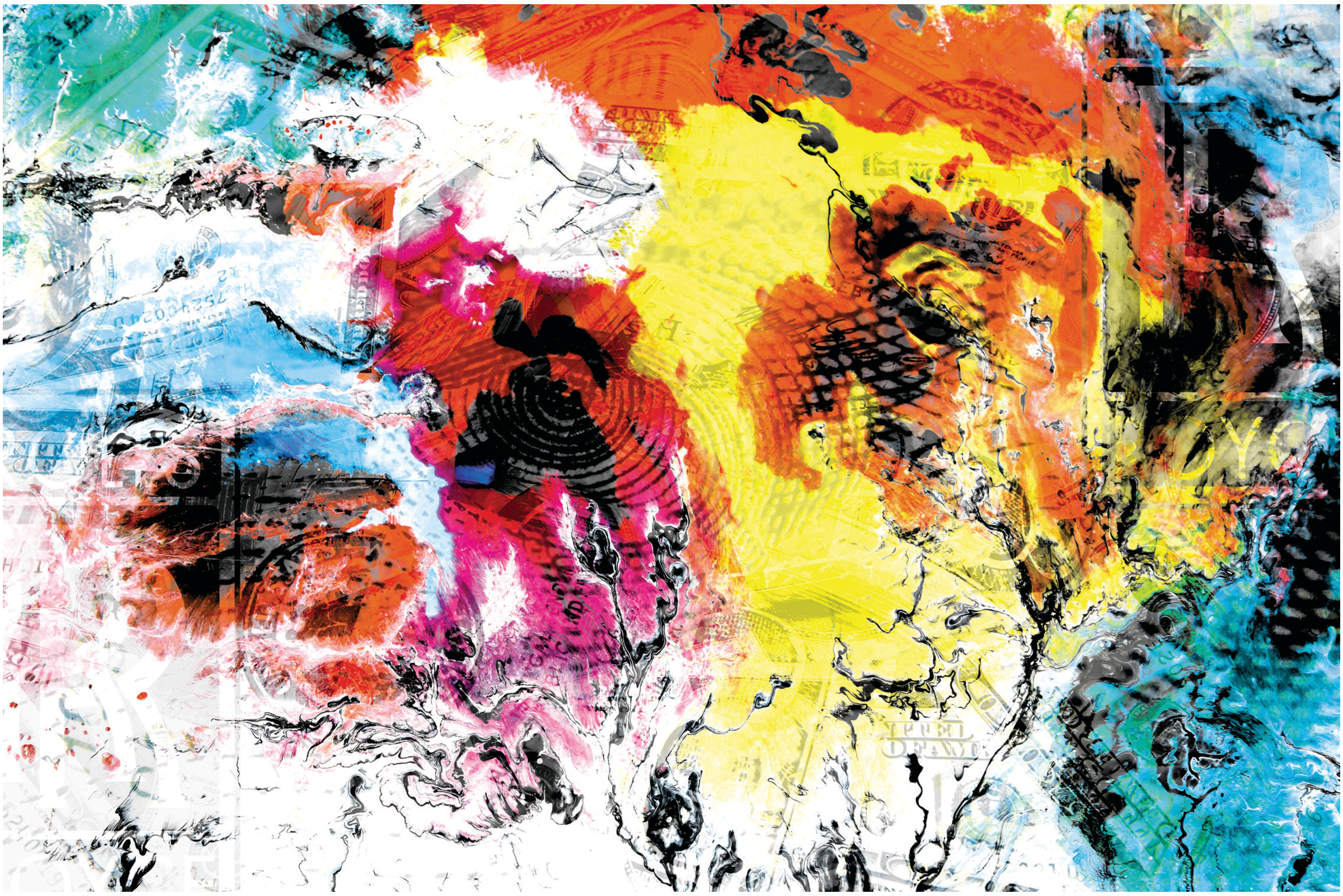
Introduction and guidance



About this free course

This free course is an adapted extract from the Open University course .

This version of the content may include video, images and interactive content that may not be optimised for your device.

You can experience this free course as it was originally designed on OpenLearn, the home of free learning from The Open University –

There you'll also be able to track your progress via your activity record, which you can use to demonstrate your learning.

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Introduction and guidance

This free badged course, *Everyday maths 2*, is an introduction to Level 2 Essential Skills in maths. It is designed to inspire you to improve your current maths skills and help you to remember any areas that you may have forgotten. Working through the examples and interactive activities in this course will help you to, among other things, run a household or make progress in your career.

You can work through the course at your own pace. To complete the course you will need access to a calculator and a notepad and pen.

The course has four sessions, with a total study time of approximately 48 hours. The sessions cover the following topics: numbers, measurement, shapes and space, and data. There will be plenty of examples to help you as you progress, together with opportunities to practise your understanding.

The regular interactive quizzes form part of this practice, and the end-of-course quiz is an opportunity to earn a badge that demonstrates your new skills. You can read more on how to study the course and about badges in the next sections.

After completing this course you will be able to:

- understand practical problems, some of which are non-routine
- identify the maths skills you need to tackle a problem
- use maths in an organised way to find the solution you're looking for
- use appropriate checking procedures at each stage
- explain the process you used to get an answer and draw simple conclusions from it.

Moving around the course

The easiest way to navigate around the course is through the 'My course progress' page. You can get back there at any time by clicking on 'Back to course' in the menu bar.

It's also good practice, if you access a link from within a course page (including links to the quizzes), to open it in a new window or tab. That way you can easily return to where you've come from without having to use the back button on your browser.

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Introduction

What is a badged course?

While studying *Everyday maths 2* you have the option to work towards gaining a digital badge.

Badged courses are a key part of The Open University's mission *to promote the educational well-being of the community.* The courses also provide another way of helping you to progress from informal to formal learning.

To complete a course you need to be able to find about 48 hours of study time. It is possible to study them at any time, and at a pace to suit you.

Badged courses are all available on The Open University's OpenLearn website and do not cost anything to study. They differ from Open University courses because you do not receive support from a tutor. But you do get useful feedback from the interactive quizzes.

What is a badge?

Digital badges are a new way of demonstrating online that you have gained a skill. Schools, colleges and universities are working with employers and other organisations to develop open badges that help learners gain recognition for their skills, and support employers to identify the right candidate for a job.

Badges demonstrate your work and achievement on the course. You can share your achievement with friends, family and employers, and on social media. Badges are a great motivation, helping you to reach the end of the course. Gaining a badge often boosts confidence in the skills and abilities that underpin successful study. So, completing this course should encourage you to think about taking other courses, for example enrolling at a college for a formal qualification. (You will be given details on this at the end of the course.)

How to get a badge

Getting a badge is straightforward! Here's what you have to do:

- read all of the pages of the course
- score 70% or more in the end-of-course quiz.

For all the quizzes, you can have three attempts at most of the questions (for true or false type questions you usually only get one attempt). If you get the answer right first time you will get more marks than for a correct answer the second or third time. Therefore, please be aware that for the end-of-course quiz it is possible to get all the questions right but not score 50% and be eligible for the OpenLearn badge on that attempt. If one of your answers is incorrect you will often receive helpful feedback and suggestions about how to work out the correct answer.

If you're not successful in getting 70% in the end-of-course quiz the first time, after 24 hours you can attempt it again and come back as many times as you like.

We hope that as many people as possible will gain an Open University badge – so you should see getting a badge as an opportunity to reflect on what you have learned rather than as a test.

If you need more guidance on getting a badge and what you can do with it, take a look at the OpenLearn FAQs. When you gain your badge you will receive an email to notify you and you will be able to view and manage all your badges in My OpenLearn within 24 hours of completing the criteria to gain a badge.

Now get started with Session 1.

Session 1: Working with numbers

Introduction

With an understanding of numbers you have the basic skills you need to deal with much of everyday life. In this session, you will learn about how to use each of the four different mathematical operations and how they apply to real life situations. Then you will encounter fractions and percentages which are incredibly useful when trying to work out the price of an item on special offer. Additionally, you will learn how to use ratio – handy if you are doing any baking – and how to work with formulas.

Throughout the course there will be various activities for you to complete in order to check your skills. All activities come with answers and show suggested working. It is important to know that there are often many ways of working out the same calculation. If your working is different to that shown but you arrive at the same answer, that's perfectly fine. If you need to refresh your written methods for addition, subtraction, multiplication or division please refer to the relevant number section in *Everyday maths 1*.

If you want more practice with your maths skills, please contact your local college to discuss the options available.

By the end of this session you will be able to:

- use the four operations to solve problems in context
- understand rounding and look at different ways of doing this
- write large numbers in full and shortened forms
- carry out calculations with large numbers
- carry out multistage calculations
- solve problems involving negative numbers
- define some key mathematical terms (multiple, lowest common multiple, factor, common factor and prime number)
- identify lowest common multiples and factors
- use fractions, decimals and percentages and convert between them
- solve different types of ratio problems
- make substitutions within given formulas to solve problems
- use inverse operations and estimations to check your calculations.

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1 Four operations

You will already be using the four operations in your daily life (whether you realise it or not). Everyday life requires you to carry out maths all the time; checking you've been given the correct change, working out how many packs of cakes you need for the children's birthday party and splitting the bill in a restaurant are all examples that come to mind.

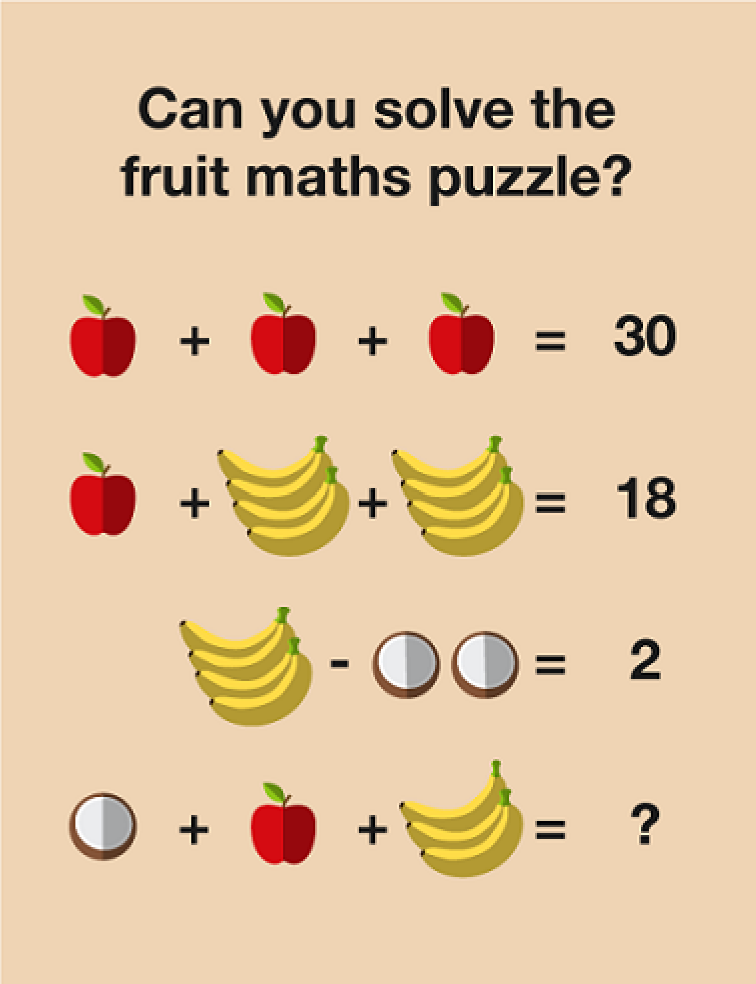

Figure 1 Fruit maths puzzle

The four operations are addition, subtraction, multiplication and division. You need to understand what each operation does and when to use it and at level 2, you will often be required to use more than one operation to answer a question.

- **Addition (+)**
  - This operation is used when you want to find the total, or sum, of two or more amounts.
- **Subtraction (−)**
  - This operation is used when you want to find the difference between two amounts or how much of something you have left after a quantity is used – for example, if you want to find the change owed after spending an amount of money.
- **Multiplication (×)**
  - This operation is used to calculate multiple amounts of the same number. For example, if you wanted to know the cost of 16 plants costing £4.75 each you would multiply £4.75 by 16:

    £4.75 × 16 = £76
- **Division (÷)**

- Division is used when sharing or grouping items. For example, if a group of 6 friends won £765 on the lottery and you wanted to know how much each person would get you would divide £765 by 6:

  £765 ÷ 6 = £127.50

If you are doing this course to prepare you for Essential Skills Wales Application of Number, remember the exam papers do not allow the use of a calculator so please attempt the calculations in this session by hand.

Activity 1: Operation choice

Each of the four questions below uses one of the four operations. Match the operation to the question.

Match each of the items above to an item below.

You need to save £306 for a holiday. You have 18 months to save up that much money. How much do you need to save per month?

Fourteen members of the same family go on holiday together. They each pay £155. What is the total cost of the holiday?

You make an insurance claim worth £18 950. The insurance company pays you £12 648. What is the difference between what you claimed and what you actually received?

You go to the local café and buy a coffee for £2.35, a tea for £1.40 and a croissant for £1.85. How much do you spend?

