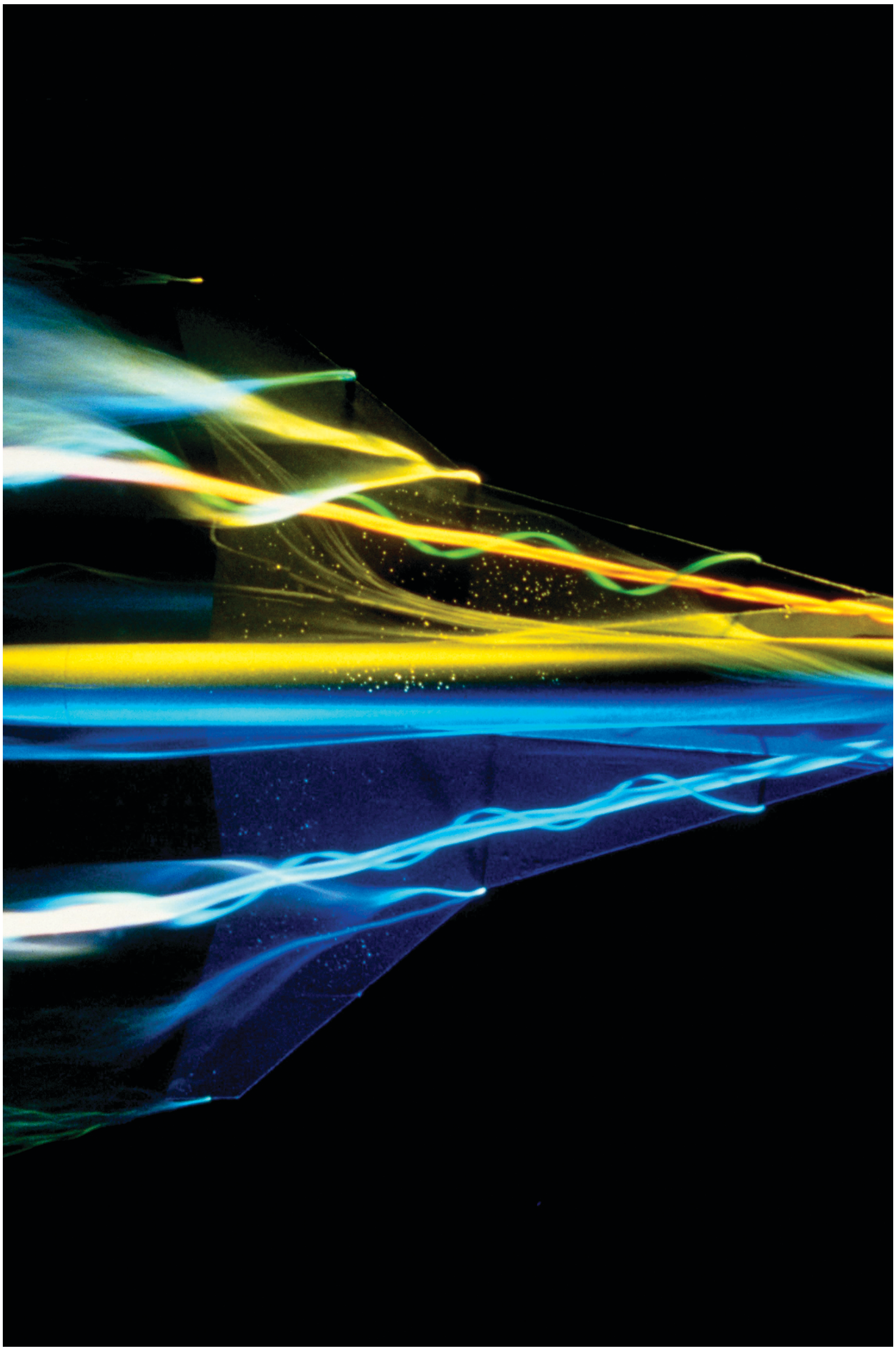
Engineering: environmental fluids



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Introduction

We all have constant daily experience of fluids, from breathing air to taking showers and checking weather forecasts, which means that we all have a great deal of intuitive knowledge about how they behave; this course will build on that knowledge.

The Earth's atmosphere and oceans are two of the most important fluids for engineers, and a working knowledge of them is required in many situations. For example, in the aeronautical and aerospace industries the behaviour and properties of the atmosphere from ground level to outer space are key aspects in the design and operation of aircraft and space vehicles. In the field of civil engineering, the simulation and study of tides and tidal flows is necessary in the design of harbours, canals, protective barrier schemes, drainage pipelines, offshore structures, etc.

In the first half of this course, you will be able to learn about the Earth's atmosphere – how properties like density, temperature and speed of sound vary with height, the effect of terrain on wind near the ground and how the Coriolis effect contributes to the formation of weather systems.

The second half of the course concerns the study of Earth's oceans, including the formation of waves and tides and how both are affected by land masses.

This free course is an adapted extract from the Open University course T229 *Mechanical engineering: heat and flow.*

Learning outcomes

After studying this course, you should be able to:

- describe the variation of fluid properties in the Earth's atmosphere between ground level and space
- understand the formation of wind, waves and tides and appreciate their significance for engineers
- calculate critical factors such as wind and wave speed, displacement amplitude and acceleration
- understand how these critical factors impact on the design of structures which interact with the ocean and sea.

1 The Earth's atmosphere and winds

Obviously, the Earth's atmosphere is hugely important in sustaining life by providing and recycling the main gases oxygen and nitrogen, recycling water from seas to rain and back again, and providing warmth and stable temperatures. It also protects life from potentially harmful effects from space such as radioactivity, heat and other radiation, and to a degree from impact with solid bodies such as meteorites and other cosmic detritus. In this section, the focus will be on its behaviour in terms of fluids, statics and dynamics.

1.1 The properties of the atmosphere

The atmosphere's characteristics are important in various aspects of engineering such as the design and operation of aircraft, road and rail vehicles, buildings and other structures, and the all-important weather forecasting. The atmosphere has also provided a source of mechanical power down the ages – for example for windmills and wind pumps, sailing vessels, etc., and now of course a source for electrical power generation with wind-driven turbines and wind farms. The atmosphere's properties and behaviour of interest in these fields include density, pressure, temperatures, wind speeds, accelerations and turbulence. These are very rarely stable being in a constant state of flux because of the rotation and other motions of the Earth with respect to the Sun and the intermittent heating and cooling cycles which result.

Although vital, the atmosphere height-wise is relatively very thin in relation to the diameter of the Earth. It has been likened to the thickness of the skin on an apple, but there is no real edge or boundary at the upper level. A common rule of thumb is for the upper limit (the Kármán line) to be 100 km from sea level, but in reality the density and pressure continue to diminish with height, with traces of atmosphere being detected at many hundreds of kilometres further up. Even at the height of some of the lower-orbit satellites (say around 150 km), there is a discernible atmosphere which will ultimately slow them down enough for them to fall back to Earth, and all but the largest will burn up before they hit the ground. The largest ones are decommissioned carefully so as to return to Earth in specified safe areas.

The density of the atmosphere at ground/sea level on a still day is taken as 1.225 kg m^{-3}. The mean pressure at this level is stated as 101.325 kPa, which can be read as a mass of air of just over 10 tonnes on each square metre on the Earth's surface. However, because the pressure reduces with height above ground level, the density decreases in proportion, hence the gradual diminishing with no definite boundary.

An illustrative statistic for the Earth's atmosphere

As a matter of comparison, if the density of the atmosphere at sea level did remain constant all the way up, what would be the height or thickness of the atmosphere to create the sea-level pressure? Give your answer in km to 3 significant figures.

Solution

From the fundamental law of hydrostatics

$$P_{\text{atm}} = \rho g h$$

so

$$
\begin{aligned}
h &= \frac{P_{\text{atm}}}{\rho g} \\
&= \frac{101.325 \times 10^3\ \text{Pa}}{1.225\ \text{kg m}^{-3} \times 9.81\ \text{m s}^{-2}} \\
&= 8431.6\ldots\ \text{m}.
\end{aligned}
$$

Therefore the height of the atmosphere would be 8.43 km (to 3 s.f.).

Activity 4

If the height of the atmosphere was 100.0 km, what would be the atmospheric pressure at sea level if the density was constant at 1.225 kg m^{-3}? Express the answer as a comparison with the standard figure of 101.325 kPa. Give your answer to 3 significant figures.

Answer

From the fundamental law of hydrostatics

$$P_{\text{atm}} = \rho g h$$

therefore the atmospheric pressure will be

$$
\begin{aligned}
P_{\text{atm}} &= 1.225\ \text{kg m}^{-3} \times 9.81\ \text{m s}^{-2} \times 100.0 \times 10^3\ \text{m} \\
&= 1201.725\ \text{kPa}.
\end{aligned}
$$

Comparing this to the standard figure,

$$\frac{1201.725\ \text{kPa}}{101.325\ \text{kPa}} = 11.860\ldots$$

which is 11.9 times (to 3 s.f.) greater than the standard figure.

In reality, the atmosphere's height and thickness are many times what they would be if the density was constant. The real density at sea level varies from a maximum of approximately 1.4 kg m^{-3} to a very low density at and above about 6 km height. The reduction of density of the atmosphere is evident at quite low levels; this limits the heights at which aircraft can generate sufficient lift and is why they have to fly so fast to climb high. Figure 1 is based on previously published data from the United Nations International Civil Aviation Organization (ICAO) regarding temperature variation with height. Other properties of interest can be deduced from their relationships with temperature.

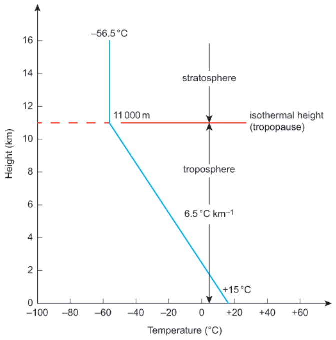


Figure 1 Temperature variation with height in standard atmosphere

Note that from sea level (zero on the vertical axis) the temperature reduces with height in a directly linear manner up to about 11 km altitude. This lower region (0–11 km) is called the **troposphere**. Above 11 km the temperature stays the same at about –56.5 °C for increasing heights. This region is the **stratosphere**, and the height at which the constant temperature starts is the **isothermal height** (sometimes isothermal level), also known as the **tropopause**. At far greater altitudes there is more variation but because the air is so thin by then the concept of atmospheric temperature does not mean very much.

The values describing the graph in Figure 1 vary a little around the Earth, but typically the sea-level mean (average) temperature is assumed to be 15 °C and the slope or gradient of the graph from zero to 11 km height in degrees per km change in height is −6.5 °C km^{-1}. Above 11 km, the gradient is of course zero as the temperature stays constant.

Calculating the height of the atmosphere at 0 °C

From the above data determine the height h at which the air temperature reaches 0° C. Give your answer to 2 significant figures.

Solution

In Figure 1, studying the proportions of the slope part of the graph by similar triangles of height (vertical) divided by temperature (horizontal) gives

$$\frac{11000\,\mathrm{m}}{56.5\,^{\circ}\mathrm{C} + 15\,^{\circ}\mathrm{C}} = \frac{z}{15\,^{\circ}\mathrm{C}}$$

so

$$h = \frac{15\,^{\circ}\mathrm{C} \times 11\,000\,\mathrm{m}}{71.5\,^{\circ}\mathrm{C}} = 2307.69\ldots\ \mathrm{m}.$$

Therefore the height at which the air temperature reaches 0 °C is 2.3 km (to 2 s.f.).

Other properties of air have been deduced or derived from known relationships with temperature and some of these are presented in Figure 2 in non-dimensionalised form so as to fit them all on one graph.

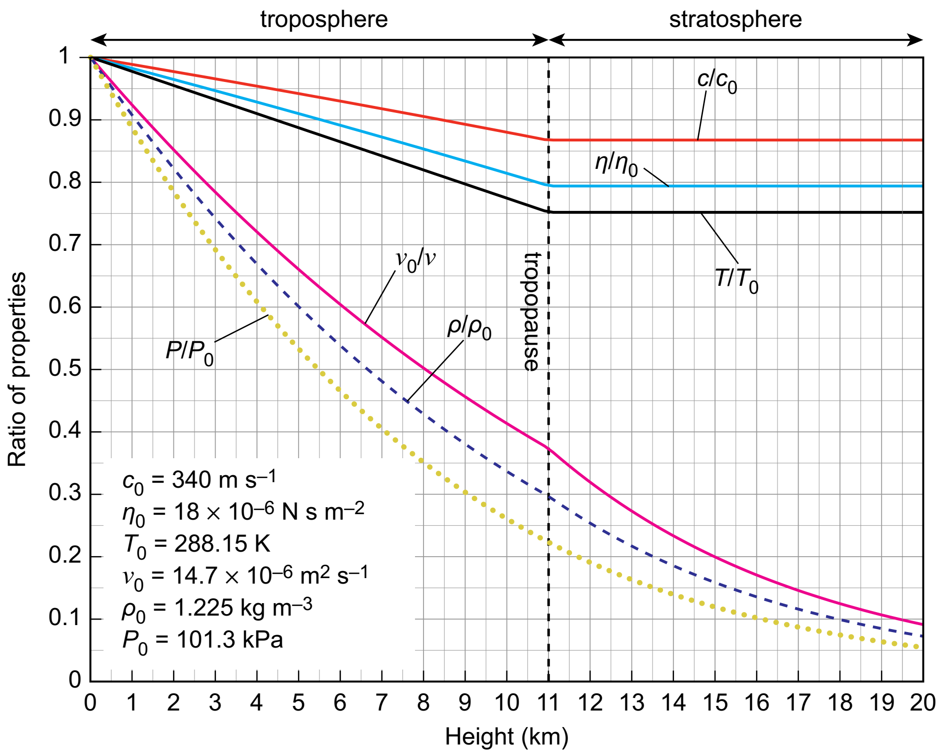


Figure 2 Properties of the standard atmosphere. Note that kinematic viscosity η increases with altitude, so the inverse ratio $\dfrac{\eta_0}{\eta}$ is shown.

The constants used to non-dimensionalise the properties are depicted on the graph with the subscript '0' and correspond to the values at sea level. Note that the height is now on the horizontal axis with the 11 km height marked as a vertical line. The properties shown are the local speed of sound (sonic velocity), c, the dynamic viscosity, η, the kinematic viscosity, ν, the density, ρ, and the pressure, P. For instance, it can be seen from the graph that the speed of sound c_0 = 340m s^{-1} at sea level and decreases linearly through the troposphere; above the tropopause it remains constant, given approximately by

$$c = c_0 \times 0.865 = 294\,\mathrm{m\,s^{-1}}.$$

In the troposphere two properties, pressure and density, can be modelled by simple expressions as follows.
For pressure:

$$\frac{P}{P_0} = \left(1 - \frac{z}{z_C}\right)^{5.256} \qquad \text{(Equation 1)}$$

where P is the absolute pressure, P_0 (= P_{atm}) is the standard sea-level value of 101.3kPa, z is the altitude under consideration and z_C is a constant value of 44 300 m.

