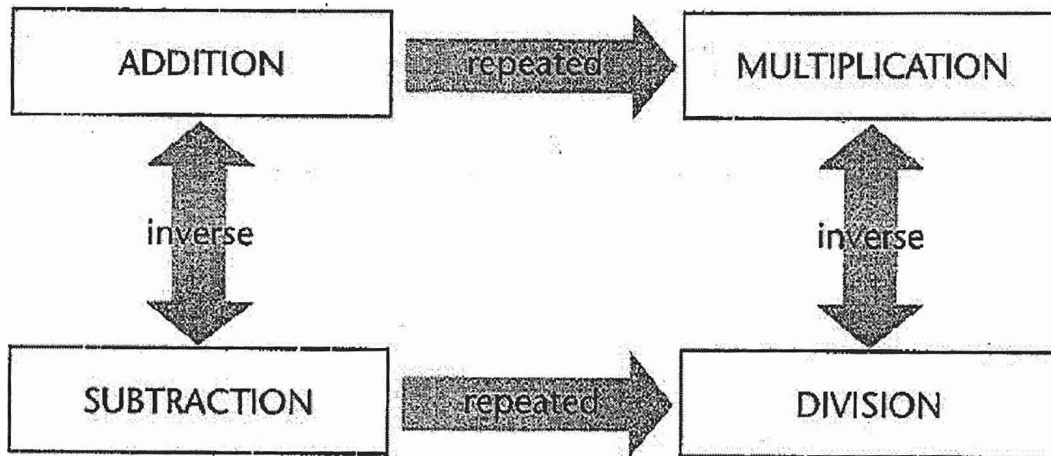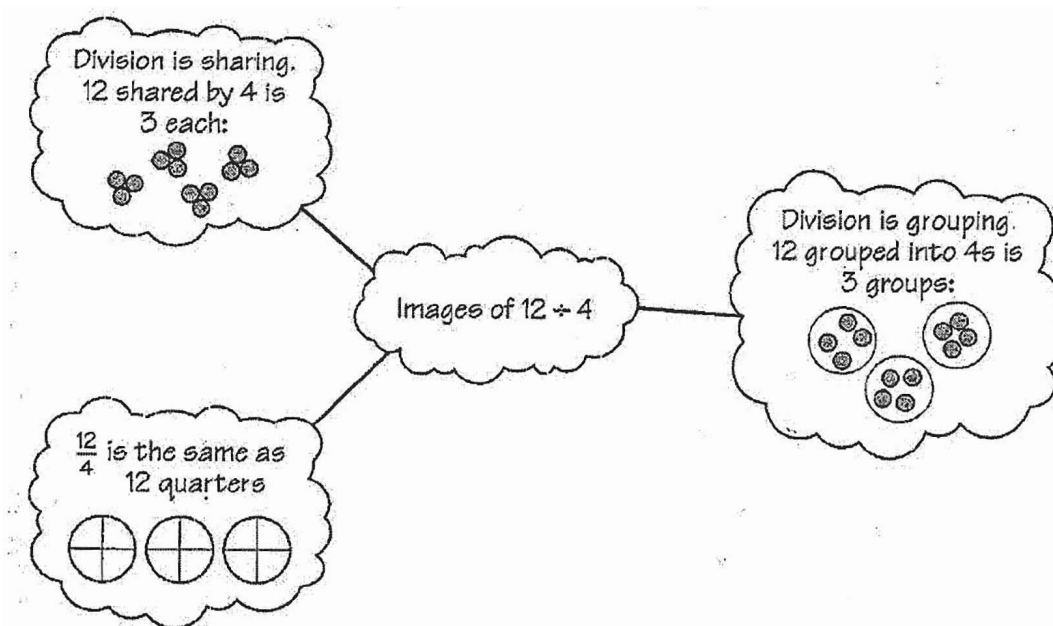**Calculating**

The 'four rules'

The concepts of addition, subtraction, multiplication and division are complex abstract ideas which are interrelated.

Since concepts are, by definition, abstract, they cannot be expressed precisely in words or diagrams. However, aspects can be illustrated by example.

The various ways of doing calculations involving the four rules can be categorized by the different means used:

- mental methods;
- methods using a calculator;
- written methods.