Try these questions to make sure that you are confident using each of the four operations.

Activity 2: Calculations with whole numbers

1. 1245 + 654
2. 187 + 65 401
3. 1060 − 264
4. 2000 − 173
5. 543 × 19
6. 1732 × 46
7. 1312 ÷ 8
8. 1044 ÷ 12
9. Stuart is saving £225 per month for a new car. How much would he have saved in:

   a. 1 year
   b. 2 years
10. A concert venue has 2190 seats arranged into 15 sections. How many seats are in each section?
11. A college sells 325 tickets to the charity ball at £12 each. How much money do they make from ticket sales?

Answer

1. 1245 + 654 = 1899
2. 187 + 65 401 = 65 588
3. 1060 − 264 = 796
4. 2000 − 173 = 1827
5. 543 × 19 = 10 317
6. 1732 × 46 = 79 672
7. 1312 ÷ 8 = 164
8. 1044 ÷ 12 = 87
9.
   a. Stuart is saving £225 per month for 1 year or 12 months so the calculation is 225 × 12 = £2700.
   b. Stuart saves £2700 in 1 year so to work out the amount saved in 2 years the calculation is 2 × £2700 = £5400. Alternatively, you could add £2700 + £2700 = £5400.
10. To calculate this you divide the number of seats by the number of sections. So, 2190 ÷ 15 = 146. So there are 146 seats in each section.
11. To calculate this you multiply the number of tickets by the cost per ticket. So, 325 × 12 = 3900. So the college will make £3900 from ticket sales.

1.1 Expressing a remainder as a decimal

To split a prize of £125 between 5 friends you would do this calculation:

£125 ÷ 5 and get the answer £25.

This is a convenient, exact amount of money. However, often when you perform calculations, especially those involving division, you do not always get an answer that is suitable for the question.

For example, if there were 4 friends who shared the same prize we would do the calculation £125 ÷ 4 and get the answer £31 remainder £1. If we did the same calculation on a calculator you would get the answer £31.25, the remainder has been converted into a decimal. Let’s look at how to express the remainder as a decimal.

You can write one hundred and twenty five pounds in two different ways: £125 or £125.00. Both ways show the same amount but the second way allows you to continue the calculation and express it as a decimal.

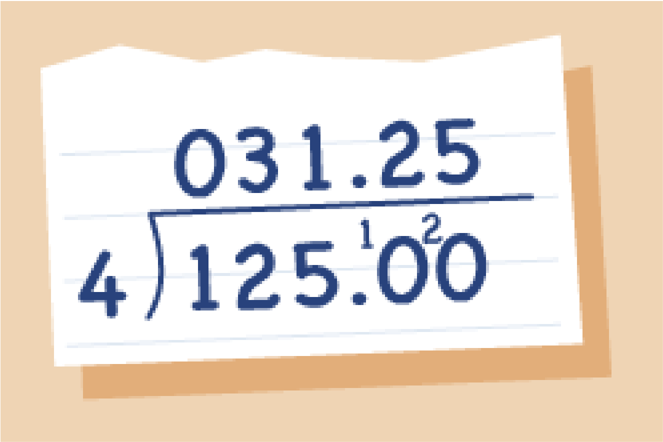

Figure 2 Expressed as a decimal: 125 ÷ 4

We can use the same principal with any whole number, adding as many zeros after the decimal point as required. Look at the following example.

A teacher wants to share 35 kg of clay between 8 groups of students. How much clay will each group get?

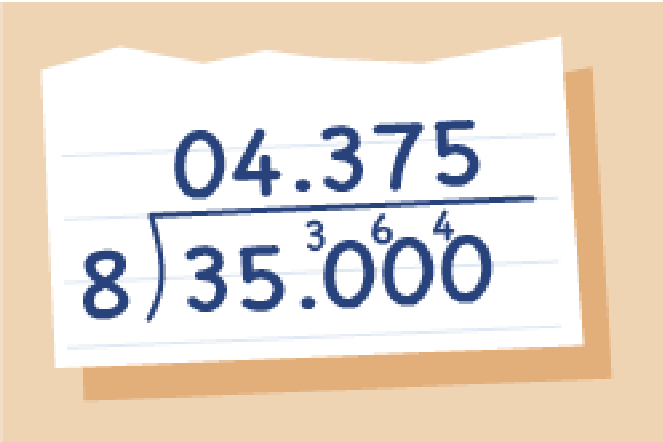

Figure 3 Expressed as a decimal: 35 ÷ 8

You can see that each group would get 4.375 kg of clay.

Activity 3: Expressing a remainder as a decimal

Work out the answers to the following without using a calculator.

1. 178 ÷ 4
2. 212 ÷ 5
3. 63 ÷ 8
4. 227 ÷ 4

Answer

1. 44.5
2. 42.4
3. 7.875

4. 56.75

1.2 Interpreting answers when dividing

After carrying out a division calculation you may not have an answer that is suitable.

For example, if you were at a restaurant and needed to split a bill of £126.49 between four people you would first calculate the division £126.49 ÷ 4 = £31.6225. Clearly you cannot pay this exact amount and so we would round it up to £31.63 to make sure the whole bill is covered.

In other situations, you may need to round an answer down. If you were cutting a length of wood that is 2 m (200 cm) long into smaller pieces of 35 cm you would initially do the calculation 200 ÷ 35. This would give an answer of 5.714…. As you will only actually be able to get 5 pieces of wood that are 35 cm long, you need to round your answer of 5.714 down to simply 5.

Note: The three full stops used in the answer above (5.714…) is a character called an ellipsis. In maths it is used to represent recurring decimal numbers so you don’t have to display them all.

Activity 4: Interpreting answers

Calculate the answers to the following. Decide whether the answers need to be adjusted up or down after calculation of the division sum.

1. Apples are being packed into boxes of 52. There are 1500 apples that need packing. How many boxes are required?
2. A bag of flour contains 1000 g. Each batch of cakes requires 150 g of flour. How many batches can you make?
3. A child gets £2.50 pocket money each week. They want to buy a computer game that costs £39.99. How many weeks will they need to save up in order to buy the game?
4. A length of copper pipe measures 180 cm. How many smaller pieces that each measure 40 cm can be cut from the pipe?

Answer

1. 1500 ÷ 52 = 28.846 which must be adjusted up to 29 boxes.
2. 1000 ÷ 150 = 6.666 which must be adjusted down to 6 batches.
3. £39.99 ÷ £2.50 = 15.996 which must be adjusted up to 16 weeks.
4. 180 ÷ 40 = 4.5 which must be adjusted down to 4 pieces.

1.3 Dealing with decimals

We often deal with decimals in everyday life, for example, calculations involving money.

Try the following (without a calculator) to check your skills. For this activity, you will need to give your answers in full and only round or adjust your answer if required. If you need a reminder about how to carry out any of the calculations, please refer back to *Everyday maths 1*.

Activity 5: Calculations with decimals

1. 54.865 + 4.965 + 23.519
2. 6.938 − 5.517
3. 25 + 0.258
4. 54 − 0.65
5. 5.632 × 2.4
6. 1.542 × 1.9
7. 42.4 ÷ 4
8. 39.45 ÷ 1.5
9. 0.48 ÷ 0.025
10. A factory orders 425 gaskets that weigh 2.3 g each. What is the total weight of the gaskets?
11. A dispenser holds 15.5 litres of water. How many full 0.2 litre cups of water can you get from one dispenser?
12. Ahmed and Lea take part in the long jump. Their results are as follows:

    Ahmed 5.501 m

    Lea 5.398 m

    Who jumped furthest and by how much?

Answer

1. 83.349
2. 1.421
3. 25.258
4. 53.35
5. 13.5168
6. 2.9298
7. 10.6
8. 26.3
9. 19.2
10. 977.5 g
11. 77.5 (77 full cups)
12. Ahmed jumped the furthest by 0.103 m

Summary

In this section you have:

- recapped how to carry out calculations with whole numbers and decimals
- learned to adjust answers where needed
- learned how to express a remainder as a decimal.

2 Dealing with large numbers

It is important to be able to carry out calculations with numbers of any size. Large numbers can be written in different ways e.g.

1 200 000 (one million, two hundred thousand) or it can be written as 1.2 million.

Here is another example:

4 250 000 000 (four billion, two hundred and fifty million) is 4.25 billion.

It is often easier to deal with very large numbers when they are written as decimals.

Notice how the decimal is placed after the whole millions or billions.

Hint: A billion is a thousand million.

Using a place value grid can help you to read large numbers as it groups the digits for you, making the whole number easier to read.

Notice how the numbers above are written in this place value grid.

Table 1

Billion	**Million**			**Thousand**					
Billions	Hundreds of millions	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	**Hundreds**	**Tens**	**Units**
			1	2	0	0	0	0	0
4	2	5	0	0	0	0	0	0	0

Sometimes when dealing with large numbers it is sensible to round them, for example, the Office for National Statistics gives the number of people unemployed in the UK in February 2019 as 1.36 million. The number of people unemployed will not be exactly 1 360 000 but, by rounding the exact value and writing it as 1.36 million, it is easier to understand.

Activity 6: Rounding large numbers

The following table gives the population of countries.

Round each population to the nearest million and write the figure in shortened form, using decimals where needed.

Table 2(a)

Country	Population
UK	66 959 016
China	1 420 062 022

Answer

Table 2(b)

Country	Population	Population rounded	Shortened form
UK	66 959 016	67 000 000	67 million
China	1 420 062 022	1 420 000 000	1.42 billion

2.1 Calculations with large numbers

The best way to get used to these types of calculations is to go straight into an example.

Example: Calculations with large numbers

Calculate the total population of Malta (0.4 million) and Cyprus (1.2 million).

Method 1
Work in shortened form:

1.2 + 0.4 = 1.6 million

Method 2
Write the numbers in full:

1 200 000 + 400 000 = 1 600 000 (1.6 million)

Activity 7: Calculations with large numbers

1. Calculate the total turnover for Cambria Trading over the first quarter (3 months).

Table 3 Cambria Trading turnover

Month	Profit (£ million)
January	1.2
February	0.9
March	0.85
April	1.1
May	1.02
June	0.87
July	1.19
August	0.98
September	1.05
October	1.08
November	1.8
December	1.65

2. Calculate the turnover of the last quarter.
3. Calculate the difference in turnover between the first and last quarters.
4.
   b. Which month had the largest turnover?
   c. Which month had the smallest turnover?
   d. What is the difference between the largest and smallest turnovers?

Answer

1. Profit in 1st quarter 1.2 + 0.9 + 0.85 = £2.95 million
2. Profit in last quarter 1.08 + 1.8 + 1.65 = £4.53 million
3. The difference is 4.53 − 2.95 = £1.58 million
4.
   a. November had the largest turnover at £1.8 million.
   b. March had the smallest at £0.85 million.
   c. The difference is 1.8 − 0.85 = £0.95 million.

Summary

In this section you have learned how to:

- write large numbers in full and shortened forms
- round large numbers
- add and subtract large numbers.

3 Rounding

Why might you want to round numbers? You may wish to estimate the answer to a calculation or to use a guide rather than work out the exact total. Alternatively, you might wish to round an answer to an exact calculation so that it fits a given purpose, for example an answer involving money cannot have more than two digits after the decimal point.

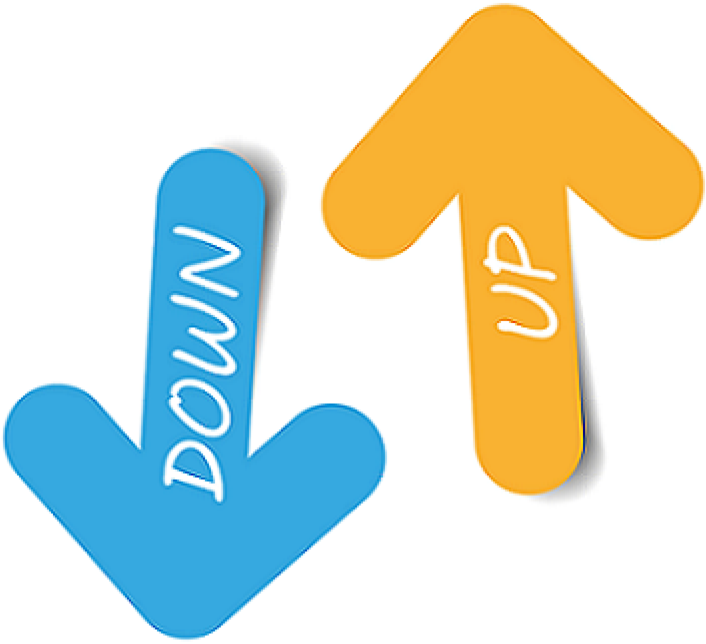

Figure 4 Rounding up and down

You will now explore each of these examples in more detail and practise your rounding skills in context.

3.1 Rounding to a degree of accuracy

Watch the short video below to see an example of how to round to 1, 2 and 3 decimal places.

