Secondary Mathematics

Reading, writing and modelling mathematics: word problems

Teacher Education through School-based Support in India
www.TESS-India.edu.in

The Open University

UKaid from the British people

TESS-India (Teacher Education through School-based Support) aims to improve the classroom practices of elementary and secondary teachers in India through the provision of Open Educational Resources (OERs) to support teachers in developing student-centred, participatory approaches. The TESS-India OERs provide teachers with a companion to the school textbook. They offer activities for teachers to try out in their classrooms with their students, together with case studies showing how other teachers have taught the topic and linked resources to support teachers in developing their lesson plans and subject knowledge.

TESS-India OERs have been collaboratively written by Indian and international authors to address Indian curriculum and contexts and are available for online and print use (http://www.tess-india.edu.in/). The OERs are available in several versions, appropriate for each participating Indian state and users are invited to adapt and localise the OERs further to meet local needs and contexts.

TESS-India is led by The Open University UK and funded by UK aid from the UK government.

Video resources

Some of the activities in this unit are accompanied by the following icon:  *. This indicates that you will find it helpful to view the TESS-India video resources for the specified pedagogic theme.*

The TESS-India video resources illustrate key pedagogic techniques in a range of classroom contexts in India. We hope they will inspire you to experiment with similar practices. They are intended to complement and enhance your experience of working through the text-based units, but are not integral to them should you be unable to access them.

TESS-India video resources may be viewed online or downloaded from the TESS-India website, http://www.tess-india.edu.in/). Alternatively, you may have access to these videos on a CD or memory card.

Version 2.0 SM13v1
All India - English

What this unit is about

Students and even adults often find mathematics very difficult because they are not able to relate mathematical facts to a real-life context and vice versa. Word problems are often seen as a way to bridge the divide between real life and the mathematics classroom. However, students across the world often perform poorly in tests involving word problems. Even when students have mastered the technical competencies of doing the mathematical operations involved in the word problems, they still can find it difficult to work out solutions to word problems require them to apply those techniques (Morales et al., 1985). Much research has focused on what the issues are with word problems in mathematics.

This unit looks at learning to work with word problems by giving you ideas to help your students to both read and write mathematical word problems. It uses the idea of mathematical modelling to help your students understand that word problems are models, and do not necessarily represent everyday life.

Many of the word problems used in this unit have been adapted from word problems in the NCERT textbooks for Class IX and X.

What you can learn in this unit

- Some ideas for teaching your students to read, write and solve word problems.
- Some suggestions to show your students how to use word problems as mathematical modelling tools.
- Ideas on how to help your students think about their learning processes and become more reflective learners.

The learning in this unit links to the NCFTE (2005, 2009) teaching requirements specified in Resource 1.

1 Issues with word problems

Students can find understanding how to work with word problems hard to do (Morales et al., 1985). They can become distracted by the narrative, which might claim to be based on real life, but is actually unrealistic. For example, consider the following word problem, of which there are many adaptations:

> Eighty-two people are queuing for the lift on the ground floor of an office building. The lift can take a maximum of nine people at a time. How many trips will it take the lift to clear the queue?

The word problem seems to be based on a real-life situation – people in an office building queuing for the lift. It does not represent reality, however: with such a long queue, there might be people who will choose to take the stairs instead of waiting for the lift, or people who will decide to wait until later to travel to the upper floors of the building.

However, in a mathematics word problem, such aspects of real life are usually not considered relevant. Hence sometimes students need to be helped to consider what is part of the context that is relevant to the mathematics and what is not – in other words, to pay attention to what is important and to disregard the rest. Students need a set of tools to:

- make sense of word problems
- notice the mathematics elements
- be aware that word problems are a case of modelling mathematical ideas.

Word problems are usually an example of mathematical modelling. It may be important to remind the students that this is how a lot of mathematics is used in careers beyond school: to model what happens (or may happen) in the world so that complex situations and often awkward numbers can be manipulated more simply and solutions to problems found. Students can become aware of this by 'decoding' word problems, and also by making up word problems themselves.

