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Real functions and graphs

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Intellectual property

Head of Intellectual Property, The Open University
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Contents

Introduction

Many problems are best studied by working with real functions, and the properties of real functions are often revealed most clearly by their graphs. Learning to sketch such graphs is therefore a useful skill, even though computer packages can now perform the task. Computers can plot many more points than can be plotted by hand, but simply ‘joining up the dots’ can sometimes give a misleading picture, so an understanding of how such graphs may be obtained remains important. The object of this course is to review the various techniques for sketching graphs that you may have met in your previous studies, and to extend these methods.

This OpenLearn course is an adapted extract from the Open Unviersity course M208: *Pure Mathematics*

Learning Outcomes

After studying this course, you should be able to:

- understand the definition of a real function
- use the notation for intervals of the real line
- recognise and use the graphs of the basic functions described in the audio section
- understand the effect on a graph of translations, scalings, rotations and reflections
- understand how the shape of a graph of a function features properties of the function such as increasing, decreasing, even and odd.

1 Overview

A fundamental concept in mathematics is that of a *function*.

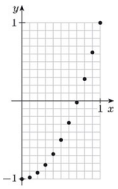
Consider, for example, the function f defined by $f(x) = 2x^2 - 1$, where $0 \leq x \leq 1$.

This is an example of a *real function*, because it associates with a given real number x the real number $2x^2 - 1$: it maps real numbers to real numbers.

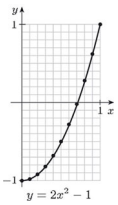
One way of picturing this function is the following. First, draw up a table of values, listing in the first row several values of x, and in the second row the corresponding values of $f(x)$; for example, $f(0.8) = 2(0.8)^2 - 1 = 0.28$.

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$f(x)$	−1	−0.98	−0.92	−0.82	−0.68	−0.5	−0.28	−0.02	0.28	0.62	1

Each column in the table is essentially an ordered pair of the form $(x, f(x))$, which can be plotted as a unique point in the plane. We refer to the set of all such points as the *graph* of f.



Of course, it is not possible to plot all the points of the graph for a function like f, which is defined for infinitely many values of x. Fortunately, for many graphs plotting a few points provides a good idea of what the graph looks like. For any so-called 'well-behaved' function, these points can be joined up by a smooth curve – and extended if necessary – to complete the picture.



No doubt you are already familiar with this method of drawing a graph. However, it is not always the most efficient method to use when knowledge about the key features of the graph is all that is wanted. For the purposes of this course, that is all that is usually required.

Please note that this course is presented through a series of downloadable PDF documents.

2 Real functions

In Section 1 we formally define *real functions* and describe how they may arise when we try to solve equations. We remind you of some basic real functions and their graphs, and describe how some of the properties of these functions are featured in their graphs.

Click the link below to open Section 1 (12 pages, 1.8MB).

Section 1

This PDF contains an audio section in which we revise the properties of some familiar graphs, and introduce some concepts which form the basis of the graph-sketching strategy given in Section 2. Please listen to the audio clips below when you are instructed to do so. Do not worry if you do not fully understand all the details at this stage, but concentrate on the main ideas.

Click play to listen to the audio clip for frames 1 to 4 (6 minutes).

Audio content is not available in this format.
Frames 1-4

Click play to listen to the audio clip for frames 5 to 9 (7 minutes).

Audio content is not available in this format.
Frames 5-9

Click play to listen to the audio clip for frames 10 to 13 (7 minutes).

Audio content is not available in this format.
Frames 10-13

Click play to listen to the audio clip for frames 14 to 17 (7 minutes).

Audio content is not available in this format.
Frames 14-17

Click play to listen to the audio clip for frames 18 to 21 (3 minutes).

Audio content is not available in this format.
Frames 18-21

Click play to listen to the audio clip for frames 22 to 25 (5 minutes).

Audio content is not available in this format.
Frames 22-25

3 Graph sketching

In Section 2 we describe how the graphs of polynomial and rational functions may be sketched by analysing their behaviour – for example, by using techniques of calculus. We assume that you are familiar with basic calculus and that its use is valid. In particular, we assume that the graphs of the functions under consideration consist of smooth curves.

Click the link below to open Section 2 (16 pages, 200KB).

Section 2

4 New graphs from old

In Section 3 we consider how to sketch the graphs of more complicated functions, sometimes involving trigonometric functions. We look at graphs which are sums, quotients and composites of different functions, and at those which are defined by a different rule for different values of x.

Click the link below to open Section 3 (7 pages, 133KB).

Section 3

5 Hyperbolic functions

In Section 4 we introduce the *hyperbolic functions* sinh, cosh and tanh, which are constructed from exponential functions. These hyperbolic functions share some of the properties of the trigonometric functions but, as you will see, their graphs are very different.

Click the link below to open Section 4 (5 pages, 104KB).

Section 4

6 Curves from parameters

In Section 5 we show how functions may be used to sketch curves in the plane, even when these curves are not necessarily the graphs of functions.

Click the link below to open Section 5 (8 pages, 151KB).

Section 5

7 Solutions to the exercises

Section 6 contains solutions to the exercises that appear throughout sections 1-5.
Click the link below to open the solutions (13 pages, 232KB).
Section 6

Conclusion

This free course provided an introduction to studying Mathematics. It took you through a series of exercises designed to develop your approach to study and learning at a distance and helped to improve your confidence as an independent learner.

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Acknowledgements

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