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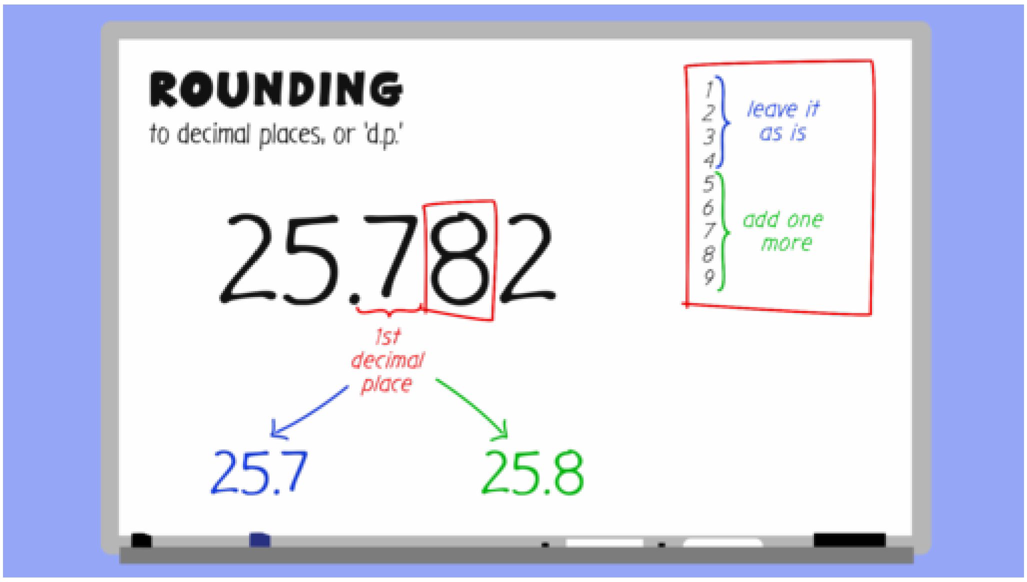

Remember this rounding rhyme to help you:

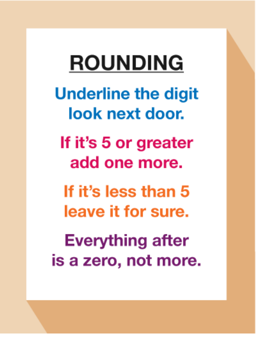

Figure 5 A rounding rhyme

Activity 8: Rounding skills

Practise your rounding skills by completing the below.

1. What is 24.638 rounded to one decimal place?
2. What is 13.4752 rounded to two decimal places?
3. What is 203.5832 rounded to two decimal places?
4. What is 345.6794 rounded to three decimal places?
5. What is 3.65 rounded to the nearest whole number?
6. What is £199.755 to the nearest penny?
7. What is £37.865 to the nearest pound?
8. What is 61.607 kg to the nearest kg?

Answer

1. 24.6
2. 13.48
3. 203.58
4. 345.679
5. 4
6. £199.76
7. £38
8. 62 kg

3.2 Rounding to approximate an answer

You might round in order to approximate an answer. At the coffee shop, you might want to buy a latte for £2.85, a cappuccino for £1.99 and a tea for £0.99. It is natural to round these amounts up to £3, £2 and £1 in order to arrive at an approximate cost of £6 for all three drinks. It is also very useful when checking calculations to make sure that your answer makes sense, especially when working with large numbers and decimals.

Activity 9: Approximation

Calculate the following using a calculator and use estimation to check your answers.

1. On 5th March 2019, 190 people matched 5 numbers and won £1650 each. What was the total prize fund?
2. Swansea AFC’s home ground is the Liberty Stadium which holds 21 088 fans. Cardiff City FC plays at the Cardiff City Stadium which holds 33 316 fans.

   What is the difference in capacity at these grounds?

Answer

1. The actual answer is £313 500 (190 × 1650)

   Estimate 200 × 1700.

   2 × 17 = 34 so 200 × 1700 = 340 000 so the answer is sensible.
2. The actual answer is 12 228 (33 316 − 21 088)

   Estimate 33 000 − 21 000 = 12 000 so the answer is sensible.

Summary

In this section you have learned:

- how and when to use rounding to approximate an answer to a calculation

- how to round an answer to a given degree of accuracy – e.g. rounding to two decimal places.

4 Multistage calculations

Often in daily life you will come across problems that require more than one calculation to reach the final answer.

Example: Multistage calculations

Four friends are planning a holiday. The table below shows the costs:

Table 4

Item	Price
Flight (return)	£305 per person
Taxes	£60 per person
Hotel	£500 per room. 2 people per room
Taxi to airport	£45

The friends will be sharing the total cost equally between them. How much do they each pay?

Method

First we use multiplication to find the cost of items that we need more than one of:

Flights = £305 × 4 = £1220
Taxes = £60 × 4 = £240
Hotel = 2 rooms required for 4 people = £500 × 2 = £1000

Now we use addition to add these totals together along with the taxi fare:

£1220 + £240 + £1000 + £45 = £2505

Finally, we need to use division to find out how much each person pays:

£2505 ÷ 4 =£626.25 each

Activity 10: Multistage calculations

1. Your current mobile phone contract costs you £24.50 per month.

   You are considering changing to a new provider. This provider charges £19.80 per month along with an additional, one off connection fee of £30.

   How much will you save over the year by switching to the new provider?
2. You are going on holiday and you have decided to stay in a cottage in North Wales for 7 nights.

   There will be 12 of you staying and the total cost of the cottage you have chosen is £460 per night. If you split the cost equally, how much will each of you pay?

Answer

1. **First** calculate the cost with the current provider

   monthly cost × 12

   £24.50 × 12 = £294 (current provider)

   Second, calculate the cost of the new provider.

   To do this you need to calculate the total monthly costs:

   £19.80 × 12 = £237.60

   and then add on the one off connection fee of £30:

   £237.60 + £30 = £267.60 (new provider)

   Finally you can calculate the difference between the two providers:

   £294 − £267.60 = £26.40 saved
2. To do your calculation, you need to work out the cost for 7 nights. Once you have done this, you can divide the total by 12:

   460 × 7 = £3220

   £3220 ÷ 12 = £268.33 (rounded to two d.p.)

   Here we would probably round this amount up to £268.34 per person to make sure the full cost is covered.

All the examples we have looked at until now have used positive numbers. However, as anyone with an overdraft will know, numbers (or bank balances) are not always positive! Our next section therefore deals with negative numbers.

Summary

In this section you have:

- applied the four operations to solve multistage calculations.

5 Negative numbers

Negative numbers come into play in two main areas of life: money and temperature. Watch the animations below for some examples.

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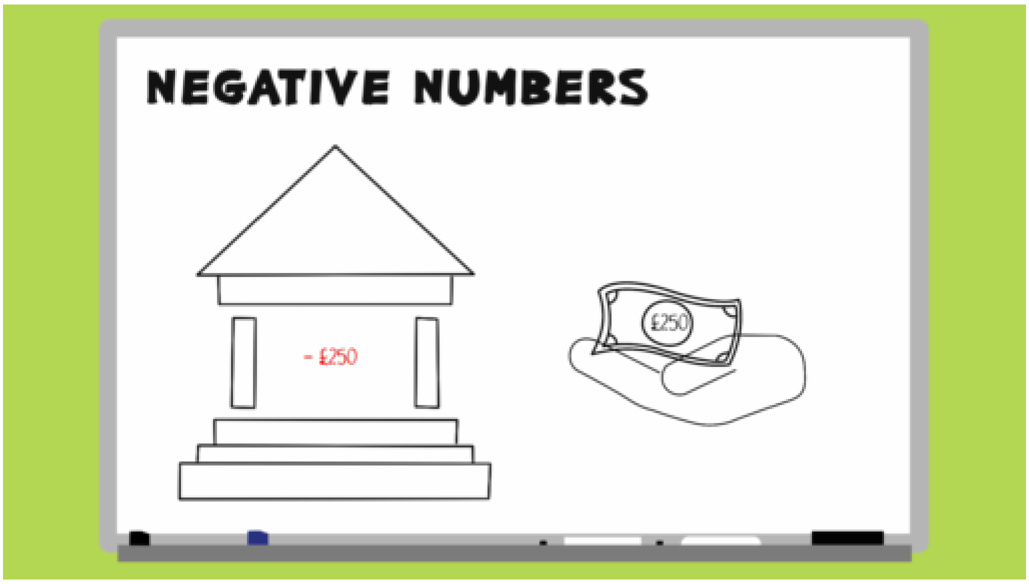

Activity 11: Negative and positive temperature

1. The table below shows the temperatures of cities around the world on a given day

Table 5

London	Oslo	New York	Kraków	Delhi
4˚C	−12C	7˚C	−3˚C	19˚C

a. Which city was the warmest?
b. Which city was the coldest?
c. What is the difference in temperature between the warmest and coldest cities?

Answer

1.
   a. Delhi was the warmest city as it has the highest positive temperature.
   b. Oslo was the coldest city as it has the largest negative temperature.
   c. The difference between the temperatures in these cities is 31˚C.

From 19˚C down to 0˚C is 19˚C and then you need to go down a further 12˚C to get to −12˚C.

2. Look at this bank statement.

Bank statement

SBH Bank
Tadworth Road
Colesdale

Miss Sonia Cedar
Account no. 23395
Summary for 1-31 October

Date	Description	Amount	Balance
09 Oct	Bank Transfer		100.00
11 Oct	Direct Debit		-20.00
15 Oct	Automated Pay In		70.00
20 Oct	Bank Transfer		100.00
21 Oct	Direct Debit		-50.00
25 Oct	Bank Transfer		200.00

Figure 6 A bank statement

b. On which days was Sonia Cedar overdrawn, and by how much?
c. How much money was withdrawn between 9 and 11 of October?
d. How much was added to the account on 15 October?

Answer

2.

b. The minus sign (−) indicates that the customer is overdrawn, i.e. owes money to the bank.

   The amount shows how much they owe. So Sonia Cedar was overdrawn on 11 October by £20 and by £50 on 21 October.

c. £120 was withdrawn on 11 October.

   The customer had £100 in the account and must have withdrawn another £20 (i.e. £100 + £20 = £120 in total) in order to be £20 overdrawn.

d. The customer owed £20 and is now £70 in credit, so £90 must have been added to the account.

3. Look at the table below showing a company's profits over 6 months.

   Hint: a negative profit means that the company made a loss.

Table 6

Month	**Profit (£000)**
January	166
February	182
March	−80
April	124
May	98
June	−46
Balance	

a. Which month had the greatest profit?
b. Which month has the greatest loss?
c. What was the overall balance for the six months?
Hint: start by calculating the total profits and the total losses.

Answer

3.

a. February had the largest profit with £182 000 (remember to look at the column heading which shows that the figures are in 000s – thousands).
b. March showed the greatest loss at £80 000.
c. To calculate the overall balance you need to first calculate the total profits and the total losses. To calculate the profits you need to do this calculation:
166 + 182 + 124 + 98 = 570

So the profit was £570 000.

Next you need to calculate the total losses; two months showed a loss so you need to add these values:

80 + 46 = 126 so the losses over the six months were £126 000.

Now you can calculate the overall balance by subtracting the losses from the profits:

£570 000 − £126 000 = £444 000

This is a positive value so it means the company made an overall profit of £444 000.

Summary

In this section you have:

- learned the two main contexts in which negative numbers arise in everyday life – money (or debt!) and temperature
- practised working with negative numbers in these contexts.

6 Mathematical terms

It is important to know the meaning of the following terms:

- multiples
- lowest common multiple
- factors
- common factors
- prime numbers

Multiples

A multiple of a number can be found by multiplying that number by any whole number e.g. multiples of 2 include 2, 4, 6, 8, 10 etc. (all are in the 2 times table).

Note: To check if a number is a multiple of another, see if it divides exactly into the multiple, e.g. to see if 3 is a multiple of 81 do 81 ÷ 3 = 27. It divides exactly so 81 *is*a multiple of 3.

Lowest common multiple

In maths, we sometimes need to find the lowest common multiple of numbers.

The lowest common multiple (LCM) is simply the smallest multiple that is common to more than one number.

Example: Lowest common multiple of 3 and 5

Hint: when looking for multiples, it is easiest to start by listing the multiples of the highest number first. This saves you going any further than you need to with the list.

The first few multiples of 5 are:

5, 10, **15**, 20, 25, 30 etc.

The first few multiples of 3 are:

3, 6, 9, 12, **15**, 18, 21 etc.

You can see that the lowest number that is a common multiple of 3 and 5 is 15.

Activity 12: Finding the lowest common multiple

Find the lowest common multiple of:

1. 6 and 12
2. 2 and 7

Answer

1. The lowest common multiple of 6 and 12 is 12:

   Multiples of 12:

   12, 24, 36, 48, 60 etc.

   Multiples of 6:

   6, **12**, 18, 24, 30 etc.

   You can see from the list that 24 is also a common multiple of 6 and 12, but 12 is the lowest common multiple.
2. The lowest common multiple of 2 and 7 is 14:

   Multiples of 7:

   7, **14**, 21, 28, 35, 42 etc.

   Multiples of 2:

   2, 4, 6, 8, 10, 12, **14** etc.