Pause for thought

Thinking about your classroom, how are word problems perceived by your students? Do they like them? Do they struggle with them? Why do you think this is?

Thinking back about your experiences as a mathematics learner, how did you perceive word problems? What helped you to understand how to approach them?

2 Reading and decoding word problems

Figure 1 Problem solving.

'Reading' word problems involves 'decoding' the information in order to understand what the mathematical model is and therefore what ideas are needed to provide a solution. Some word problems contain irrelevant information and others do not. It is important to train your students' minds to spot the relevance (or degree of relevance) of mathematical information in a word problem.

The very first step in this process involves the student scanning the problem to identify key words. The student then has to decide which information is relevant or irrelevant. The student then moves on to reorganise the word problem into smaller parts to start solving the problem. Some of your students may not always understand all the words in a word problem, particularly if it is not written in their first language. You will need to give them confidence to try the maths problem, even if they are not always sure of all the Hindi or English words.

In Activity 1 your focus is on training the mind of your students to spot relevant information and then to work at 'chunking' the word problems into smaller parts. This activity works particularly well when students work in pairs or small groups.

Before attempting to use the activities in this unit with your students, it would be a good idea to complete all (or at least part) of the activities yourself. It would be even better if you could try them out with a colleague, as that will help you when you reflect on the experience. Trying the activities yourself will mean that you get insights into learners' experiences that can in turn influence your teaching and your

experiences as a teacher. When you are ready, use the activities with your students. After the lesson, think about the way that the activity went and the learning that happened. This will help you to develop a more learner-focused teaching environment.

Activity 1: Key words and chunking problems

Tell your students to work with one or two other students. Make sure that every student has someone to work with. Then tell your students to read each word problem below and make a list of the key words or phrases that can help them solve the problem. They should then set out the problems in simpler chunks of information.

1. A square garden has a walking track that is eight feet wide along its sides. If one side of the garden is ten metres long, find the distance travelled by Hamid if he walks around the garden twice.
2. Kavita was given some money on her 16th birthday by her uncle. She used the money to buy two pairs of jeans for Rs. 950 each from a store that was offering a 20 per cent discount. After the purchase, she still had Rs. 150 left. How much money did her uncle give her?

Figure 2 A row of jeans.

3. Rita buys 3 kg of mangos and 12 bananas for Rs. 280 on Tuesday. Three days later, from the same shop, Rahul buys 2 kg of mangos and 18 bananas for Rs. 300. Write the equations that can help find the cost of 1 kg of mangos and a dozen bananas.

When they have completed this, tell your students to answer these questions:

- For each key word or phrase you think is useful, provide your reason why.
- What helped you decide which numerical data was useful and which was not? Did you miss any useful information?
- Was there some information that you found difficult to classify as 'useful' or 'not useful'? How did you go about accepting or rejecting this information?
- What did you find easy or difficult about chunking a word problem into smaller pieces of information?
- Did you visualise any of the above problems before identifying key words or chunking? How was visualising helpful?

Before the end of the lesson, ask the students to bring together their answers and try to jointly construct good general ideas on how to solve these types of problems.

Case Study 1: Mrs Chakrakodi reflects on using Activity 1

This is the account of a teacher who tried Activity 1 with her secondary students.

My class has found using word problems very difficult. The students like to answer mathematics questions quickly and get lots done, and word problems always slow them down. Since being able to answer word problems is so important in examinations, I decided to spend some time helping my students learn to solve them. First I asked them what skills they needed when solving word problems. They could not answer at first, so I asked them to discuss the question in pairs. Their answers were that you have to:

- read the problem carefully
- recognise what is the context and what is important for solving the mathematics
- note down important words, numbers and information, perhaps on a diagram
- ignore irrelevant words and numbers
- think what mathematics must be used to solve it, then use it
- check and decide whether the answer makes sense
- give the answer in a way that relates it to the problem asked – for example, using appropriate units of measurement.