Task 11   Choosing a method

Consider the following calculations:

(a) $361 - 261$ (b) 361×261 (c) $361 + 261$

What method would you use for each of these: mental, written, or a calculator?

Comment

The methods you use will depend on how adept you are at each of them. You may have worked out $361 - 261 + 100$ mentally, whereas it is more likely that you used a calculator to work out $361 \times 261 - 94\,221$. When working out $361 + 261 = 622$ it is possible that you used any one of the methods or indeed all three. It is quite common for people to use a combination of all three methods.

When you are manipulating numbers you are aiming for fluency as well as accuracy, but not necessarily great speed. There are some skills and strategies that help when using each of these methods.

This section looks at the various methods of calculating – mental methods, ones using a calculator, and written methods. It then discusses calculating with fractions, decimals, percentages and ratios. For such calculations, most people use a mixture of methods depending on the numbers involved.

Mental methods

In recent times there has been a much increased emphasis on mental arithmetic in schools, and most adults sometimes use mental methods for informal calculations where the numbers are simple or a calculator is not to hand. Quick recall of number facts and fast mental calculation has been taken to be evidence of understanding and high mathematical ability. Equally, poor recall and slow calculation has been taken as evidence of low ability. But these are very dubious assertions.

This section helps you think about the methods you use when calculating mentally (you may find that you use more than you are aware of). Useful mental calculating strategies are then looked at.

There are several important aspects to mental calculation:

- For any mental calculation there can be a wide variety of methods – some are more efficient than others for *particular numbers,* while some are more efficient for the *particular person* doing the calculation. Also, there is variation in the need for accuracy, determined by the purpose of the calculation in the context in which it arose.
- There is a need to understand ('have a feel for') numbers and operations in order to *devise* an appropriate method.
- Mental methods commonly take a different form from the standard written methods.
- Mental methods can be shared; they do not need to be re-invented by every individual.

Use the following task as a way of finding out some of the methods you use.

Task 12   Buying stamps

Mentally work out how much it would cost to buy the following postage stamps (do each part without using the solution to the previous one). Think about what method you use.

(a)   50 at 28p each   (b)   27 at 50p each

(c)   25 at 75p   (d)   28 at 75p

Comment

Did you always use the same method? If not, why not? The following are some which you might have used for part (d):

- 75p = £¾, 28 × ¾ = 28 ÷ 4 × 3 = 7 × 3 = £21
- 2 × 75p = £1.50, 14 × 1.50 = (10 × 1.50) + (4 × 1.50) = 15 × 6 = £21
- 20 × 0.75 = 15, 8 × 0.75 = 6, 15 + 6 = £21
- 2 stamps cost £1.50 so 4 cost £3; 28 is 7 groups of 4, 7 × £3 = £21.

Mental strategies

With mental calculations there are not set methods – what you use depends on your personal strategies and the actual numbers. You use a **method** with particular numbers, but a **strategy** is more like a generalization of a collection of methods. This creates difficulties for some people who feel lost without a standard method to hold on to. However, there is plenty of evidence that you can improve your mental arithmetic by being more aware of strategies you use and deliberately adopting new ones. For example, one strategy is to

break down a number into parts that are easier to deal with. This strategy is often used by people who can work out VAT in their heads. At the time of writing VAT is payable at 17.5%. At first glance this appears to be a bizarre amount. But you can break down 17.5% as follows:

17.5% = 10% + 5% + 2.5%

To work out 17.5% of £48 you would calculate these three percentages (10% is easy, and the others are each half of the previous one) and add them:

17.5% of £48 = £4.80 + £2.40 + £1.20 = £8.40

Try this method on £64.

You may have found that you had difficulty in 'holding' three different figures in your head. Mental methods do not need to be entirely 'in the head'; it is often useful to jot down intermediate numbers.

Here is another example: 506 – 309.

Method 1: Taking 300 from 506 gives 206. Taking 9 from 206 gives 197.
Method 2: Taking 300 from 500 gives 200. Taking 6 from 9 gives 3. Taking 3 from 200 gives 197.
Method 3: Make the 309 up to 400 which gives 91, then add 100, then add the 6, giving 197.

These three methods share a common feature in that they work with the bigger numbers, the hundreds, before the units. In the standard written methods, the rule is to start with the units for addition, subtraction and multiplication, but not for division!

Below are some common strategies that you might use.