Factors, common factors and prime numbers

Factors of a number divide into it exactly. Factors of all numbers include 1 and the number itself. However, most numbers have other factors as well. If you think of all of the numbers that multiply together to make that number, you will find all of the factors of that number.

Example: What are the factors of 8?

$8 \times 1 = 8$

$2 \times 4 = 8$

So the factors of 8 are 1, 2, 4 and 8.

Activity 13: Finding factors

1. What are the factors of 54?
2. What are the factors of 165?

Answer

1. The factors of 54 are 1, 2, 3, 6, 9, 18, 27 and 54
2. The factors of 165 are 1, 3, 5, 11, 15, 33, 55 and 165

A common factor is a factor that goes into more than one number. For example, 4 is a common factor of 8 and 12 because it divides exactly into both numbers.

Prime Numbers

A prime number is a number which only has 2 factors: 1 and itself.
The prime numbers between 1 and 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

Note:

- 1 is *not* a prime number as it only has one factor.
- 2 is the only even prime number.

You have now learned how to use all four operations, how to work with negative numbers and learned some important mathematical terms. Every other mathematical concept hinges around what you have learned so far; so once you are confident with these, you'll be a success!

Summary

In this section you have:

- learned some key mathematical terms: multiple, lowest common multiple, factor, common factor and prime number
- identified the lowest common multiple
- identified factors.

7 Fractions

You will be used to seeing fractions in your everyday life, particularly when you are out shopping or scouring the internet for the best deals. It's really useful to be able to work out how much you'll pay if an item is on sale or if a supermarket deal really is a good deal!

Figure 7 A poster advertising a sale

There are several different elements to working with fractions. First you will look at simplifying fractions.

7.1 Simplifying fractions

Watch the video below which looks at how to simplify fractions before having a go yourself in Activity 14.

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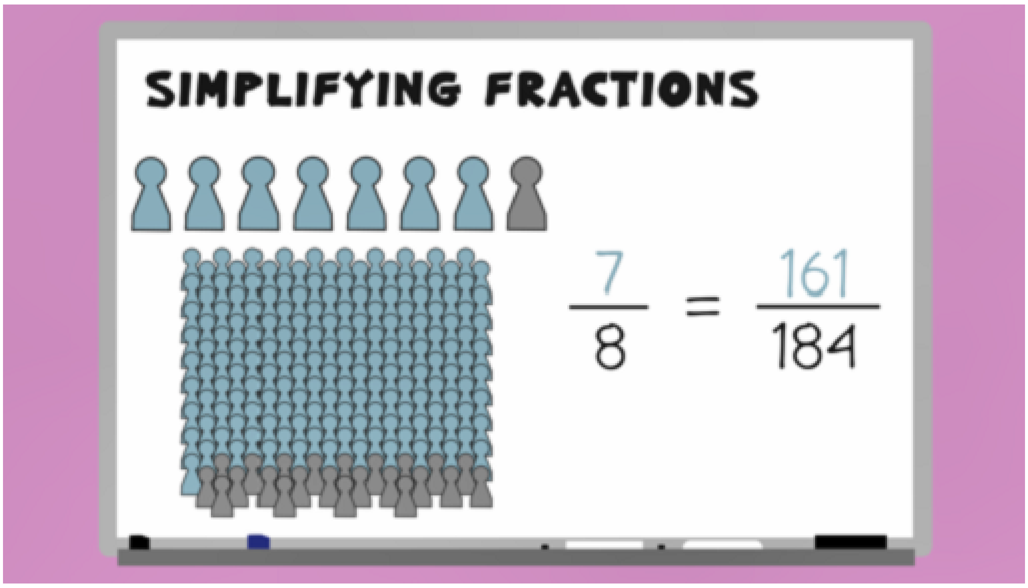

Activity 14: Simplifying fractions

Show the following fractions in simplest form, where possible:

a. $\frac{25}{75}$

b. $\frac{12}{36}$

c. $\frac{72}{96}$

d. $\frac{32}{48}$

e. $\frac{5}{126}$

f. $\frac{164}{256}$

Answer

a. $\frac{25}{75} = \frac{1}{3}$

b. $\frac{12}{36} = \frac{1}{3}$

c. $\frac{72}{96} = \frac{3}{4}$

d. $\frac{32}{48} = \frac{2}{3}$

e. $\frac{5}{126}$ can't be simplified

f. $\frac{164}{256} = \frac{41}{64}$

Next you'll look at expressing a quantity of an amount as a fraction.

7.2 Writing a quantity of an amount as a fraction

Sometimes you will need to show one amount as a fraction of another. This might sound complicated, but it's actually very logical. Look at the examples below.

Example 1: Fraction of an amount

In Figure 8, what fraction of Smarties are red?

Figure 8 Smarties in different colours

$$\frac{number\ of\ red\ smarties}{total\ number\ of\ smarties} = \frac{4}{30}$$

To express the fraction of Smarties that are red, you simply need to count the red Smarties (4) and the total number of Smarties (30). Since there are 4 red Smarties out of 30 altogether, the fraction is $\frac{4}{30}$. It is worth noting here that this could also be written as 4/30.

You may well be asked to give your answer as a fraction in its simplest form, so always check to see if you can simplify your answer. In this case $\frac{4}{30}$ will simplify to $\frac{2}{15}$.

Example 2: Fraction of an amount

250 g of flour is taken from a 1 kg bag. What fraction is this?

Hint: there are 1000 g in a kg.

To express quantities as fractions, the top and bottom numbers need to be in the same units, so here you need to make sure that you express both the top and bottom values in grams:

The flour removed is already expressed in grams: 250 g
The total amount is in kilograms so you need to convert to grams: 1 kg = 1000 g

Now write the amount taken over the total amount to express as a fraction:

$\frac{250}{1000}$ (250 g of flour out of the 1000 g bag has been taken or used)

Then cancel down (or simplify) if possible:

$$\frac{250}{1000} = \frac{1}{4}$$

So $\frac{1}{4}$ of the flour has been used.

Activity 15: Expressing one number as a fraction of another

1. What fraction of a kilogram is:
   a. 100 g
   b. 750 g
   c. 640 g
   d. 20 g

Answer

1. 100 g = 1000 g, so:
   a. 100 g $\frac{100}{1000}$ = $\frac{1}{10}$ of a kilogram

   b. 750 g $\frac{750}{1000}$ = $\frac{3}{4}$ of a kilogram

   c. 640 g $\frac{640}{1000}$ = $\frac{16}{25}$ of a kilogram

   d. 20 g = $\frac{20}{1000}$ = $\frac{1}{50}$ of a kilogram

2. What fraction of an hour is:
   b. 15 minutes
   c. 20 minutes
   d. 35 minutes
   e. 48 minutes

Answer

2. 1 hour = 60 minutes so:
   b. 15 minutes = $\frac{15}{60}$ = $\frac{1}{4}$ of an hour.

   c. 20 minutes = $\frac{20}{60}$ = $\frac{1}{3}$ of an hour.

   d. 35 minutes = $\frac{35}{60}$ = $\frac{7}{12}$ of an hour.

   e. 48 minutes = $\frac{48}{60}$ = $\frac{4}{5}$ of an hour.

3. A farmer takes 120 eggs to the local farmer's market. She has 24 eggs left at the end of the day. What fraction of the eggs are left?

Answer

3. $\frac{24}{120}$ are left. This cancels down to $\frac{1}{5}$, so $\frac{1}{5}$ of the eggs are left.

4. A class of students sit a test. 18 pass and 12 fail. What fraction passed the test?

Answer

4. Work out the total number of students by adding the number who passed to those who failed 18 + 12 = 30. Now work out the fraction that passed:

   $\frac{18}{30}$ (18 out of 30 students passed)

   Now cancel down:

   $\frac{18}{30} = \frac{3}{5}$.

   So $\frac{3}{5}$ passed the test.

5. Mary bought her car for £12 500. When she goes to trade it in she is offered £8750. What fraction of the original price is this?

Answer

5. $\frac{8750}{12500} = \frac{7}{10}$

6. 30 people entered a raffle. 6 of these people won a prize. What fraction of people did not win a prize? Give your answer as a fraction in its simplest form.

Answer

6. As this question wants the number of people who did not win a prize we must first do:

   30 − 6 = 24 people did not win a prize.

   As a fraction this becomes $\frac{24}{30}$ which simplifies to $\frac{4}{5}$.

Sometimes fractions will not cancel down easily. When this happens, you estimate the fraction by rounding the numbers to values that will cancel. Sometimes this means breaking the ‘rules’ of rounding.

Example: Estimating fractions

$\frac{1347}{2057}$ will not cancel.

By rounding 1347 up to 1400 and 2057 to 2100 we can cancel down the fraction to get:

$$\frac{1400}{2100} = \frac{2}{3}$$

Note: Round to numbers that are easy to cancel, but if you round off too much, you will lose the accuracy of your answer.

Now that you can express a quantity as a fraction, estimate and simplify fractions, the next step is to be able to work out fractions of amounts. For example, if you see a jacket that was priced at £80 originally but is in the sale with $\frac{2}{5}$ off, it's useful to be able to work out how much you will be paying.

7.3 Fractions of amounts

Fractions of amounts can be found by using your division and multiplication skills. To work out a fraction of any amount you first divide your amount by the number on the bottom of the fraction – the denominator. This gives you 1 part.

You then multiply that answer by the number on the top of the fraction – the numerator.

It is worth noting here that if the number on the top of the fraction is 1, multiplying the answer will not change it so there is no need for this step. Take a look at the examples below.

Example: Divide by the denominator

Method

To find $\frac{1}{5}$ of 90 we do 90 ÷ 5 = 18.

Since the number on the top of our fraction is 1, we do not need to multiply 18 by 1 as it will not change the answer.

So $\frac{1}{5}$ of 90 = 18.

Example: Multiply by the numerator

Method

To find $\frac{4}{7}$ of 42 we do 42 ÷ 7 = 6.

This means that $\frac{1}{7}$ of 42 = 6.

Since you want $\frac{4}{7}$ of 42, we then do 6 × 4 = 24.

So $\frac{4}{7}$ of 42 = 24.

Let's go back to the jacket that used to cost £80 but is now in the sale with $\frac{2}{5}$ off. How do you find out how much it costs? Firstly, you need to find $\frac{2}{5}$ of 80. To calculate this you do:

£80 ÷ 5 = £16 and then £16 × 2 = £32

This means that you save £32 on the price of the jacket. To find out how much you pay you then need to do £80 − £32 = £48.

You will have practised finding fractions of amounts in *Everyday maths 1*, but have a go at the following activity to recap this important skill.

Activity 16: Finding fractions of amounts

Work out the following without using a calculator. You may double-check on a calculator if you need to and remember to check your answers against ours.

1. You are looking to buy house insurance and want to get the best deal. Put the following offers in order, from cheapest to most expensive, after the discount has been applied.

Table 7

Company A	Company B	Company C
£120 per year	£147 per year	£104 per year
Special offer: $\frac{1}{3}$ off!	Special offer: $\frac{2}{7}$ off!	Special offer: $\frac{1}{4}$ off!

Answer

1. Company C is cheapest:

   $\frac{1}{4}$ of £104 = £104 ÷ 4 = £26 discount

   £104 − £26 = £78

   Company A is second cheapest:

   $\frac{1}{3}$ of £120 = £120 ÷ 3 = £40 discount

   £120 − £40 = £80

   Company B is most expensive:

$\frac{2}{7}$ of £147 = £147 ÷ 7 × 2 = £42 discount

£147 − £42 = £105

2. A cinema sells 2400 tickets over a weekend. They review their ticket sales and find that $\frac{2}{3}$ of the weekend ticket sales were to adults. How many adult tickets were sold?

Answer

2. 1600 tickets sold to adults:

   2400 ÷ 3 = 800 to give $\frac{1}{3}$

   2 × 800 = 1600 to give $\frac{2}{3}$

3. A college has raised $\frac{3}{5}$ of its £40 000 charity fundraising target. How much money does the college need to raise to meet its target?

Answer

3. £16 000 needed to meet target.

   40 000 ÷ 5 = 8000 to give $\frac{1}{5}$

   8000 × 3 = 24 000 to give $\frac{3}{5}$ (the amount raised)

   But the question asks how much is needed to meet its target so we need to subtract the amount raised from the target:

   40 000 − 24 000 = £16 000

Discounts and special offers are not always advertised using fractions. Sometimes, you will see adverts with 10% off or 15% off. Another common area where we see percentages in everyday life would be when companies apply VAT at 20% to items or when a restaurant adds a 12.5% service charge. The next section looks at what percentages are, and how to calculate them.

Summary

In this section you have:

- learned how to express a quantity of an amount in the form of a fraction
- learned how to, and practised, simplifying fractions
- revised your knowledge on finding fractions of amounts.

8 Percentages

There are different ways of working out percentages of amounts. We will take a look at the most common methods now.

Figure 9 Percentage discounts in a sale

Note: You may use a different method to these ones. You may even use different methods depending on the percentage you are calculating. Do whatever works for you.

8.1 Calculating a percentage of an amount

Method 1

Percentages are just fractions where the number on the bottom of the fraction must be 100. If you wanted to find out 15% of 80 for example, you work out $\frac{15}{100}$ of 80, which you already know how to do!