For density:

$$\frac{\rho}{\rho_0} = \left(1 - \frac{z}{z_C}\right)^{4.256} \qquad \text{(Equation 2)}$$

where ρ is the required density, ρ_0 is the standard sea-level value of 1.225kg m^{-3}, z is the altitude under consideration and z_C is a constant value of 44 300 m.

Above the isothermal level of the tropopause, the pressure and density are based on the values at this isothermal level, as:

$$\frac{P}{P_1} = \frac{\rho}{\rho_1} = \exp\left(-\frac{z - z_1}{z_\mathrm{d}}\right) \qquad \text{(Equation 3)}$$

where z_d is a constant value of 6377 m and P_1, ρ_1 and z_1 are the values of pressure, density and altitude at the isothermal level, and which will be, respectively, 22.6 kPa, 0.364 kg m^{-3} and 11000 m.

Reductions in atmospheric pressure with height

What is the percentage reduction in atmospheric pressure at the height when the air temperature drops to 0 °C? Give your answer to 3 significant figures.

Solution

Rearranging Equation 1 to find pressure P gives

$$P = \left(1 - \frac{z}{z_\mathrm{C}}\right)^{5.256} \times P_0.$$

From Example 2, the height of the atmosphere at which the air temperature reaches 0 °C was found to be $z = 2307.69\,\mathrm{m}$, so

$$\begin{aligned} P &= \left(1 - \frac{2307.69\,\mathrm{m}}{44\,300.0\,\mathrm{m}}\right)^{5.256} \times \left(101.3 \times 10^3\ \mathrm{Pa}\right) \\ &= 76.47\ldots \times 10^3\ \mathrm{Pa}. \end{aligned}$$

Therefore the percentage reduction in pressure is

$$\frac{(101.3 - 76.47\ldots)\ \mathrm{kPa}}{101.3\,\mathrm{kPa}} \times 100\% = 24.5\%\ (\text{to 3 s.f.}).$$

Activity 5

What is the percentage reduction in atmospheric density at the height when the air temperature reaches 0 °C, compared with the value at sea level? Give your answer to 3 significant figures.

Answer

Equation 2,

$$\frac{\rho}{\rho_0} = \left(1 - \frac{z}{z_C}\right)^{4.256}$$

can be rearranged to find density

$$\rho = 1.225\,\mathrm{kg\,m^{-3}} \times \left(1 - \frac{2307.69\,\mathrm{m}}{44\,300.0\,\mathrm{m}}\right)^{4.256}$$
$$= 0.9755\ldots\ \mathrm{kg\,m^{-3}}.$$

Therefore the percentage reduction in density is

$$\frac{(1.225 - 0.9755\ldots)\ \mathrm{kg\,m^{-3}}}{1.225\,\mathrm{kg\,m^{-3}}} = 20.4\%\ (\text{to 3 s.f.}).$$

Atmospheric model equations

The troposphere denotes the part of Earth's atmosphere from an altitude of zero to 11 000 m. In this region the local atmospheric pressure can be evaluated from the expression in equation (4) as

$$\frac{P}{P_0} = \left(1 - \frac{z}{z_\mathrm{C}}\right)^{5.256}$$

where P_0 is the standard sea-level value of 101.3 kPa, z is the altitude in metres under consideration and z_C is a constant value of 44 300 m.

Also in the troposphere, the local air density can be evaluated from the expression in equation (5) as

$$\frac{\rho}{\rho_0} = \left(1 - \frac{z}{z_\mathrm{C}}\right)^{4.256}$$

where ρ is the required density, ρ_0 is the standard sea-level value of 1.225kg m^{-3}, z is the altitude under consideration and z_C is a constant value of 44300 m.

The top level of the troposphere is known as the tropopause, and the height of the lower boundary of the tropopause, 11 000 m, is known as the isothermal height. Above this height, the pressure and density are both related by the expression in equation (6) as

$$\frac{P}{P_1} = \frac{\rho}{\rho_1} = \exp\left(-\frac{z - z_1}{z_\mathrm{d}}\right)$$

where z_d is a constant value of 6377 m and P_1, ρ_1 and z_1 are the values of the pressure, density and altitude at the isothermal level, and hence will be respectively 22.6 kPa, 0.364kg m^{-3} and 11000 m.

1.2 Upper-atmosphere winds and air movements

Although the upper atmosphere is directly first in line with radiation arriving from the Sun, the mechanism by which it is heated is less direct. A good deal of the solar radiation passes through the atmosphere and warms up the more massive and dense land masses and oceans below. These absorb and retain heat energy which is then re-radiated at infrared wavelengths that do not pass so easily through the air as the incoming radiation. This is not a uniform process, however.

The oceans have a temperature variation which is less (i.e. more steady) than that of land masses, and the clouds reflect heat back and forth in a varying manner. Combine these effects with the daily rotation of the Earth causing heat cycles and the longer heat cycles due to the seasons caused by the Earth’s axis being tilted (**obliquity**) as it orbits the Sun, and all of this adds up to a highly complex pattern of temperature variations in the atmosphere. It is the temperature variation which causes changes in density and pressure, which in turn cause air movements and winds that affect weather patterns.

Trade winds

Winds may be considered in two groups. First are the more regular settled patterns on a global scale as shown in Figure 3. This represents an overall sustained pattern of regular winds which were important in the days of sailing ships, assisting their sojourns around the world delivering and collecting goods. The term **trade winds**, which originally referred to the old English expression of ‘tracking winds’, became associated with this commercial context. These and other steady winds arise because of the uneven but regular heating of the Earth. Near the equator where the Sun’s radiant heat is most powerful, the air is heated, expands, reduces in density and rises. In rising, especially over the ocean, it cools, and water vapour condenses, giving up latent heat. This sustains the upwards motion which spills outwards North and South away from the tropics.

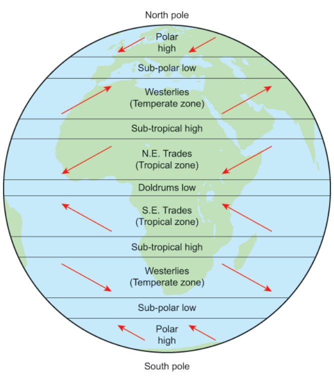


Figure 3 Regular global winds

Meanwhile cooler higher-pressure air moves towards the equatorial region because of the now lower-pressure there. These movements are deflected laterally, however, due to the

Coriolis effect of the Earth’s rotation to form the trade winds. The green box on the Coriolis acceleration outlines how this effect arises. A similar situation arises near the poles of the Earth, where the colder higher-pressure air is induced to flow away from the poles towards the lower-pressure air in the temperate zones. Again the Coriolis effect deflects these to form the so called **westerlies** in both hemispheres.

Coriolis accelerations

Any object moving on a straight path north of the equator appears to an observer on the ground to be deflected to the right. Conversely, any object moving on a straight path south of the equator appears to be deflected to the left.

This apparent deflection, resulting from movement towards or away from an axis of rotation, is a manifestation of the Coriolis acceleration or the Coriolis effect, named after Gaspard-Gustave de Coriolis (1792–1843) who pioneered the study of rotating frames of reference.

The Coriolis acceleration

The Coriolis acceleration arises in any situations when an object or body travels towards or away from the axis of a rotating frame of reference. The combination of rotation and distance from the axis results in a linear velocity, so if the distance changes the velocity also changes and the result is an acceleration at right angles to both the direction of travel and the axis of rotation. This is the Coriolis acceleration, denoted by a_{cor}.

For a simple visualisation, imagine a fairground worker walking outwards along a radius of a rotating roundabout in order to get tickets (Figure 4). Although everything on the platform is rotating at the same angular velocity about the central vertical axis, the horses further away from the centre have a higher linear velocity relative to the stationary ground. Thus, a worker moving radially outwards on the platform will need to accelerate to keep up with the horses. Equally if travelling radially inwards towards the axis of rotation the worker will need to decelerate in order to match the lower velocities of the inner horses.

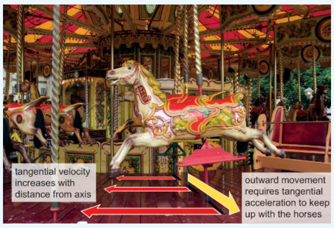


Figure 4 A typical fairground ride and manifestation of Coriolis acceleration

Figure 5 summarises the situation for different combinations of inward or outward movement and clockwise or anticlockwise rotation. The component of velocity towards or away from the axis, u_{S}, is itself rotating about a vertical axis through O.

Note the directions of u_{S}, the rotating reference angular velocity, ω, and the resulting acceleration a_{cor}.

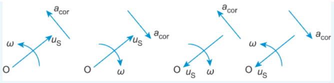


Figure 5 Directions of the Coriolis acceleration

The magnitude of the Coriolis acceleration experienced by the body is given by the simple equation:

$$a_{\text{cor}} = 2\omega u_{\text{S}}.$$

Figure 6 shows an idealised picture of the Earth viewed from far above the equator. Neglecting the tilt angle (obliquity), the Earth rotates from west to east about the imaginary vertical axis joining the North and South Poles. The angular velocity of rotation, , is of course one revolution or 2π radians per full day (24 hours). (It actually takes the Earth 23 hours, 56 minutes and 4 seconds to make a full 360° rotation. The other 3 minutes and 56 seconds is needed to account for the Earth’s rotation round the sun and can be ignored for most engineering purposes.)

Consider an object or element of something moving on or near the Earth’s surface directly from north to south, shown red in Figure 6. The something could be a chunk of sea, air, a ship, artillery shell, etc.

The object has a velocity u and is at a latitude angle of θ, so the component of velocity parallel to the axis of rotation is $u\cos\theta$ and the component of velocity perpendicular to the axis of rotation is $u\sin\theta$. The Coriolis acceleration

$$a_{\text{cor}} = 2\omega u_{\text{S}}$$

where

$$u_{\text{S}} = u\sin\,\theta$$

therefore

$$a_{\text{cor}} = 2\omega u\sin\,\theta. \qquad \text{(Equation 7)}$$

The same formula applies to travel east or west: in these cases the full Coriolis acceleration of $2\omega u$ is directed outwards from the axis and the component tangential to the surface is $2\omega u\sin\theta$. Since it applies to both north–south and east–west movement, Equation 7 can be applied to any movement on the surface of the earth.

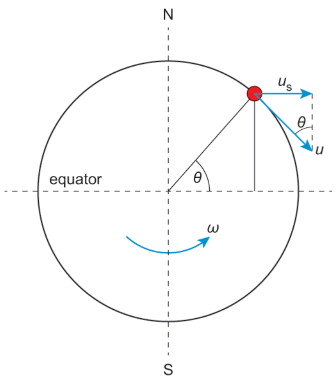


Figure 6 Schematic view of Earth from above the equator

The Coriolis effect on an Earth scale

What is the Coriolis acceleration experienced by a body on the surface of the Earth in terms of its latitude, and its velocity u? Give your answer to 3 significant figures.

Solution

As the earth makes a full rotation (2π radians) every 24 hours, a rotating reference frame moving with it will have an angular velocity given by

$$\omega = \frac{2\pi}{24 \times 60 \times 60} = 72.72\ldots \times 10^{-6}\ \text{rad}\ \ \text{s}^{-1}.$$

Using equation (8), an object travelling with a velocity u will be subject to a Coriolis acceleration of

$$\begin{aligned} a_{\text{cor}} &= 2\omega u \sin\theta \\ &= 2 \times 72.72\ldots \times 10^{-6} \times u \sin\theta \\ &= \left(145 \times 10^{-6} \times u \sin\theta\right)\ \text{m}\,\text{s}^{-2}\ (\text{to } 3\text{ s.f.}). \end{aligned}$$

This is obviously very small compared to u, but in weather systems and tidal currents over many hours or even days, these accelerations can result in major weather effects. Alternatively, if the value of u is very high, such as in artillery shells or missiles, the effect has to be allowed for in setting aim coordinates. (This is how Coriolis came into the picture.)

Activity 6

Determine the Coriolis acceleration and the accompanying lateral accelerating force on a cubic metre of air at ground level if the wind speed north to south is 80.0 km h^{-1} at a latitude of 60.0°. Use ρ_0 as the standard sea-level value of 1.225 kg m^{-3}. Give your answer to 3 significant figures.

Answer

The wind speed is

$$u = 80\ \mathrm{km\ h^{-1}} = \frac{80 \times 10^3\ \mathrm{m}}{60\,\mathrm{s} \times 60} = 22.22\ldots\ \mathrm{m\ s^{-1}}.$$

The angular velocity of the Earth is

$$\omega_{\mathrm{Earth}} = \frac{2\pi}{24\,\mathrm{s} \times 60 \times 60} = 72.72\ldots \times 10^{-6}\ \mathrm{s^{-1}}.$$

At 60 degrees the effective radial velocity is

$$\begin{aligned} u_{\mathrm{s}} &= u \sin 60^\circ \\ &= 22.22\ldots\ \mathrm{m\,s^{-1}} \times \sin 60^\circ \\ &= 19.24\ldots\ \mathrm{m\,s^{-1}}. \end{aligned}$$

So the Coriolis acceleration is

$$\begin{aligned} a_{\mathrm{cor}} &= 2\omega u_{\mathrm{s}} \\ &= 2 \times \left(72.72\ldots \times 10^{-6}\ \mathrm{s^{-1}}\right) \times \left(19.24\ldots\ \mathrm{m\,s^{-1}}\right) \\ &= 2.799\ldots \times 10^{-3}\,\mathrm{m\,s^{-2}}. \end{aligned}$$

The Coriolis force is found from $F = ma$ therefore

$$\begin{aligned} F_{\mathrm{cor}} &= ma_{\mathrm{cor}} \\ &= 1.225\,\mathrm{kg} \times \left(2.799\ldots \times 10^{-3}\,\mathrm{m\,s^{-2}}\right) \\ &= 3.43 \times 10^{-3}\,\mathrm{N}\ (\text{to 3 s.f.}). \end{aligned}$$

Cyclones and anticyclones

The second group of winds comprises the more adhoc erratic fluctuations which can turn into vortices thousands of kilometres across that last for a limited period, usually measured in days. These are the **cyclones** and **anticyclones** mentioned in weather reports. A cyclone is often referred to as a **depression** or a **low** because it centres on a region of low pressure. The low pressure will be at most around 10 or 12 per cent below standard atmospheric pressure, but on the scale of a cyclone even a drop that small can provoke huge air movements. The air begins to move radially inwards towards the low pressure, and is then deflected by the Coriolis effect as the Earth rotates.

