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Understanding risk versus return in portfolio theory

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Managing My Investments

Understanding risk versus return in portfolio theory

The theory

The expected return of portfolio P, written as $E(R_p)$ is the weighted average of the constituent expected returns, so that we can write:

$$E(R_p) = W_X E(R_X) + W_Y E(R_Y)$$

where

$E(R_p)$ = expected return on portfolio P

$E(R_X)$ = expected return on share X

$E(R_Y)$ = expected return on share Y

W_X = proportion or weight of portfolio invested in share X in value terms

W_Y = proportion or weight of portfolio invested in share Y in value terms

$$W_X + W_Y = 1$$

The risk of the portfolio – that is, its standard deviation, S_p – is not the weighted average of the constituent risks, but a more complex formula, involving something called the correlation coefficient between the two constituent shares which will now be discussed.

The risk of portfolio, S_p , can be written as:

$$S_p = \sqrt{W_X^2 S_X^2 + W_Y^2 S_Y^2 + 2W_X W_Y S_X S_Y \text{Corr}_{X,Y}}$$

where

S_p = standard deviation of returns on portfolio P

S_X = standard deviation of returns on share X

S_Y = standard deviation of returns on share Y

$\text{Corr}_{X,Y}$ = correlation coefficient of returns between share X and share Y

Applying the theory to Gelato and HotChoc

The return from a portfolio is simply the proportion of the portfolio taken by each share multiplied by the return for each share.

So a portfolio of 62% Gelato and 38% Hotchoc gives a return of:

$$(0.62 \times 12.7\%) + (0.38 \times 4.4\%) = 9.5\%$$

The risk of a portfolio is measured by a complex formula which looks at the risk of each share (in proportion to its percentage of the total portfolio) and adjusts for the relationship between the directions of share price movements of shares in the portfolio.

Hotchoc and Gelato are negatively correlated (as the animations show) and for the example we assumed a correlation of minus 0.3 (-0.3).

The portfolio's risk measure is then

$$\sqrt{[(0.62^2) \times (13.8^2)] + [(0.38^2) \times (8.2^2)] + [2 \times 0.62 \times 0.38 \times 13.8 \times 8.2 \times -0.3]} = 8.2\%$$

Why is the risk return trade-off between Gelato and Hotchoc not a straight line?

The reason the trade-off between A and B is not a straight line relates to the trade-off of risk as the portfolio alters (by contrast the trade-off of return is linear). When you mix the two shares the aggregate of risk is not just a weighted average of the risk of the two shares since the risks associated with the two shares is negatively correlated – so combining the two shares makes the portfolio more stable than just being 100% concentrated in Gelato or Hotchoc.

So this explains why D has a higher return and a lower risk than A. The higher return is because high returns from Gelato are replacing low returns from Hotchoc. Lower risk as the combination of two shares, that tend to move in opposite directions is more stable and hence less risky, than having a portfolio of just Hotchoc.