Working out the percentage of an amount requires a similar method to finding the fraction of an amount. Take a look at the examples below to increase your confidence.

Example 1: Finding 17% of 80

Method

17% of 80 = $\frac{17}{100}$ of 80, so here we do:

80 ÷ 100 = 0.8

0.8 × 17 = 13.6

Another way of thinking about this method is that you are dividing by 100 to find 1% first and then you are multiplying by whatever percentage you want to find.

Alternatively, you could multiply the value by the top number first and then divide by 100:

17 × 80 = 1360

1360 ÷ 100 = 13.6

The answer will be the same.

Example 2: Finding 3% of £52.24

Method

3% of 52.24 = $\frac{3}{100}$ of 52.24, so we do:

52.24 ÷ 100 = 0.5224.

0.5224 × 3 = £1.5672 (£ 1.57 to two d.p.)

Or

52.24 × 3 = 156.72

156.72 ÷ 100 = £1.5672 (£1.57 to two d.p.)

This is a good method if you want to be able to work out every percentage in the same way. It can be used with and without a calculator. Many calculators have a percentage key, but different calculators work in different ways so you need to familiarise yourself with how to use the % button on your calculator.

Method 2

To use this method you only need to be able to work out 10% and 1% of an amount. You can then work out any other percentage from these.

Let’s just recap how to find 10% and 1%.

10%

To find 10% of an amount divide by 10:

10% of £765 = 765 ÷ 10 = £76.50

10% of £34.50 = 34.50 ÷ 10 = £3.45

Hint: remember to move the decimal point one place to the left to divide by 10.

1%

To find 1% of an amount divide by 100:

1% of £765 = 765 ÷ 100 = £7.65

1% of £34.50 = 34.50 ÷ 100 = £0.345 (£0.35 to two d.p.)

Hint: remember to move the decimal point two places to the left to divide by 100.

Once you know how to work out 10% and 1%, you can work out any other percentage.

Example 1: Finding 24% of 60

Find 10% first:

60 ÷ 10 = 6

10% = 6

20% is 2 lots of 10% so:

6 × 2 = 12

20% = 12

Now find 1%:

60 ÷ 100 = 0.6

4% is 4 lots of 1% so:

0.6 × 4 = 2.4

4% = 2.4

Now add the 20% and 4% together:

12 + 2.4 = 14.4

Example 2: Finding 17.5% of £328

17.5% can be broken up into 10% + 5% + 2.5%, so you need to work out each of these percentages and then add them together.

Find 10% first:

328 ÷ 10 = 32.8

10% = 32.8

5% is half of the 10% so:

32.8 ÷ 2 = 16.4

5% = 16.4

2.5% is half of the 5% so:

16.4 ÷ 2 = 8.2

2.5% = 8.2

Now add the 10%, 5% and 2.5% figures together:

32.8 + 16.4 + 8.2 = £57.40

This is a good method to do in stages when you do not have a calculator.

Note: There are some other quick ways of working out certain percentages:

50% – divide the amount by two.

25% – halve and halve again.

These quick facts can be used in combination with method 2 to make calculations, e.g. 60% could be worked out by finding 50%, 10% and then adding the 2 figures together. You just need to look for the easiest way to split up the percentage to make your calculation.

Activity 17: Finding percentages of amounts

Use whichever method/s you prefer to calculate the answers to the following:

1. Find:

   a. 45% of £125
   b. 15% of 455 m
   c. 52% of £677
   d. 16% of £24.50
   e. $2\frac{1}{2}$% of 4000 kg
   f. 82% of £7.25
   g. $37\frac{1}{2}$% of £95

2. The Cambria Bank pays interest at 3.5%. What is the interest on £3000?
3. Sure Insurance offer a 30% No Claims Bonus. How much would be saved on a premium of £345.50?
4. Sunshine Travel Agents charge 1.5% commission on foreign exchanges. What is the charge for changing £871?

Answer

1.
   a. £56.25
   b. 68.25 m
   c. £352.04
   d. £3.92
   e. 100 kg
   f. £5.945 (£5.95 to 2 d.p.)
   g. £35.625 (£35.63 to two d.p.)
2. £105
3. £103.65
4. £13.065 (£13.07 to two d.p.)

Just as with fractions you will often need to be able to work out the price of an item after it has been increased or decreased by a given percentage. The process for this is the same as with fractions; you simply work out the percentage of the amount and then add it to, or subtract it from, the original amount.

Activity 18: Percentages increase and decrease

1. You earn £500 per month. You get a 5% pay rise.
   a. How much does your pay increase by?
   b. How much do you now earn per month?

Answer

1.
   a. £25
   b. £525 per month.

2. You buy a new car for £9500. By the end of the year its value has decreased by 20%.
   b. How much has the value of the car decreased by?
   c. How much is the car worth now?

Answer

2.
   b. The car has decreased by £1900.
   c. The car is now worth £7600.

3. You invest £800 in a building society account which offers fixed-rate interest at 4% per year.
   c. How much interest do you earn in one year?
   d. How much do you have in your account at the end of the year?

Answer

3.
   c. £32 interest earned.
   d. £832 in the account at the end of the year.

4. Last year Julie's car insurance was £356 per annum. This year she will pay 12% less. How much will she pay this year?

Answer

4. She will pay £42.72 less so her insurance will cost £313.28

5. A zoo membership is advertised for £135 per year. If Tracy pays for the membership in full rather than in monthly installments, she receives a 6% discount. How much will she pay if she pays in full?

Answer

5. She will save £8.10 so she will pay £126.90.

6. A museum had approximately 5.87 million visitors last year. Visitor numbers are expected to increase by 4% this year. How many visitors is the museum expecting this year?

Answer

6. 5.87 million = 5 870 000.
   4% of 5 870 000 = 234 800
   5 870 000 + 234 800 = 6 104 800 people

Next you'll look at how to express one number as a percentage of another.

8.2 Expressing one number as a percentage of another

Sometimes you need to write one number as a percentage of another. You have already practised writing one number as a fraction of another; this just takes it a bit further.

Example 1: What percentage are women?

A class is made up of 21 women and 14 men, what percentage of the class are women?

To work this out, you start by expressing the numbers as a fraction. You then multiply by 100 to express as a percentage.

The formula is:

$$\frac{amount\ we\ need}{total} \times 100$$

In this case, 21 out of a total of 35 people are women so the sum we do would be:

$$\frac{21}{35} \times 100$$

The fraction line is also a divide line, so if you were doing this on a calculator you would do:

21 ÷ 35 × 100 = 60%

How would you work this out without a calculator?

There are different ways you can make the calculation. Two methods are shown below.

Method 1

$$\frac{21 \times 100}{35}$$

You start by multiplying the top number in the fraction by 100. The bottom number will stay the same:

$$\frac{21 \times 100}{35} = \frac{2100}{35}$$

Now you need to cancel the fraction down as much as possible:

$\frac{2100}{35}$ ÷ top and bottom by 5 = $\frac{420}{7}$, then, ÷ top and bottom by 7 = $\frac{60}{1}$

Anything over 1 is a whole number so the answer is 60.

So 60% of the class are women.

Note: When using this method, if you cancel as far as possible and you do not end up with an answer over 1, you will need to divide the top number by the bottom number to work out the final answer, e.g. the fraction $\frac{15}{4}$ cannot cancel any further, so:

15 ÷ 4 = 3.75

Method 2

The other method involves expressing the fraction as a decimal first and then converting it to a percentage. This means that you multiply by 100 at the very end of the calculation.

A local attraction sold 150 tickets last bank holiday, 102 of which were full price. What percentage of the tickets sold were at the concessionary price?

Work out the number of concessionary tickets sold:

150 – 102 = 48

Write the number of concessionary tickets sold as a fraction of the total number sold:

$\frac{48}{150}$

Cancel down your fraction:

$\frac{48}{150}$ ÷ top and bottom by 6 = $\frac{8}{25}$

Once you cannot cancel any further, you need to divide the top number by the bottom number to express as a decimal:

8 ÷ 25 = 0.32

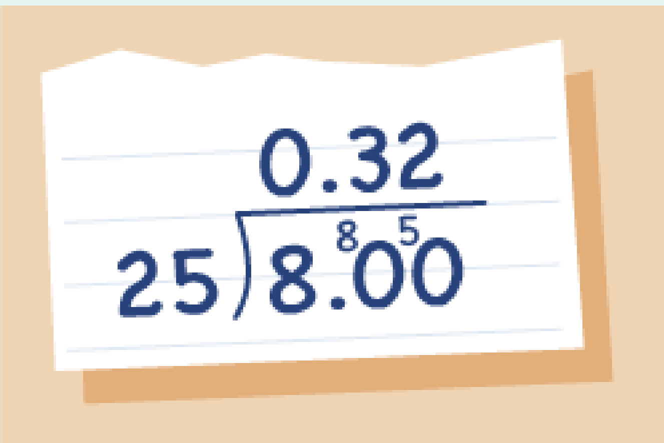

Figure 10 Expressed as a decimal: 8 divided by 25

Finally, multiply the decimal answer by 100 to express as a percentage:

$0.32 \times 100 = 32\%$

So 32% of the tickets were sold at the concessionary price.

Activity 19: Expressing one number as a percentage of another

Use whichever method you prefer to calculate the answers to the following. Give answers to two d.p. where appropriate.

Hint: make sure your units are the same first.

1. What percentage:
   a. of 1 kg is 200 g?
   b. of an hour is 48 minutes?
   c. of £6 is 30p?

Answer

1.
   a. 1 kg = 1000 g

      $\frac{200}{1000} \times 100 = 20\%$

   b. 1 hour = 60 minutes

      $\frac{48}{60} \times 100 = 80\%$

   c. £1 = 100 p

      $\frac{30}{600} \times 100 = 5\%$

2. Bea swam 50 laps of a 25 m swimming pool in a charity swim. A mile is almost 1600 m. What percentage of a mile did Bea swim?

Answer

2. $50 \times 25 = 1250$ m

   $\frac{1250}{1600} \times 100 = 78.13\%$ (to two d.p.)

3. A student gets the following results in the end of year tests:

Table 8

	Maths	English	Science	Art
Mark achieved	64	14	72	56
Possible total mark	80	20	120	70

Calculate her percentage mark for each subject.

Answer

Maths: $\frac{64}{80} \times 100 = 80\%$

English: $\frac{14}{20} \times 100 = 70\%$

Science: $\frac{72}{120} \times 100 = 60\%$

Art: $\frac{56}{70} \times 100 = 80\%$

4. Susan is planting her flower beds. She plants 13 yellow flowers, 18 white flowers and 9 red ones. What percentage of her flowers will not be white?

Answer

4. Number not white = 13 + 9 = 22

   Total number she is planting = 13 + 18 + 9 = 40

   $\frac{22}{40} \times 100 = 55\%$

   55% of the flowers will not be white.

5. A building society charges £84 interest on a loan of £1200 over a year. What percentage interest is this?

Answer

5. $\frac{84}{1200} \times 100 = 7\%$

   The interest rate is 7%.

Next you will look at percentage change. This can be useful for working out the percentage profit (or loss) or finding out by what percentage an item has increased or decreased in value.

8.3 Percentage change

Watch the video below on how to calculate percentage change, then complete Activity 20.

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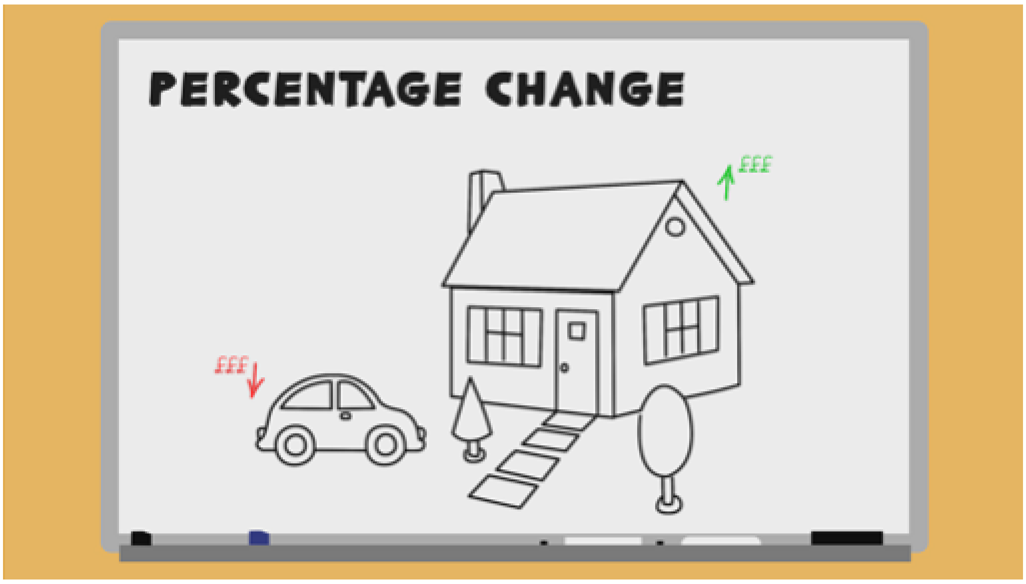

Activity 20: Percentage change formula

Practise using the percentage change formula which you learned about in the video above on the four questions below. Where rounding is required, give your answer to two decimal places.

