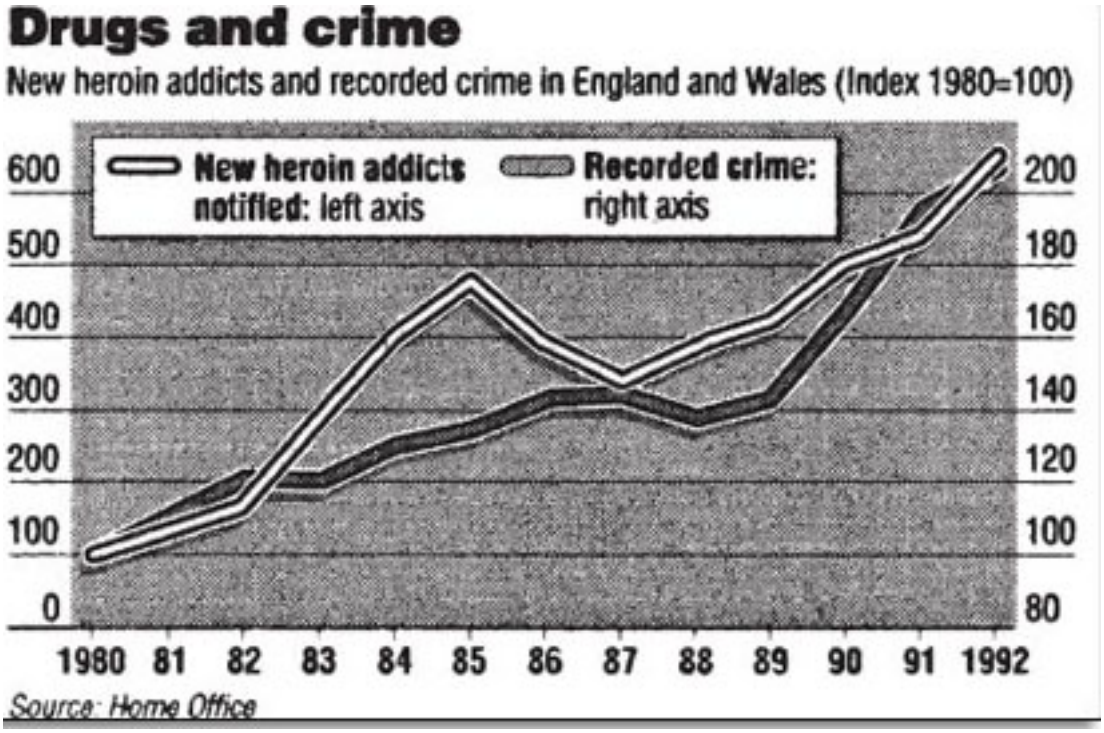


Figure 1 Crime and drug abuse

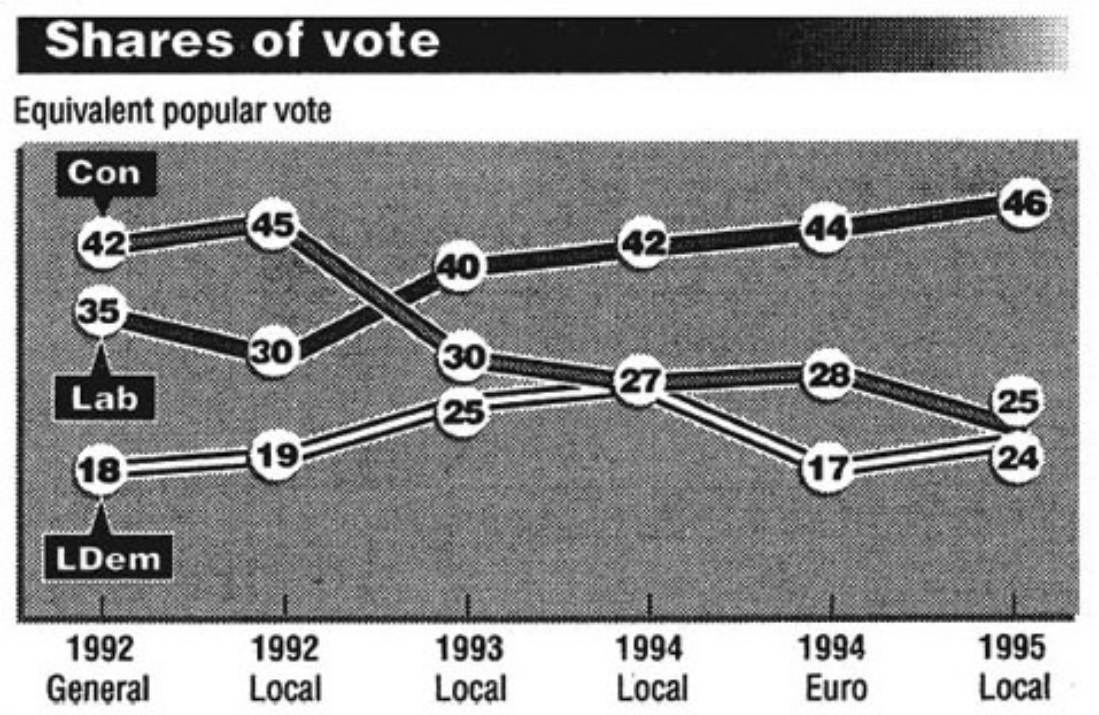


Figure 2 Support for political parties

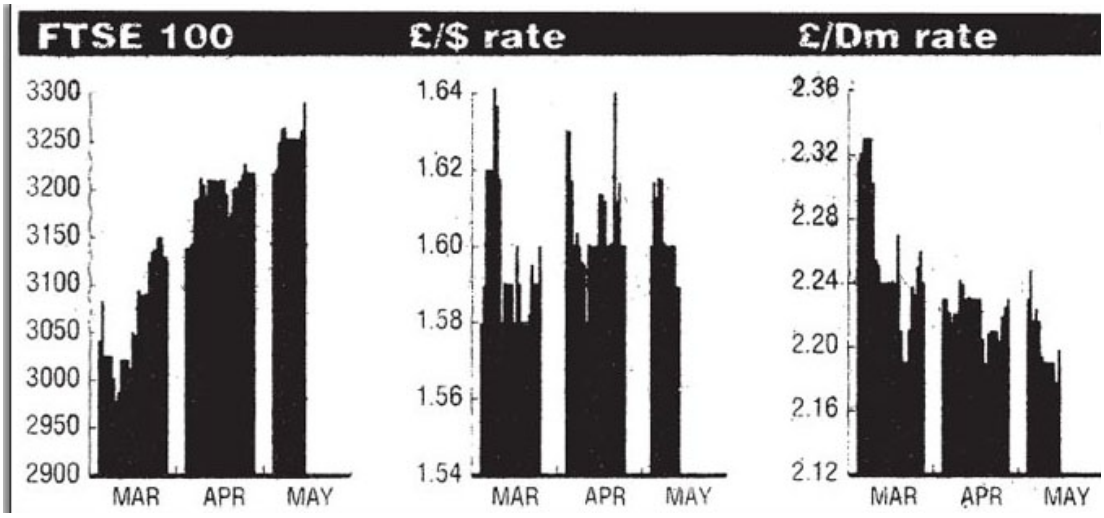


Figure 3 Financial indicators

Time-series graphs should not be looked at in isolation from the context in which they appear. If a graph appears in a newspaper article, for example, it usually has a very specific role to play in supporting the argument being put forward by the writer. Remember that someone has chosen to include that particular graph; someone has chosen how the graph should look; and someone has chosen what information should be included on the graph and what should be ignored. Like the text in which it is embedded, the time-series graph does not appear by accident. It is there to reinforce visually the story the author wants to tell.

One use of time-series graphs is to keep track of how a quantity is changing so that an assessment or prediction can be made. In health care, for example, graphs of how a person's temperature varies with time can give indications both of the general health of the individual and act as a guide to specific physiological events.

1.3.2 Time-series graphs: an example

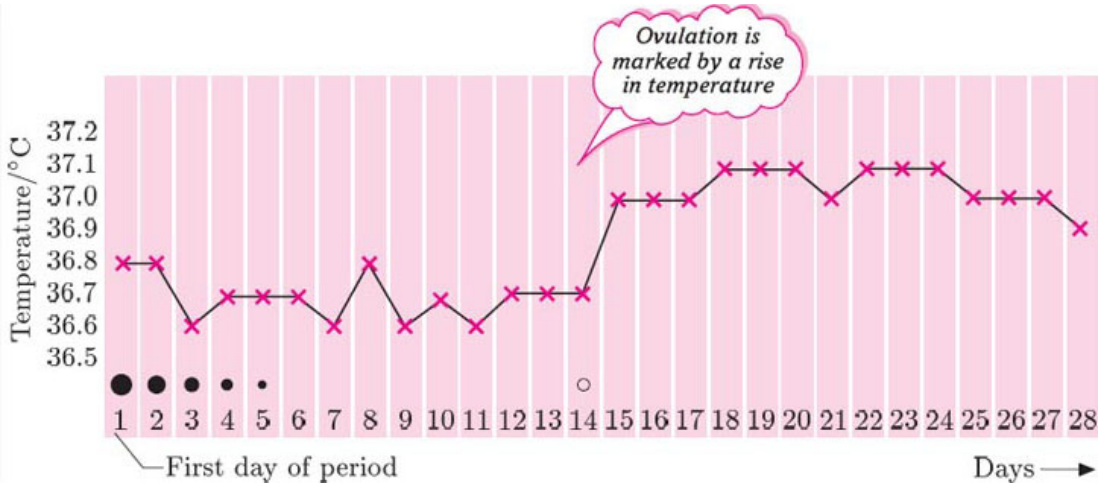


Figure 4 Temperature over the monthly menstrual cycle

Figure 4 shows a time-series graph of a woman's temperature over her menstrual cycle, published in a pregnancy guide. Each point on the graph represents the temperature taken first thing in the morning. The points are joined by straight lines to give an overall visual indication of the temperature variation over the month. These lines contain no extra *information* about temperature, however. Since temperature is measured only once each day, the lines do not represent the woman's actual temperature during the intervening 24 hours, but instead reflect an assumption that the temperature does not vary wildly between measurements.