Addition and subtraction strategies

- Count on or back in repeated steps of 10, 2 or 5.
- Identify near doubles and adjust.
- Separate into tens and units and add tens first.
- Add/subtract 9,19, 29, ... or 11, 21, 31, ... by adding/subtracting 10, 20, 30, ... then adjusting by 1.
- Add several small numbers and look for pairs that total 10.

Multiplication and division strategies

- To multiply by 10, shift digits one place to the left.
- To divide by 10, shift digits one place to the right.
- Use the relationship between multiplication and division as inverses.
- Use related facts of doubling and halving.

- Use closely related facts already known.
- Split numbers into parts that are easily dealt with.

Calculators

Calculator key sequences are shown with the numbers in **bold** and the operation keys in a box.

While it is possible to use calculators for simple calculations, they are not always the most efficient method.   For example, the calculation 22 + 23 requires six key presses to get the answer:

22 $\boxed{+}$ 23 $\boxed{=}$

Doing a sum like this mentally is usually faster than using a calculator; you need to use your judgement about when to reach for a calculator.

Many people are sometimes unsure exactly which keys to press to work out a calculation: for example, £12.99 less 5% discount.   This is partly because people are often unsure how to use particular kinds of keys, and partly because calculators do not always give the outcome you might expect.   This section examines the following issues:

- unexpected answers on calculators
- calculator logics
- the use of the calculator constant facility.

 The next task draws your attention to some of the features of a calculator operation.   The intention is to raise issues through your being surprised, perhaps, at the answers the calculator gives.

Task 13   Guess and press

Although this task can be carried out on your own, it is better done with a small group of people working and discussing together.

a) Look at each key sequence below.   Before you use the calculator, write down what answer you would expect it to produce on the calculator display.

b) For each key sequence, press the keys in the given order on your calculator and write down the result you actually got.   Clear the calculator display before moving to the next key sequence.

(i) 7 $\boxed{\div}$ 4   (ii) 3 $\boxed{\div}$ 4 $\boxed{=}$ (iii) 2 $\boxed{\div}$ 3 $\boxed{=}$

(iv) 2 ⊞ 3 ⊠ 4 ⊟   (v) 2 ⊞ ⊞ 3 ⊟

(vi) 2 ⊞ ⊞ 3 ⊟ ⊟

Comment

Some examples of the sorts of issues that this task was designed to raise are given below.

(i)   7 ⊡ 4

As written, most calculators will show 4.   Two points arising from this are, first, that most calculator key sequences require the press of ⊟ to complete them, and second, that it is worth clearly distinguishing between a textbook 'sum' and a calculator key sequence.

(ii)   3 ⊡ 4 ⊟   and   (iii)   2 ⊡ 3 ⊟

Although the first division (3 ÷ 4) gives an exact answer, the second (2 ÷ 3) produces a string of digits which (in theory) goes on for ever.   This reveals two interesting features of your calculator.   First, does it 'round' the final digit up to 7, or does it 'cut' the string, leaving a 6 as the final digit displayed? Second, how many digits can your calculator display – eight, ten or twelve?

(iv)   2 ⊞ 3 ⊠ 4 ⊟

This sequence will produce the answer 14 or 20, depending on the operating system of the calculator you are using.   This is an important thing to know and is explained more fully in the next task, 'Calculator logic'.

(v)   2 ⊞ ⊞ 3 ⊟   and   (vi)   2 ⊞ ⊞ 3 ⊟ ⊟

These sequences are likely to raise questions about the calculator's constant facility.   The constant facility is such a useful feature of calculators that it is also explored later in this section.

One of the problems of working with calculators is that they do not all work in the same way.   You have to get to know your particular calculator and its nuances.

Task 14   Calculator logic

Imagine that someone asks you to do the following calculation in your head. Read it out loud before doing it.

What is 2-plus-3 (pause) times 4?

Now try the following calculation, again reading it out loud first.

What is 2 plus (pause) 3-times-4?

Finally, here is an exercise in algebra to try.

Find the value of the expression $2 + 3a$, when $a = 4$.

Comment

It is likely that for the first sentence, when read as written above, you will produce the answer 20. For the second calculation, although the basic words are the same, the answer is less certain. If you read the '3-times-4' part all in a rush, as written above, the answer 14 may seem more appropriate (i.e. as 2 + 12).