1. Last year your season ticket for the train cost £1300. This year the cost has risen to £1450. What is the percentage increase?

Answer

1. Difference: £1450 − £1300 = £150

   Original: £1300

   Percentage change = $\frac{150}{1300}$ × 100

   Percentage change = 0.11538... × 100 = 11.54% increase (rounded to two d.p.)

2. You bought your house 10 years ago for £155 000. You are able to sell your house for £180 000. What is the percentage increase the house has made?

Answer

2. Difference: £180 000 − £155 000 = £25 000

   Original: £155 000

   Percentage change = $\frac{25000}{155000}$ × 100

   Percentage change = 0.16129... × 100 = 16.13% increase (rounded to two d.p.)

3. You purchased your car 3 years ago for £4200. You sell it to a buyer for £3600. What is the percentage decrease of the car?

Answer

3. Difference: £4200 − £3600 = £600

   Original: £4200

   Percentage change = $\frac{600}{4200}$ × 100

   Percentage change = 0.14285... × 100 = 14.29% decrease (rounded to two d.p.)

4. Stuart buys a new car for £24 650. He sells it 1 year later for £20 000. What is the percentage loss?

Answer

4. Difference: £24 650 − £20 000 = £4650

   Original: £24 650

   Percentage change = $\frac{4650}{24650}$ × 100

   4650 ÷ 24 650 × 100 = 18.86% loss (rounded to two d.p.)

Congratulations, you now know everything you need to know about percentages! As you have seen, percentages come up frequently in many different areas of life and having completed this section, you now have the skills to deal with all kinds of situations that involve them.

You saw at the beginning of the section that percentages are really just fractions. Decimals are also closely linked to both fractions and percentages. In the next section you will see just how closely related these three concepts are and also learn how to convert between each of them.

Summary

In this section you have:

- found percentages of amounts
- calculated percentage increase and decrease
- calculated percentage change using a formula
- expressed one number as a percentage of another.

9 Fractions, decimals and percentages

You have already worked with decimals in this course and many times throughout your life. Every time you calculate something to do with money, you are using decimal numbers. You have also learned how to round a number to a given number of decimal places.

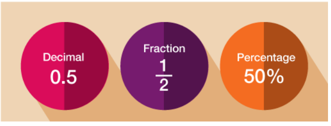

Figure 11 Equivalent decimals, fractions and percentages

9.1 Converting between percentages, decimals and fractions

Since fractions, decimals and percentages are all just different ways of representing the same thing, we can convert between them in order to compare. Take a look at the video below to see how to convert fractions, decimals and percentages.

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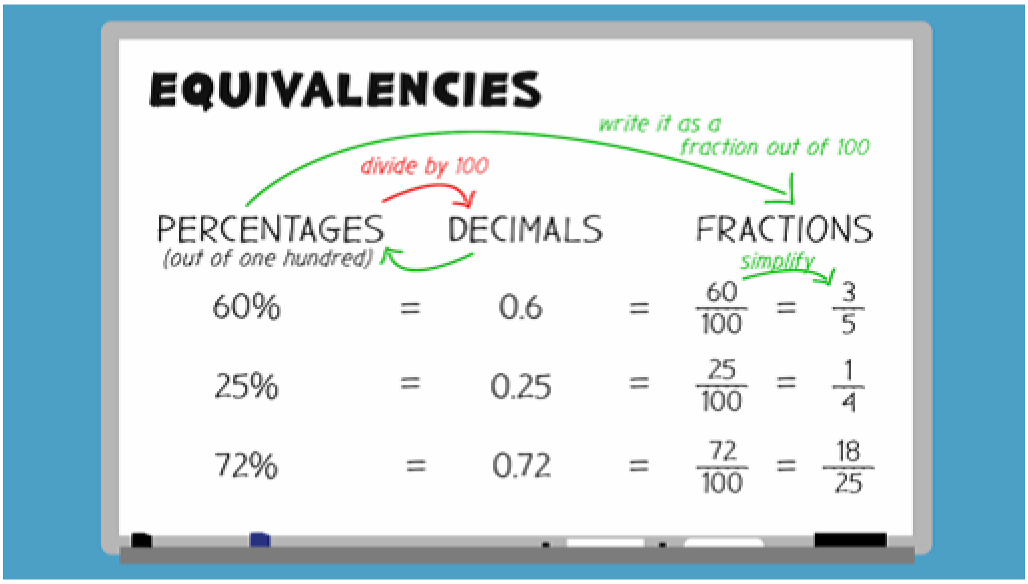

Lets look in more detail at changing a percentage to a fraction.

Example: 50% is $\frac{50}{100}$

As you can see this percentage is essentially a fraction of 100. However you can simplify it to $\frac{1}{2}$.

To change a percentage to a fraction, put the percentage over 100 and simplify if possible.

Sometimes we might see a percentage like this: 12.5%.

If we use the method above we get $\frac{12.5}{100}$ but we can't have a decimal in a fraction.

To get rid of the decimal in the fraction we must multiply the top and bottom of the fraction, the numerator and denominator, by any number that will give us whole numbers. In this case 10 or 2 both work well (12.5 × 10 = 125 and 12.5 × 2 = 25):

Method 1: × 10

$\frac{12.5}{100}$ × top and bottom by 10 = $\frac{125}{1000}$ = $\frac{1}{8}$

Method 2: × 2

$\frac{12.5}{100}$ × top and bottom by 2 = $\frac{25}{200}$ = $\frac{1}{8}$

Activity 21: Converting between percentages, decimals and fractions

1. Express these percentages as decimals:
   a. 62%
   b. 50%
   c. 5%
2. Express these decimals as percentages:
   a. 0.02
   b. 0.2
   c. 0.752
   d. 0.055
3. Express these percentages as fractions:
   a. 15%
   b. 2.5%
   c. 37.5%

Answer

1.
   a. 0.62
   b. 0.5
   c. 0.05
2.
   a. 2%
   b. 20%
   c. 75.2%
   d. 5.5%

3.

a. $\frac{15}{100} = \frac{3}{20}$

b. $\frac{2.5}{100}$ × top and bottom by 10 = $\frac{25}{1000} = \frac{1}{40}$

c. $\frac{37.5}{100}$ × top and bottom by 10 = $\frac{375}{1000} = \frac{3}{8}$

You may have multiplied by different numbers to get rid of the decimal in the last two questions. However, your final answers should still be the same as ours.

Now have a go at matching these fractions to decimals and percentages.

Activity 22: Matching fractions, decimals and percentages

Choose the correct fraction for each percentage and decimal.

$\frac{7}{20}$

$\frac{2}{5}$

$\frac{2}{25}$

$\frac{5}{8}$

Match each of the items above to an item below.

35% = 0.35 =

40% = 0.4 =

8% = 0.08 =

62.5% = 0.625 =

Next you'll look in more detail at how to change a fraction to a percentage.

9.2 Changing a fraction to a percentage

There are two ways you can do this.

Method 1

To change a fraction into a percentage, multiply it by $\frac{100}{1}$ (essentially, you are just multiplying the top number by 100 and the bottom number will stay the same).

Example: Change $\frac{3}{4}$ into a percentage

$\frac{3}{4} \times \frac{100}{1} = \frac{300}{4}$

This cancels to $\frac{75}{1}$ = 75%

Note: Remember anything over 1 is a whole number. If you do not end up with a 1 on the bottom, you will have to divide the top number by the bottom one to get your final answer.

Method 2

Divide the top of the fraction by the bottom (to express the fraction as a decimal) and then multiply the answer by 100.

Example: $\frac{3}{4}$ = 3 ÷ 4 = 0.75

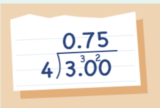

Figure 12 Expressed as a decimal: 3 ÷ 4

0.75 × 100 = 75%

Activity 22: Changing a fraction to a percentage

1. Express these fractions as percentages:

   a. $\frac{3}{8}$

b. $\frac{9}{10}$

c. $\frac{4}{5}$

Answer

1.
   a. 37.5%
   b. 90%
   c. 80%

Now you'll look at changing a fraction to a decimal.

9.3 Changing a fraction to a decimal

Again there are two ways to do this, both based on the two methods just shown for changing a fraction to a percentage.

Method 1

Example: Change the fraction into a percentage and divide by 100

$\frac{1}{4} \times \frac{100}{1} = \frac{100}{4}$ which cancels to $\frac{25}{1}$ = 25%

Now convert to a decimal by dividing by 100:

25 ÷ 100 = 0.25

Method 2

Example: Divide the top of the fraction by the bottom

$\frac{1}{4}$ = 1 ÷ 4 = 0.25.

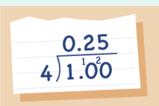

Figure 13 Expressed as a decimal: 1 ÷ 4

Activity 23: Changing a fraction to a decimal

Express these fractions as decimals:

1. $\frac{2}{5}$
2. $\frac{1}{8}$
3. $\frac{3}{10}$

Answer

1. 0.4
2. 0.125
3. 0.3

Fractions and percentages deal with splitting numbers into a given number of equal portions, or parts. When dealing with the next topic, ratio, you will still be splitting quantities into a given number of parts, but when sharing in a ratio, you do not share evenly. This might sound a little complicated but you'll have been doing it since you were a child.

Summary

In this section you have:

- learned about the relationship between fractions, decimals and percentages and are now able to convert between the three.

10 Ratio

As you can see from Figure 14, ratio is an important part of everyday life.

Figure 14 Day-to-day ratio

It is important to understand how to tell which part of the ratio is which. If for example, you have a group of men and women in the ratio of 5:4, as the men were mentioned first, they are the first part of the ratio.

The order of the ratio is very important. Consider the following:

> Julia attends a drama club where 100 members are men and 150 members are women. What is the ratio of women to men at the drama club?

Notice how the information that you need to answer the question is given in the opposite order to that required in the answer. It is very important that you give the parts of the ratio in the correct order.

The ratio of women to men is 150:100

If you were asked for the ratio of men to women it would be 100:150

10.1 Simplifying ratios

Sometimes you need to work out the ratio from the quantities you have.

If we refer back to the example we discussed earlier, we said that the ratio of women to men at the drama club is 150:100. However, you can simplify this ratio by dividing all parts by the same number. This is similar to simplifying fractions, which you have done.

With 150:100, we can divide each side of the ratio by 50 (you could also divide by 10 and then by 5), so the ratio will simplify to 3:2. Therefore, the ratio of women to men at the club is 3:2. Having it written in its simplest form makes it easier to think about and to use for other calculations. For every 2 men you have, there are 3 women.

Let’s look at another example.

Example: Recipes and ratio

Look at this recipe for a mocktail:

Sunset Smoothie

- 50 ml grenadine
- 100 ml orange juice
- 150 ml lemonade

The ratio of the ingredients is:

grenadine:orange juice:lemonade
50 : 100 : 150

To simplify this ratio you can divide all of the numbers by 50 (or by 10 and then 5).

This gives the ratio of grenadine to orange juice to lemonade as 1:2:3.

Activity 24: Simplifying ratios

Simplify the following ratios:

1. The ratio of women to men in a class is 15:20.
2. The ratio of management to staff in a warehouse is 10:250.
3. The ratio of home to away supporters is 24 000 to 8000.
4. The ratio of votes in a local election was candidate A 1600, candidate B 800, Candidate C 1200.
5. The ratio of fruit in a bag of mixed dried fruit is 150 g currants, 100 g raisins, 200 g sultanas and 50 g mixed peel.

Answer

1. Women to men is 3:4 (divide both sides by 5).
2. Management to staff is 1:25 (divide both sides by 10).
3. Home to away supporters is 3:1 (divide both sides by 8000 *or* by 1000 and then by 8).
4. A to B to C is 4:2:3 (divide each part of the ratio by 400 *or* by 100 and then by 4).
5. Currants to raisins to sultanas to mixed peel is 3:2:4:1 (divide by 50 *or* by 10 and then 5).

Ratio questions can be asked in different ways. There are three main ways of asking a ratio question. Take a look at an example of each below and see if you can identify the differences.

Type 1

A recipe for bread says that flour and water must be used in the ratio 5:3. If you wish to make 500 g of bread, how much flour should you use?

Type 2

You are growing tomatoes. The instructions on the tomato feed say ‘Use 1 part feed to 4 parts water’. If you use 600 ml of water, how much tomato feed should you use?

Type 3

Ishmal and Ailia have shared some money in the ratio 3:7. Ailia receives £20 more than Ishmal. How much does Ishmal receive?

In questions of type 1, you are given the total amount that both ingredients must add to, in this example, 500 g. In questions of type 2 however, you are not given the total amount but instead are given the amount of one part of the ratio. In this case you know that the 4 parts of water total 600 ml.

The final type of ratio question does not give us either the total amount or the amount of one part of the ratio. Instead, it gives us just the difference between the first and second part of the ratio. Whilst neither type of ratio question is more complicated than the others, it is useful to know which type you are dealing with as the approach for solving each type of problem is slightly different.