The important function of this graph is to show the way a woman's temperature *changes* over her monthly cycle. You can see that the temperature does not stay at the 'normal' body temperature of 37°C, but varies from a minimum of about 36.6°C at the beginning of the cycle to a peak of about 37.1°C near the end, a range of about 0.5°C. Since the variation is small compared with the average temperature, the vertical axis of the graph does not start from zero but covers only the temperature range that normally occurs.

Plotting the graph at this scale shows up variations as small as 0.1°C. which would be lost if the scale covered a wider range. At this level of detail, the time-series graph shows quite clearly the rapid rise (relatively) in temperature that signals that ovulation has taken place. Since a causal link between ovulation and temperature rise has been established by medical research, a woman plotting her daily temperature as a time-series graph can use it to determine when ovulation has occurred. Thus, she can *read out* the temperature rise from the graph, and bring other knowledge to bear to *read in* the interpretation that an egg has been released.

This interpretation of the graph also relies on the accuracy with which the temperature readings have been taken and plotted. In this case, the thermometer must be a special 'fertility' thermometer which will give readings to an accuracy and resolution sufficient to respond to the relatively small changes taking place. If the measurements were accurate only to within say, 0.5 degrees, or if the graph scale itself did not allow you to plot points any more accurately than to within 0.5 degrees, then the fine detail of the temperature variations would be lost and a relatively reliable indication of ovulation would be less likely.

