1 Using exponent notation

This section offers background explanation about the exponent notation which is used in Sections 4 and 9 of Block 1.

1.1 Positive exponents

The exponent notation is simply a notation that makes it easier to cope with a number being multiplied by itself several times. It does this by replacing the rather clumsy notation

$$3 \times 3 \times 3 \times 3 \times 3$$

or

$$7 \times 7 \times 7 \times 7$$

by a more compact notation. For the five 3s multiplied together, this notation is

$$3^5$$

and for the four 7s multiplied together it is

$$7^4$$

There is a pattern here. Can you spot it? *Five* 3s multiplied together are written as

$$3^5$$

and *four* 7s multiplied together are written as

$$7^4$$

Similarly in

$$2 \times 2 \times 2$$

there are *three* 2s, and this is written as

$$2^3$$

and in

$$10 \times 10 \times 10 \times 10 \times 10 \times 10$$

there are *six* 10s, and this is written as

$$10^6$$

The terms 'base' and 'exponent' are used to refer to the two components of numbers such as 3^5 or 7^4 or 2^3 or 10^6. Figure 1 shows which of the two numbers is called the base and which the exponent.

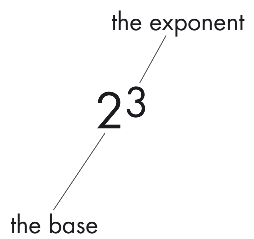

Figure 1

Reading exponent notation

It is very important for you to be able to say expressions like 2^3 or 4^9 aloud if you need to talk about the notation. It is also important for you to say it to yourself as you read: if you don't do so then you will find it difficult if not impossible to work with the notation. People say expressions like this in slightly different ways. I will introduce you to all of these ways, and will suggest which you might like to use yourself.

I'll look at exponents 2 and 3 in a moment, as these are normally said differently from other exponents. From the exponent 4 upwards, all numbers in exponent notation are said in the same way. For example,

7^4

is said as:

'seven to the fourth power' or 'seven to the fourth' or 'seven to the power four' or 'seven to the four'.

I suggest you use one of the two shorter versions: 'seven to the fourth' or 'seven to the four'.

Similarly,

10^6

is said as

'ten to the sixth power' or 'ten to the sixth' or 'ten to the power six' or 'ten to the six'.

Again, I suggest you use one of the shorter versions, 'ten to the sixth' or 'ten to the six'.

Try saying each of the following to yourself over and over until the words come naturally:

7^5, 10^4, 2^8, 3^9, 10^7, 2^{16}

It is possible to say

2^3

as 'two to the third power', etc., but it is much more common to say 'two cubed'. Similarly, it is possible to say

10^2

as 'ten to the second power', etc., but it is much more common to say 'ten squared'.

Practise using this 'squared' and 'cubed' terminology on the following:

3^2, 4^3, 10^3, 2^3, 7^2, 8^2

In case you are wondering where the terms 'squared' and 'cubed' come from, think about finding the area of a square of side 3 metres. You'd multiply 3 by 3 to get 3 'squared'. Similarly, to find the volume of a cube of side 2 metres you'd multiply 2 by 2 by 2 to get 2 'cubed'.

Activity 1

(a) In 5^9, what is the base and what is the exponent?

(b) How many times is 5 multiplied together in 5^9? Write this out fully using multiplication signs.

(c) Write the following in exponent notation:

(i) $4 \times 4 \times 4$

(ii) $6 \times 6 \times 6 \times 6 \times 6 \times 6$

(iii) $2 \times 2 \times 2 \times 2$

(iv) 10×10

(v) $2 \times 2 \times 2 \times ... \times 2 \times 2$ (twenty 2s in all)

Comment

The answers are at the end of this booklet.

Notice that the *same* numbers must be multiplied together before this notation can be used. So, for example,

$$2 \times 2 \times 2 \times 3 \times 3 \times 3$$

cannot be written as

$$2^6 \text{ or as } 3^6$$

but it *can* be written as

$$2^3 \times 3^3$$

1.2 Negative and zero exponents

Study note: The material in this section assumes that you are comfortable with the use of negative numbers and fractions. If this is not the case, you might like to refer to a book which deals with these topics, for example Countdown to Mathematics vol. 1 *by L. Graham and D. Sargent, published by Addison-Wesley, Slough, in 1981. (Don't be put off by its publication date; it's still very useful!) The ISBN number is 0 201 13730 5.*

I'll start this section by looking at some patterns. As you already know,

$$5 \times 5 = 5^2$$

If I multiply by one more 5, I get

$$5 \times 5 \times 5 = 5^3$$

Notice that the exponent has gone *up* by 1. If I multiply by one more 5, I get

$$5 \times 5 \times 5 \times 5 \quad 5^4$$

and once again the exponent has gone up by 1.

