Maths skills: resource 1

Scientific notation

Going up: powers of ten for large numbers

It is estimated that the total volume of water stored on the Earth is 1,460,000,000 km³. When dealing with large numbers like one thousand four hundred and sixty million (1,460,000,000), it becomes tedious to write out the number in words or to keep writing out all of the zeros. Worse still, it is very easy to lose some of the zeros or add extra ones by mistake. Fortunately, we can refer to large numbers without having to write out all of the zeros. The powers of ten notation is less prone to errors and tedium because it removes the zeros. We will introduce the powers of ten notation with some numbers more manageable than 1,460,000,000, though.

One thousand is ten times ten times ten:

\[ 10 \times 10 \times 10 = 1000 \]

We can use powers notation to write 1000 = \(10^3\).

<table>
<thead>
<tr>
<th>Question</th>
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<tbody>
<tr>
<td>How do you think you would write 100 in powers of ten?</td>
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<table>
<thead>
<tr>
<th>Answer</th>
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<tr>
<td>Two tens are multiplied together to give one hundred ((10 \times 10 = 100)) so the superscript after the 10 must be 2. That’s (10^2).</td>
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When expressing 100 and 1000 in powers of ten, there are no great savings on writing zeros, but what about one million (1000000)? One million is the product of multiplying together six tens:

\[ 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1000000 \]

so it is written as \(10^6\). Now you begin to see the benefit of the powers of ten notation.

One thousand is often written not just as \(10^3\) but as \(1 \times 10^3\). Spoken aloud, this would be expressed, ‘one times ten to the power three’ or just ‘one times ten to the three’. Likewise one million is either \(1 \times 10^6\) or simply \(10^6\). Now we can give two alternative explanations that may help you to get to grips with powers of ten. The power of ten shows how many times 1 has been multiplied by 10. Taking \(1 \times 10^3\) as an example, 1000 is seen to be \(1 \times 10 \times 10 \times 10\). In a second view, the power of ten shows how many places the decimal point has to move to the right to give the actual number. If we write 1 as 1.0 to remind ourselves where the decimal point is, then one move to the right would turn 1.0 into 10.0, a second move would give 100.0 and a third move would give 1000.0, that is, one thousand.

\[ \ʃʃʃʃ \]
\[ 1.000 \]

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You do not have to recall both of these ways of understanding powers of ten; just use
the one that suits you best, or develop your own way of fixing the idea in your
armoury of mathematical techniques.

Let’s go back to the total amount of water on the Earth. Using the powers of ten
notation, 1,460,000,000 could be written as $1.46 \times 10^9$. A significant saving on zeros!
The complete number would be spoken as ‘one point four six times ten to the power
9’ or just ‘one point four six times ten to the nine’. The power of 9 tells us how many
times 1.46 has been multiplied by 10 to give the final number of 1,460,000,000. It is
nine times. That is, our number is comprised of:

\[
1.46 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10
\]

To see clearly that this expression is still one thousand four hundred and sixty million,
it helps to begin with 1.46 and work our way to the number we want by multiplying
each time by ten:

1.46

\[
1.46 \times 10 = 14.6 = 1.46 \times 10^1
\]

\[
1.46 \times 10 \times 10 = 146 = 1.46 \times 10^2
\]

\[
1.46 \times 10 \times 10 \times 10 = 1460 = 1.46 \times 10^3
\]

If we carry on doing this, we end up with:

\[
1.46 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,460,000,000\]

\[
= 1.46 \times 10^9
\]

Alternatively, you can think of each increase by one in the power of ten as moving the
decimal point one place to the right. That is, if you multiply 1.46 by 10 the decimal
point moves one place to the right, giving 14.6.

\[
1.46
\]

Likewise, to multiply 1.46 by one thousand, the decimal point moves three places to
the right, giving 1460.0. In the powers of ten notation, this is written $1.46 \times 10^3$.

\[
1.46 \times 10^3
\]

There is a convention called scientific notation that is used when writing a number
with a power of ten. Scientific notation requires the number accompanying the power
of ten to be less than 10 but equal to or greater than 1. Let’s take the example of one
million. It could be correctly expressed as $1 \times 10^6$, $10 \times 10^5$, $100 \times 10^4$, $1000 \times 10^3$,
and so on, or even as $0.1 \times 10^7$, but only the first of these obeys the convention of
scientific notation and this is the one that should be used. As a second example, it is
quite correct mathematically to write 85,000 as $85 \times 10^3$, or $0.85 \times 10^5$, but correct
scientific notation would demand $8.5 \times 10^4$.  