I then asked the class to look at the problems in Activity 1. They wanted just to answer the problems, but I wanted them to think about the process and use the problem solving skills we had come up with. So we looked together at the second question of Kavita going shopping for jeans.

First I asked them to chunk the information. We came up with the following chunks of important information:

- Kavita was given some money.
- She bought two pairs of jeans.
- Each pair of jeans cost Rs. 950.
- She still had Rs. 150.
- How much money did her uncle give her?

Then we looked at the mathematics involved at each step. I asked the class to come up with the meaning of each word or phrase, and the mathematical meaning it is trying to convey. I had to help them with the first one, but after that they got the idea:

- 'Kavita was given some money.' This is the 'unknown' – say, x.
- 'She buys two pairs of jeans.' Whatever she spent has to be subtracted from x.
- 'Each pair of jeans cost Rs. 950.' So she spent 2 × 950 = Rs. 1900.
- 'She still had Rs. 150.' This should be x – 1900.
- 'How much money did her uncle give her?' This is x.

Once we had done this, the students tackled the other two word problems in the same way, chunking them and trying to write down the mathematical expressions for what they were doing. They realised fairly soon that to answer the first question they needed to know where Hassan was walking on the path, and that the measurements were in feet and metres, so some conversion would be needed. The third question also resulted in a debate over whether it mattered that the two visits to the shop were three days apart, so we talked about irrelevant information in word problems.

For homework I asked the students to have some fun devising word problems of their own that contained some good mathematics and some irrelevant information that was not obvious.

Video: Using pair work

http://tinyurl.com/video-usingpairwork

Read Resource 2, 'Using pair work', for more ideas.

Reflecting on your teaching practice

When you do such an activity with your class, reflect afterwards on what went well and what went less well. Consider the questions that led to the students being interested and being able to get on, and those where you needed to clarify. Such reflection always helps with finding a 'script', which helps you engage the students to find mathematics interesting and enjoyable. If they do not understand and cannot do something, they are less likely to become involved. Use this reflective exercise every time you undertake the activities, noting as Mrs Chakrakodi did some quite small things that made a difference.

Pause for thought

Good questions to trigger such reflection are:

- How did it go with your class?
- What responses from students were unexpected? Why?
- What questions did you use to probe your students' understanding?
- Did you modify the task in any way? If so, what was your reasoning for this?

3 From words to algebra to words

One of the difficult ideas in word problems is about translating the words into algebra and algebra into words. There are two parts to the next activity. The first part gives your students the opportunity to practise matching words and algebraic expressions and vice versa in an enjoyable way. The second part asks the students to make their own word problems from some algebraic equations.

Activity 2: Words and algebra

Part A: Flash cards

Preparation

Make flash cards in two different colours (or with different shading, as in Figure 3). Leave one side of the flash card plain so that the students can write on that side. On the green cards, write an arithmetic statement in English (or the language of instruction followed in your school). On the orange cards, write the corresponding statement using mathematical symbols and operations. Make the cards relevant to the kinds of work that your students are engaged with. This could be trigonometry, circles or any other aspect of mathematics – see the examples in Figure 3.

5 more than x	5 less than x	x less than 5	5 times x
$x + 5$	$x - 5$	$5 - x$	$5x$
Quotient of 5 and x	Quotient of x and 5	5 by x	x by 5
$5/x$	$x/5$	$5/x$	$x/5$
Sum of 5 and x	Difference between 5 and x	Product of 5 and x	5 raised to x
$5 + x$	$\|5 - x\|$	$5x$	5^x
x raised to 5	5 squared	5 more than 5 times x	Ratio of 5 and 5 more than x
x^5	5^2	$5x + 5$	$5/(x + 5)$

Figure 3 Words and algebra flashcards.