In the northern hemisphere the Coriolis effect will tend to move the airflow direction clockwise away from the centre, as shown in Figure 7. In this figure the airflow was heading towards the central low pressure, but was then deflected away from it to an extent by the Coriolis effect. The pressure gradient inwards can counteract or overcome this tendency of deflection to the point that an equilibrium flow is set up with just enough inward force remaining to provide the centripetal acceleration for overall circular flow to develop in an anticlockwise direction, as shown in Figure 7. (In the southern hemisphere the overall flow direction is clockwise.)

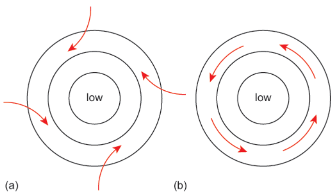


Figure 7 Development of a cyclone (northern hemisphere), (a) showing the beginnings of Coriolis deflection and (b) the balanced equilibrium flow condition

An anticyclone is often referred to as a high because it centres on a region of high pressure. The high pressure will be at most only around 5 or 6 per cent above standard atmospheric pressure with lower pressure gradients than for cyclones, leading to steadier and more gentle air movements. The air begins to move radially outwards towards the surrounding lower-pressure regions. As with cyclones, the air is then deflected by the Coriolis effect as the Earth rotates, this time inducing an overall flow direction clockwise in the northern hemisphere. Figure 8 shows the idea. (In the southern hemisphere the overall flow direction is anticlockwise.)

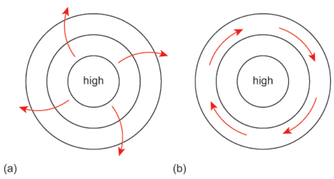


Figure 8 Development of an anticyclone (northern hemisphere), (a) showing the beginnings of Coriolis deflection and (b) the balanced equilibrium flow condition

Generally speaking, because of the lower pressure differences compared with cyclones, anticyclones invoke fewer clouds and lighter winds. In the summer this can lead to extensive exposure to sunlight and rising warm air from which moisture can condense into thunderclouds or to form morning mists. In the winter, more radiant heat escapes from the ground, leading to lower temperatures both night and day with fogs and frosts at night as well as ice and freezing temperatures. Cyclones, on the other hand, lead to cooler weather in summer due to cloudy and wet conditions, and slightly warmer winter days than with anticyclones but accompanied again by clouds and possibly snow, and importantly, strong winds, as they are driven by higher pressure differences than anticyclones.

1.3 Ground-level winds and air movements

Section 2.2 dealt mainly with the general picture of air movements in the main troposphere (meaning above about 1000 m altitude). These are known as the **geostrophic winds** or **gradient winds**. The lower sub-region below 1000 m comprises the **atmospheric boundary layer**. This has a complex wind profile in terms of wind speed versus height because of the variability of heating effects and geographic features. In this region, the wind speed is modelled from a reference wind speed at a **ground level to reference height**, z, of 10 m. In this layer the flow is mixed and can be modelled as turbulent.

In engineering applications – for example, in the design of buildings and other external structures such as radio masts, or monitoring likely winds around airports – the profile is usually represented by a power model of the form:

$$\left(\frac{u}{u_{\mathrm{r}}}\right)=\left(\frac{z}{z_{\mathrm{r}}}\right)^{p} \quad \text{(Equation 9)}$$

where u is the required **design wind speed** at height z, u_{r} is the wind speed at the 10 m reference height z_{r}, and the exponent p is related to the surface roughness. Table 1 shows some values for p.

Table 1 Ground-level wind speed model exponents

Feature	Model exponent, p
Flat lands and open water	0.10
Open varied terrain	0.15
Suburban	0.25
City centres	0.35

Selecting design wind speeds

It is proposed to erect a 50.0 m tall radio mast in the flatlands of Norfolk where the reference wind speed at 10.0 m height is 23.0 m s^{-1}. What would be the design wind speed for the top of the mast? Give your answer to 3 significant figures.

Solution

The design wind speed can be found using equation (10):

$$\left(\frac{u}{u_{\mathrm{r}}}\right)=\left(\frac{z}{z_{\mathrm{r}}}\right)^{p}.$$

First, identify the relevant values:

$$u_{\mathrm{r}}=23.0\,\mathrm{m\,s^{-1}},\ z_{\mathrm{r}}=10.0\,\mathrm{m},\ z=50.0\,\mathrm{m},\ p=0.1.$$

Rearranging equation (10), the design wind speed is

$$\begin{aligned} u &= u_{\mathrm{r}} \times \left(\frac{z}{z_{\mathrm{r}}}\right)^{p} \\ &= 23.0\,\mathrm{m\,s^{-1}} \times \left(\frac{50.0\,\mathrm{m}}{10.0\,\mathrm{m}}\right)^{0.1} \\ &= 27.0\,\mathrm{m\,s^{-1}}\ (\text{to } 3\text{ s.f.}). \end{aligned}$$

Activity 7

What would be the design wind speed at the top of an offshore wind turbine of height 245 m? The reference wind speed at reference height 10.0 m is 25.0 m s−1. Give your answer to 3 significant figures.

Answer

Considering equation (10),

$$u_{\mathrm{r}} = 25.0\ \mathrm{ms}^{-1},\ z_{\mathrm{r}} = 10.0\ \mathrm{m}\ ,\ z = 245\ \mathrm{m}\ ,\ p = 0.1.$$

So

$$u = 25.0\,\mathrm{m\,s^{-1}} \times \left(\frac{245\,\mathrm{m}}{10.0\,\mathrm{m}}\right)^{0.1} = 34.4\,\mathrm{m\,s^{-1}}\ (\text{to 3 s.f.}).$$

Wind speeds and their effects have been categorised in the **Beaufort wind force scale**, which will be familiar to listeners of broadcast shipping and weather forecast bulletins (and students of previous modules). It is named after the Irish hydrographer Sir Francis Beaufort, (1774–1857). It is not an exact scale, being based originally on subjective visual observations noted from sailing ships at sea. Table 2 summarises its main features as applied now to effects on land and sea.

Table 2 Beaufort scale and wind effects on land, adapted from Meteorological Office data

Beaufort scale number	Wind type	Wind speed limits		Effects on land	Effects out at sea
		(m s^{-1})	(km h^{-1})		
0	Calm	< 1	< 3.6	Smoke rises vertically	Flat mirror-like surface
1	Light air	1–2	3.6–7.2	Smoke drifts, weather vanes not indicating	Ripples like scales, no foam crests visible
2	Light breeze	2–3	7.2–10.8	Leaves rustle, weather vanes indicating, wind felt on face	Small wavelets not breaking
3	Gentle breeze	4–5	14.4–18.0	Leaves and twigs moving, light flags extended	Large wavelets, a few beginning to break
4	Moderate breeze	6–8	21.6–28.8	Dust raised, paper and small branches moved	Small longer waves, frequent ‘white horses’

5	Fresh breeze	9–11	32.4–39.6	Small trees sway, crested wavelets on lakes	Moderate waves getting longer, some spray and many ‘white horses’
6	Strong breeze	11–14	39.6–50.4	Large branches move, telegraph wires hum, umbrellas difficult to manage	Large waves, widespread longer-length foam crests
7	Near gale	14–17	50.4–61.2	Whole trees sway, walking into wind difficult	Waves breaking, foam being blown, spindrift starting
8	Gale	17–21	61.2–75.6	Twigs break off trees, walking into wind more difficult	High long waves, crests breaking into spindrift, prominent foam streaks
9	Strong gale	21–24	75.6–86.4	Chimney pots, tiles breaking loose, walkers blown over	High waves, dense foam affecting visibility
10	Storm	25–28	90.0–100.8	Trees uprooted, considerable structural damage	Very high waves, long overhanging crests, whole surface becoming white. Poor visibility
11	Violent storm	29–32	104.4–115.2	Widespread damage	Exceptionally high waves hiding small ships, dense white foam
12	Hurricane	33+	118.8+	Devastation	Air filled with foam and spray, sea completely white, very poor visibility

Referred to in the table, spindrift is spray blown from the cresting waves in the direction of the gale, while white horses is a colloquial term for short lengths of foaming white water.

Because the wind motion at ground level is in a turbulent boundary layer, the wind speed at any point varies erratically. The higher-gradient wind can also come into the picture at any time due to the ad hoc nature and development of large-scale weather events. Thus, when designing buildings and outdoor structures, the consideration of wind loads likely to be experienced has to rely on statistical methods using published data.

For buildings and other civil engineering works, these wind loads are covered by a British Standard (BSI, 2010) which in turn is based on a Eurocode (CEN, 2005) with a National Annex for the UK. The Eurocodes are Europe-wide standards for incorporation into national legislation, but because of the vagaries of weather and wind patterns (in particular, for example, the exposed nature of the UK), each country will have its own national specifications based on localised data.

The basic idea is to establish for a particular location a maximum value of mean wind velocity that is sustained for a 10-minute period, and that is only likely to occur with an annual probability of 0.02, i.e. once in every 50 years. This will be chosen from a wind speed map such as that in Figure 9. The wind speeds are shown on each contour in m s^{-1} and the grid squares labelled NA to TW are 100 km × 100 km each. For a location between contour lines, a value can be interpolated or the higher value of the two adjacent contours can be used. The process then is described in the standard as the calculation of characteristic values of overall wind actions. In brief, this can comprise more than 20 stages considering a number of issues, such as the location (distance from coast), the land terrain, the proximity of other buildings, the height of the building, the altitude of the building from sea level, the shape of the building, the orientation of the building, the proximity of any cliffs, ridges or escarpments, etc.

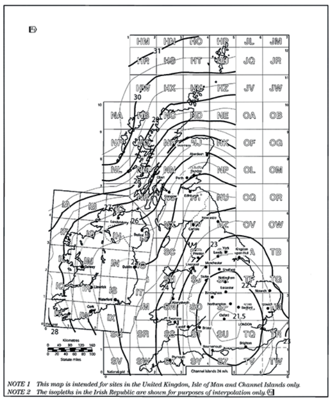


Figure 9 Wind map for the UK from British Standards (2010)

These considerations are quantified as individual factors and coefficients, which are then applied to the chosen worst-case mean wind velocity to determine a peak velocity pressure (usually written q_p even though it is a pressure, with units of pascals) which can be used for design load cases. Equation 11 gives an example:

$$q_\mathrm{p} = 0.613\,(u_\mathrm{map} \times c_\mathrm{alt} \times c_\mathrm{dir})^2 \times c_\mathrm{e} \times c_\mathrm{e,T} \qquad \text{(Equation 11)}$$

where u_map is the wind speed, c_alt is the altitude coefficient, c_dir is the direction coefficient, c_e is the exposure coefficient and $c_\mathrm{e,T}$ is a town location coefficient. Equation 11 is basically the equation for dynamic pressure in air of density ρ = 1.225 kg m^{-3} i.e.

$$P_\mathrm{dyn} = \frac{1}{2}\rho u^2 = 0.613u^2$$

but with a series of correction factors for location. When designing real structures, the appropriate values for these factors must be determined from the original standards. As the load cases determined by this method are based on statistics and probability, there is always a possibility, however small, that they will be exceeded in an exceptional storm. There is, however, an optimum design where the cost of further construction for greater safety is not justifiable and the calculated failure probability is extremely small.

Activity 8

Using the simple model in Equation 9 and the British Standards wind speed map (a larger PDF version can be found here), determine the wind speed in km h^{-1} allowed for in the design of a 30.0 m-high building in the centre of Carlisle. What would it be in the countryside surrounding the city, assuming that the countryside comprises open varied terrain? Give your answers to 3 significant figures.

Answer

From the wind speed map shown in Figure 9, the wind speed for Carlisle is 24 m s^{-1}.

From Equation 9 and Table 1

$$\frac{u}{24\,\mathrm{m\,s^{-1}}} = \left(\frac{30\,\mathrm{m}}{10\,\mathrm{m}}\right)^{0.35}$$

so

$$u = 24\,\mathrm{m\,s^{-1}} \times \left(\frac{30\,\mathrm{m}}{10\,\mathrm{m}}\right)^{0.35} = 35.25\ldots\ \mathrm{m\,s^{-1}}$$

or

$$u = \frac{35.25\ldots\ \mathrm{m\,s^{-1}} \times 3600\,\mathrm{s\,h^{-1}}}{1000\,\mathrm{m\,km^{-1}}} = 127\,\mathrm{km\,h^{-1}}\ (\text{to 3 s.f.})$$

In the countryside

$$u = 24\,\mathrm{m\,s^{-1}} \times \left(\frac{30\,\mathrm{m}}{10\,\mathrm{m}}\right)^{0.15} = 28.29\ldots\ \mathrm{m\,s^{-1}}$$

or

$$u = \frac{28.29\ldots\ \mathrm{m\,s^{-1}} \times 3600\,\mathrm{s\,h^{-1}}}{1000\,\mathrm{m\,km^{-1}}} = 102\,\mathrm{km\,h^{-1}}\ (\text{to 3 s.f.}).$$

Design wind speeds from wind maps

A notional design wind speed is obtained from official maps and then modified with factors to take account of local features and height.

If u_r is the reference wind speed at a reference height z_r of 10 m, z is the height under consideration and p is a factor related to local features as indicated in Table 1, the design wind speed can be obtained from equation (10):

$$\frac{u}{u_\mathrm{r}} = \left(\frac{z}{z_\mathrm{r}}\right)^{p}.$$

1.4 Spacecraft re-entry considerations

Figure 10 The Soyuz spacecraft

The Soyuz spacecraft pictured in Figure 10 comprises three sections: a spherical orbital module, a blunt-ended descent (sometimes called re-entry) module and a service module (see Figure 11).

