9.E4 Charles Hutton on Johannes Kepler's construction of logarithms

Kepler here, first of any, treats of logarithms in the true and genuine way of the measures of ratios, or proportions, as he calls them, and in a very full and scientific manner: and this method of was afterwards followed and abridged by Nercator, Halley, Cotes, and others, as we shall see in the proper places. Kepler forst erects a regular and purely mathematical system of proportions, and the measures of proportions, treated at considerable length in a number of propositions, which are fully and chastely demonstrated by genuine mathematical reasoning, and illustrated by examples in numbers. This part contains and demonstrates both the nature and the principles of the structure of logarithms. And in the second part he applies those principles in the actual construction of his table, which contains only 1000 numbers and their logarithms, in the form as we before described: and in this part he indicates the various contrivances employed in deducing the logarithms of proportions one from another, after a few of the leading ones has been first formed, by the general and more remote principles. He uses the name *logarithms*, given them by the inventor, being the most proper, as expressing the very nature and essence of those artificial numbers and containing as it were a definition in the very name of them; but without taking any notice of the inventor, or the origin of those useful numbers.

As this tract [of1625] is very curious and important in itself, and is besides very rare and little known, instead of a particular description only, I shall here give a brief translation of both the parts, omitting only the demonstrations of the propositions, and some rather long illustration of them. The book is dedicated to Philip, landgrave of Hesse, but is without either preface or introduction, and commences immediately with the subject of the first part, which is entitled *The Demonstration of the Structure of Logarithms*; and the contents of it are as follow:

Postulate 1 That all proportions equal among themselves by whatever variety of couplets of terms they may be denoted, are measured or expressed by the same quantity.

Axiom 1 If there be any number of quantities of the same kind, the proportion of the extremes is understood to be composed of all the proportions of every adjacent couplet of terms, from the first to the last.

Proposition 1 The mean proportional between two terms, divides the proportion of those terms into two equal proportions.

Axiom 2 Of any number of quantities regularly increasing, the means divide the proportion f the extremes into one proportion more than the number of the means.

Postulate 2 That the proportion between any two terms is divisible into any number of parts, until those parts become less than any proposed quantity.

An example of this section is then inserted in a small table, in dividing the proportion which is between 10 and 7 into 1073741824 equal parts, by as many mean proportionals wanting one, namely, by taking the mean proportional between 10 and 7, than the mean between 10 and this mean, and the mean between 10 and the alst, and so on for 30 mean, or 30 extractions of the square root, the last or 30th of which roots is 99999999966782056900; and the 30th power of 2, which is 1073741824, shows into how many parts the proportion between 60 and 7 o, or between 1000&c, and 700&c, is divided by 1073741824 means, each of which parts is equal to the proportion between 1000&c, and the 30th mean 999&c, that is, the proportion between 1000&c, and 999&c, is the 1073741824th part of the proportion between 10 and 7. Then by assuming the small difference 00000000033217943100, for the measure of the very small element of the proportion of 10 to 7, or for the measure of the proportion of 1000&c, to 999&c, or for the logarithm of this last term, and multiplying it by 1073741824, the number of parts, the product gives 35667.49481.37222.14400, for the logarithm of the less term 7 or 700&c.

Postulate 3 The extremely small quantity or element of the proportion may be measured or denoted by any quantity whatever; as, for instance, by the difference of the terms of that element.

Proposition 2 Of three continued proportionals, the difference of the two first has to the difference of the latter two, the same proportion which the first term has to the 2d, or the 2d to the 3d.

[…]

Proposition 20 When four numbers are proportional, the first to the second as the third to the fourth, and the proportions of 1000 to each of the three former are known, there will also be known the proportion of 1000 to the fourth number.

Corollary 1 By this means other chiliads are added to the former.

Corollary 2 Hence arises the method of performing the Rule-of-Three, when 1000 is not one of the terms. Namely, from the sum of the measures of the proportions of 1000 to the second and third, take that of 1000 to the first, and the remainder is the measure of the proportion of 1000 to the fourth term.

Definition The measure of the proportion between 1000 and any less number as before described, and expressed by a number, is set opposite to that less number in the chiliad, and is called its *logarithm*, that is, the number (*arithmos*) indicating the proportion (*logos*) which 1000 bears to that number, to which the logarithm is annexed.