If you have 30 students in your class, you will need 15 pairs of cards.

The activity

Randomly distribute the cards to the students. Tell your students to find the student who has the card that completes your pair.

Part B: Writing own word problems

Preparation

Modify the following equations to make them relevant to your class and write them on the blackboard:

- $y = 3x$
- $x + y = 150$
- $3x - y = 22$
- $2x + 3y = 88$
- $A = 16p$
- $32 = x(y + 2)$

The activity

Tell your students the following:

- For each equation, write as many context-based word problems as you can. For example, for the equation $y = 3x$ you could write 'Kavita's feet are three times as long as her baby brother's'.

- Which equations did your students find writing word problems for the most difficult? Why do you think this was?
- For each equation, which one of your word problems was most realistic? Why? Can you try to make the other word problems more realistic?

At the end of the activity, ask your students to select the most interesting word problems for each equation and display them on the classroom wall.

Video: Storytelling, songs, role play and drama

http://tinyurl.com/video-ssrpd

You may also want to have a look at the key resource 'Storytelling, songs, role play and drama' (http://tinyurl.com/kr-ssrpd).

Case Study 2: Mrs Chakrakodi reflects on using Activity 2

My students had enjoyed using Activity 1 and their confidence was growing, but they still had difficulties writing down the algebra for ideas such as 'She still had Rs. 150' in the problem that we did together. Therefore, I decided to use the cards from Activity 2 and made up some more.

There were 64 students in my class, so I formed 32 pairs so that everyone had a partner. I took the students outside to the playground so they would have space to find each other. There was a lot of noise while they moved about trying to find their pair, but it was over quickly. Once they had all paired up, I got them to sit down and work together on two context-based word problems equivalent to the algebra on their cards, which they wrote on the back of their cards. Each pair then joined another pair and gave one another their context-based word problems to write the mathematical phrase in their mother tongue and then to write the algebra. When the students disagreed about anything, we discussed the ideas as a class and then I asked for some 'really good' word problems to share with the class.

Everyone seemed to get a lot out of using this idea. They had people to ask if they were not sure and they all heard and worked with lots of examples connecting algebra with words and with contexts.

For Part B, I asked the students to continue to work in the groups of four that they had formed earlier and to write at least four problems for each equation. When each group had come up with something for at least four equations, I stopped their working. At this point some of the groups had finished all the problems and I realised perhaps I should have had some more problems for the more mathematically confident students.

Then I used the follow-up questions in a class discussion. Asking 'Which was most difficult and why?' meant that the students had to think about their thinking – that is called 'metacognition', I think. Asking them to take this overview also meant I became more aware of what they found difficult and therefore where more practice would be needed; for this class it was using brackets. The question about whether they were realistic or not was also useful, I felt. They had to think about what they could model with this level of mathematics and saw why word problems can be unrealistic at times.

Pause for thought

- What questions did you use to probe your students' understanding?
- Did you feel you had to intervene at any point?
- Did you modify the task in any way? If so, what was your reasoning for this?

4 Modelling a context mathematically

Word problems are often essentially mathematical problems dressed up in everyday language. They can help students understand that mathematics does indeed relate to the real world, and that they themselves are acting like mathematicians when they decode word problems. This is why it is important that students grasp the concept that mathematics in real-world problems models complex situations and they need to be confident and knowledgeable enough to extract the essential elements from them.

Focusing on the process of making sense of a complex situation and modelling it mathematically can also help students to focus on the 'making sense' aspect of word problems.

Identifying the mathematical ideas that will be needed to solve word problems, or modelling the context mathematically, can be difficult for students. The word problems they encounter in textbooks usually only need the mathematics that has just been studied – so thinking more broadly about what other mathematics may be needed to model the situation is seldom required, except in examinations. The next activity focuses on students thinking about what mathematics is needed and how to express the problem in a way that mathematics can be used to solve the problem. Read the activity and case study, and then plan and carry out the activity with your class.