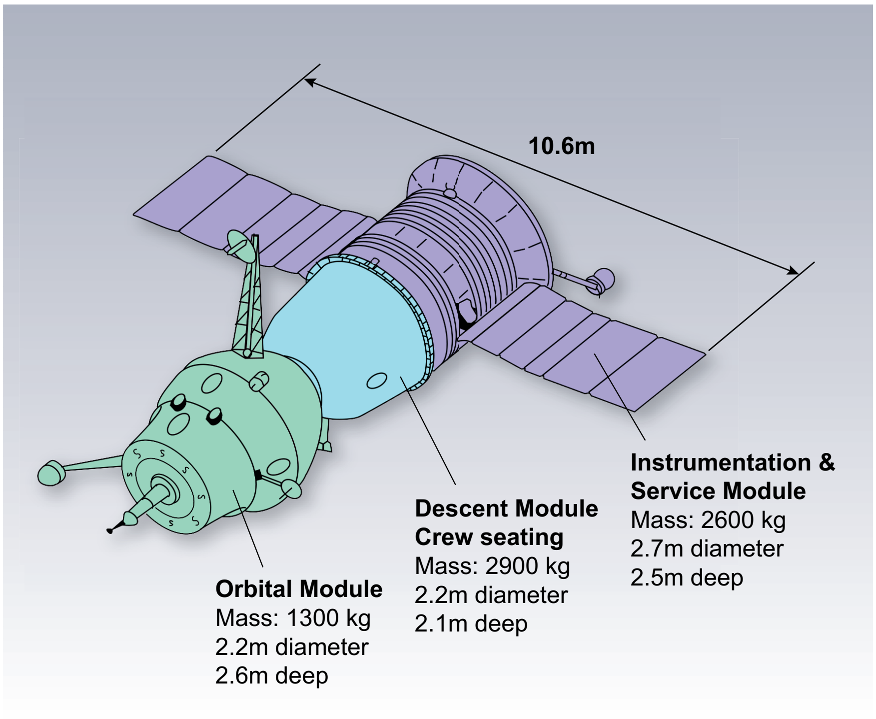
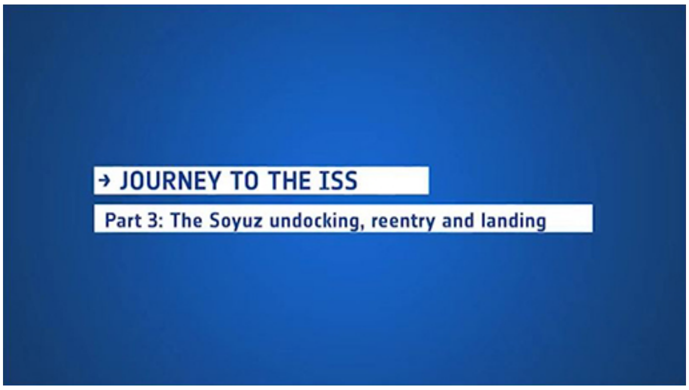


Figure 11 The service module of the Soyuz spacecraft

Before starting the next activity, you may find it interesting to watch the video about Soyuz re-entry produced by the European Space Agency but note that it is not necessary to watch it to undertake the activity.

Video content is not available in this format.

Video 1 The Soyuz spacecraft



Activity 9

The Soyuz descent module parachute is activated at an altitude of around 10 km, when the capsule has a velocity of around 900 kph (250 m/s). Using Figure 2 (reproduced here for convenience):

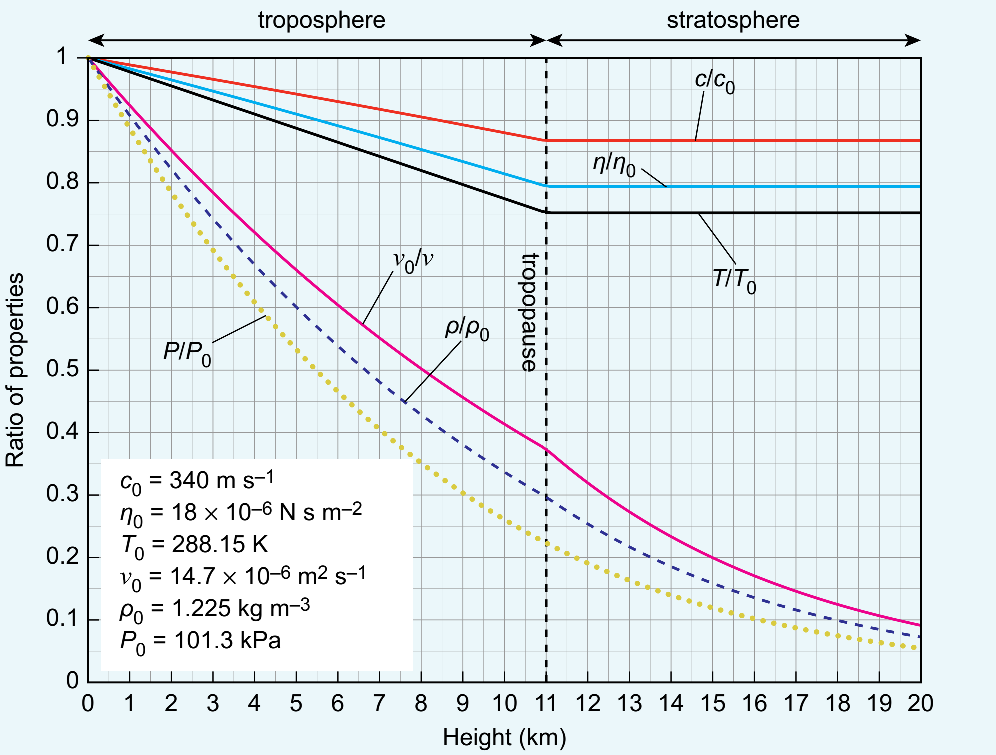


Figure 2 (repeated) Properties of the standard atmosphere. Note that kinematic viscosity η increases with altitude, so the inverse ratio $\frac{\eta_0}{\eta}$ is shown.

Question 1

(a) The local speed of sound at that height (10 km), to 2 s.f. (Note: the line c/c_o is the ratio of the speed of sound at a given height, c, to the speed of sound at sea level, c_o).

Find the following:

Answer

From the figure it can be seen that at 10 km the speed of sound ration $\frac{c}{c_0} = 0.87$.and it is stated that $c_0 = 340\,\mathrm{m\,s^{-1}}$ so the local speed of sound is:

$$\begin{aligned} c &= c_0 \times 0.87 \\ &= 340\,\mathrm{ms^{-1}} \times 0.87 \\ &= 295.8\,\mathrm{ms^{-1}} \\ &= 30\,\mathrm{ms^{-1}}\ (\text{to 2 s.f}). \end{aligned}$$

(b) The Mach number, Ma (the ratio of speed to speed of sound) of the capsule when the parachute is deployed.

Answer

Speed of capsule = 250 m/s

$$\mathrm{Ma} = \frac{u}{c} = \frac{250}{295.8} = 0.85\ (\text{to 2 s.f.}).$$

Question 2

The capsule is assumed to be at its terminal velocity when the parachute is activated (at an altitude of 10 km). At terminal velocity the aerodynamic drag, F_d, on the capsule is equal to its weight, W, so:

F_d = W = mg, where g is the acceleration due to gravity (another quantity that varies with altitude, but you can assume to be 9.8 $m.s^{-2}$)

Aerodynamic drag on capsule is given by

$$F_d = \tfrac{1}{2} C_d \rho u^2 A$$

Where C_d is the drag coefficient, ρ is the local air density, u is the velocity of the capsule and A is the cross sectional area of the capsule.

Using data from Figure 2 and from Figure 11, calculate the value of the drag coefficient of the capsule.

Answer

At terminal velocity the weight of the capsule is exactly balanced by its aerodynamic drag, so

by its aerodynamic drag, so

$$mg = \frac{1}{2} C_{\mathrm{D}} \rho u^2 A$$

which can be rearranged to find the drag coefficient

$$C_{\mathrm{D}} = \frac{2mg}{\rho u^2 A}.$$

From Figure 11, the diameter of the capsule is 2.2 m, so the area is

$$\begin{aligned} A &= \frac{\pi d^2}{4} \\ &= \pi \times \frac{(2.2\,\mathrm{m})^2}{4} \\ &= 3.801\ldots\ \mathrm{m}^2 \\ &= 3.8\,\mathrm{m}^2\ (\text{to } 2\ \text{s.f.}). \end{aligned}$$

Also from Figure 11, the mass of the re-entry module is 2900 kg and from Figure 2, the density ratio at 10 km is about 0.34. The density, therefore, is given by,

$$\rho = 0.34 \times 1.225\,\mathrm{kg\,m^{-3}} = 0.416\,\mathrm{kg\,m^{-3}}.$$

Substituting in the values gives a drag coefficient of

$$\begin{aligned} C_d &= \frac{2mg}{\rho u^2 A} \\ &= \frac{2 \times 2900\,\mathrm{kg} \times 9.81}{0.416 \times (250\,)^2 \times 3.801\ldots} \\ C_d &= 0.58\ (\text{to } 2\ \text{s.f.}). \end{aligned}$$

Question 3

A relatively small braking parachute is initially deployed to slow the capsule down. When the braking parachute has reduced the speed of the capsule to around 80 m/s at a height of 7.5km, the main parachute, which has an area of 1000 m^2, is deployed and reduces the capsule speed to a steady 25 kph (6.9 m/s).

Assuming a drag coefficient of 1.7 (this is fairly standard for parachutes), estimate the altitude at which this new terminal velocity will be established. Neglect the contribution to drag of the capsule itself and give your answer to 2 significant figures.

Answer

As before, at terminal velocity the weight of the capsule is exactly balanced by its aerodynamic drag, so

$$mg = \frac{1}{2} C_{\mathrm{D}} \rho u^2 A$$

but this time the air density is the unknown and the area is the area of the parachute, so rearranging and substituting known values,

$$\rho = \frac{2mg}{C_\mathrm{D} u^2 A}$$
$$= \frac{2 \times 2900\,\mathrm{kg} \times 9.81\,\mathrm{m\,s^{-2}}}{1.7 \times (6.9\,\mathrm{m\,s^{-1}})^2 \times 1000\,\mathrm{m}^2},$$
$$= 0.702\ldots\ \mathrm{kg\,m^{-3}}.$$

Since the standard sea level density of air is , this is density ratio of $\rho_0 = 1.225\,\mathrm{kg\,m^{-3}}$

$$\frac{\rho}{\rho_0} = \frac{0.702\ldots\ \mathrm{kg\,m^{-3}}}{1.225\,\mathrm{kg\,m^{-3}}} = 0.57\ (\text{to 2 s.f.}).$$

Referring to Figure 2, this corresponds to an altitude of 5.5 km

Question 4

The same parachute is carried to just above ground level, where retro-rockets cushion the final landing. Estimate the velocity of the capsule just before the rockets fire. Give your answer to 2 significant figures.

Answer

Assuming that the capsule descends at local terminal velocity to ground level, the total drag must remain constant all the way down, that is,

$$\tfrac{1}{2}C_d\rho u^2\ \left(\text{at deployment}\right) = \tfrac{1}{2}C_d\rho u^2\ \left(\text{at ground level}\right)$$

Since neither the drag coefficient nor the parachute area are changing, then this equation can be simplified to,

$$\rho_\text{deploy}\, u^2_\text{deploy} = \rho_{\text{ground}\times}\ u^2_\text{ground}$$

Which can be rearranged as,

$$u^2_\text{ground} = \frac{\rho_\text{deploy}}{\rho_\text{ground}} \times u^2_\text{deploy}.$$

and therefore

$$u_\text{ground} = \sqrt{\frac{\rho_\text{deploy}}{\rho_\text{ground}} x \times u_\text{deploy}}$$

$$^{u}ground = \sqrt{0.57} \times 6.9\,\mathrm{ms} = 5.2\,\mathrm{m\,s^{-1}}\ (\text{to 2 s.f.}).$$

2 The Earth's oceans and seas

An ocean is a vast body of salt water of considerable depth. On Earth there are reckoned to be five main oceans; all of them are connected and together they form the World Ocean. Two of the oceans, the Pacific and Atlantic, are bisected on maps by the Earth's equator, so sometimes it is said that there are seven oceans. Assuming the Earth to be a perfect sphere with a constant radius from its centre to a smoothed-out sea level (known as a geoid), the surface areas of the oceans can be compared with each other and the total surface area of Earth (i.e. land and water combined) as in Table 3.

Table 3 Comparative sizes of Earth's oceans

Ocean	Average depth (m)	Surface area ($\times 10^3$ km^2)	% of Earth's surface
North Pacific + South Pacific	3970.0	168 723.0	33.1
North Atlantic + South Atlantic	3646.0	85 133.0	16.7
Indian	3741.0	70 560.0	13.8
Antarctic (a.k.a. Southern)	3270.0	21 960.0	4.3
Arctic	1205.0	15 558.0	3.1

Thus the oceans make up around 70% of the Earth's surface. The expression 'sail the seven seas' actually refers to the oceans. A sea in itself is also a body of water somewhat smaller in surface area and shallower than an ocean and bounded fully or partially by land masses – to a greater extent than oceans at any rate. Table 4 shows a partial list of well-known seas.

Table 4 Comparative sizes of Earth's seas (not a full list)

Sea	Average depth (m)	Surface area ($\times 10^3$ km^2)	% of Earth's surface
Mediterranean	1429.0	2966.0	0.58
Caribbean	2647.0	2718.0	0.53
South China	1652.0	2319.0	0.45
Bering	1547.0	2292.0	0.45
Gulf of Mexico	1486.0	1593.0	0.31
East China	188.0	1249.0	0.25
Hudson Bay	128.0	1232.0	0.24
North Sea	95.0	750.0	0.15

2.1 Wave motions in water

The word 'sea' is also sometimes used to describe waves and currents whipped up by local winds – as in 'a sea was running'. The surfaces of oceans and seas are rarely still, owing to their interactions with the atmosphere and the interchanges of energy from air movements or wind. As is well known, when a wind blows over a stretch of water, **waves** are formed. The area of water over which wind is blowing is called a **fetch**. In such a situation, it only takes a small random variation in the air pressure normal to the surface to create a disturbance on the surface of the water.

Small waves of only a few millimetres in height and separation (**wavelength**) may develop. These are called **ripples**. If the wind persists over the fetch of water, the ripples become larger, turning into waves. The wind transfers some of its kinetic energy to the waves, so the pressure differences in the air increase, feeding more wave growth. If the wind dies down, ripples are restored to a flat surface by the surface tension of the water, but gravity continues to feed the waves.

If the wind continues, a series of waves is set up which can actually travel faster than the wind speed itself. The waves will have a repeating motion with a frequency and wavelength. Water and air are not too good at damping large vibrations, so whilst the smaller shorter-wave energy is soon dissipated, waves with longer wavelengths can and do travel many thousands of kilometres. In this situation, the waves are known as a **swell**. Swells are often created by strong winds and storms many thousands of kilometres away.

It is important to note that the water is not moving along with the wave, apart from the relatively slow tidal movements or any underlying currents. An individual particle of water more or less stays where it is as a wave passes. The particle will move up and down in a roughly circular path as each wave passes through. This effect can be seen by holding the end of a long rope and flicking back and forth. A half-loop or wave will travel along the rope, but each bit of the rope stays in position in terms of its distance from the end. On the other hand, a **tsunami** (often called a **tidal wave**) really is a physical displacement of water caused by a single event like an earthquake; out at sea it might travel as if it were a normal but high-speed wave, but this time the body of water really is moving along with it.

