2.5 Binary addition

These questions relate to Sections 14.1 and 14.2 of Block 1. Question 6 relates to part (a) of Activity 41; Question 7 refers to part (b) of Activity 41 and to 2s complement overflow.

Question 6

Add the following unsigned 4-bit binary integers:

* (a) 0111 + 1000
  (b) 0101 + 1010
* (c) 0101 + 0101
† (d) 0110 + 0011
* (e) 0111 + 0111

Question 7

(a) Add the following unsigned 8-bit binary integers:

(i) 1000 1111 + 0011 0000
* (ii) 1000 0001 + 1101 0011
† (iii) 0111 0000 + 0111 1111
(iv) 0111 0000 + 0110 1011
(v) 0101 0101 + 0101 0101

(b) Suppose that instead all of the binary integers in part (a) had been *2's complement* integers. In which additions would 2's complement overflow have occurred?

2.6 Binary subtraction

These questions relate to Section 14.3 of Block 1. Question 8 relates to Activity 42 and Question 9 to Activity 43.

Question 8

Find the 2's complement of each of these signed integers:

* (a) 0111 1011
* (b) 1011 1000
  (c) 0101 1111
  (d) 1100 0111
  (e) 0111 0101

Question 9

Carry out the following subtractions by first finding the additive inverse of the number to be subtracted. All numbers are in 2's complement representation.

† (a) 0101 0000 – 0100 1101
(b) 1011 0111 – 0000 1111
(c) 0101 1100 – 1111 0001
(d) 0111 1000 – 1101 1100
(e) 1001 1110 – 0111 1011

Question 6

(a) First write the two numbers to be added one under the other:

$$\begin{array}{ccccc} & 0 & 1 & 1 & 1 \\ + & 1 & 0 & 0 & 0 \\ \hline \end{array}$$

Then start by adding the two rightmost bits. This is 1 + 0, which is 1:

$$\begin{array}{ccccc} & 0 & 1 & 1 & 1 \\ + & 1 & 0 & 0 & 0 \\ \hline & & & & 1 \end{array}$$

Next add the two next-to-rightmost bits. This is again 1 + 0, which is 1:

$$\begin{array}{ccccc} & 0 & 1 & 1 & 1 \\ + & 1 & 0 & 0 & 0 \\ \hline & & & 1 & 1 \end{array}$$

Repeat this for the two next-to-leftmost bits. Once again, this is 1 + 0, which is 1:

$$\begin{array}{ccccc} & 0 & 1 & 1 & 1 \\ + & 1 & 0 & 0 & 0 \\ \hline & & 1 & 1 & 1 \end{array}$$

And finally add the two leftmost bits. This is 0 + 1, which is 1:

$$\begin{array}{ccccc} & 0 & 1 & 1 & 1 \\ + & 1 & 0 & 0 & 0 \\ \hline & 1 & 1 & 1 & 1 \end{array}$$

So the answer is 1111.

(b) The sum of 0101 and 1010 is 1111.

(c) First write the two numbers to be added one under the other:

$$\begin{array}{ccccc} & 0 & 1 & 0 & 1 \\ + & 0 & 1 & 0 & 1 \\ \hline \end{array}$$

Then start by adding the two rightmost bits. This is 1 + 1, which is 0 and carry 1 (notice that the carried 1 goes under the *next* column):

	0	1	0	1	
+	0	1	0	1	
				0	
			1		(carry)

Next add the two next-to-rightmost bits plus the carry bit. This is 0 + 0 + 1, which is 1:

	0	1	0	1	
+	0	1	0	1	
			1	0	
			1		(carry)

Now add the two next-to-leftmost bits. This is 1 + 1, which is 0 and carry 1:

	0	1	0	1	
+	0	1	0	1	
		0	1	0	
	1		1		(carry)

And finally add the two leftmost bits plus the carry bit. This is 0 + 0 + 1, which is 1:

	0	1	0	1	
+	0	1	0	1	
	1	0	1	0	
	1		1		(carry)

So the answer is 1010.

(d)

	0	1	1	0	
+	0	0	1	1	
	1	0	0	1	
	1	1			(carry)

So the answer is 1001.

(e) First write the two numbers to be added one under the other:

	0	1	1	1
+	0	1	1	1

Then start by adding the two rightmost bits. This is 1 + 1, which is 0 and carry 1:

	0	1	1	1	
+	0	1	1	1	
				0	
			1		(carry)

Next add the two next-to-rightmost bits plus the carry bit. This is 1 + 1 + 1, which is 1 and carry 1:

	0	1	1	1	
+	0	1	1	1	
			1	0	
		1	1		(carry)

Now add the two next-to-leftmost bits plus the carry bit. This is 1 + 1 + 1, which is 1 and carry 1:

	0	1	1	1	
+	0	1	1	1	
		1	1	0	
	1	1	1		(carry)

And finally add the two leftmost bits plus the carry bit. This is 0 + 0 + 1, which is 1:

	0	1	1	1	
+	0	1	1	1	
	1	1	1	0	
	1	1	1		(carry)

So the answer is 1110.