But now suppose that I start with

$$5 \times 5 \times 5 \times 5 \quad 5^4$$

and *divide* by 5. Then I get

$$\frac{5 \times 5 \times 5 \times 5}{5} = 5 \times 5 \times 5 = 5^3$$

Notice that in dividing by 5 the exponent has gone *down* by 1.

If I continue dividing by 5 I get

$$\frac{5\times5\times5}{5} = 5\times5 = 5^2$$

and the exponent goes down by 1 again. What happens if I divide by 5 again? I get

$$\frac{5\times5}{5} = 5$$

If I continue the pattern of exponents going down by 1 for each division by 5 then I'd have to write this as 5^1. And indeed, 5^1 is taken to be 5, and similarly for any other base: 2^1 is 2, 3^1 is 3, and so on.

So I have arrived at the result that any base to the power 1 equals itself. This is something you need to remember.

But what happens if I divide by 5 once more? I get

$$\frac{5}{5} = 1$$

Again, using the pattern of the exponent going down by 1 for each division by 5, I'd have to write this as 5^0. And indeed, 5^0 is taken to be 1, and similarly for any other base: 2^0 is 1, 3^0 is 1, and so on.

So I have arrived at the result that any base to the power zero equals 1. This is again something you need to remember.

Suppose now that I divide by 5 once more. I get

$$\frac{1}{5}$$

and if again I use the pattern of exponents going down by 1, I now have an exponent of 1 less than 0, which is –1. I'd have to write this as 5^{-1}. And indeed 5^{-1} is taken to be 1/5.

Similarly,

$$2^{-1} = \frac{1}{2}$$

and

$$10^{-1} = \frac{1}{10}$$

and

$$7^{-1} = \frac{1}{7}$$

and so on.

If I continue the division by 5 process, I get

$$\frac{1}{5 \times 5} = \frac{1}{5^2} = 5^{-2}$$

then

$$\frac{1}{5 \times 5 \times 5} = \frac{1}{5^3} = 5^{-3}$$

and so on. From this comes the general rule that

$$\frac{1}{2^4} = 2^{-4}$$

$$\frac{1}{10^9} = 10^{-9}$$

and so on.

Activity 2

Write each of these expressions using a negative exponent.

(a) $\frac{1}{2^3}$

(b) $\frac{1}{10^6}$

(c) $\frac{1}{6^5}$

(d) $\frac{1}{2 \times 2}$

(e) $\frac{1}{10 \times 10 \times 10}$

Comment

The answers are at the end of this booklet.

More about reading exponent notation

The notation I have just introduced is said as follows, using base 2 as an example:

2^1 is read as ‘two to the one’

2^0 is read as ‘two to the zero’ or ‘two to the nought’

2^{-1} is read as ‘two to the minus one’

2^{-2} is read as ‘two to the minus two’

and so on.

1.3 Summary

In an expression like 2^4, 2 is called the base and 4 the exponent. This expression can be said as 'two to the fourth' or 'two to the four'.

The exponents 2 and 3 are said as 'squared' and 'cubed' respectively.

Using the example of 2 as the base,

$$2^2 = 2 \times 2$$

$$2^3 = 2 \times 2 \times 2$$

$$2^4 = 2 \times 2 \times 2 \times 2$$

and so on.

Also

$$2^1 = 2$$

$$2^0 = 1$$

For negative exponents,

$$2^{-2} = \frac{1}{2^2} = \frac{1}{2 \times 2}$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{2 \times 2 \times 2}$$

and so on.

Answers

Answers to activities

Activity 1

(a) In 5^9, the base is 5 and the exponent is 9.

(b) In 5^9, 5 is multiplied together 9 times. This is

$$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$$

(c) (i) 4^3

(ii) 6^6

(iii) 2^4

(iv) 10^2

(v) 2^{20}

Activity 2

(a) 2^{-3}

(b) 10^{-6}

(c) 6^{-5}

(d) 2^{-2}

(e) 10^{-3}