2.2 Wave speed, amplitude and displacement

The interaction of a wind over the surface of water to produce waves is complex. On the surface of deep water the **wave speed** c_w of typical waves can be modelled as

$$c_\mathrm{w} = \sqrt{\frac{Lg}{2\pi}} \qquad \text{(Equation 12)}$$

where L is the wavelength from peak to peak between two waves following each other and g is the acceleration due to gravity. The **peak** is the highest point of the wave, and is also known as the **crest**. Thus the wave speed is greater for longer wavelengths. Consequently a swell may comprise long and fast waves, which can also be very high if the initiating wind speed itself is both high and sustained for a significant time. The height, H, of a wave is taken to be the distance from the lowest level of the surface to the top of the wave. The lowest level is called the **trough**. The shape of a wave – its cross section or side view – depends upon its height and wavelength. At lower heights, it tends to be sinusoidal, so the **amplitude** of this type of wave will be half the height from trough to crest.

Wave speeds

The action of wind over deep water is to create a disturbance on the surface layers of the water. This disturbance takes the form of a wave which travels more or less in the same direction as the wind but at a speed which is given by

$$c_\mathrm{w} = \sqrt{\frac{Lg}{2\pi}}$$

where c_w is the wave speed, L is the wavelength and g is the acceleration due to gravity.

Note that the wave is a disturbance which moves along and through the water – the water itself does not move along, except in the case of a tsunami.

Higher waves tend to have a narrower crest and a wider and shallower trough, as sketched in Figure 12.

At a value of $\frac{H}{L} = \frac{1}{7}$ the crests become more pronounced and sharp-edged in profile and the top edges **break** into foaming white water (white horses). This foaming dissipates energy, which effectively stops further growth in height, meaning that the ratio $\frac{H}{L}$ stays at a maximum value of $\frac{1}{7}$.

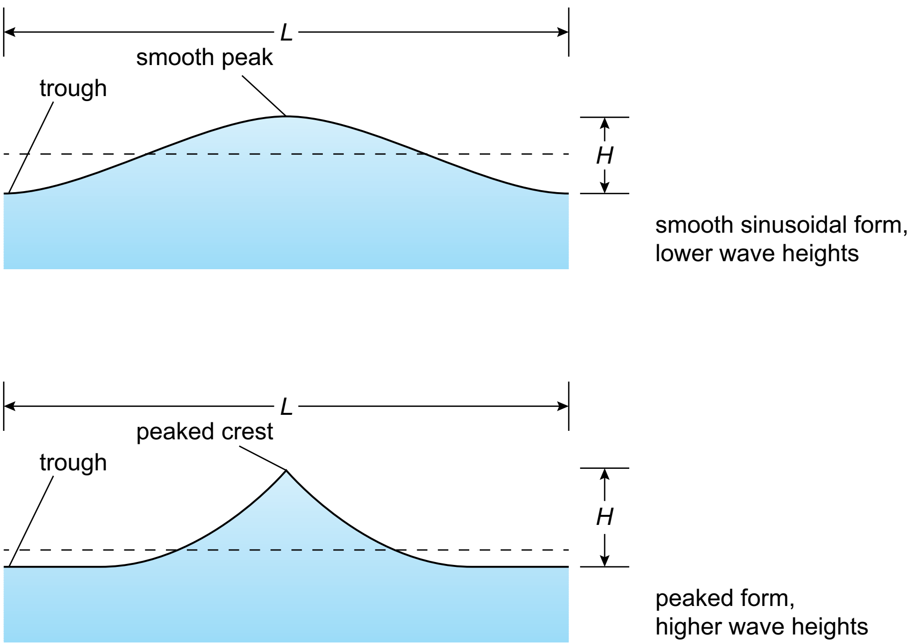


Figure 12 Cross sections of wave profiles

The breaking of waves is of course most evident in shallow water at the beach. Once the depth of water has reduced to about $\frac{L}{2}$, the shape of the wave profile alters again. If the slope of the beach is small (e.g. less than about 1 in 30 or 3.3%) the wave will break progressively as it rolls in, and the water itself does travel along in this case. If the slope is much greater, the wave is effectively slowed down and cannot adjust; instead, it becomes unstable, growing in height and then breaking by plunging over in a dramatic fashion. This still contains a lot of energy and can impart high forces on anything in its path. Even a non-breaking wave can cause large forces owing to the energy it contains, as the speed causes drag forces on anything it flows past.

A typical wave can be modelled quite easily. Figure 13 shows a wave of sinusoidal form with a relatively small surface displacement amplitude $\frac{H}{2}$ in comparison to the wavelength and depth d of the water. The sea bed is assumed to be flat and smooth (with negligible friction), and there is a steady series of waves flowing to the right with speed c_{w}. The wave depth is H, which in this model will be twice the amplitude. The **wave periodic time**, T, is the time taken for one complete wavelength to pass through and is given by

$$T = \frac{L}{c_{\mathrm{w}}} \qquad \text{(Equation 13)}$$

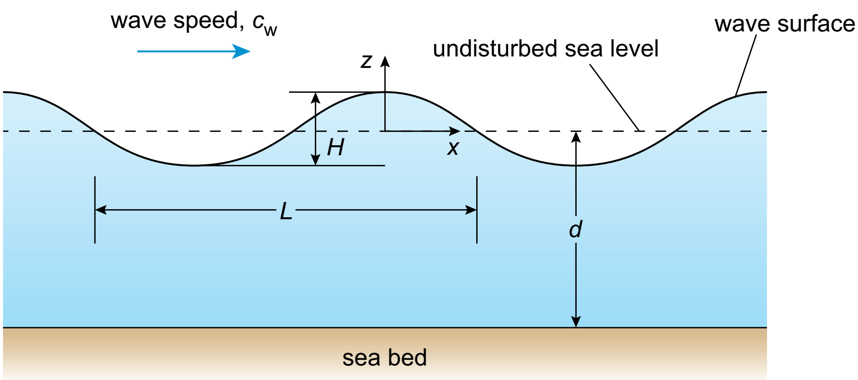


Figure 13 Cross section of a sinusoidal wave profile

As mentioned above, each water particle will move in an approximately circular **orbit** as the wave disturbance passes through. Figure 14 shows the shapes of an individual water particle orbits for shallow, intermediate depth and deep water in schematic form; the relative sizes are not to scale. In shallow water, the orbit is elliptical in cross section and reaches to the sea bed. In the intermediate depth, the orbit is more circular, and in deep water the orbit is completely circular and does not extend to the sea bed.

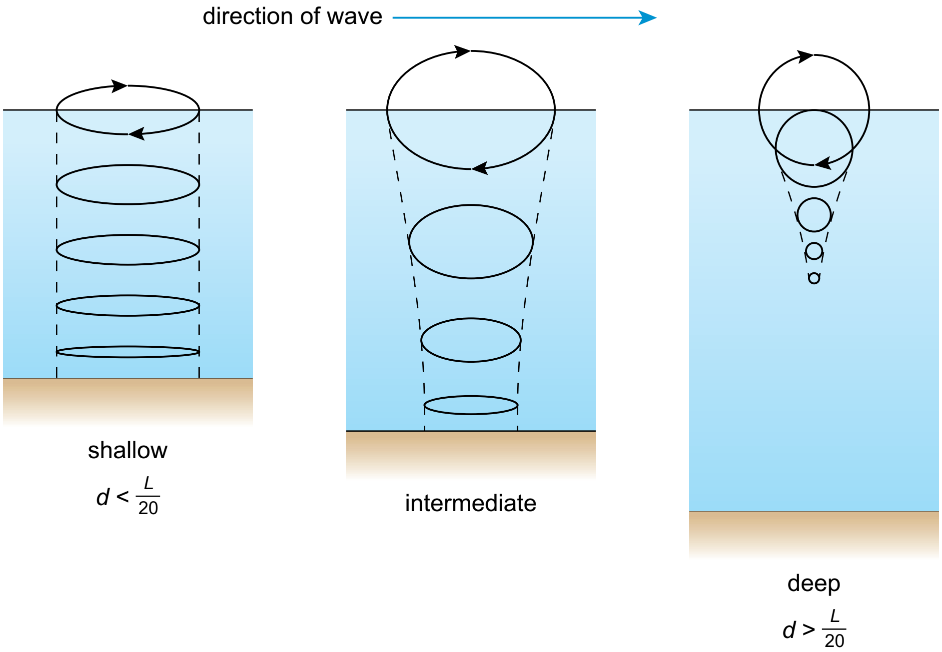


Figure 14 Wave water particle orbits (not to scale)

Generally at a water depth equivalent to half the wave length, $d = \dfrac{L}{2}$, the amplitude of wave motion is barely 4% of that at the surface. This forms a useful rule of thumb in defining a ‘deep water’ wave. Assuming a deep water situation, the wave motion of an individual particle of water as a wave passes is near enough circular. Taking a stationary reference axis set at the flat sea level, as the depth increases with z, the wave motion amplitude reduces by a factor of $\mathrm{e}^{\left(\frac{2\pi z}{L}\right)}$, where numerically z will be negative.

In other words, if A_s is the surface amplitude and A_z is the amplitude at depth z (where z is a negative number) then

$$A_\mathrm{z} = A_\mathrm{s}\mathrm{e}^{\left(\frac{2\pi z}{L}\right)}. \quad \text{(Equation 14)}$$

If the radius of the circular motion is r, the particle speed will be given by $u = \omega r$ (as with any circular motion), and its acceleration will be

$$a = \omega^2 r \quad \text{(Equation 15)}$$

where ω is the radian circular frequency. Again, as with all circular orbiting motion

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{(Equation 16)}$$

and

$$L = c_\mathrm{w} T$$

which is a rearranged version of Equation 13.

Deep water wave study

For a wave in deep water of wavelength 200.0 m and height 6.0 m, calculate:

a. the wave speed
b. the periodic time
c. the displacement amplitude at the surface
d. the displacement amplitude at 50.0 m depth
e. the maximum horizontal acceleration at 25.0 m depth.

Give your answers to 3 significant figures.

Solution

a. Using Equation 12:

$$\begin{aligned} c_{\mathrm{w}} &= \sqrt{\frac{Lg}{2\pi}} \\ &= \sqrt{\frac{200.0\,\mathrm{m} \times 9.81\,\mathrm{m\,s^{-2}}}{2\pi}} \\ &= 17.67\ldots\ \mathrm{m\,s^{-1}} \\ &= 17.7\,\mathrm{m\,s^{-1}}\ (\text{to 3 s.f.}). \end{aligned}$$

b. Using Equation 13;

$$\begin{aligned} T &= \frac{L}{c_{\mathrm{w}}} \\ &= \frac{200.0\,\mathrm{m}}{17.67\ldots\ \mathrm{m\,s^{-1}}} \\ &= 11.3\,\mathrm{s}\ (\text{to 3 s.f}). \end{aligned}$$

c. The amplitude at the surface is

$$A_{\mathrm{s}} = \frac{H}{2} = \frac{6\,\mathrm{m}}{2} = 3\,\mathrm{m}.$$

d. To find the displacement amplitude, Equation 14 is used:

$$A_{\mathrm{z}} = A_{\mathrm{s}}\mathrm{e}^{\left(\frac{2\pi z}{L}\right)}.$$

First calculate the factor at 50 m depth:

$$\begin{aligned} \mathrm{e}^{\left(\frac{2\pi z}{L}\right)} &= \mathrm{e}^{\left(\frac{2\pi(-50\ \mathrm{m})}{200\ \mathrm{m}}\right)} \\ &= \mathrm{e}^{-1.571\ldots} \\ &= 0.2078\ldots. \end{aligned}$$

This gives a displacement amplitude

$A_Z = 3\,\mathrm{m} \times 0.2078\ldots = 0.624\,\mathrm{m}$ (to 3 s.f.).

e. First calculate the displacement amplitude at 25 m depth, since this is also the radius of the particle’s circular motion:

$$\begin{aligned} \mathrm{e}^{\left(\frac{2\pi z}{L}\right)} &= \mathrm{e}^{\left(\frac{2\pi(-25\ \mathrm{m})}{200\ \mathrm{m}}\right)} \\ &= \mathrm{e}^{-0.785\ldots} \\ &= 0.4559\ldots \end{aligned}$$

so the radius of circular motion (or displacement amplitude) is

$$\begin{aligned} r &= A_{\mathrm{s}}\mathrm{e}^{\left(\frac{2\pi z}{L}\right)} \\ &= 3\,\mathrm{m} \times 0.4559\ldots \\ &= 1.367\ldots\ \mathrm{m}. \end{aligned}$$

Now, using Equation 16:

$$\begin{aligned} \omega &= \frac{2\pi}{T} \\ &= \frac{2\pi}{11.3\ldots\ \mathrm{s}} \\ &= 0.5551\ldots\ \mathrm{s}^{-1}. \end{aligned}$$

Therefore, from Equation 15, the acceleration amplitude is

$$\begin{aligned} a &= \omega^2 r \\ &= \left(0.5551\ldots\ \mathrm{s}^{-1}\right)^2 \times 1.367\ldots\ \mathrm{m} \\ &= 0.422\,\mathrm{m\,s^{-2}}\ \text{(to 3 s.f.)}. \end{aligned}$$

Activity 1

The wave as described above has decayed such that its height is reduced by 50%. Determine the same parameters, noting the changes in values:

a. the wave speed
b. the periodic time
c. the displacement amplitude at the surface
d. the displacement amplitude at 50.0 m depth
e. the maximum horizontal acceleration at 25.0 m depth.

Give your answers to 3 significant figures.

