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Using visualisation in maths teaching

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The Open University, Walton Hall, Milton Keynes, MK7 6AA

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Introduction

This course looks at visualisation as it relates to mathematics, focusing upon how it can be used to improve learning. It will also identify ways in which to make more use of visualisation within the classroom.

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Learning Outcomes

After studying this course, you should be able to:

- demonstrate an engagement in a number of activities that involve visualisation and learning from personal experiences what it means
- understand the views of a well-known mathematics educator talking about visualisation and find out how personal views compare with those of some other secondary-school mathematics teachers
- identify some ways that visualising could be incorporated into a classroom and consider a number of resources that might be useful.

1. A powerful force for perception and understanding

> ‘Imagery is a powerful force for perception and understanding. Being able to “see” something mentally is a common metaphor for understanding it. An image may be of some geometrical shape, or of a graph or diagram, or it may be some set of symbols or some procedure.’
>
> (Open University, 1988, p. 10)

This course uses the word visualisation synonymously with mental imagery. It happens as we articulate our thoughts and as we understand what we are doing.

Each of the four sections in this course should further develop your thinking on visualisation.

Starters

The main purpose of this course is to encourage you to form images and be aware of your experiences of visualisation.

What does visualisation mean?

In ‘What does visualisation mean?’, you will find quotes and readings about visualisation, followed by an opportunity to compare your views with those of some other secondary-school mathematics teachers. By the end of this section you should be confident that you know what visualisation means.

In the classroom

‘In the classroom’ describes a lesson in which visualising provides an introduction for the students, another worthwhile and interesting activity, and some ways to use resources to promote and stimulate visualisation.

Conclusion

The conclusion points to some recommended books that will be useful if you want to think a little more seriously about visualisation, as well as highlighting where visualisation is incorporated into the framework for mathematics school teaching.

2. Starters

We all have pictures in our heads but some people use them more than others.

'Doing' can often be the most powerful way to learn. Before discussing other people's thoughts on visualisation, it is probably worthwhile to spend some time exploring some visualisation activities with your colleagues. This should enable you to consider the next section from an experiential perspective.

Activity 1

Please click on the 'view document' links below to explore a range of visualisation activities. Try at least two of these.

Click 'view document' below to download Arithmetic

View document

Click 'view document' below to download Circles

View document

Click 'view document' below to download Cutting paper

View document

Click 'view document' below to download Locus

View document

Click 'view document' below to download Multiples on grids

View document

Click 'view document' below to download Nets

View document

Click 'view document' below to download TV

View document

1. For each activity you attempt:
   - **Try to do them in your heads** before you look at anything.
   - **Shut your eyes** if it helps.
   - Try them individually and then **talk in a group**, but do not give any answers away – help others to see what you can see.
   - **No drawing**, only talking.
   - **No gesticulating** either.
2. When you have experienced a number of visualisation activities, write down some of your own thoughts about visualisation and what it means.

3. What does visualisation mean?

> ‘Imagery is a powerful force for perception and understanding. Being able to “see” something mentally is a common metaphor for understanding it. An image may be of some geometrical shape, or of a graph or diagram, or it may be some set of symbols or some procedure.
>
> Visualising means summoning up a mental image of something – seeing it in your mind. Some people can actually close their eyes and “see” a picture, but for others it has much more to do with imagining, than seeing. Try to picture a cube, the seven-times table, a graph of sin *x*. Describe what you “see”.
>
> The point of this is that if you really want to grasp a concept or idea, struggling to visualise it is worthwhile. There are many aids to visualisation. Diagrams or symbols on paper often help, or physical apparatus. Trying to “say what you see” (or cannot see!) can be helpful, too. Visualisation and articulation go hand in hand.’
>
> (Open University, 1988, p. 10)

Click on the link below and read ‘Mental mathematics’ by Chris Bills. This text suggests a number of mathematical visualisation activities for you to try out. You are asked to think about the mental strategies and processes you have used (metacognition) in engaging with these activities.

Click 'view document' below to download Mental mathematics.

View document

Activity 2: Questionnaire

- For each of the questions below write few very brief notes of your own.

1. What do we mean by the term mental imagery?
2. What do we think is the value (for the learner) of working with images?
3. Does mental imagery lend itself more easily to shape and space topics?
4. How can we use it in number work?
5. Can this work have a place within the School Curriculum?
6. Do pupils improve their mental imagery as they are given more opportunities to use it?
7. Is mental imagery part of the problem-solving process?
8. How do you think it improves concepts and skills?

9. Describe any classroom-management issues and any useful hints for the use of this approach for the first time.
10. Can students of different abilities cope with this approach?

- Talk to at least one other person about your responses.
- Click on the link below to read the various components of the questionnaire and compare your views with those of other teachers.

Answer

(Source: These responses were collected by Brenda Stevens and Aled Williams, of Oxford Brookes University. bjstevens@brookes.ac.uk). The responses here were made by teachers attending a course at Oxford Brookes University.