Activity 3: Identifying the mathematical model for a word problem

Tell your students the following:

Here are three word problems:

1. Yamini and Fatima, two Class IX students, together contributed Rs. 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation that matches this text.
2. Mary wants to decorate her Christmas tree. She wants to place the tree on a wooden box covered with coloured paper with picture of Santa Claus on it. She must know the exact quantity of paper to buy for this purpose. If the box is 80 cm long, 40 cm wide and 20 cm high, how many 40 cm square sheets of paper would she require?
3. The Shanti Sweets Stall was placing an order for making cardboard boxes for packing its sweets. Two sizes of boxes were required: a larger box with dimensions of 25 cm × 20 cm × 5 cm, and a smaller box with dimensions of 15 cm × 12 cm × 5 cm. For all the overlaps, 5 per cent of the total surface area is required extra. If the cost of the cardboard is Rs. 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.

For each word problem given:

- draw an illustration for the problem
- identify the unknowns in the problems
- identify what you know
- find the relationship between the unknowns and knowns
- represent the relationship mathematically.

Case Study 3: Mrs Rawool reflects on Activity 3

I showed the class the three problems from Activity 3 and asked them not to solve them, but to work through the five steps in modelling the mathematics in the situation. Of course, several of the students set about solving the problems: when they put up their hands to give the answer, I asked them if they had a question and to stand up so that the whole class could hear it and help answer it. That confused them a bit until I reminded them about what they were supposed to be doing!

I noticed that when Jagadev put up his hand to answer, he stopped working and stopped thinking. I do not want that to happen. I remembered then that I had read about using a 'no hands up' policy in class [Black et al., 2003]. I decided there and then to make it a rule in class that 'hands up' was only to be used for asking a question. I would expect everyone to continue to think about and discuss the work with others until I told them to stop, not to compete to be the first to say 'I've finished'. When I want an answer I ask a specific person, because then everyone would have to continue thinking. I think when we all remember we are a 'hands up only to ask' class, there is a lot more thinking going on. If Jagadev or others finish their work, they are now more likely to find someone to discuss the work with further as there is no need to compete to be the first to answer or finish. This has made all the class more cooperative and confident as well.

All this focus on word problems means that my students now seem more happy to tackle them. A phrase we now use repeatedly in the classroom is 'Could we model that like this?'

Pause for thought

Think about the pedagogical approach of 'no hands up': did your students put their hands up with an answer to the problem when you had asked them to think about the process? Do you think a 'no hands up' rule except to ask a question, would help encourage your students to work more collaboratively and therefore do more thinking and learning?

5 Summary

This unit focused on learning to work with word problems in their widest sense. It has discussed the relationship between contexts and the mathematics that can model that context, and some of the barriers that students encounter in understanding this. To overcome these barriers this unit suggested some ways to help your students to read and write mathematical problems. Using the idea of mathematical modelling helps your students understand that word problems are models and not necessarily real representations of everyday life.

Working with word problems in this way can give meaning to mathematics based on the student's personal experience and can therefore make them active participants in their own learning as they use mathematics to model their own situations.

Pause for thought

Identify three ideas that you have used in this unit that would work when teaching other topics. Make a note now of two topics you have to teach soon where those ideas can be used with some small adjustments.

Resources

Resource 1: NCF/NCFTE teaching requirements

This unit links to the following teaching requirements of the NCF (2005) and NCFTE (2009), and will help you to meet those requirements:

- View students as active participants in their own learning and not as mere recipients of knowledge; to encourage their capacity to construct knowledge; to ensure that learning shifts away from rote methods.
- View learning as a search for meaning out of personal experiences and knowledge generation as a continuously evolving process of reflective learning.
- Connect school knowledge with community knowledge and life outside the school.