Answer

a. Using Equation 12,

$$\begin{aligned} c_{\mathrm{w}} &= \sqrt{\frac{Lg}{2\pi}} \\ &= \sqrt{\frac{200.0\,\mathrm{m} \times 9.81\,\mathrm{m\,s^{-2}}}{2\pi}} \\ &= 17.67\ldots\ \mathrm{m\,s^{-1}} \\ &= 17.7\,\mathrm{m\,s^{-1}}\ (\text{to 3 s.f.}). \end{aligned}$$

The value is the same.

b. Using Equation 13,

$$\begin{aligned} T &= \frac{L}{c_{\mathrm{w}}} \\ &= \frac{200.0\,\mathrm{m}}{17.67\ldots\ \mathrm{m\,s^{-1}}} \\ &= 11.31\ldots\ \mathrm{s} \\ &= 11.3\,\mathrm{s}\ (\text{to 3 s.f.}). \end{aligned}$$

The value is the same.

c. The amplitude at the surface is

$$A_{\mathrm{s}} = \frac{H}{2} = \frac{3\,\mathrm{m}}{2} = 1.5\,\mathrm{m}.$$

The surface amplitude has reduced by 50%.

d. To find the displacement amplitude Equation 14 is used:

$$A_{\mathrm{z}} = A_{\mathrm{s}}\mathrm{e}^{\left(\frac{2\pi z}{L}\right)}.$$

The factor at 50 m depth will be the same:

$$\begin{aligned} \mathrm{e}^{\left(\frac{2\pi z}{L}\right)} &= \mathrm{e}^{\left(\frac{2\pi(-50\ \mathrm{m})}{200\ \mathrm{m}}\right)} \\ &= \mathrm{e}^{-1.570\ldots} \\ &= 0.2078\ldots. \end{aligned}$$

This gives a displacement amplitude of

$$\begin{aligned} A_{\mathrm{z}} &= 1.5\,\mathrm{m} \times 0.2078\ldots \\ &= 0.3118\ldots\ \mathrm{m} \\ &= 0.312\,\mathrm{m}\ (\text{to 3 s.f.}). \end{aligned}$$

This shows a reduction of 50%.

e. First calculate the displacement amplitude at 25 m depth, since this is also the radius of the particle’s circular motion. The factor will be the same at

$$\begin{aligned} \mathrm{e}^{\left(\frac{2\pi z}{L}\right)} &= \mathrm{e}^{\left(\frac{2\pi(-25\ \mathrm{m})}{200\ \mathrm{m}}\right)} \\ &= \mathrm{e}^{-0.785\ldots} \\ &= 0.4559\ldots. \end{aligned}$$

The radius of circular motion (or displacement amplitude) is

$$\begin{aligned} r &= A_{\mathrm{s}} \mathrm{e}^{\left(\frac{2\pi z}{L}\right)} \\ &= 1.5\,\mathrm{m} \times 0.4559\ldots \\ &= 0.683\ldots\ \mathrm{m}. \end{aligned}$$

This is a reduction of 50%. Now, using Equation 16, angular velocity will be the same at

$$\begin{aligned} \omega &= \frac{2\pi}{T} \\ &= \frac{2\pi}{11.3\ldots\ \mathrm{s}} \\ &= 0.555\ldots\ \mathrm{s}^{-1}. \end{aligned}$$

The acceleration amplitude is

$$\begin{aligned} a &= \omega^2 r \\ &= \left(0.555\ldots\ \mathrm{s}^{-1}\right)^2 \times 0.683\ldots\ \mathrm{m} \\ &= 0.211\,\mathrm{m\,s^{-2}}\ (\text{to 3 s.f.}). \end{aligned}$$

This is a reduction of 50%.

Wave models

For typical wind-provoked waves over deep water the following relationships can be used to model the wave properties:

The speed of the wave is

$$c_{\mathrm{w}} = \sqrt{\frac{Lg}{2\pi}}$$

where L is the wavelength and g is the acceleration due to gravity.

The periodic time for one complete wave to pass by is

$$T = \frac{L}{c_{\mathrm{w}}}$$

and the wavelength in terms of velocity and periodic time is $L = c_{\mathrm{w}} T$.

The frequency of the waves in relation to the periodic time is

$$\omega = 2\pi f = \frac{2\pi}{T}$$

where ω is the radian circular frequency and f is the frequency in Hertz or cycles per second.

At a depth z (a negative numerical value) the amplitude of a wave is

$$A_z = A_s e^{\frac{2\pi z}{L}}$$

where A_s is the amplitude at the surface.

2.3 Waves and winds

A critical factor in the design of structures which interact with the sea is the likely worst-case wave height, which is known as a **design wave**. It is considered that even this might be exceeded once every 50 years. In the absence of any real wave height records, the 50-year design wave may be predicted using records of severe wind and weather conditions for the area of concern. The more frequent smaller waves that might over time have an effect on the fatigue life of structures must also be considered. Figure 15 shows a historical map for the waters surrounding the UK for the late 1970s; the kind of map that would have been consulted in the design of offshore oil rigs. This was based on maps from the Institute of Oceanographic Sciences.

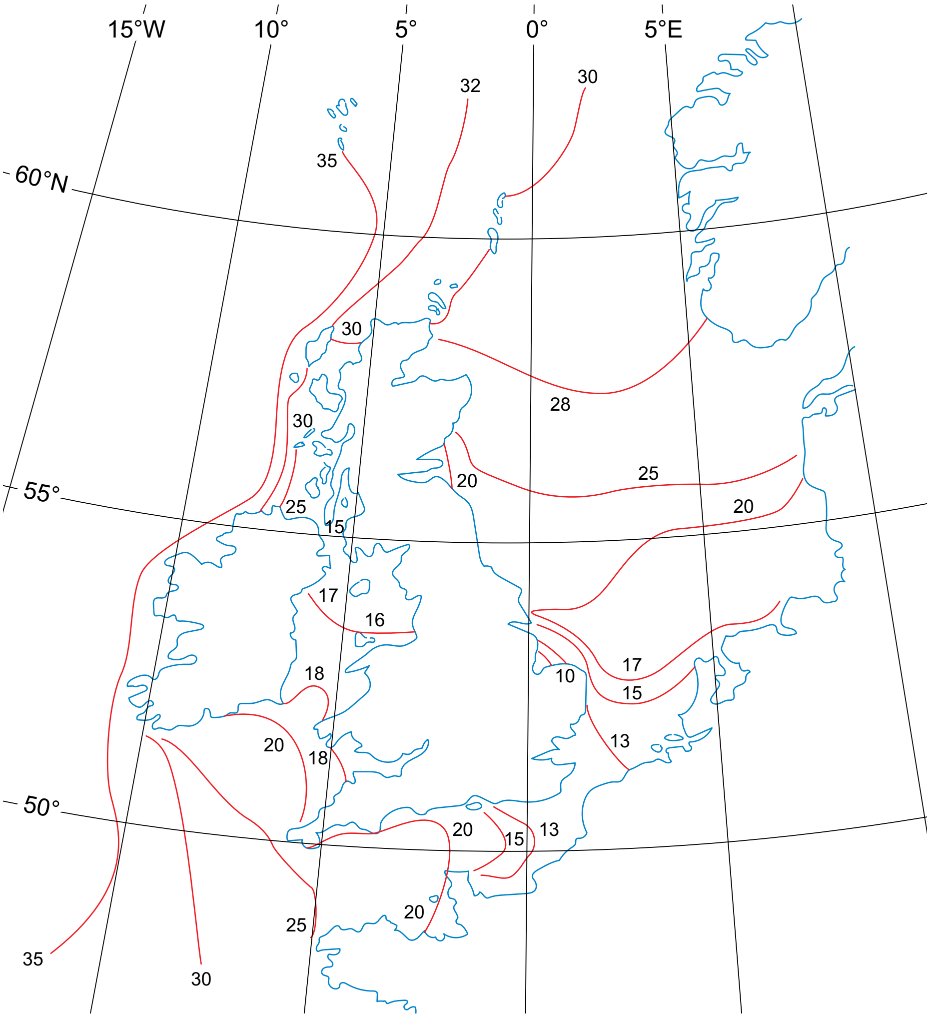


Figure 15 50-year wave heights (in metres) around the UK

Winds, of course, have a direct effect on the size and speed of waves beyond causing swells a long distance away. Table 2 related the Beaufort scale of wind speeds to the direct effect on the oceans and seas. Conversely, the state of the seas is one way of estimating the Beaufort wind rating.

2.4 Tides and tidal currents

A tide is the flow away (**ebb**) and flow back or return (**flood**) of something. The most obvious are those of the oceans and seas, in which there are regular **high tides** and **low tides**. The difference in heights is known as the **tidal range**. These tides are caused chiefly by the gravitational attraction forces of the Moon, and partly by those of the Sun, acting on the Earth. The gravitational pull of the Sun is overall much stronger but, as it is much further away, they are weaker on Earth than those of the relatively nearby Moon. The effects on the Earth are about 70% from the Moon and 30% from the Sun. Tides affect shipping – progress, mooring, loading and departures – and influence the design, build and maintenance of coastal and offshore infrastructure such as estuary bridges, harbour walls, drainage outlets, gas and oil rigs, etc. They also contain and cycle huge amounts of energy, some of which is diverted through turbines to generate useful power.

The tide is a lift and then release of huge bodies of water in the form of a **tidal bulge** on a regular basis as the Earth rotates beneath the gravitational pulls of the Moon and Sun. When near to a coast, the bulge turns into physical flows of water towards and away from the shoreline as the Earth rotates. When the effects of Moon and Sun occur in phase (together), the flows and heights increase the tidal ranges in what are called **spring tides**, as in the phrase 'spring forth' – nothing to do with the season. About six days later the relative positions of the Sun and Moon mean that they are pulling at right angles to one another and the result is a smaller tidal range called **neap tides**, from an Anglo-Saxon word meaning 'without the power'. Figure 16 illustrates the effects of spring and neap tides.

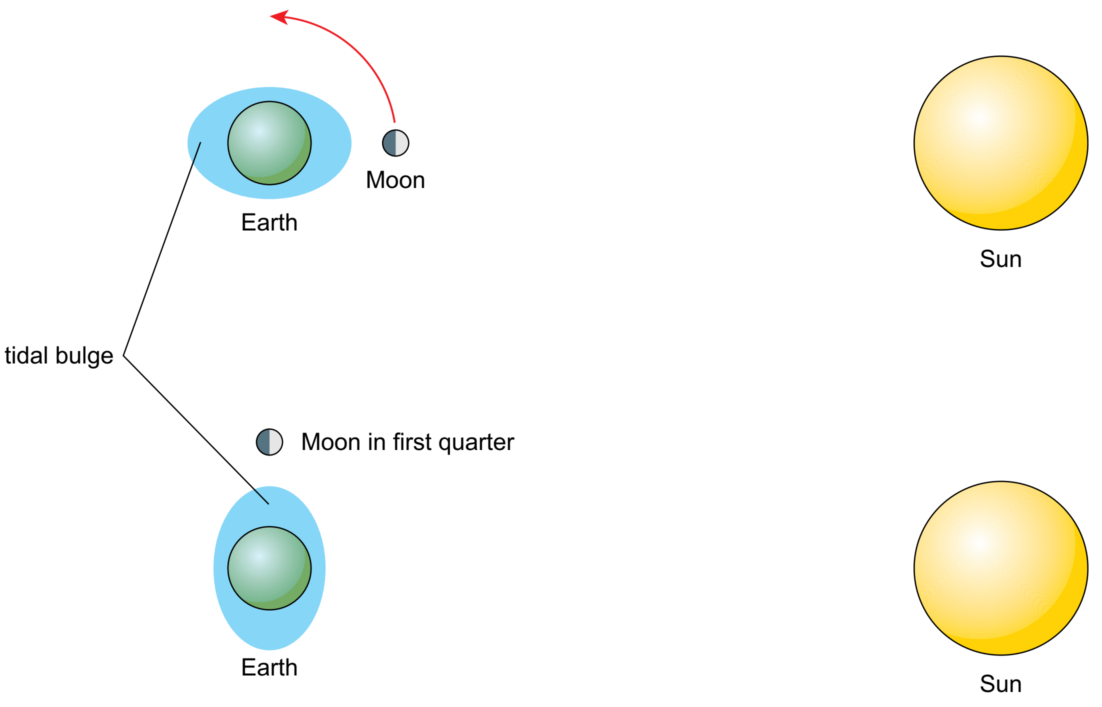


Figure 16 Upper: Earth, Moon and Sun in line – spring tides; lower: Moon and Sun pulling at right angles – neap tides (note: figure not to scale and highly exaggerated)

In the upper part of Figure 16, the Moon is new but is in line with the Sun, and so produces spring tides on Earth. The same thing occurs when the Moon is in its second quarter about two weeks later on the opposite side of the Earth. It is then full but is still in line with the Sun and produces the next spring tides. Meanwhile, in between, the Moon in its first quarter as shown in the lower part of Figure 16 is pulling at right angles to the Sun’s pull, resulting in the lower-range neap tides. The same thing occurs when the moon is in its third quarter. Either way, as the Earth rotates once every 24 hours, it will pass through two tidal bulges; the tides are approximately twelve hours apart. The tidal range takes about a week to go from the largest spring tides to the smallest neap tides, then back again in the next week.

Tidal rise and fall can be predicted as tidal curves. A typical curve (for Hestan Island in the Solway, in October 2019) is shown in Figure 17. The blue peaks represent the twice daily rise and fall of the tides. The graph covers the week that it takes to change from neap to spring tides.

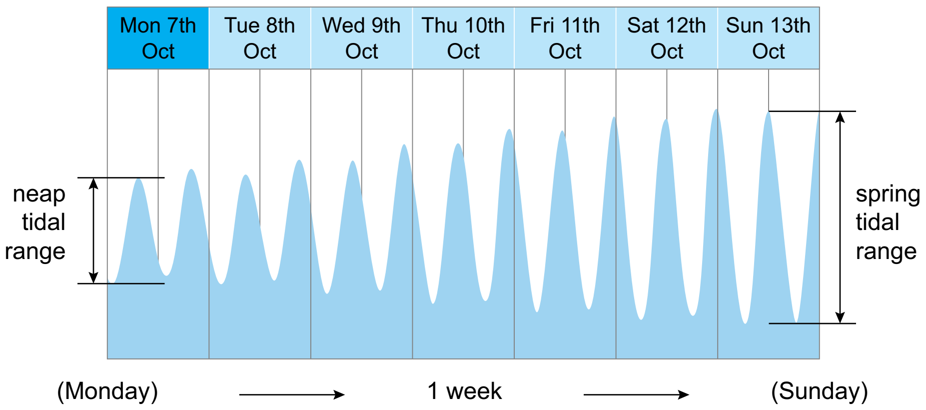


Figure 17 Hestan Island tidal curve

The Earth effectively rotates beneath the tidal bulges and influences the speeds of the tidal flood and ebb streams in an almost sinusoidal way. For many estuaries and other areas subject to high tidal ranges, the 50/90 rule is observed regarding tidal stream speeds, as illustrated in Table 5. From a **slack water** period (i.e. when the tide changes from ebb to flood and vice versa), the relative speeds of the tidal stream are approximately as in Table 5.

Table 5 Relative speeds of tidal streams

Hour after slack water	Per cent of maximum speed
0	0%
1	50%
2	90%
3	100%
4	90%
5	50%
6	0%

Note that the highest speeds occur at mid-flow, in hours 3 and 4. Local features, however, can create anomalous variations that can catch out the unwary.

This is a useful overlying model as to the causes of ocean and sea tides, but, as might be expected, there are some other issues that affect both the overall and local patterns. Without going into too much detail, these can be summarised as follows:

- **Astronomical effects**: The gravitational pull of the Moon and Sun vary with the distance from these bodies to Earth; so therefore do the tidal effects they cause. These effects are global, but there are also more local effects depending on how far above or below the equator the Moon and Sun are in the sky at that particular location.
- **Physical obstructions**: The presence of land masses, coastlines, shallows, etc., in addition to physical obstruction can cause tidal flows to be reflected, interfered with and otherwise modified.
- **Reflections and interference**: The presence of land masses, coastlines, shallows, etc., can cause tidal flows to reflect and otherwise be modified so as to interfere with the incoming bulge. These can either amplify or detract from a local tidal range.