10.2 Solving ratio problems where the total is given

The best way for you to understand how to solve these problems is to look through the worked example in the video below.

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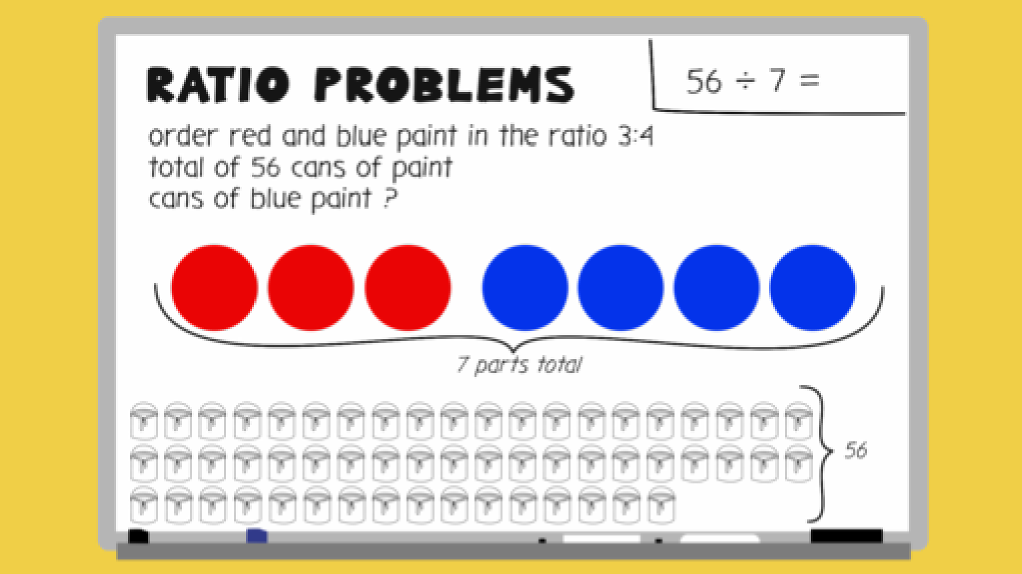


Activity 25: Ratio problems where the total is known

Try solving these ratio problems:

1. To make mortar you need to mix soft sand and cement in the ratio 4:1. You need to make a total of 1500 g of mortar.

   How much soft sand will you need?

Answer

1. Add the parts of the ratio:

   4 + 1 = 5

   Divide the total amount required by the sum of the parts of the ratio:

   1500 g ÷ 5 = 300 g

   Since soft sand is 4 parts, we do 300 g × 4 = 1200 g of soft sand.

   Check by working out how much cement you need. Cement is 1 part so you would need 300 g:

   1200 g + 300 g = 1500 g which is the correct total.

2. To make the mocktail 'Sea Breeze' you need to mix cranberry juice and grapefruit juice in the ratio 4:2.

   You want to make a total of 2700 ml of mocktail. How much grapefruit juice should you use?

Answer

2. Add the parts of the ratio:

   4 + 2 = 6

   Divide the total amount required by the sum of the parts of the ratio:

2700 ml ÷ 6 = 450 ml

Since grapefruit juice is 2 parts, we do 450 ml × 2 = 900 ml of grapefruit juice.

Check by working out how much cranberry juice you would use:

4 × 450 = 1800

1800 ml + 900 ml = 2700 ml

You may have simplified the ratio to 2:1 before doing the calculation, but you will see that your answers are the same as ours.

3. The instructions to mix Misty Morning paint are mix 150 ml of azure with 100 ml of light grey and 250 ml of white paint.

   How much light grey paint would you need to make 5 litres of Misty Morning?

Answer

3. Start by expressing and then simplifying the ratio:

   150:100:250 which simplifies to 3:2:5 = 10 parts

   5 litres = 5000 ml (converting to ml makes your calculation easier.)

   Divide the total amount required by the sum of the parts of the ratio:

   5000 ÷ 10 = 500 so 1 part = 500 ml

   Light grey is 2 parts:

   2 × 500 = 1000 ml or 1 litre

   Check:

   azure is 3 parts: 3 × 500 = 1500 ml or 1.5 litres

   white is 5 parts: 5 × 500 = 2500 ml or 2.5 litres

   1000 + 1500 + 2500 = 5000 ml or 5 litres

4. You want to make 14 litres of squash for a children's party. The concentrate label says mix with water in the ratio of 2:5.

   How much concentrate will you use?

Answer

4. Add the parts of the ratio:

   2 + 5 = 7

   Divide the total amount required by the sum of the parts of the ratio:

   14 litres ÷ 7 = 2 litres so 1 part = 2 litres

   (Note: this calculation was straightforward so there was no need to convert to ml.)

   Since the concentrate is 2 parts you will need 2 litres × 2 = 4 litres of concentrate.

   Check:

   Water is 5 parts:

   5 × 2 litres = 10 litres

   4 + 10 = 14 litres.

5. A man leaves £8400 in his will to be split between 3 charities:

   Dogs Trust, RNLI and MacMillan Research in the ratio 3:2:1.

   How much will each charity receive?

Answer

5. Add the parts of the ratio:

   3 + 2 + 1 = 6

   Divide the total amount required by the sum of the parts of the ratio:

   £8400 ÷ 6 = 1400

   - The Dogs Trust receives 3 parts: 3 × £1400 = £4200
   - The RNLI receives 2 parts: 2 × £1400 = £2800
   - MacMillan Research receives 1 part so: £1400

   Check:

   4200 + 2800 + 1400 = £8400

Next you'll look at ratio problems where the total of one part of the ratio is known.

10.3 Solving ratio problems where the total of one part of the ratio is given

Take a look at the worked example below:

You are growing tomatoes. The instructions on the tomato feed say:

Use 1 part feed to 4 parts water

If you use 600 ml of water, how much tomato feed should you use?

These questions make much more sense if you look at them visually:

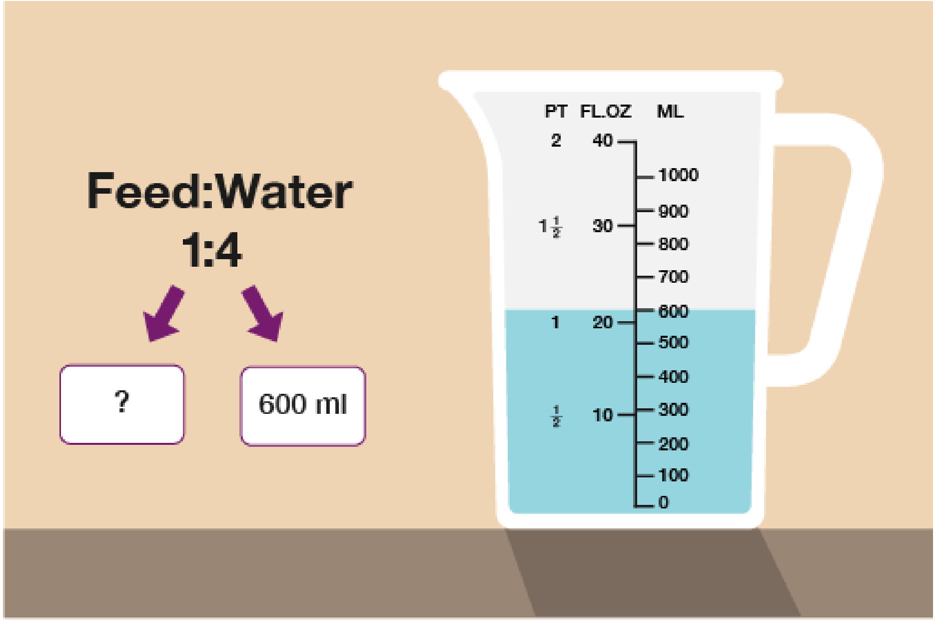


Figure 15 Solving ratio problems to grow tomatoes

You can now see clearly that 600 ml of water is worth 4 parts of the ratio. To find one part of the ratio you need to do:

600 ml ÷ 4 = 150 ml

Since the feed is only 1 part, feed must be 150 ml. If feed was more than one part you would multiply 150 ml by the number of parts.

!Warning! Calibri not supportedJust as with the previous type of question, you need to try to work out the value of 1 part. The value of any other number of parts can be worked out from this.

Activity 26: Ratio problems with one part given

Practise your skills by tackling the ratio problems below:

1. A recipe requires flour and butter to be used in the ratio 3:5. The amount of butter used is 700 g.
   How much flour will be needed?

Answer

1. **Flour:Butter**

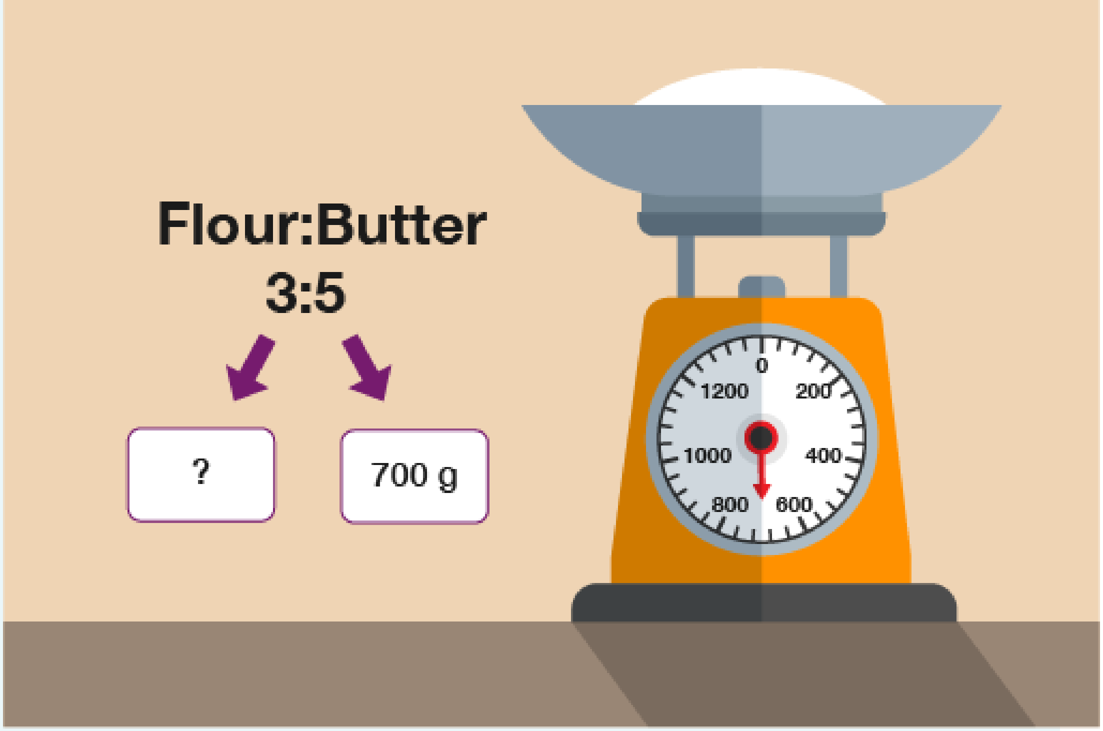


Figure 16 Using ratios in recipes

To find the amount of flour needed you then do 140 g × 3 = 420 g flour.

2. When looking after children aged between 7 and 10, the ratio of adults to children must be 1:8.
   b. For a group of 32 children, how many adults must there be?
   c. If there was one more child in the group, how would this affect the number of adults required?

Answer

2. **Adults:Children**
   b.

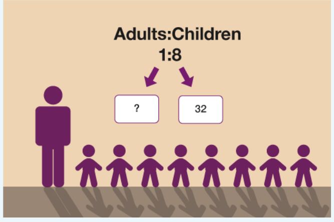


Figure 17 Working out the ratio of adults to children

To find one part you do 32 ÷ 8 = 4.

Since adults are only 1 part, you need 4 adults.

c. If there were 33 children, one part would be 33 ÷ 8 = 4.125.

Since you cannot have 4.125 adults, you need to round up to 5 adults so you would need one more adult for 33 children.

3. A shop mixes bags of muesli using oats, sultanas and nuts in the ratio 6:3:1. If the amount of sultanas used is 210 g, how heavy will the bag of muesli be?

Answer

3. **Oats:Sultanas:Nuts**

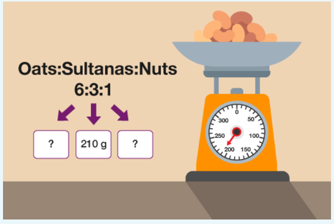


Figure 18 Working out the ratio of oats, sultanas and nuts

Sultanas are 3 parts so to find 1 part you do 210 g ÷ 3 = 70 g.
Oats are 6 parts so 6 × 70 = 420 g.
Nuts are only 1 part so they are 70 g.
The total weight of the bag would be 210 g + 420 g + 70 g = 700 g.

Next you'll look at ratio problems where only the difference in amounts is given.

10.4 Solving ratio problems where only the difference in amounts is given

Earlier in the section you came across the question below. Let's have a look at how we could solve this.