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Scientific notation requires the number accompanying the power of ten to be less than 10 but equal to or greater than 1.

**Question**
Express the following numbers in scientific notation:
(a) 100,000,000
(b) 400,000,000,000
(c) 35,000
(d) $95 \times 10^5$
(e) $0.51 \times 10^3$

**Answer**
(a) $100,000,000 = 1.0 \times 10^8$ when expressed in scientific notation.
(b) $400,000,000,000 = 4.0 \times 10^{11}$ when expressed in scientific notation.
(c) $35,000 = 3.5 \times 10^4$ when expressed in scientific notation.
(d) $95 \times 10^5 = 9.5 \times 10^6$ when expressed in scientific notation.
(e) $0.51 \times 10^3 = 5.1 \times 10^2$ when expressed in scientific notation.

**Question**
Write out in full the numbers corresponding to:
(a) $7.3 \times 10^4$
(b) $3.6 \times 10^6$
(c) $4.44 \times 10^5$
(d) $6.05 \times 10^3$

**Answer**
(a) $7.3 \times 10^4 = 73,000$ when written out in full
(b) $3.6 \times 10^6 = 3,600,000$ when written out in full
(c) $4.44 \times 10^5 = 444,000$ when written out in full
(d) $6.05 \times 10^3 = 6050$ when written out in full
Question
The average distance of the Earth from the Sun is 150,000,000,000 metres. Express this number in a more concise form that obeys the convention of scientific notation.

Answer

150,000,000,000 = 1.5 × 10^{11}.

There are two ways of doing this. Starting with 1.5, the decimal point has to be moved 11 places to the right to produce 150,000,000,000. Therefore the power must be 11.

An alternative approach is to recognise that 1.5 has to be multiplied by 10 eleven times to obtain 150,000,000,000. Again, this tells us that the power term must be 10^{11}.

Going down: powers of ten for small numbers

Let’s see how the powers of ten notation can be extended to cover small numbers, such as 0.00000000025 (the diameter of a water molecule).

Write down the next two numbers in each of the following two sequences.

\[ \begin{array}{cccccccc}
10000 & 1000 & 100 & \cdots & \cdots \\
1 \times 10^4 & 1 \times 10^3 & 1 \times 10^2 & \cdots & \cdots \\
\end{array} \]

In the first sequence, each successive number is divided by 10 (i.e. had one zero taken off the end) so the number that follows 100 is \( \frac{100}{10} = 1 \). The next number in that sequence must result from another division by 10. That is, we must divide 10 by 10 and \( \frac{10}{10} = 1 \). Therefore, the second answer is 1. In the second sequence of numbers, each successive number has 1 subtracted from its power, so the first answer is \( 1 \times 10^1 \) because \( 2 - 1 = 1 \). For the second answer, we must subtract 1 from the power 1.

Because \( 1 - 1 = 0 \), the next answer is \( 1 \times 10^0 \).

In fact, both sequences are the same because 10,000 is \( 1 \times 10^4 \), 1000 is \( 1 \times 10^3 \), 100 is \( 1 \times 10^2 \), and 10 is \( 1 \times 10^1 \). The implication is that \( 1 = 1 \times 10^0 \) and hence \( 10^0 = 1 \). This makes perfectly good sense if you recall that, in the second sequence given above, the power is the number of times that 1 is multiplied by ten (e.g. \( 10^2 = 1 \times 10 \times 10 \)). For \( 1 \times 10^0 \), 1 is multiplied by 10 no times at all, leaving it as 1.