The tidal bulge in the open ocean is at most still less than a metre above stationary sea level, and the direct effect of the tidal forces on smaller seas and inland lakes is much smaller than this. Nevertheless, an ocean tidal bulge is a huge quantity of water when it encounters a shoreline (or, strictly speaking, when the rotating Earth shoreline encounters the bulge). The effects of the depth and shape of the sea bed, the orientation and shapes of the shoreline, etc., can create substantial changes in the local sea level. On the other hand, in some areas these features in conjunction with the Coriolis effect can create a **tidal node** region or system in which all effects cancel each other out such that there is no regular change in sea level. This is also known as an **amphidromic point** around which there may be strong currents in the **amphidromic system** but no net change in the sea level.

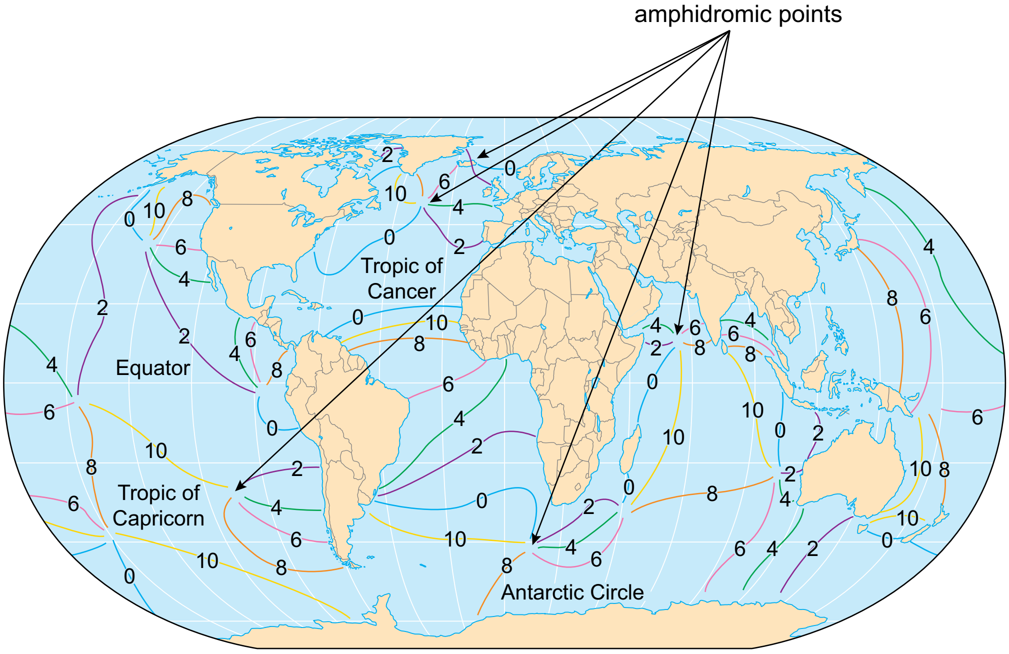


Figure 18 Sample of amphidromic points. There are 140 known such points

Figure 18 shows some of the 140 known amphidromic points distributed around the world's oceans. By definition, the tidal range at amphidromic points is zero, but it increases with distance away from the point. Due to the Coriolis effect, lifting or incoming tides tend to circulate around amphidromic points, anticlockwise in the northern hemisphere and clockwise in the southern hemisphere. This has the effect of creating high tides at the same time in different locations, shown by cotidal lines or contours; some examples of these are shown in Figure 19.

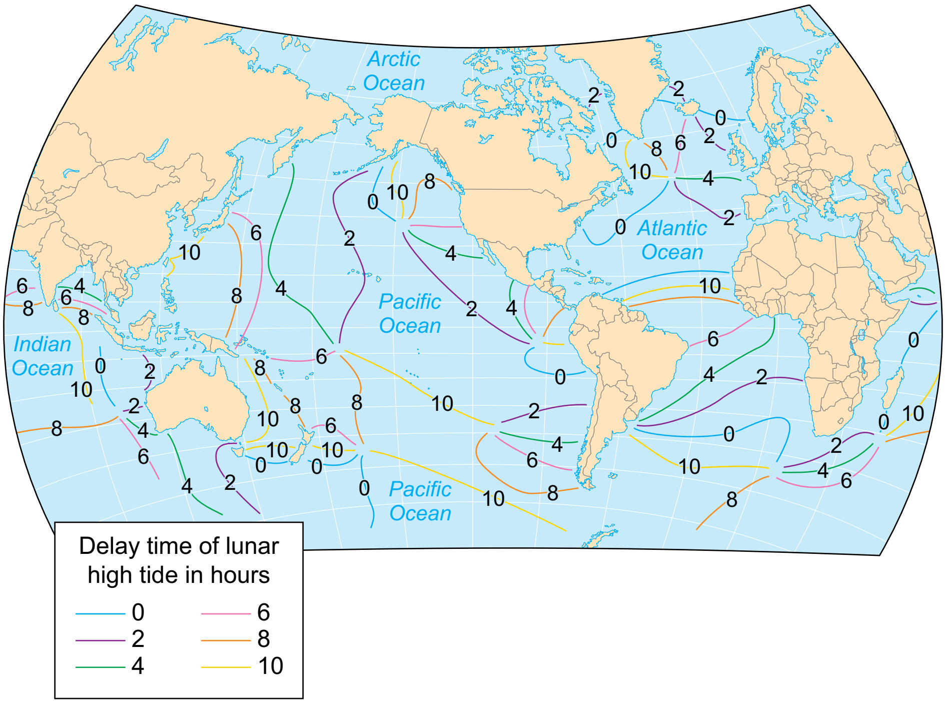


Figure 19 Co-tidal lines indicating high tides occurring at the same times

Tides can be funnelled to stream around islands, promontories and other features both large and small to create regular **surges**, **currents** and amplified sea-level changes. The effect of shallow water and projecting spits of land create the aforementioned wave reflections and interferences, setting up **tidal currents** which appear to have little direct relationship with the oncoming open ocean tidal bulge. Such currents can give rise to **double tides** like those around Southampton, where the ebb tide of the English Channel running through Spithead creates a local high tide in addition to the 'normal' flood tide up the river Solent.

For the British Isles, the main stream of the Atlantic bulge flood tide approaches from the west, and on approaching the southern part of Ireland it splits into three main current streams. One follows the west coast of Ireland travelling north. Another enters and travels northwards up the Irish Sea, meeting up with the first one to the north of Ireland; both of these combine to continue flowing around the north of Scotland and back down the east coast of Britain and the North Sea towards Dover. Meanwhile, the third current stream flows into the English Channel, meeting the North Sea stream off Dover. In other words, the currents swirl both clockwise and anticlockwise around the island of Great Britain. These North Sea currents and surges can cause large tidal ranges, particularly when accompanied by strong winds, but there is an amphidromic point on the eastern side of the North Sea, off Denmark, and another midway between Norfolk and the Netherlands.

Coriolis effect on a tidal current stream

A tidal current stream is flowing due north at a speed of 10.0 km h^{-1}. What would be the Coriolis acceleration forces on a body of 100.0 tonnes of seawater at latitudes 30 and 75 degrees respectively? Give your answers to 3 significant figures.

Solution

The reference frame angular velocity is that of the Earth, i.e.

$$\omega = \frac{2\pi}{(24 \times 60 \times 60)\ \mathrm{s}} = 72.72\ldots \times 10^{-6}\ \mathrm{s}^{-1}.$$

The effective radial velocity from the axis of rotation is given by

$$u_\mathrm{s} = u \sin\theta.$$

So at latitude 30 degrees

$$u_\mathrm{s} = u \sin 30 = u \times 0.5$$

and at latitude 75 degrees

$$u_\mathrm{s} = u \sin 75 = u \times 0.965\ldots.$$

The velocity itself is

$$u = \frac{10 \times 10^3\ \mathrm{m}}{(60 \times 60)\ \mathrm{s}} = 2.777\ldots\ \mathrm{m\,s^{-1}}.$$

The Coriolis accelerations are: for 30 degrees $a_\mathrm{cor} = 2\omega u_\mathrm{s} = 2\omega u \sin 30^\circ$ so

$$\begin{aligned} a_\mathrm{cor} &= 2 \times \left(72.72\ldots \times 10^{-6}\ \mathrm{s}^{-1}\right) \times \left(2.777\ldots\ \mathrm{m\,s^{-1}} \times 0.5\right) \\ &= 2.02\ldots \times 10^{-4}\ \mathrm{m\,s^{-2}} \end{aligned}$$

and for 75 degrees $a_\mathrm{cor} = 2\omega u_\mathrm{s} = 2\omega u \sin 75^\circ$ so

$$\begin{aligned} a_\mathrm{cor} &= 2 \times \left(72.72\ldots \times 10^{-6}\ \mathrm{s}^{-1}\right) \times \left(2.777\ldots\ \mathrm{m\,s^{-1}} \times 0.965\ldots\right) \\ &= 3.902\ldots \times 10^{-4}\ \mathrm{m\,s^{-2}}. \end{aligned}$$

The Coriolis forces are given by $F = ma$. So for 30 degrees

$$\begin{aligned} F_\mathrm{cor} &= m a_\mathrm{cor} \\ &= \left(100 \times 10^3\ \mathrm{kg}\right) \times \left(2.02\ldots \times 10^{-4}\ \mathrm{m\,s^{-2}}\right) \\ &= 20.20\ldots\ \mathrm{N} \end{aligned}$$

and for 75 degrees

$$\begin{aligned} F_{\mathrm{cor}} &= ma_{\mathrm{cor}} \\ &= \left(100 \times 10^3\,\mathrm{kg}\right) \times \left(3.902\ldots \times 10^{-4}\,\mathrm{m\,s^{-2}}\right) \\ &= 39.02\ldots\ \mathrm{N}. \end{aligned}$$

Thus at latitude 30 degrees the Coriolis force is 20.2 N and at latitude 75 degrees it is 39.0 N (both to 3 s.f.). Note how the force nearly doubles with the 45 degree increase in latitude. Also, although these are relatively small forces, 100 tonnes of water represents the mass of a cube of water with sides of only about 4.6 m, which is tiny compared with a sizeable chunk of ocean.

Activity 2

A tidal current stream is flowing north to south at a speed of 8.0 km h^{-1}. Referring to the approach presented in Figure 19, estimate the lateral Coriolis acceleration force on a cubic metre of seawater of density 1025.0 kg m^{-3} at latitude 60 degrees. What would be the lateral speed due to the Coriolis acceleration of the same volume of water after 3 hours, neglecting the change in latitude? Give your answer to 2 significant figures.

Answer

The angular velocity of the Earth is

$$\omega_{\mathrm{Earth}} = \frac{2\pi}{(24 \times 60 \times 60)\ \mathrm{s}} = 72.72\ldots \times 10^{-6}\ \mathrm{s^{-1}}$$

and the wind speed is

$$u = 8\ \mathrm{km\ h^{-1}} = \frac{8 \times 10^3\ \mathrm{m}}{(60 \times 60)\ \mathrm{s}} = 2.222\ldots\ \mathrm{m\ s^{-1}}.$$

At 60 degrees the effective radial velocity north to south is

$$\begin{aligned} u_{\mathrm{s}} &= u \sin 60^\circ \\ &= 2.222\ldots\ \mathrm{m\,s^{-1}} \times \sin 60^\circ \\ &= 1.924\ldots\ \mathrm{m\,s^{-1}}. \end{aligned}$$

So the Coriolis acceleration is

$$\begin{aligned} a_{\mathrm{cor}} &= 2\omega u_{\mathrm{s}} \\ &= 2 \times \left(72.72\ldots \times 10^{-6}\,\mathrm{s^{-1}}\right) \times \left(1.924\ldots\ \mathrm{m\,s^{-1}}\right) \\ &= 0.279\ldots \times 10^{-3}\,\mathrm{m\,s^{-2}}. \end{aligned}$$

The lateral Coriolis force is

$$\begin{aligned} F_{\mathrm{cor}} &= m \times a_{\mathrm{cor}} \\ &= 1025.0\,\mathrm{kg} \times \left(0.279\ldots \times 10^{-3}\ \mathrm{m\,s^{-2}}\right) \\ &= 0.29\,\mathrm{N}\ (\text{to 2 s.f.}) \end{aligned}$$

and after three hours the lateral speed will be

$$\begin{aligned} u &= a_{\text{cor}}t, \\ &= \left(0.279\ldots \times 10^{-3}\,\text{m}\,\text{s}^{-2}\right) \times (3 \times 60 \times 60) \\ &= 3.022\ldots\ \text{m}\,\text{s}^{-1} \end{aligned}$$

or $11\,\text{km}\,\text{h}^{-1}$ (to 2 s.f.).

2.5 Force on a floating tunnel

A fixed rail/road link has been proposed in the North Channel between south-west Scotland and Northern Ireland (see Figure 20).

Figure 20 UKHO Chart 2198: North channel, southern part

Although Portpatrick in Scotland to Donaghadee in Northern Ireland is only 35 km (for comparison, the Channel Tunnel is 50 km long and the Lake Pontchartrain Causeway viaduct is 38 km long) the location poses a series of problems.

- Since the whole of the northern half of the Irish Sea has to fill and drain twice a day through the North Channel, the tidal streams are strong: at maximum (spring tides) the tide flows at 3 knots (1.5 m s^{-1}) mid-channel and at 4.5 knots (2.25 m s^{-1}) near the Irish coast (see Figure 20). Note that tides are normally expressed in knots: 1 knot is 1 nautical mile per hour, equal to 0.514 m s^{-1}.
- The weather in the North Channel is notoriously wild, and the combination of strong winds blowing across fast tidal flow in the opposite direction regularly produces huge waves of up to 20 m peak to trough (10 m amplitude). A breakwater at Portpatrick constructed in 1836 by John Rennie the Younger using techniques developed by his father for building lighthouses lasted less than three years before it was destroyed by a winter storm.
- There is significant shipping traffic, for which a route must be left clear. Any crossing solution must also be able to withstand a collision with a ship.
- In the middle of the channel, slightly towards the Scottish side, is Beaufort’s Dyke, a glacial valley 45 km long, 3 km wide and up to 300 m deep. On its own it would pose

a significant challenge, but to make matters worse it was used as a dumping ground for hazardous waste after World War II and contains many thousands of tonnes of high explosives, incendiary bombs, poison gas and some nuclear waste, all poorly contained.