Resource 2: Using pair work

In everyday situations people work alongside, speak and listen to others, and see what they do and how they do it. This is how people learn. As we talk to others, we discover new ideas and information. In classrooms, if everything is centred on the teacher, then most students do not get enough time to try out or demonstrate their learning or to ask questions. Some students may only give short answers and some may say nothing at all. In large classes, the situation is even worse, with only a small proportion of students saying anything at all.

Why use pair work?

Pair work is a natural way for students to talk and learn more. It gives them the chance to think and try out ideas and new language. It can provide a comfortable way for students to work through new skills and concepts, and works well in large classes.

Pair work is suitable for all ages and subjects. It is especially useful in multilingual, multi-grade classes, because pairs can be arranged to help each other. It works best when you plan specific tasks and establish routines to manage pairs to make sure that all of your students are included, learning and progressing. Once these routines are established, you will find that students quickly get used to working in pairs and enjoy learning this way.

Tasks for pair work

You can use a variety of pair work tasks depending on the intended outcome of the learning. The pair work task must be clear and appropriate so that working together helps learning more than working alone. By talking about their ideas, your students will automatically be thinking about and developing them further.

Pair work tasks could include:

- **'Think–pair–share':** Students think about a problem or issue themselves and then work in pairs to work out possible answers before sharing their answers with other students. This could be used for spelling, working through calculations, putting things in categories or in order, giving different viewpoints, pretending to be characters from a story, and so on.
- **Sharing information:** Half the class are given information on one aspect of a topic; the other half are given information on a different aspect of the topic. They then work in pairs to share their information in order to solve a problem or come to a decision.

- **Practising skills such as listening:** One student could read a story and the other ask questions; one student could read a passage in English, while the other tries to write it down; one student could describe a picture or diagram while the other student tries to draw it based on the description.
- **Following instructions:** One student could read instructions for the other student to complete a task.
- **Storytelling or role play:** Students could work in pairs to create a story or a piece of dialogue in a language that they are learning.

Managing pairs to include all

Pair work is about involving all. Since students are different, pairs must be managed so that everyone knows what they have to do, what they are learning and what your expectations are. To establish pair work routines in your classroom, you should do the following:

- Manage the pairs that the students work in. Sometimes students will work in friendship pairs; sometimes they will not. Make sure they understand that you will decide the pairs to help them maximise their learning.
- To create more of a challenge, sometimes you could pair students of mixed ability and different languages together so that they can help each other; at other times you could pair students working at the same level.
- Keep records so that you know your students' abilities and can pair them together accordingly.
- At the start, explain the benefits of pair work to the students, using examples from family and community contexts where people collaborate.
- Keep initial tasks brief and clear.
- Monitor the student pairs to make sure that they are working as you want.
- Give students roles or responsibilities in their pair, such as two characters from a story, or simple labels such as '1' and '2', or 'As' and 'Bs'). Do this before they move to face each other so that they listen.
- Make sure that students can turn or move easily to sit to face each other.

During pair work, tell students how much time they have for each task and give regular time checks. Praise pairs who help each other and stay on task. Give pairs time to settle and find their own solutions – it can be tempting to get involved too quickly before students have had time to think and show what they can do. Most students enjoy the atmosphere of everyone talking and working. As you move around the class observing and listening, make notes of who is comfortable together, be alert to anyone who is not included, and note any common errors, good ideas or summary points.

At the end of the task you have a role in making connections between what the students have developed. You may select some pairs to show their work, or you may summarise this for them. Students like to feel a sense of achievement when working together. You don't need to get every pair to report back – that would take too much time – but select students who you know from your observations will be able to make a positive contribution that will help others to learn. This might be an opportunity for students who are usually timid about contributing to build their confidence.

If you have given students a problem to solve, you could give a model answer and then ask them to discuss in pairs how to improve their answer. This will help them to think about their own learning and to learn from their mistakes.