Why stop at 1 or \( 10^0 \)? Using the same rules, write down the next number in each of these sequences.

\[ \begin{array}{cccccccc}
100 & 10 & 1 & \cdots & \cdots \\
1 \times 10^2 & 1 \times 10^1 & 1 \times 10^0 & \cdots & \cdots \\
\end{array} \]

In the first sequence, dividing 1 by 10 gives \( \frac{1}{10} \) or 0.1 as the next number. In this box, we’re keeping to decimals, so the answer we want is 0.1. But what about the second sequence? The answer is more straightforward than it may seem. We continue to subtract 1 from the powers of ten so that the next number in the sequence has a negative power of ten (\( 1 \times 10^{-1} \)) because \( 0 - 1 = -1 \). Remembering that the two
sequences are equivalent, it seems that $1 \times 10^{-1} = 0.1$. This is exactly right! We could equally write $10^{-1} = 0.1$.

Just as a positive power of ten denotes how many times a number is multiplied by 10, so a negative power of ten denotes how many times a number is divided by 10. For $10^{-1}$, we must divide 1 by 10 just once and we end up with 0.1.

**Question**

What is the meaning of $10^{-2}$?

**Answer**

Because the power is now $-2$, we must divide 1 by 10 twice. That is, $1 \div 10 \div 10 = 0.01$.

Another way to think about powers of ten for very small numbers involves shifting the decimal point. A negative power of ten denotes the number of places that the decimal point moves to the left. For example, think of $1 \times 10^{-2}$, which we will write as $1.0 \times 10^{-2}$ to remind us of the position of the decimal point. Starting with the number 1.0, the power of $-2$ requires us to move the decimal point two places to the left. One place to the left gives 0.1 and two places 0.01.

```
0 0 1 . 0
```

We therefore have $10^{-2} = 0.01$.

Let’s try an example. Suppose a raindrop has a breadth of about 0.002. This distance could be given in scientific notation as $2 \times 10^{-3}$ m. This is clear from the following series.

Start with: 2

Divide by ten: $2 \div 10 = 0.2 = 2.0 \times 10^{-1}$

Divide by ten again: $2 \div 10 \div 10 = 0.02 = 2.0 \times 10^{-2}$

And again: $2 \div 10 \div 10 \div 10 = 0.002 = 2.0 \times 10^{-3}$

Alternatively, in considering the meaning of ‘two times ten to the power minus three,’ you may wish to start with the number 2.0 and move the decimal point three places to the left to give 0.002.

When expressing large numbers in scientific notation, the power of ten (which is positive) denotes the number of places that the decimal point moves to the right. Similarly, when expressing small numbers in scientific notation, a negative power of ten denotes the number of places that the decimal point moves to the left.

You have seen that a negative power of ten tells you how many times you need to divide by ten, so that

$0.001 = 10^{-3} = \frac{1}{1000}$

But, of course, $1000 = 10^3$, and so
0.001 = 10\(^{-3}\) = \frac{1}{1000} = \frac{1}{10^3} \quad \text{and so } 10^{-3} = \frac{1}{10^3}

This relationship between positive and negative powers of ten is quite general, so

\(10^{-6} = \frac{1}{10^6}, \quad 10^{-8} = \frac{1}{10^8}, \quad 10^{-13} = \frac{1}{10^{13}},\) and so on.

Convention requires that, when writing large numbers in scientific notation, the power of ten should be accompanied by a number that is equal to or greater than 1 but less than 10. The same convention is used when dealing with small numbers and hence negative powers of ten. This is why 0.002, the breadth of the raindrop, is given in scientific notation as \(2 \times 10^{-3} \text{ m},\) and not as \(0.2 \times 10^{-2} \text{ m}\) or \(20 \times 10^{-4} \text{ m}.

**Question**

Express the following measurements in scientific notation:

(a) A water molecule, about 0.00000000025 across.
(b) An average-sized sand grain on a gently sloping beach, about 0.00025 across.
(c) The size of one particle of clay, the main constituent of mud, about \(1/1000000 \text{ m}\) across.
(d) The average size of a hailstone, 0.0035 m across.