Time-series graphs give information only at the points that have been explicitly plotted. Even if a line has been drawn joining the plotted points, it can, at best, only represent an informed guess of what is going on between the points. *Trends* are expressions of

confidence that a set of data conforms to some recognisable pattern, rather than being just a set of random and unrelated numbers. A lot of mathematical effort has gone into developing techniques both for *interpolation* – estimating the values that lie between known points – and *extrapolation* – estimating values that lie beyond the range of plotted data. But whatever assumptions are made about long-and short-term trends, the measured data are all there is to go on. This point is brought out by Figure 5.

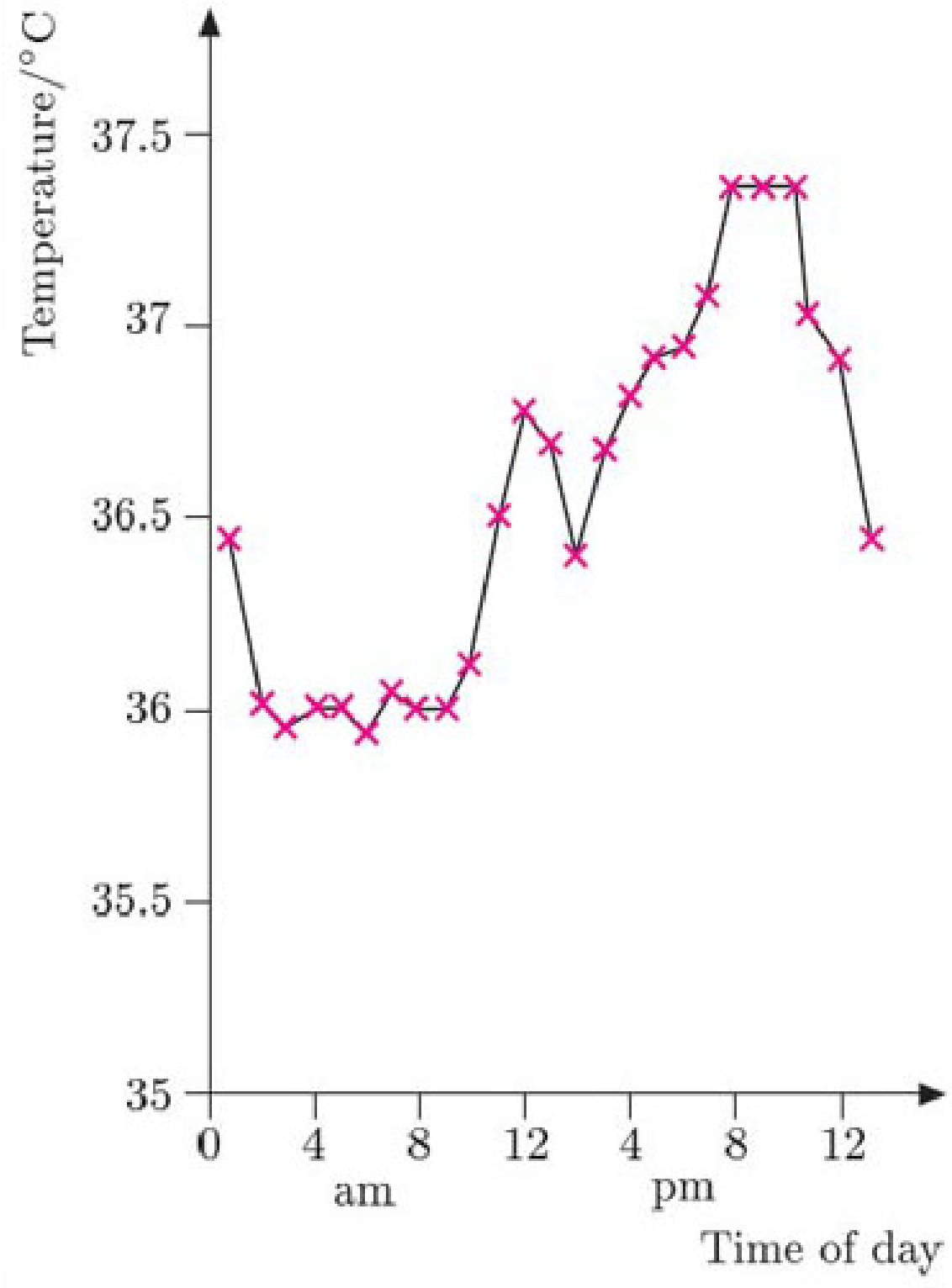


Figure 5 Temperature variation over a day

The daily, or *diurnal*, temperature variations are similar in men and women.

Temperature data can be plotted as a time series, but this time the graph characterises the daily rather than the monthly rhythm of a woman's temperature variation. After waking, a woman's temperature rises rapidly, followed by a dip in the afternoon, a gradual climb to a peak in the evening and then a sharp drop during sleep. The variation over a day is about 1.5°C, significantly greater than the monthly variation. In between the adjacent points on the monthly graph, therefore, there is a lot of hidden activity. But notice that the notion of 'rapid' and the visual perception of steepness is dramatically altered by the scale chosen for the vertical axis.

So there is a trade-off. The time-series graph based on a single daily measurement gives no indication of a daily cycle, particularly as it was deliberately taken at the same time each day. On the other hand, readings taken throughout the day will give a more detailed record, but the extent of the daily fluctuations may obscure the single small temperature rise that signals ovulation. Because of the variation of temperature over the course of a day, temperature measurements must be taken at the same time each day for the monthly series to be meaningful. Taking a morning reading one day and an afternoon reading the next could indicate a misleading temperature rise. Similarly, temperatures quoted on their own can be a misleading guide to a woman's general state of health; a temperature higher than 37°C may indicate a slight fever if it is recorded in the morning, but not if it is recorded in the evening or after ovulation. This is, of course, one of the difficulties – body temperature rises for a variety of reasons.