The algebra example, when the a is replaced by 4, produces the same calculation, namely $2 + 3 \times 4$. This time the answer of 14 is inescapable, because the rules of algebra dictate that the multiplication of 3×4 must be completed before the 2 is added.

So, what will your calculator make of $2 + 3 \times 4$?

Calculators, unfortunately, are not programmed to interpret dramatic pauses and changes of intonation when given instructions to perform a calculation. They will simply calculate according to the rules with which they have been programmed.

Most calculators have either an arithmetic (this is pronounced arithmetic, with the stress on the third syllable) or an algebraic operating system. Calculators which perform the operations from left to right in the order in which they are keyed in (i.e. ones which will give the answer 20 to the above sequence) are said to have an 'arithmetic operating system'. Calculators which conform to algebraic rules, such as that multiplication and division must be performed before addition and subtractions (i.e. which will give the answer 14 to the above sequence) are said to have an 'algebraic operating system'.

Most of the calculators found in primary classrooms are often termed 'four-function' machines. These usually have arithmetic logic and will carry out addition, subtraction, multiplication and division of numbers (the four functions referred to in the name). On the other hand, scientific calculators tend to conform to an algebraic operating system.

The calculator constant facility

As especially useful feature of a calculator is the 'constant' facility: the method by which you can carry out repeat operations.

The constant facility is a way of setting up the calculator to do a particular calculation and, thereafter, it will continue to perform the same calculation each time the ☐= key is pressed.   The constant facility can be thought of as a form of 'function machine', where you input a number into the calculator, press ☐= to apply the function, and the output value then appears in the display.   The constant facility and the ☐% key are two aspects of a simple calculator that many people find the most puzzling.

Setting up the constant facility

Almost all calculators have a constant facility, but unfortunately they do not all operate with the constant in the same way.   The two most common methods of setting up and operating the constant, the 'automatic' and the 'double-press' constant, are described below.   Check which way works for your machine.   If neither works, track down the calculator's manual and see whether it can be set up by some other means (for example, some calculator constants are based on a key marked Ⓚ).

Task 15   Checking the constant facility

This will enable you to check whether your calculator has an automatic or a double-press constant.

(a)   Carry out this sequence:

2 ⊞ ⊞ ⊟ ⊟ ⊟ ...

For calculators with a double-press constant, this key sequence should produce the 2 times table.   (Many calculators show a small 'к' in the display to indicate that the constant has been activated.)   For calculators with an automatic constant, '2' will remain in the display.

(b)   Carry out this sequence:

8 ⊞ 2 ⊟ ⊟ ⊟ ⊟ ⊟ ...

If successive presses of ☐= have the effect of changing the number on the display, then you know that your calculator has an automatic constant facility.   Such calculators will produce a sequence of numbers.   (Other calculators will show '10' on the display throughout.)

(c)   (Only if you have a calculator with an automatic constant.)   There is more than one type of automatic constant.   You need to discover whether the constant operation is applied to the number *before* the

operation each time (i.e. the **8**) or the number *after* it (the **2**).   This needs to be checked out carefully for any new calculator you use, as it varies from model to model.   To find out which type you have, carry out the following sequences:

8 ⊟ 2 ▣ ▣ ▣ ▣ …

8 ⊠ 2 ▣ ▣ ▣ ▣ …

8 ⊡ 2 ▣ ▣ ▣ ▣ …

Using the constant facility

Below are some key sequences for you to try.   They assume the double-press constant.   If your calculator constant operates differently, the key sequences will need to be adapted.

(a)   Pressing 10 ⊞ ⊞ 5 ▣ ▣ ▣ ▣ … should give 15, 25, 35, 45, …

(b)   Pressing 1 ⊟ ⊟ 11 ▣ ▣ ▣ ▣ … should give 10, 9, 8, 7,

This method is used later is converting units of measurement.

The constant facility can be of use to a teacher.   Below is a method of converting a set of class results, marked out of a total of 65, into percentages.

Set the adjustment factor: press 100 ⊡ 65 ▣
Set up the constant: press ⊠ ⊠

Adjust the first mark: enter mark 1 and press ▣
(Don't press the 'Clear' key before entering the next mark.)

Adjust the second mark: enter mark 2 and press ▣
And so on.

One problem with the constant facility is that it is rather fragile, in that it will vanish if any operation key or the 'Clear' key is pressed.