**Answer**

(a) The starting point for quoting 0.00000000025 in scientific notation is 2.5 (the number that lies between 1.0 and 9.9). The decimal point has to be moved ten places to the left to reach 0.00000000025, so the power of ten must be –10 and the answer \(2.5 \times 10^{-10} \text{ m}.
\)
(b) \(2.5 \times 10^{-4} \text{ m}.
\)
(c) First of all convert the fraction \(\frac{1}{1,000,000}\) into a decimal. This is 0.000001. In scientific notation this is \(1 \times 10^{-6} \text{ m}.\) Alternatively,

\[
\frac{1}{1,000,000} \text{ m} = \frac{1}{10^6} \text{ m} = 1 \times 10^{-6} \text{ m}
\]
(d) \(3.5 \times 10^{-3} \text{ m}.
\)

**Question**

Write out in full the decimal numbers corresponding to:

(a) \(7.3 \times 10^{-4}\)
(b) \(2.9 \times 10^{-7}\)

**Answer**

(a) To find the decimal number corresponding to \(7.3 \times 10^{-4},\) the decimal point in 7.3 has to be moved four places to the left to give 0.00073. The alternative approach is to think of, and work out, \(7.3 \div 10 \div 10 \div 10 \div 10.\)
Question
Use powers of ten notation to answer the following questions:
(a) How many millimetres are there in one kilometre?
(b) How many kilometres is one millimetre equal to?

Answer
(a) From the definition of the prefixes, 1 km = 1000 m and 1 m = 1000 mm. So
\[1 \text{ km} = 1000 \times 1 \text{ m}\]
\[= 1000 \times (1000 \text{ mm})\]
\[= 1,000,000 \text{ mm}\]
\[= 10^6 \text{ mm}\]
(b) Since \(10^6 \text{ mm} = 1 \text{ km}\), then
\[1 \text{ mm} = \frac{1 \text{ km}}{10^6} = \frac{1}{10^6} = 10^{-6} \text{ km}\]

Using a calculator for scientific notation
In your future studies, you are likely to be doing many calculations with numbers in scientific notation, so it is important that you know how to input them into your calculator efficiently and how to interpret the results.

First of all, make sure that you can input numbers in scientific notation into your calculator. This might be labelled as EXP, EE, E or EX, but there is considerable variation between calculators. Make sure that you can find the appropriate button on your calculator. Using a button of this sort is equivalent to typing the whole of \('\times 10\) to the power’. So, on a particular calculator, keying 2.5 EXP 12 enters the whole of \(2.5 \times 10^{12}\).

To enter a number such as \(5 \times 10^{-16}\) into your calculator, you may need to use the button labelled something like \(\pm\) in order to enter the negative power.

To enter a number such as \(10^9\) into your calculator using the scientific notation button, it is helpful to remember that \(10^9\) is written as \(1 \times 10^9\) in scientific notation, so you will need to key in something like \(1 \text{ EXP} \ 9\).

In addition to being able to enter numbers in scientific notation into your calculator, it is important that you can understand your calculator display when it gives an answer in scientific notation. Enter the number \(2.5 \times 10^{12}\) into your calculator and look at the display. Again there is considerable variation from calculator to calculator, but it is likely that the display will be similar to one of those shown in Figure 1. The 12 at the right of the display is the power of ten, but notice that \(\text{the ten itself is frequently not displayed}\). If your calculator is one of those which displays \(2.5 \times 10^{12}\) as shown in Figure 1e, then you will need to take particular care; this \(\text{does not mean } 2.5^{12}\) on this occasion. You should be careful not to copy down a number displayed in this way on
your calculator as an answer to a question; this could cause confusion at a later stage. No matter how scientific notation is entered and displayed on your calculator or computer, when writing it on paper you should always use the form exemplified by $2.5 \times 10^{12}$.

![Calculator displays](image)

**Figure 1**: Examples of how calculators might display the number $2.5 \times 10^{12}$.

### Question
To check that you can use your calculator for scientific notation, do the following calculations:

(a) $(4.5 \times 10^4) \times (4.0 \times 10^{11})$
(b) $(6.5 \times 10^{-27}) \times (2.0 \times 10^{-14})$
(c) $10^8 \div (2 \times 10^{-17})$

### Answer
(a) $1.8 \times 10^{16}$
(b) $1.3 \times 10^{-40}$
(c) $5 \times 10^{24}$

If you obtained the incorrect answer $5 \times 10^{25}$, it is likely that you entered $10 \times 10^8$ instead of $10^8$ into your calculator. Remember that $10^8$ can be written as $1 \times 10^8$. © The Open University 2012