3.3 Time-series graphs: summing up

So time-series graphs must be read with care. Adopt a questioning attitude when you are faced with a graph. Look carefully at the vertical axis to see just what the range of variation is, and at the horizontal axis to see what time intervals have been chosen. Ask yourself about the significance of this choice – what might be going on between each plotted point?

You might question whether the plotted variation is significant or whether it is the result of expected fluctuations. What about the accuracy of the figures, and the accuracy with which they have been plotted? Look at the line of the graph itself, if there is one. Points will not always be joined by straight lines – ask yourself what assumptions have been made about the progression from one time point to the next.

Now try the next activity. You are asked to perform a mathematical task in another example of an application of graphs. Tasks such as this are common choices for psychological experiments.

Activity 3: Study patterns

In 1916, a study conducted on a class of psychology students at the University of California revealed a variation of study efficiency over the day. A group of 165 students were asked to give their preferred hours of study, and then undertook five repetitive memory tasks at one-hour intervals over three consecutive days. The tasks tested their ability to remember short sequences of numbers, to substitute numbers for symbols, to recognise geometrical figures, and to remember simple ideas.

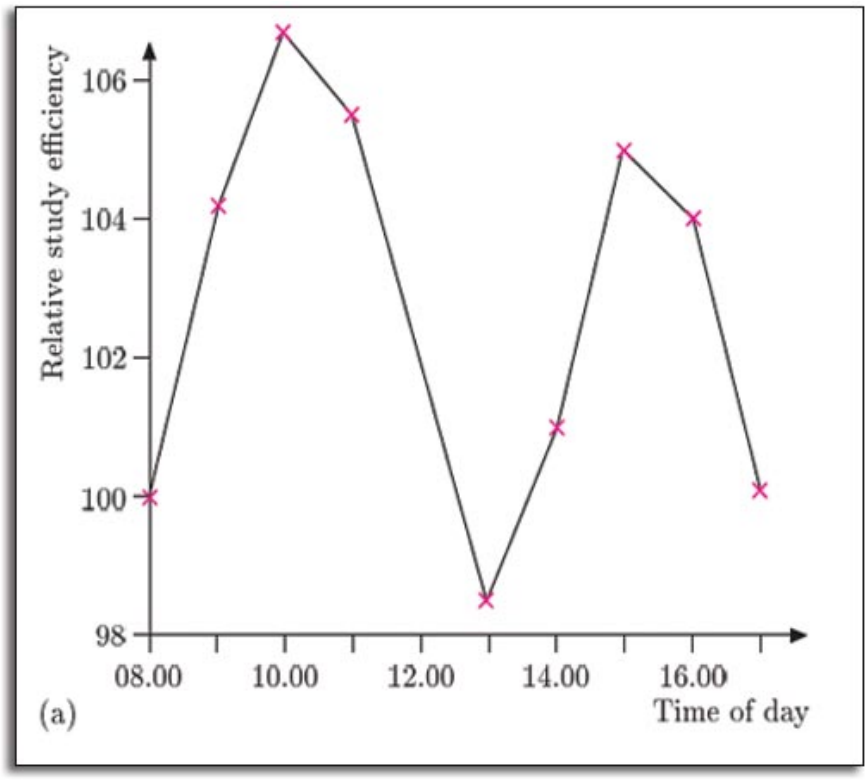
A composite measure of efficiency was calculated for the group each hour. The results are given in Table 1. Column 1 gives the time of day, column 2 gives the corresponding

relative efficiency measure, and column 3 lists the numbers of students preferring to study at that time of day.

Using your calculator, display the study efficiency data as a time-series graph. Which plotting window is appropriate to display this data?

Enter and display the study-time preference data as a time-series graph on your calculator.

What conclusions can you draw from these graphs about the study patterns of the students? Would your own study pattern fit into this picture? Note down your response.



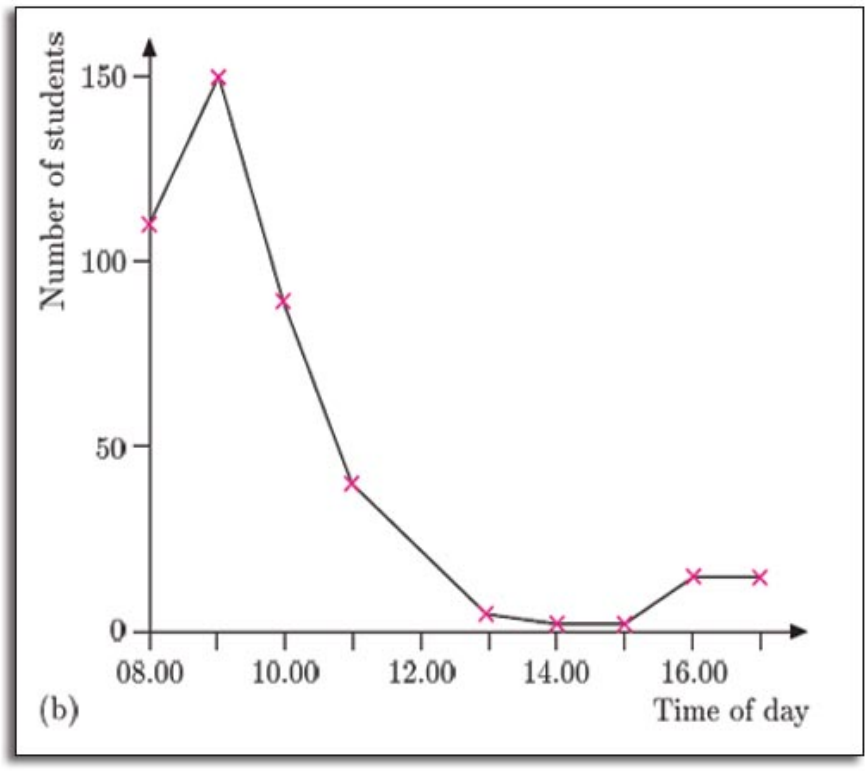


Figure 6a shows the study efficiency data and Figure 6b shows the preferred study time, with each plotted against the time of day (using the 24-hour clock).