Question 7

(a) The method here is exactly the same as in Question 6.

(i) The sum of 1000 1111 and 0011 0000 is 1011 1111.

(ii) First, write the two numbers one under the other:

	1	0	0	0	0	0	0	1	
+	1	1	0	1	0	0	1	1	

Then add the two rightmost bits. 1 + 1 is 0 and carry 1:

	1	0	0	0	0	0	0	1	
+	1	1	0	1	0	0	1	1	
								0	
							1		(carry)

Next, add the two next-to-rightmost bits and the carry bit. 0 + 1 + 1 is 0 and carry 1:

	1	0	0	0	0	0	0	1	
+	1	1	0	1	0	0	1	1	
							0	0	
						1	1		(carry)

Working towards the left, the next sum is 0 + 0 + 1, which is 1:

	1	0	0	0	0	0	0	1	
+	1	1	0	1	0	0	1	1	
						1	0	0	
						1	1		(carry)

The next bit towards the left in the sum is found from 0 + 0, and so is 0:

	1	0	0	0	0	0	0	1	
+	1	1	0	1	0	0	1	1	
					0	1	0	0	
						1	1		(carry)

The next bit towards the left is 0 + 1 and so is 1:

	1	0	0	0	0	0	0	1	
+	1	1	0	1	0	0	1	1	
				1	0	1	0	0	
						1	1		(carry)

The next bit towards the left is 0 + 0 and so is 0:

	1	0	0	0	0	0	0	1	
+	1	1	0	1	0	0	1	1	
			0	1	0	1	0	0	
						1	1		(carry)

The next-to-leftmost bit is 0 + 1, which is 1:

	1	0	0	0	0	0	0	1	
+	1	1	0	1	0	0	1	1	
		1	0	1	0	1	0	0	
						1	1		(carry)

And finally the leftmost bit is found from 1 + 1, which is 0 and carry 1:

	1	0	0	0	0	0	0	1	
+	1	1	0	1	0	0	1	1	
	0	1	0	1	0	1	0	0	
1						1	1		(carry)

So this sum needs 9 bits to hold it, and is 1 0101 0100.

(iii)

	0	1	1	1	0	0	0	0	
+	0	1	1	1	1	1	1	1	
	1	1	1	0	1	1	1	1	
	1	1	1						(carry)

So the answer is 1110 1111.

(iv) 0111 0000 plus 0110 1011 is 1101 1011.

(v) 0101 0101 plus 0101 0101 is 1010 1010.

(b) The discussion of 2's complement overflow in the Block 1 text (Section 14.2) indicates that it is only a possibility if two negative numbers are being added, or if two positive numbers are being added. If the numbers in part (a) above are 2's complement values, then the sum in part (i) represents a positive and a negative value being added together and there will be no 2's complement overflow. In all the rest of (ii) to (v) it is necessary to compare the sign of the result with the signs of the two numbers being added. In all of them the sign of the answer is *different* from the signs of the two numbers being added. (Remember that in part (ii) the extra, ninth, bit should be ignored in 2's complement addition; it is the value of the leftmost of the eight bits that matters.) Hence 2's complement overflow has occurred in all of parts (ii) to (v).

Question 8

In each case, the 2's complement is formed by first finding the complement (1's complement) and then adding 1. The complement is formed by changing each 1 to 0 and each 0 to 1.

(a) The complement of 0111 1011 is 1000 0100. Adding 1 to this gives 1000 0101. So this is the 2's complement.

(b) The complement of 1011 1000 is 0100 0111. Adding 1 to this gives 0100 1000. So this is the 2's complement.

(c) The 2's complement of 0101 1111 is 1010 0001.

(d) The 2's complement of 1100 0111 is 0011 1001.

(e) The 2's complement of 0111 0101 is 1000 1011.

Question 9

In each case, the 2's complement of the number to be subtracted must be found, as in Question 8. Then this can be added to the other number, as in Question 7.

(a) The 2's complement of the number to be subtracted, 0100 1101, is 1011 0011. Hence the sum is:

	0	1	0	1	0	0	0	0	
+	1	0	1	1	0	0	1	1	
	0	0	0	0	0	0	1	1	
1	1	1	1						(carry)

A carry to a ninth bit can be ignored in 8-bit 2's complement arithmetic, and so the answer is 0000 0011.

(b) The answer is 1010 1000.

(c) The answer is 0110 1011.

(d) The answer is 1001 1100.

(e) The answer is 0010 0011.

Notice that in parts (d) and (e) 2's complement overflow has occurred. In (d) adding two positive numbers has produced an apparently negative result, and in (e) adding two negative numbers has produced an apparently positive result.