If you are new to pair work, it is important to make notes on any changes you want to make to the task, timing or combinations of pairs. This is important because this is how you will learn and how you will

improve your teaching. Organising successful pair work is linked to clear instructions and good time management, as well as succinct summarising – this all takes practice.

Additional resources

- A newly developed maths portal by the Karnataka government: http://karnatakaeducation.org.in/KOER/en/index.php/Portal:Mathematics
- Class X maths study material: http://www.zietmysore.org/stud_mats/X/maths.pdf
- National Centre for Excellence in the Teaching of Mathematics: https://www.ncetm.org.uk/
- National STEM Centre: http://www.nationalstemcentre.org.uk/
- OpenLearn: http://www.open.edu/openlearn/
- BBC Bitesize: http://www.bbc.co.uk/bitesize/
- Khan Academy's math section: https://www.khanacademy.org/math
- NRICH: http://nrich.maths.org/frontpage
- Mathcelebration: http://www.mathcelebration.com/
- Art of Problem Solving's resources page: http://www.artofproblemsolving.com/Resources/index.php
- Teachnology: http://www.teach-nology.com/worksheets/math/
- Maths is Fun: http://www.mathsisfun.com/
- National Council of Educational Research and Training's textbooks for teaching mathematics and for teacher training of mathematics: http://www.ncert.nic.in/ncerts/textbook/textbook.htm
- LMT-01 *Learning Mathematics*, Block 1 ('Approaches to Learning') Block 2 ('Encouraging Learning in the Classroom'), Block 6 ('Thinking Mathematically'): http://www.ignou4ublog.com/2013/06/ignou-lmt-01-study-materialbooks.html
- *Learning Curve* and *At Right Angles*, periodicals about mathematics and its teaching: http://azimpremjifoundation.org/Foundation_Publications
- Central Board of Secondary Education's books and support material (also including the *Teachers Manual for Formative Assessment – Mathematics (Class IX)*) – select 'CBSE publications', then 'Books and support material': http://cbse.nic.in/welcome.htm

References/bibliography

Black, P., Harrison, C., Lee, C., Marshall, B. and Wiliam, D. (2003) *Assessment for Learning: Putting it into Practice*. Buckingham: Open University Press.

Morales, R.V., Shute, V.J. and Pellegrino, J.W. (1985) 'Developmental differences in understanding and solving simple mathematics word problems', *Cognition & Instruction*, vol. 2, no. 1, p. 41.

National Council of Educational Research and Training (2005) *National Curriculum Framework (NCF)*. New Delhi: NCERT.

National Council of Educational Research and Training (2009) *National Curriculum Framework for Teacher Education (NCFTE)*. New Delhi: NCERT

National Council of Educational Research and Training (2012a) *Mathematics Textbook for Class IX*. New Delhi: NCERT.

National Council of Educational Research and Training (2012b) *Mathematics Textbook for Class X*. New Delhi: NCERT.

Nunes, T. (1993) 'Learning mathematics: perspectives from everyday life', in Davis, R. and Maher, C. (eds) *Schools, Mathematics, and the World of Reality*, pp. 61–78. Needham Heights, MA: Allyn and Bacon.

Polya, G. (1957) *How to Solve It*. New York, NY: Anchor.

Polya, G. (1962) *Mathematical Discovery: On Understanding, Learning and Teaching Problem Solving*, combined edn. New York, NY: Wiley.

Watson, A., Jones, K. and Pratt, D. (2013) *Key Ideas in Teaching Mathematics*. Oxford: Oxford University Press.

Acknowledgements

Grateful acknowledgement is made to the following sources for permission to reproduce the material in this unit:

Figure 1: © almagami/iStockphoto.com.

Figure 2: © Chris Rubber Dragon/iStockphoto.com.

Every effort has been made to contact copyright owners. If any have been inadvertently overlooked the publishers will be pleased to make the necessary arrangements at the first opportunity.

Video (including video stills): thanks are extended to the teacher educators, headteachers, teachers and students across India who worked with The Open University in the productions.