Students in the study preferred to work between 8 am and 10 am. The time-series graph shows that efficiency dropped markedly in the early afternoon. This dip also appears in other studies and seems to be independent of whether or not the subjects had a midday meal. The points on the graphs are connected by straight lines (they also

could be connected by a smoothly curving line), indicating that no sudden changes are expected between the data points. From your own experience is this a reasonable assumption?

The graphs suggest that the preferred hours of work do not reflect the times of greatest ability (in terms of study efficiency), since the afternoon is unused. This may mean that students were unaware of the potential of the afternoons for studying, or simply that they preferred to do something else. What are your study patterns and preferences?

Table 1 Preferred hours and study performance for the student group

Time of day	Relative study efficiency	Study time preference
08.00	100	110
09.00	104.2	150
10.00	106.7	90
11.00	105.5	40
12.00	no data	no data
13.00	98.5	5
14.00	100.8	4
15.00	105	4
16.00	104	15
17.00	100.1	15

(Based on Palmer, J.D. (1976) *An Introduction to Biological Rhythms*, Academic Press, London, p. 142)

Here you have been reading about graphs; you were asked to consider ways of making interpretations from them and have used your calculator to display a graph. Look back to Activity 2 and note briefly which methods you feel are most useful in learning about graphs.

1.4 Graphical conversions

1.4.1 Introduction

The term ‘conversion graph’ describes a graph used to convert a quantity measured in one system of units to the same quantity measured in another. For example, you can draw up a conversion graph to convert temperatures expressed in degrees Celsius to temperatures expressed in degrees Fahrenheit; to convert liquid volumes expressed in pints to the same volumes expressed in litres; to convert a sum of money expressed in one currency to the same amount expressed in a different currency.

One way to convert one measurement into another is by using mathematics. In this case you need to use the map scale to convert between distances and areas on the map and on the ground. The relationship you need has the general form

measurement on the ground = some number x measurement on the map.

For your 1 : 25 000 map, the relationships between distances and areas are

distance on the ground = 25 000 x distance on the map

and

area on the ground = $(25\ 000)^2$ x area on the map

where, in each case, the map and ground measurements used the same units.

Now these relationships are expressed as formulas, but you can represent the same information graphically. Representing things in a different way can offer a new perspective and a new way of thinking about a relationship. Often it is helpful to move between different viewpoints and different representations when you are trying to understand a problem. Figure 7 shows the relationship between distances in the form of a graph.

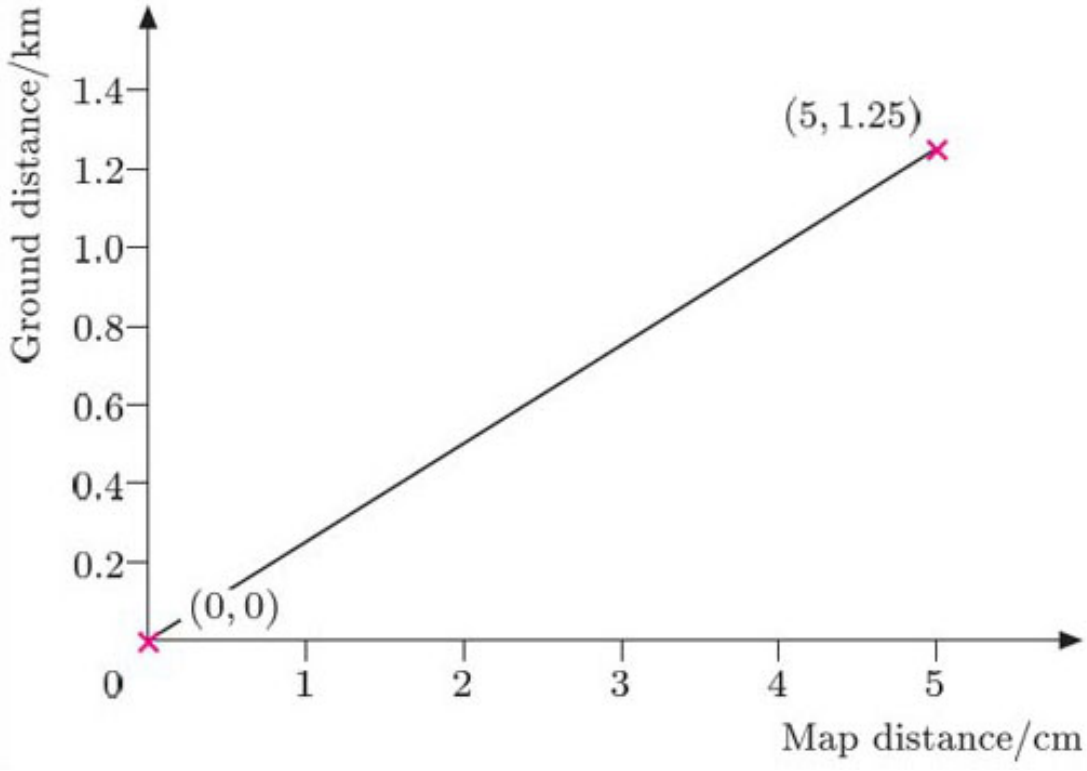


Figure 7 Graph of ground distance against map distance

Look at the way Figure 7 has been constructed. By convention, the map distance – which is what you know – is plotted along the horizontal axis, and the ground distance – which is what you want to find out – is plotted along the vertical axis. The axis of a graph is rather like a map. The axis represents a quantity (such as temperature, distance, time). The scale of the axis, like the scale of a map, relates the distance along the axis from the origin to the amount of the quantity. Notice the scales and the labels on the axes. Map distances are conveniently measured in centimetres, but ground distances are more conveniently quoted in kilometres. So in this case the scale on the horizontal axis relates to a measurement in centimetres and the scale on the vertical axis relates to a measurement in kilometres.