1. What do we mean by the term mental imagery?
   - Learning by example – analogy – except that the analogies are visual pictures. Then instead of having objects in front of us we imagine it.
   - Any mathematics done just in your head: for example, imagining shapes/ pictures and moving objects/lines/angles around.
   - Whenever we use pictures or diagrams, whether moving or still, imagined (straightaway) or initially drawn to aid understanding.
   - When any form of problem is tackled using only the imagination.
   - Anything which requires a 'picture' – mental or drawn(?)
2. What do we think is the value (for the learner) of working with images?
   - Because we can say 'it is similar to this' and the image in particular can usually be less ambiguous than words and have more meaning.
   - It helps to think more about other topics and apply the idea themselves.
   - Sometimes it is a lot easier than doing something practically.
   - Visual imagery should start with images with which the learner is familiar, thus providing confidence.
   - The visual is very powerful and young people are already more comfortable with television and computers than with abstract concepts and procedures.
   - It provides analogies for concepts and procedures that can connect a new concept with an old one, or elaborate the existing concepts. It can therefore be more memorable and aids understanding.

- The skill of being able to manipulate mental images improves spatial abilities.
- Once the problem has been visualised, not only does the problem appear easier to comprehend, but any solutions found will tend to be remembered longer.
- It strengthens and/or creates ideas and concepts; variety leads to fun.

3. Does mental imagery lend itself more easily to shape and space topics?
   - Not necessarily. We may have to look harder for algebra and number, but worth doing because it makes it more fun. Probably not appropriate in data handling.
   - Yes, as this is the main topic that it would be used for if it has been previously taught by another teacher. Also, when pupils (and teachers) are new to mental maths this is generally the easiest topic to grasp.
   - It has a different role in other topics in providing analogies and contexts for concepts. There are probably more ways of using it for shape and space.
   - Yes, as it is easier to imagine a picture or object.
   - I think so although algebra can be spatial
4. How can we use it in number work?
   - For example, ratios, equal ratios. I found it useful to use images of cakes being split up etc. I suppose division is always a good one for mental imagery (and division stands in a good relationship to number).
   - Fractions, decimals, percentages and percentage change.
   - Four operations.
   - Ratio; sharing and mixing.
   - Adding – climbing up; subtraction – falling down.
   - Fractions – imagine an object divided up.
   - Imagine a number line.
   -

Fractions, decimals, percentages, ratio, four rules of number, solving problems.

5. Can this work have a place within the National Curriculum?
   - It should because it makes learning fun and memorable and methods can be recalled/replicated in the future.
   - Could be mentioned, but is more a method of delivery.
   - A teaching method rather than a separate topic.
   - Definitely.
   - Of course – can teaching have a place in the National Curriculum?
6. Do pupils improve their mental imagery as they are given more opportunities to use it?
   - Pupils seem to get better at understanding your way of communicating whatever style you use (within reason) so of course they would get more skilled at using this 'tool' in particular.
   - Yes, but they find it very difficult.
   - I believe that it is like a muscle in the brain that can improve like any other with exercise. So, yes.
   - Yes.
7. How do you think it improves concepts and skills?
   - It facilitates communication of meaning. A good natural 'pointer' at the concepts you want to get across. It is a fairly universal medium of communication. (Sometimes language – words, writing and reading are obstacles, but 'imagining' is easy for most of us.)
   - It can strengthen existing concepts, or provide a new way of understanding the concept by clarifying and extending it. It provides a justification for procedures and it is easier to remember.
   - If they have a mental image it is easier to recall.
   - By visualising concepts you can generally better see the whole picture.
   - Lots of different 'pictures' – overall view – generalise.
   -

More likely to remember having constructed your own idea of what it is.

- Don't need to remember – it's just there.

8. Is mental imagery part of the problem-solving process?

- I don't know. It isn't for me, not naturally. Maybe my problem-solving skills would benefit from trying to use it. For those brought up using it, so it comes naturally. Why not?
- To solve fully any problem you need to be able to have an image or at least some of it. In creating this image, you are starting to solve the problem.
- Yes, but often pupils don't use it and find questions more difficult because of it.
- Yes – finding a picture to fit the situation is an invaluable tool in problem solving. It is a way of understanding the situation.
- Yes.

9. Describe any classroom-management issues and any useful hints for the use of this approach for the first time?

- I strongly believe that complete silence must be achieved.
- I have found it useful to get the class to 'travel' to a different place where they can imagine all of this happening and are allow to relax. (I even gave a two-minute meditation lesson on one occasion with some excellent results.)
- Try to empty any images by imagining a blank sheet of paper. Focusing on the picture helps them to concentrate (e.g. ask them what colour their shape is).
- Make it accessible to all by transferring it to paper so everyone knows what they should have seen and then building on the image so it has some kind of purpose – use it as an introduction rather than an end in itself so that they can see it as a tool to be used again. Differentiate so that all can visualise something.
- Little and often. Use to help explanations or setting up problems. Clarifying ideas. Finding out misconceptions. Classes like imagining things so it could be used to settle a class at times, though it depends on what is being imagined.
- Need silence; need to trust each other, not mock. I told them that we were going to try something new and some of them might not feel comfortable, but

that didn't matter. If it didn't work we'd stop and do something else. If it was going to work they needed to be quiet and mustn't laugh at each other – needed to support each other. Then settled down by 'imagine you are walking through a desert … (first time – one off of 15 minutes; bottom set Year 8, 24 pupils).

