Transcript

Out for the Count: The Mathematics of Voting Systems

Which method was the ideal voting system?

Andrew Potter: Did you manage to return a winner? You should have found that Candidate D was the winner! Let's see why.

There were 6 different ways to pair the candidates against one another. We did the first one as an example, which showed that between A and B, A was the preferred candidate. You should have found that, no matter who you put D up against, D is the preferred candidate. This means that, under the Condorcet method, D is declared the winner!

What did you think about the Condorcet method? Did it occur to you that there might not be a winner who wins against all the other candidates? If you did, well done, because that's a key drawback to this method, and is one of the reasons that it's not really used in practice. Although we managed to declare a winner in our example, it's not guaranteed that there will always be a Condorcet winner. Nevertheless, the Condorcet criterion is a useful principle to use when assessing the fairness of a voting system. We can see that if there is a Condorcet winner, then it is "fair" in one sense at least: that they are the preferred candidate when run against each of the other opponents.

This brings us on to the question of whether there is an ideal voting system? Unfortunately, the answer is no, but we can build "a wish list" of properties we'd like to see in a voting system, so that we can assess its fairness. For example, the Condorcet criterion is one of them – unfortunately we've seen that First Past the Post, Alternative Vote and Borda all fail to ensure that the Condorcet winner always wins.

What else would we want a fair system to have? We might ask for it to satisfy the "majority criterion". That is, if one candidate is ranked first by a majority of voters, then that candidate must win. You can show that both First Past the Post and Alternative Vote satisfy this criterion, but the Borda system doesn't.

Then there's the "participation criterion". We'd like for it to be the case that voting should always be preferable to not voting. This is true for First Past The Post and the Borda system, but in Alternative Vote, there are cases where you increase the chances of your preferred candidate winning by not bothering to vote at all!

And lastly, there's the criterion of "independence of irrelevant alternatives". This says that the result should not be changed by the inclusion or removal of a candidate who has no chance of winning. All of the systems we've looked at in this activity fail this criterion!

One of the most powerful contributions of mathematics to this area is that we can actually prove that there is no "holy grail" of voting systems, which ticks all the boxes of fairness that we might want a voting system to have. We can use mathematics to hone in on precise, rigorous definitions of the properties we consider to be "fair" and use these to make informed choices. Different systems have different properties of fairness and we might decide that one type of fairness is more important than another and make a trade-off in selecting a voting system.

I hope this activity has helped you think about how mathematics can help us in deciding what is fairness when it comes to making democratic decisions. Thank you very much for listening!