The labels on each axis tell you about the thing that is measured, and the units it is measured in. So the horizontal axis is labelled ‘Map distance/cm’, and the vertical axis is labelled ‘Ground distance/km’. Notice that there is an oblique line between the quantity and its units: ground distance divided by kilometres, and map distance divided by

centimetres. This means that the numbers along the horizontal and vertical axis are ratios – pure numbers – which can be added, subtracted, multiplied or divided without worrying about their units.

Remember that quantities with units are the result of measurements taken in the real world. When you draw a graph you are not dealing with actual kilometres or centimetres but with their representations as lengths along the axes of the graph. You can perform calculations with the numbers to make predictions about the material world. But when you go back to that world, remember to restore the units, so that the numbers relate once again to actual measurable distances.

Returning to the formula for the conversion graph, recall that the starting point is a scale relationship which links map distance to ground distance, when both are measured in the same units. If the units are centimetres, you have

ground distance in centimetres = 25 000 x map distance in centimetres

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But how to convert from a map distance measured in centimetres to a ground distance measured in kilometres? This changes the formula slightly.

There are 100,000 centimetres in a kilometre, so to convert from centimetres to kilometres divide by 100,000. The formula becomes:

$$\begin{aligned}\text{ground distance in kilometres} &= \frac{\text{ground distance in centimetres}}{100\,000} \\ &= \frac{25\,000 \times \text{map distance in centimetres}}{100\,000} \\ &= 0.25 \times \text{map distance in centimetres}\end{aligned}$$

Whatever the map distance is in centimetres, the ground distance in kilometres can be found by multiplying by 0.25. Mathematically, the ground distance is said to be *directly proportional* to the map distance. This means that ground distance is found simply by multiplying the corresponding map distance by some fixed number, known as the *constant of proportionality*. Another way of expressing this is the fact that the ratio of ground distance to map distance is constant (and equal to this constant of proportionality).

ground distance (in km) / map distance (in cm) = 0.25

1.4.2 Graphical conversions: drawing a straight-line graph

This means, for instance, that if you double one value, the effect is to double the other, and if you third one value, the upshot is that the other is divided by three as well. And the fact that the relationship is directly proportional has an important consequence for the graph-it will necessarily be a straight line.

Only two points are needed to draw a straight-line graph. Choosing one of the points is straightforward, it is the origin of the graph. Why? Because zero distance on the map corresponds to zero horizontal distance on the ground. So one point of the graph here must have the coordinates (0, 0). But this will not always be where the two axes meet. Remember from Figure 4, that not all graphs are drawn with the vertical axis scale starting at zero. This is the same for any distance or length conversion (and others such as area, or weight, or volume).

The other point can be chosen so that it fits conveniently into the range you want the graph to cover.Figure 6 shows map distances up to 5 cm, corresponding to ground distances up to 0.25 × 5=1.25 km. So the second point can be placed at the top end of the

scale at (5,1.25). In fact, the further the second point can be placed from the origin the better, because inaccuracies in drawing the graph are reduced.

Look at Figure 8a. Here the second point is very close to the origin. Any inaccuracy in plotting this point or in drawing the line through the point will be magnified significantly at the end of the range. As a result, the graph will become less accurate the further you move from the origin. If the second point is put as far away as possible, as in Figure 8b, any drawing inaccuracy will result in lower errors over the range of the graph.

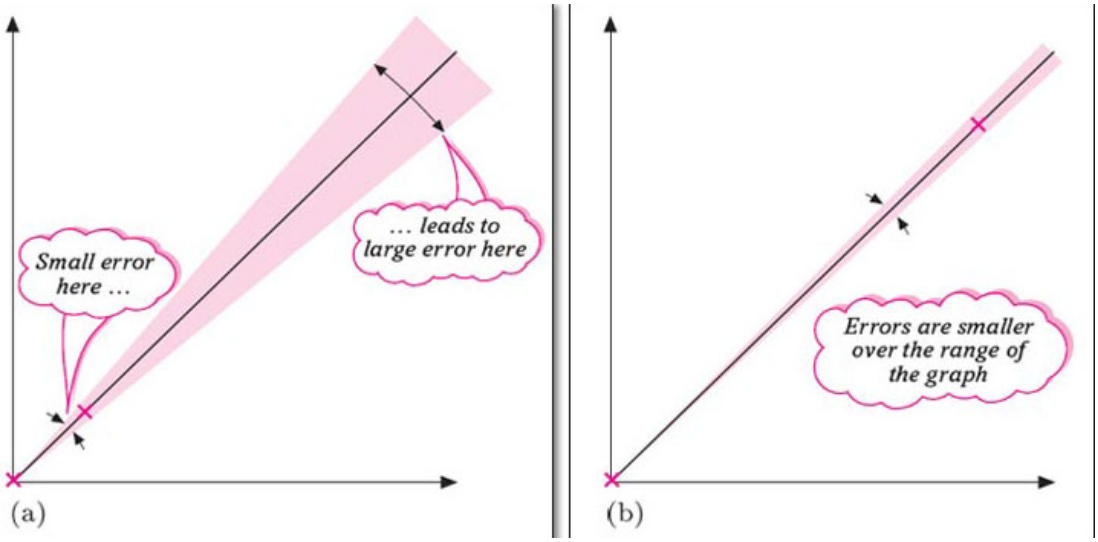

Figure 8 Effect on accuracy of the position of the second point of a straight-line graph

A straight line links the points (0,0) and (5,1.25) on Figure 7 and represents the proportional relationship between map and ground distances.

4.3 Graphical conversions: How do you use the graph?

Look at Figure 9. Start with the map distance on the horizontal scale, move vertically up until you reach the line, then move horizontally until you reach the vertical axis. The number at that point will give you the corresponding ground distance in kilometres.