- It worked well; some wanted to tell what their pictures were. Asked if I would hypnotise them again next week!

10. Can pupils of different abilities cope with this approach?

- Yes, but the images for lower abilities must be kept simple or related to a real-life situation.

- I see absolutely no reason why not; although obviously it should be recognised that not all pupils will be able to follow the activity fully right through to the end.

4. In the classroom

There are many possible strategies for making more use of visualisation within the mathematics classroom. There are several visualisation activities for you to experiment with in Activity 3. One teacher's approach to incorporating visualisation is given in the following case study and three-part video clip.

Case study

In a secondary-school classroom, the teacher, Peter Gates, is working with fourteen-year-old students on a topic in which they are exploring aspects of circles. This particular activity involves working out distances around a race-track.

The school is Brindley Hall, at Stantonbury Campus, Milton Keynes. Peter has a class of 27 mixed-ability students. They are his own class for pastoral purposes and he teaches them mathematics. They work on an individualised scheme, the Kent Mathematics Project (KMP) for roughly half of their mathematics lessons and participate in whole-class projects, such as the one on circles, for the other lessons. These projects help to prepare students for their GCSE coursework that they will have to undertake in subsequent years.

The video clip has a three-part structure; view the three parts now by clicking on the links in the list below.

- Introducing the problem.

Click on the 'play' button below to view the video

Video content is not available in this format.
Video 1

- Two groups of students working on the problem.

Click on the 'play' button below to view the video

Video content is not available in this format.
Video 2

- Reporting back.

Click on the 'play' button below to view the video

Video content is not available in this format.
Video 3

The students are encouraged to devise their own approaches to problems and to follow their own ideas and questions. Peter is concerned to involve the students in the mathematics they encounter, so that it becomes as meaningful as possible for them. He uses a number of devices in order to achieve this, one of which invokes their mental

imagery to help them ‘see’ into a problem. Use of practical apparatus and calculators is also important to their way of working.

Activity 3

Visualisation is an important facet in all areas of the curriculum. Many people argue that we visualise what we have seen, but all our senses may be important. There are many resources that actively promote visualisation in the classroom.

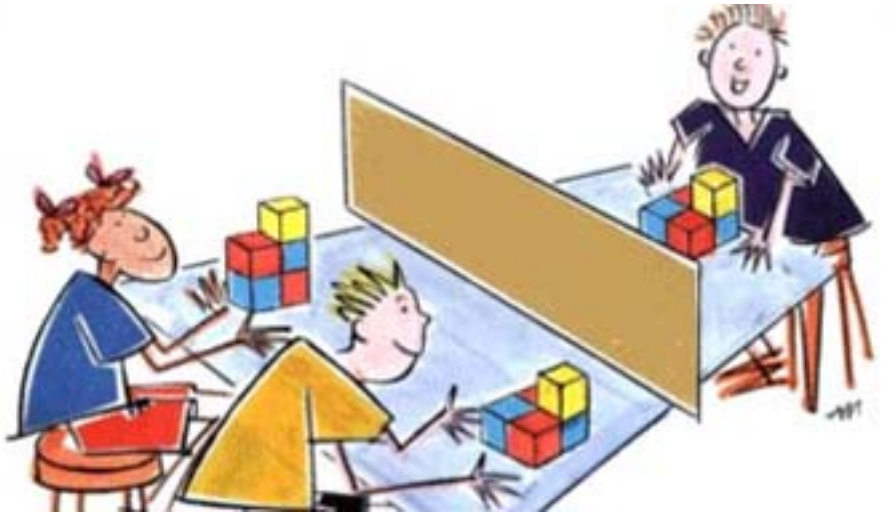

Figure 3

In the classroom activity illustrated here the students design a solid from a few coloured cubes and then describe it. The others in the group then try to visualise the configuration and make their own.

You might like to try this activity in your classroom.

Further classroom activities for visualisation are available by clicking on the ‘view document’ and website links below. Explore some of these with colleagues and identify two or three to try out in class.

Click 'view document' below to download Diagrams.

View document

Click 'view document' below to download The feely box.

View document

Click 'view document' below to download The hundred square.

View document

Click 'view document' below to download Number lines.

View document

Click 'view document' below to download People activities.

View document

Click 'view document' below to download Photographs.

View document

Click 'view document' below to download Tracing paper.

View document

The Key Stage 3 National Strategy website.

Multiples and factors: assessing pupils' work website.

Conclusion

I hope you now have a better idea of what it means to visualise a piece of mathematics.

Visualising is a critically important process when mathematicians and others actually do mathematics. Unfortunately, the process of visualising does not appear in publications, which all tend to be displayed very formally and are mostly restricted to the final results.

As teachers we need to ensure that we are very aware of all the processes of mathematics and so we must always attempt to know what our students are visualising.

Activity 4: Further exploration

Click 'View document' below to read Mathematics books for teachers

View document

The book list provides a selection of further reading if you're starting to think more seriously about visualisation and how to help it happen effectively in your classroom.

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