7.B2 Nicolas Chuquet on exponents

A Boethius says in his first book and in the first chapter, the science of numbers is very great, and among the sciences of the quadrivium it is one in the pursuit of which every man ought to be diligent. And elsewhere he says that the science of numbers ought to be preferred as an acquisition before all others, because of its necessity and because great secrets and other mysteries which there are in the properties of numbers. All sciences partake of it, and it has need of none. […]

To understand the reason why denomination of number is added to denomination, and to have knowledge of the order of numbers which was mentioned in the first chapter, it is necessary in a continuous sequence, like 1, 2, 4, 8, 16, 32, etc, or 3, 9, 27 etc.

Numbers	Denomination
1	0
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9
1 024	10
2 024	11
4 096	12
8 192	13
16 384	14
32 768	15
65 536	16
131 072	17
262 144	18
524 288	19
1 048 576	20

Now it is necessary to know that 1 represents and is in the place of numbers, whose denomination is 0.2 represents […] the first terms, whose denominations is 1.4 holds the place of the second terms, whose denomination is 2. And 8 is the place of the third terms, 16 holds the place of the fourth terms, 32 represents the fifth terms, and so for the others. Now whoever multiplies 1 by 1, it comes to 1, and because 1 multiplied by 1 does not change at all, neither does any other number when it is multiplied by 1 increase or diminish, and for this consideration, whoever multiplies a number by a number, it comes to a number, whose denomination is 0. And whoever adds 0 to 0 makes 0. Afterwards, whoever multiplies 2, which is the first number, by 1, which is a number, the multiplication comes to 2: then afterwards, whoever adds their denominations, which are 0 and 1,, it makes 1; thus the multiplication comes to 2^1. And from this it comes that when multiplies numbers by first terms or vice versa, it comes to first terms. Also whoever multiplies 2^1 by 2^1, it comes to 4 which is a second number. Thus the multiplication amounts to 4^2. For 2 multiplied by 2 makes 4 and adding the denominations, that is 1 with 1, makes 2. And from this it comes that whoever multiplies first terms by first terms, it comes to second terms. Likewise who ever multiplies 2^1 by 4^2, it comes to 8^3. For 2 multiplied by 4 and 1 added with 2 makes 8^3. And thus whoever multiplies first terms by second terms, it comes to third terms. Also, whoever multiplies 4^2 by 4^2, it comes to 16 which is a fourth number, and for this reason whoever multiplies second terms by second terms, it comes to fourth terms. Likewise whoever multiplies 4 which is a second number by 8 which is a third number makes 32 which is a fifth number. And thus whoever multiplies second terms by third terms or vice versa, it comes to fifth terms. And third terms by fourth terms comes to 7^{th} terms, and fourth terms by fourth terms, it comes to 8^{th} terms, and so for the others. In this discussion there is manifest a secret which is in the proportional numbers. It is that whoever multiplies a proportional number by itself, it comes to the number of the double of its denomination, as, whoever multiplies 8 which is a third number by itself, it comes to 64 which is a sixth. And 16 which is a fourth number multiplied by itself should come to 256, which is an eighth. And whoever multiplies 128 which is the 7^{th} proportional by 512 which is the 9^{th}, it should come to 65 536 which is the 16^{th}.