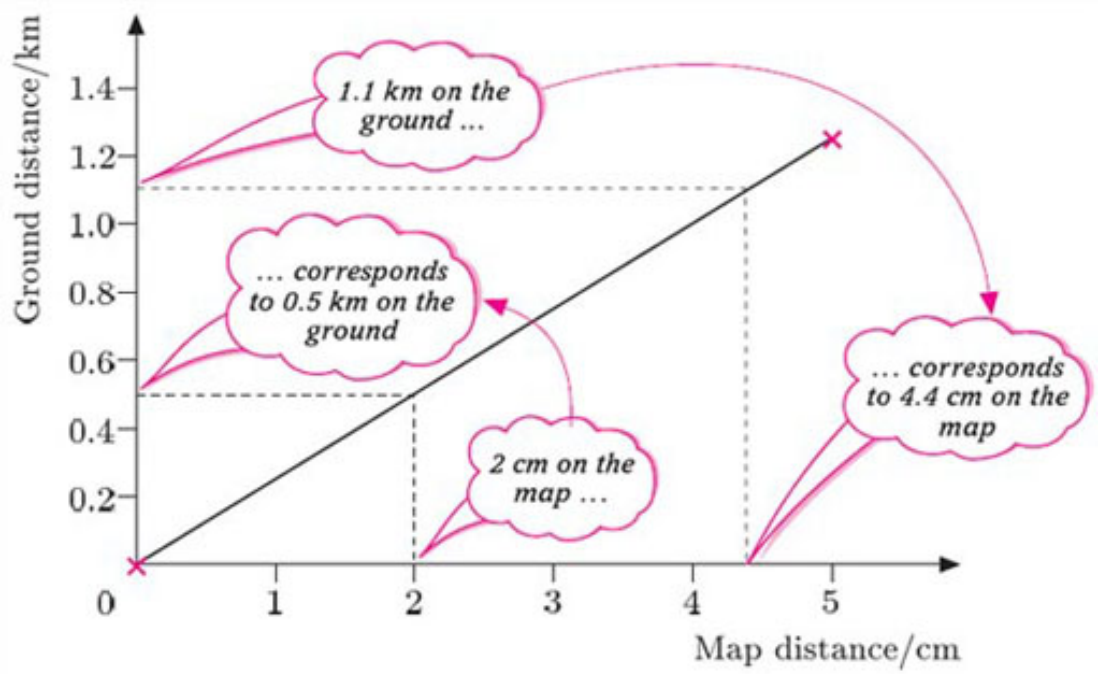

Figure 9 Converting between map distances and ground distances

You can also use the graph to go from ground distance to map distance. Find the particular ground distance on the vertical scale, move horizontally across to the line and then move vertically down until you reach the horizontal scale. You can then read off the corresponding map distance in centimetres. If you could be bothered to do it – and had unlimited time (like forever!) – you could have drawn the same graph by working out every possible ground distance for every possible map distance between 0 and 5 cm, and plotting each coordinate pair. But the results of all those countless, individual, specific calculations are automatically included in just one straight line drawn using the knowledge that the relationship is a directly proportional one. A mathematical formula generalises a relationship by containing markers – words or symbols – which you replace by numbers for specific calculations. Likewise, a graph generalises the results of individual calculations, indicating by means of its shape the corresponding values in a relationship. A graph, therefore, represents the general form of a directly proportional relationship as well as allowing you to handle specific examples.

Activity 4: A litre of water …

When the metric system of weights and measures was introduced in the UK, the government information campaign included the rhyme: ‘A litre of water’s a pint and three-quarters’, to help people remember the conversion factor. The conversion constant is not specific to water (it merely helped the rhyme) and can be used for any fluid.

1 With the campaign slogan in mind, draw a graph on graph paper to convert between pints and litres. Use your graph to find (a) how many litres correspond to 3 pints, and (b) how many pints correspond to 4 litres.

2 Using the fact that there are 8 pints to 1 gallon, find the conversion factor between gallons and litres, and draw up a table of gallon/litre equivalents. Do you think it would be more helpful if you used a conversion graph rather than a table? Write down any advantages or disadvantages you can think of.