One possible solution to the problems is a floating tunnel (see Figure 21). At the time of writing this technology is under development in Norway as a possible solution to the similar problem of fjord crossings needed for the coastal highway project.

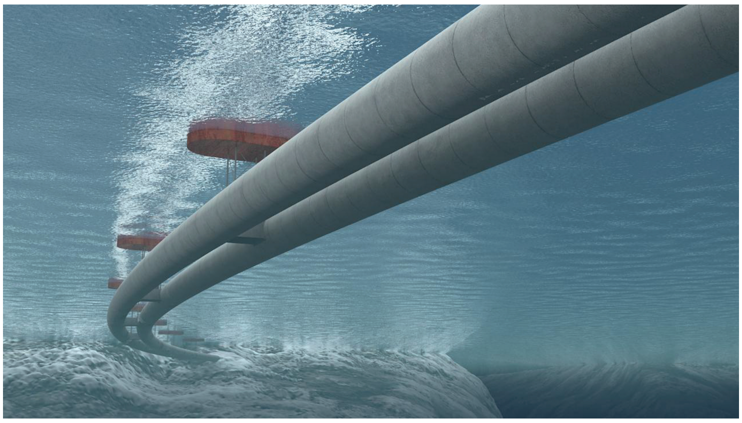


Figure 21 An artist’s impression of a proposed floating tunnel for Norway

A floating tunnel may be held up by its own buoyancy and anchored to the sea bed, or it may have negative buoyancy (i.e. a tendency to sink) and be suspended in position by surface pontoons.

In Activities 3.3a–3.3c you will investigate whether a floating tunnel in the North Channel will be able withstand the wave and tidal forces at this location.

Before you attempt Activity 3, it may be necessary to review your understanding of two dimensionless quantities which often occur when analysing and describing fluids: drag coefficient, Cd, and Reynolds number, Re.

Before you attempt Activity 3, it may be necessary to review your understanding of two dimensionless quantities which often occur when analysing and describing fluids: drag coefficient, Cd, and Reynolds number, Re. Start of Activity

Reynolds number

In a fluid flow situation, the Reynolds number is an important dimensionless parameter which characterises the nature of the flow. It is effectively the ratio of inertial forces to viscous forces in the fluid, both of which are resisting changes to velocity (i.e. accelerations of an object or fluid). For a cylinder of circular cross-section placed at right angles to a fluid flow, the equation for Reynolds number Re can be stated as

$$\mathrm{Re} = \frac{U_\infty d}{\nu}$$

Where U_∞ is the transverse fluid flow velocity some distance away i.e. not disturbed by the cylinder), d is the cylinder diameter and v (Greek letter nu) is the kinematic viscosity of the fluid.

Drag coefficient

From the definition of drag force presented in the earlier part of the course on the atmosphere, you know that drag force, F_d can be stated as

$$F_{\mathrm{D}} = \frac{1}{2} C_{\mathrm{D}} \rho u^2 A$$

Where C_d is the dimensionless parameter, the drag coefficient. The drag coefficient is an experimentally determined value which varies characteristically with Reynolds number (Re) for a given flow situation.

Activity 3

Tidal forces: Question 1

Interactive content is not available in this format.

Assuming that a single tunnel is 10 m in external diameter, find the Reynolds number based on diameter for a maximum tidal stream of 2.5 m s^{-1}. Assume that the density of seawater is $\rho = 1020\,\mathrm{kg\,m^{-3}}$ and kinematic viscosity is

$\nu = 1.38 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}$. Give your answer to 2 significant figures.

Answer

Using the tidal stream velocity of $u = 2.5\,\mathrm{m\,s^{-1}}$, external diameter $l = 10\,\mathrm{m}$ and kinematic viscosity of seawater of $\nu = 1.38 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}$, the Reynolds number is

$$\begin{aligned}
\mathrm{Re} &= \frac{ul}{\nu} \\
&= \frac{2.5\,\mathrm{m\,s^{-1}} \times 10\,\mathrm{m}}{1.38 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}} \\
&= 1.8 \times 10^7 \text{ (to 2 s.f.).}
\end{aligned}$$

Tidal forces: Question 2

Interactive content is not available in this format.

Using the following graph, estimate the drag coefficient at the Reynolds number found in Question 1 and hence the expected lateral force per kilometre on the tunnel. Give your answer to 2 significant figures.

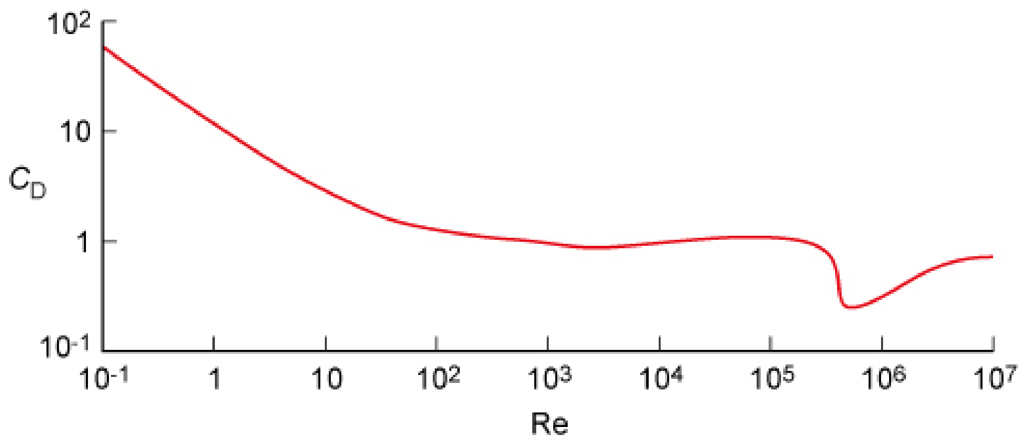


Figure 22

Answer

Flow is completely turbulent and the drag coefficient will be close to that shown for Re = 10^7, so from the graph $C_\mathrm{D} = 0.9$.

A 1 km length of pipe has a transverse area of $1000\,\mathrm{m} \times 10\,\mathrm{m} = 10\,000\,\mathrm{m}^2$, so the drag force per km is

$$\begin{aligned} F_\mathrm{D} &= \tfrac{1}{2} C_\mathrm{D} \rho u^2 A \\ &= \tfrac{1}{2} \times 0.9 \times 1020\,\mathrm{kg\,m^{-3}} \times \left(2.5\,\mathrm{m\,s^{-1}}\right)^2 \times 10\,000\,\mathrm{m}^2 \\ &= 29\,\mathrm{MN}\ (\text{to } 2\text{ s.f.}). \end{aligned}$$

This is a substantial force, to put it mildly. However you may care to compare it to the buoyant force exerted by the sea on the same length of tunnel when the centreline is submerged by only 20 m. The buoyant force F_b on an immersed object is equal to the weight of fluid displaced by it (Archimedes' principle), so

$$F_\mathrm{b} = \rho g V$$

$$F_\mathrm{b} = 1020 \times 9.81 \times \frac{\pi \times (10\,\mathrm{m})^2}{4} \times 1000\,\mathrm{m}$$

$$F_d = 790\,MN (\text{to } 2 \text{ s.f.}$$

This is around 27 times greater than the tidal force, which is therefore not particularly large by the standards of the project.

A winter storm creates deep water waves of amplitude 10 m and wavelength 100 m in the middle of the North Channel. Work through the following questions to find whether the tunnel will be able to withstand the wave forces in winter.

Give your answer to 2 significant figures where appropriate.

Wave forces: Question 3

Interactive content is not available in this format.

How fast will the waves travel?

Answer

Using the formula for deep water waves in Equation 12, the wave speed is

$$\begin{aligned} c_{\mathrm{w}} &= \sqrt{\frac{Lg}{2\pi}} \\ &= \sqrt{\frac{100\,\mathrm{m} \times 9.81\,\mathrm{m\,s^{-2}}}{2\pi}} \\ &= 12.49\ldots\,\mathrm{m\,s^{-1}} \\ &= 12\,\mathrm{m\,s^{-1}}\ (\text{to 2 s.f.}). \end{aligned}$$

Wave forces: Question 4

What is the period of the waves if the centre of the tunnel is at a depth of 50 m?

Answer

Using Equation 13, at 50.0 m depth the period of the waves is

$$\begin{aligned} T &= \frac{L}{c_{\mathrm{w}}} \\ &= \frac{100\,\mathrm{m}}{12.49\ldots\,\mathrm{m\,s^{-1}}} \\ &= 8.0\,\mathrm{s}\ (\text{to 2 s.f.}). \end{aligned}$$

Wave forces: Question 5

Will the waves cause significant forces?

Answer

According to Section 1.2, deep-water, wave-induced motion at half the wavelength is around 4% of the surface value. That is the case here, so the wave motion at the tunnel centre line will be only 4% × 10 m = 40 cm. Since tidal flow in the centre regularly reaches 150 cm s^{-1}, the extra wave displacement of 40 cm there-and-back every 4 seconds will not add significant additional forces.

Conclusion: Question 5

Is a floating tunnel a viable solution to the problem from the fluid dynamics point of view?

Answer

Conclusion: A floating tunnel should easily be able to withstand both tidal and wave forces in this location.

Conclusion

This course has focused on two of the most important environmental fluids for engineers: the atmosphere and the oceans. It has been shown that a thorough understanding of the mechanics of these fluids is vital to the success of engineering projects as diverse as spacecraft re-entering the earth's atmosphere and floating tunnels providing road/rail links between land masses.

This free course is an adapted extract from the Open University course T229 *Mechanical engineering: heat and flow*.

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Figure 10: NASA

Figure 11: adapted from https://en.wikipedia.org/wiki/Soyuz_(spacecraft)#

Figure 15: based on maps of the Institute of Oceanographic Sciences, May 1977

Figure 17: taken from https://www.tidetimes.org.uk/hestan-island-tide-times

Figure 18: taken from http://slideplayer.com/slide/3761217/13/images/20/Amphidromic+Points+Cotidal+map+shows+tides+rotate+around+amphidromic+points.+There+are+140+amphidromic+points+in+the+world%E2%80%99s+oceans..jpg

Figure 19: taken from http://slideplayer.com/slide/3761217/13/images/21/Cotidal+Lines+Cotidal+lines+show+where+high+tides+occur+at+the+same+time.jpg

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Glossary

Beaufort Wind Force Scale

A thirteen-step scale of wind speeds (Force 0 to Force 12) based on observations of the effects at sea and on land.

Coriolis effect

The apparent tangential acceleration of an object moving towards or away from an axis around which it is moving.

Reynolds number

A dimensionless number that indicates the relative importance of inertial and viscous forces and also the likelihood of turbulent flow. Reynolds numbers are frequently used to compare flow patterns.

amphidromic point

A point with no tidal rise or fall. Also called a tidal node.

amphidromic system

A system (e.g. the North Sea or the Sound of Jura) containing an amphidromic point.

amplitude

The maximum extent of a vibration, oscillation or periodic function, measured from the position of equilibrium or centre line. The amplitude of a sinusoidal curve is half the difference between the maximum and minimum values of the curve.

anticyclones

The large-scale atmospheric rotation around an area of high pressure. Anticyclonic rotation is clockwise in the northern hemisphere and anti-clockwise south of the equator.

atmospheric boundary layer

The region of the atmosphere, up to about 1000 m above ground level, in which interaction with the ground significantly affects wind speed and direction.

break

A water wave breaks when it changes from approximately sinusoidal in shape to hooked prior to the crest collapsing. Waves may break as a result of wind action, moving into shallower water or a combination of both.

crest

The top of a water wave.

currents

Bulk flow in the atmosphere or oceans.

cyclones

The large-scale atmospheric rotation around an area of low pressure. Cyclonic rotation is anti-clockwise in the northern hemisphere and clockwise south of the equator. A cyclone is also the name for a hurricane when south of the equator.

depression

An area of atmospheric low pressure. Also called a low.

design wave

The likely worst-case wave height, used as a factor in the design of structures that interact with the sea.

design wind speed

The likely highest wind speed, used as a factor in the design of structures that interact with the atmosphere.

double tides

Two high tides in close succession with a small drop in between (or two low tides in close succession with a small rise in between).

drag coefficient

A non-dimensional form of drag: the drag force produced as a fraction of the product of stagnation pressure and a characteristic area. See also **lift coefficient**.

ebb

The receding or downward-going tide.

fetch

The distance over which wind can build up waves at sea.

flood

The approaching or upward-going tide.

geostrophic winds

Winds above the atmospheric boundary layer, driven by pressure gradients and Coriolis forces. Also called gradient winds.

gradient winds

See **geostrophic winds**.

ground level to reference height

The reference height of 10 m used for modelling wind variation in the atmospheric boundary layer.

high tides

The time at which the sea reaches its greatest depth in a particular tidal cycle; also the depth at that time.

isothermal height

See **tropopause**.

low

See **depression**.

low tides

The time at which the sea reaches its smallest depth in a particular tidal cycle; also the depth at that time.

neap tides

The time of lowest tidal range, when moon and sun work in opposition.

obliquity

The tilt of the Earth's axis relative to a normal to the plane in which it orbits the sun.

orbit

The path followed by a body moving round another under the influence of gravity.

peak

The highest point of a wave.

ripples

Surface waves in water (or any other liquid) with an amplitude much smaller than the undisturbed depth.

slack water

A time when tidal currents are zero, usually coinciding with high or low tide.

spring tides

The time of highest tidal range, when moon and sun work together.

stratosphere

The upper part of the atmosphere.

surges

Rapid changes of sea level due to tidal effects.

swell

Long-wavelength oceanic waves.

tidal bulge

One of two areas of increased water depth, caused by the gravitational attraction of the sun and moon, which travel around the world and cause tides.

tidal currents

Horizontal flows of water caused by tidal depth changes.

tidal node

See **amphidromic point**.

tidal range

The difference in height between a high tide and the preceding or following low tide.

tidal wave

An ocean wave, normally caused by an undersea earthquake, which on approaching land causes a sea level change comparable to that caused by tides.

trade winds

The relatively constant east–west winds that blow in the tropical zones north and south of the equator.

tropopause

The top of the troposphere, above which the atmospheric temperature (in the stratosphere) is effectively constant. Also called isothermal height.

troposphere

The lower part of the atmosphere, in which most weather systems exist.

trough

The lowest part of a surface wave.

tsunami

Japanese term for a tidal wave, derived from ‘tsu’ (harbour) + ‘nami’ (wave).

wave periodic time

The time it takes for any point to experience a full wave cycle.

wave speed

The speed at which a travelling wave advances.

wavelength

The spatial distance over which a periodic waveform repeats (e.g. the distance between successive peaks or successive troughs).

waves

Any regular oscillation of a continuous medium.

westerlies

The relatively constant west–east winds that blow in the temperate zones further north and south of the equator than the tropical zones.