

Number and algebra

Introduction

For many people, the word ‘algebra’ conjures up a forest of letters, numbers and symbols, and a memory of apparently pointless rules for doing things with expressions and solving equations. This is similar to having the impression that cooking is only about following complicated recipes using special techniques. It misses the point about algebra, its power and simplicity.

Algebra is about generalized mathematical thinking arising from seeing patterns and relationships. ‘Algebra is an attribute, a fundamental power of the mind. Not of mathematics only.’ (Gattegno, 1877) It is about expressing those patterns in words, symbols, diagrams and graphs, and interpreting what is observed. The important patterns are ones that are not just particular to one situation, but are generalizations that apply to many different situations that have underlying similarities. Expressing these perceived generalities is the root of algebra, which is why algebra is also concerned with:

- expressing the same thing in different forms;
- ‘doing and undoing’.

This section is in five main parts, all of which are aspects of algebra:

- finding and using pattern – saying and recording what is seen in different ways;
- generalized arithmetic – becoming explicitly aware of the rules for manipulating numbers;
- finding the unknown – using a symbol to stand for an as-yet-unknown quantity, which can be used to form and solve equations;
- using formulas – formulas as shorthand, and using them to find values;
- picturing functions – representing functions as equations, tables and graphs.

Finding and using pattern

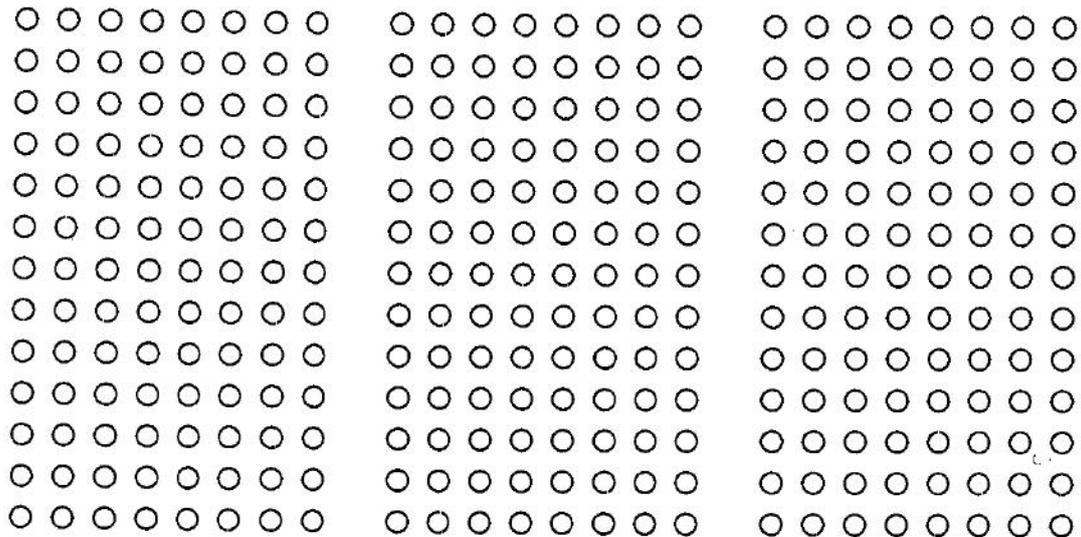
Generality is an important part of mathematics. Its most basic and clear instance is in pattern spotting, in seeing links and connections between things. This section takes a simple situation and uses it to demonstrate several of the processes of algebraic thinking, including becoming aware of a pattern and expressing it in words, pictures and symbols.

Identifying and expressing generality

Although we often talk about ‘seeing’ a pattern, visualizing a pattern in your head is not the only way of being aware of it. You may find a pattern more obvious when spoken; or you may prefer to draw diagrams or use words and symbols. These are all important ways of expressing patterns and you can improve your algebraic thinking by being able to move between them. The following tasks are designed to help you gain confidence in doing this.

Task 1 Shady circles

In the first block below, shade in every third circle starting from the second, counting from left to right, row by row.



Comment

Many people find that at some point they shift from counting to following a pattern. At first they are not confident of the pattern, and so keep counting, but then the pattern takes over. If you did not notice this happening, try another version on the second block: shade in every fifth circle starting from the third.

Task 2 More shady circles

On a fresh block, shade in every third circle starting from the second, but this time counting from left to right in the first row, then right to left in the second, and so on, weaving your way down the rows. Look out for when your brain moves from counting to having confidence in the pattern. Try to say what happened.

Comment

These tasks illustrate the power we all have for seeking out and employing patterns. The tasks started with a given counting rule, 'shade in every ...' but

in doing the task the brain simplified it to something like 'lines are developing' and 'just continue the lines...'.

Tasks 1 and 2 were concerned with *seeing* a pattern. To reach generality you need to be able to *express* a pattern in words and symbols. How you do this is closely connected with the way in which you see the pattern. The following tasks take you through the process of expressing generality arising from a sequence of diagrams.