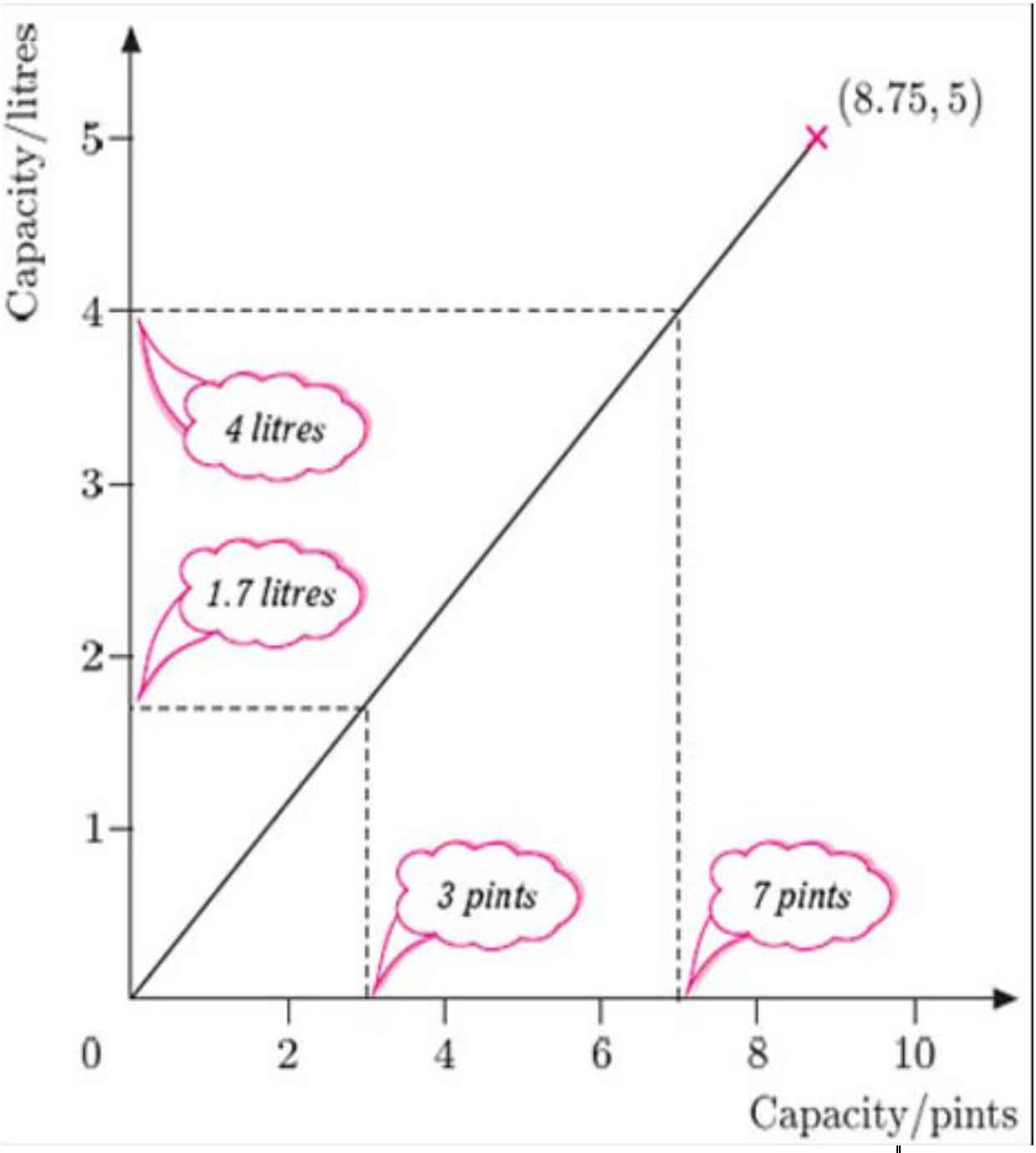

Figure 10 Graph to convert between pints and litres

Figure 10 shows a graph to convert between pints and litres. 1 litre is about 1.75 pints (actually, a litre is closer to 1.76 pints, but stick with the rhyme!), so to be able to convert 4 litres, the scale along the horizontal axis of the graph needs to extend to at least 4×1.75=7. The graph has been drawn slightly bigger than the minimum size with the horizontal scale representing 0 to 10 pints and the vertical scale representing 0 to 5 litres. The graph was constructed by drawing a straight line from the origin (0,0) to the point (8.75, 5), representing the equivalence between 5 litres and 8.75 pints. You may have chosen different point and/or scale. Make sure that your graph has a title, and that the axes are scaled and labelled clearly and correctly.

From the graph, 3 pints is equivalent to just over 1.7 litres, and 4 litres is equivalent to 7 pints.

8 pints is equivalent to about 4.6 litres. So the conversion relationship is 1 gallon=4.6 litres.

Here are some ideas about the pros and cons of tables and graphs. Tables are easy to use and hold a lot of information in a compact way. But they do not give any visual impression of the general mathematical form of the relationship in the way that graphs do. In principle, graphs drawn from formulas can be used to convert between any two equivalent values, provided that the values lie within the range of the graph. Tables give only selected pairs of values, intermediate values have to be estimated. You may have thought of some other points.

Recall that two quantities being directly proportional to each other – such as map and ground distances, or pints and litres – means the amount of one is found simply by multiplying the other by a fixed conversion number. In Activity 4, for example, the formula describing the relationship (according to the rhyme) is:

volume in pints = 1.75 x volume in litres.

The number 1.75 is the constant of proportionality. It relates a volume measured in pints to the same volume measured in litres, and strictly it has units itself. In this case, the units are ‘pints per litre’.

1.4.4 Graphical conversions: How is the constant of proportionality represented on a graph?

One of the main features of a straight-line graph is that the line has a constant slope. The gradient of the slope is numerically equal to the constant of proportionality. For a 1 : 25 000 map, the constant of proportionality between ground distances in kilometres and map distances in centimetres is 0.25 km per cm. So the gradient of the corresponding graph is 0.25.

A similar relationship holds for a 1 : 50 000 map. In this case, 1 cm on the map corresponds to 0.5 km on the ground, so the constant of proportionality is 0.5 km per cm and the gradient of the corresponding graph is 0.5. The steeper gradient says, in effect, that you get more kilometres for your centimetres on this map.

In general, therefore, if the line passes through the origin of a straight-line graph, then the gradient of the graph links the values on the horizontal and vertical axes. The relationship is:

value on vertical axis = gradient x value on horizontal axis

Different constants of proportionality give straight-line graphs with different gradients. The steeper the gradient, the greater the value on the vertical axis for a given value on the horizontal axis. Changing the scale on the vertical axis has an effect on the visual perception of steepness. Choice of scale can have a profound effect on the visual impact of a graph. But *numerically*, the gradient of the graph is unchanged by a simple change of axis scale.

Activity 5: Converting to metric

If you were to draw graphs to convert from (a) pounds to kilograms (1 pound = 454 grams), or (b) miles to kilometres (1 km = 0.621 miles), what would be the gradient in each case?

Make a quick sketch of the conversion graph in each case.

Did you find making the sketches helped you to answer the question? Or did you find they made the task more difficult? Make a few brief notes to record your response.

One pound is equivalent to 0.454 kg. So the conversion graph will go through the points (0,0) and (1,0.454), as in Figure 11a. The gradient is

$$\frac{\text{change along vertical axis}}{\text{change along horizontal axis}} = \frac{0.454 - 0}{1 - 0} = 0.454$$

A distance of 1 km is equivalent to 0.621 miles. So the conversion graph will pass through the points (0,0) and (0.621,1), as in Figure 11b. The gradient is

$$\frac{1 - 0}{0.621 - 0} = \frac{1}{0.621} = 1.61$$