Example: Solving ratio amounts from the difference

Ishmal and Ailia have shared some money in the ratio 3:7.

Ailia receives £20 more than Ishmal. How much does Ishmal receive?

Ishmal:Ailia
3:7

You know that the difference between the amount received by Ishmal and the amount received by Ailia is £20. You can also see that Ailia gets 7 parts of the money whereas Ishmal only gets 3.

The difference in parts is therefore 7 − 3 = 4. So 4 parts = £20.

Now this is established, you can work out the value of one part by doing:

£20 ÷ 4 = £5

As you want to know how much Ishmal received you now do:

£5 × 3 = £15

As an extra check, you can work out Ailia's by doing:

£5 × 7 = £35

This is indeed £20 more than Ishmal.

Activity 27: Ratio problems where difference given

Now try solving this type of problem for yourself.

1. The ratio of female to male engineers in a company is 2:9. At the same company, there are 42 more male engineers than females.

   How many females work for this company?
2. A garden patio uses grey and white slabs in the ratio 3:5. You order 30 fewer grey slabs than white slabs.

   How many slabs did you order in total?

Answer

1. The difference in parts between males and females is 9 − 2 = 7 parts.

   You know that these 7 parts = 42 people.

   To find 1 part you do:

   42 ÷ 7 = 6

   Now you know that 1 part is worth 6 people, you can find the number of females by doing

   6 × 2 = 12 females

   Check:

   The number of males is 6 × 9 = 54. The difference between 54 and 12 is 42.
2. The difference in parts between grey and white is 5 − 3 = 2 parts.

   These 2 parts are worth 30. To find 1 part you do:

   30 ÷ 2 = 15

   To find grey slabs do:

   15 × 3 = 45

   To find white slabs do:

   15 × 5 = 75

   Check:

   The difference between the number of grey and white slabs is 30 (75 − 45).

   Now you know both grey and white totals, you can find the total number of slabs by doing:

   45 + 75 = 120 slabs in total.

Even though there are different ways of asking ratio questions, the aim of any ratio question is to determine the value of one part. Once you know this, the answer is simple to find!

Ratio can also be used in less obvious ways. Imagine you are baking a batch of scones and the recipe makes 12 scones. However, you need to make 18 scones rather than 12. How do you work out how much of each ingredient you need? The final ratio section deals with other applications of ratio.

10.5 Other applications of ratio

A very common and practical use of ratio is when you want to change the proportions of a recipe. All recipes state the number of portions they will make, but this is not always the number that you wish to make.You may wish to make more or less than the actual recipe gives. If you wanted to make 18 scones but only have a recipe that makes 12, how do you know how much of each ingredient to use?

To make 12 scones

400 g self-raising flour

1 tablespoon caster sugar

80 g butter

250 ml milk

Figure 19 Scones on a plate

As you already know the ingredients to make 12 scones, you need to know how much of each ingredient to make an extra 6 scones. Since 6 is half of 12, if you halve each ingredient, you will have the ingredients for the extra 6 scones. To find the total for 18 scones you need to add together the ingredients for the 12 scones and the 6 scones.

Table 9

12 scones	6 scones	18 scones
400 g flour	400 g ÷ 2 = 200 g flour	400 g + 200 g = 600 g flour
1 tablespoon caster sugar	1 ÷ 2 = $\frac{1}{2}$ tablespoon caster sugar	1 + $\frac{1}{2}$ = 1 $\frac{1}{2}$ tablespoons caster sugar
80 g butter	80 g ÷ 2 = 40 g butter	80 g + 40 g = 120 g butter
250 ml milk	250 ml ÷ 2 = 125 ml milk	250 ml + 125 ml = 375 ml milk

Have a go at the activity below to check your skills.

Activity 28: Ratio and recipes

1. This recipe makes 18 biscuits:

220 g self-raising flour
150 g butter
100 g caster sugar
2 eggs

How much of each ingredient is needed for 9 biscuits?

Answer

1. Since 9 is half of 18, you need to halve each ingredient to find the amount required to make 9 biscuits.

   220g ÷ 2 = 110 g flour
   150g ÷ 2 = 75 g butter
   100g ÷ 2 = 50 g sugar
   2 ÷ 2 = 1 egg

2. To make strawberry milkshake you need:

   630 ml milk
   3 scoops of ice cream
   240 g of strawberries
   The recipe serves 3

   How much of each ingredient is needed for 9 people?

Answer

2. You know the ingredients for 3 but want to know the ingredients for 9. Since 9 is three times as big as 3, you need to multiply each ingredient by 3.

   630 ml × 3 = 1890 ml milk
   3 × 3 = 9 scoops of ice cream
   240 g × 3 = 720 g of strawberries

3. Angel Delight recipe:

   Add 60 g powder to 300 ml cold milk
   Serves 2 people

   How much of each ingredient is needed to serve 5 people?

Answer

3. You could work this out in 2 different ways.

   Method 1

   You know the ingredients for 2 people. You can find ingredients for 4 people by doubling the ingredients for 2. You then need ingredients for an extra 1 person. Since 1 is half of 2, you can halve the ingredients for 2 people.

   60 g + 60 g + 30 g = 150 g powder
   300 ml + 300 ml + 150 ml = 750 ml milk

   Method 2

You know the ingredients for 2 people so you can find the ingredients for 1 person by halving them. You can then multiply the ingredients for 1 person by 5.

60g ÷ 2 = 30 × 5 = 150 g powder

300ml ÷ 2 = 150 × 5 = 750 ml milk

The final practical application of ratio can be very useful when you are out shopping. Supermarkets often try and encourage us to buy in bulk by offering larger 'value' packs. But how can you work out if this is actually a good deal? Take a look at the example below.

Example: Ratio and shopping

Which of the boxes below offers the best value for money?

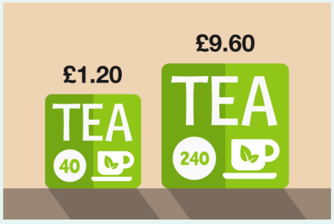


Figure 20 Shopping options: tea

There are various ways of comparing the prices.

Method 1

To work out which is the best value for money we need to find the price of 1 teabag.

If 40 teabags cost £1.20 then to find the cost of 1 teabag you do:

£1.20 ÷ 40 = £0.03, or 3p

If 240 teabags cost £9.60 then to find the cost of 1 teabag you do:

£9.60 ÷ 240 = £0.04, or 4p

The box containing 40 teabags is therefore better value than the larger box.

Method 2

The ratio of teabags is

40 : 240 which you can simplify to 1:6

If we use the price for the small box you can see that 1 part is £1.20

You can then use this value to calculate the price of the large box. At this price, the bigger box would be £1.20 × 6 = £7.20 so we can see that the small box is better value.

Activity 29: Practical applications of ratio

Use whichever method you prefer to work out the best deal in each case.

1. Work out which deal is the best value for money.

   a.

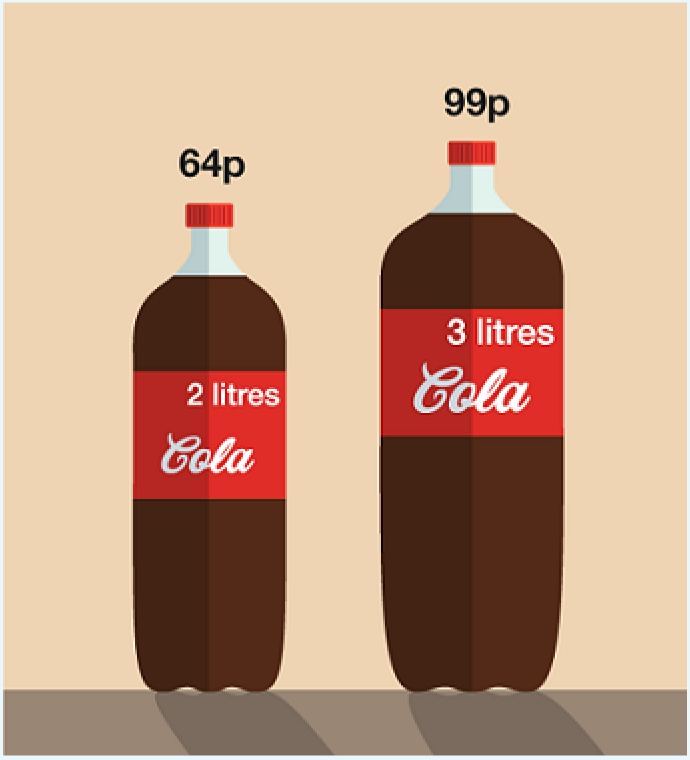


Figure 21 Cola options

Answer

1.

   a. 2 litres cost 64 p, so 1 litre costs 64 p ÷ 2 = 32p.
   3 litres cost 99p, so 1 litre costs 99p ÷ 3 = 33p.
   Comparing the cost of 1 litre in each case, we see that the 2-litre bottle is the best buy.

   b.

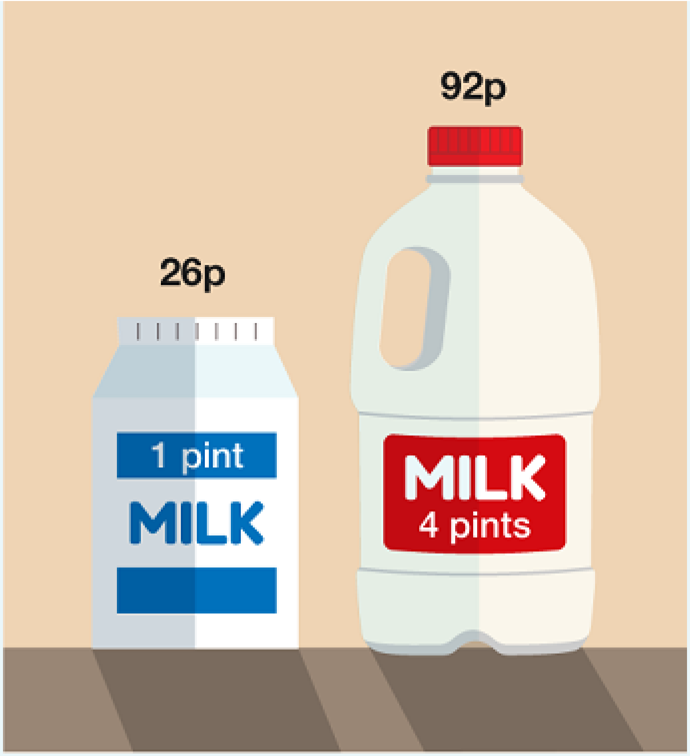


Figure 22 Milk options

Answer

1.

b. 1 pint costs 26p.

4-pint carton costs 92p, so 1 pint costs 92p ÷ 4 = 23p.

Comparing the cost of 1 pint of milk in each case, we see that the 4-pint carton is the best buy.

c.

Figure 23 Washing powder options

Answer

1.
   c. 5 kg costs £10, so 1 kg costs £10 ÷ 5 = £2.

      2 kg cost £3, so 1 kg costs £3 ÷ 2 = £1.50.

      Comparing the cost of 1 kg of powder in each case, we see that the 2 kg box is the best buy.

2. Two supermarkets sell the same brand of juice. Shop B is offering ‘buy one get second one half price’ for apple juice and ‘buy one get one free’ for orange juice.

   For each type of juice which shop is offering the best deal?

   b.

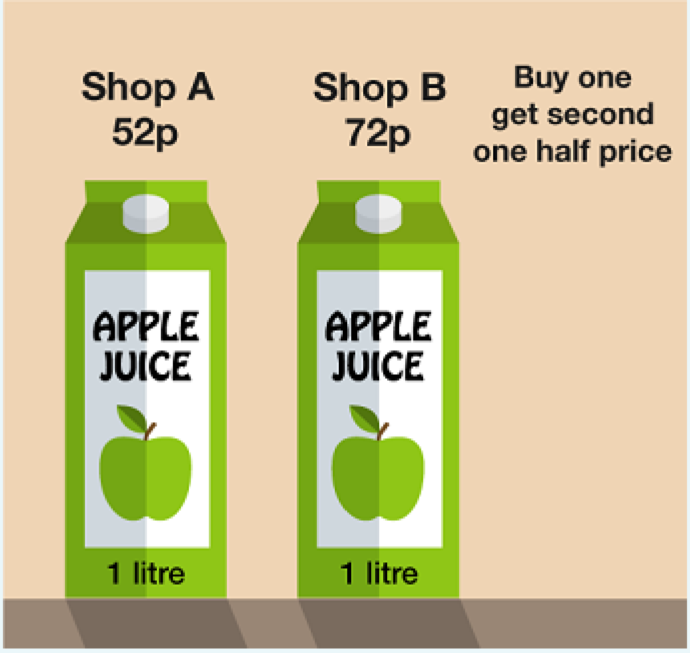


Figure 24 Apple juice options

Answer

2.

b. Shop A: 1 litre costs 52p

Shop B: 2 litres cost 72p + 36p = 108p (here we pay 72p for the first litre and 36p for second litre), so 1 litre costs 108p ÷ 2 = 54p.

Comparing the cost of 1 litre of apple juice in each case, we see that Shop A offers the better deal.

b.

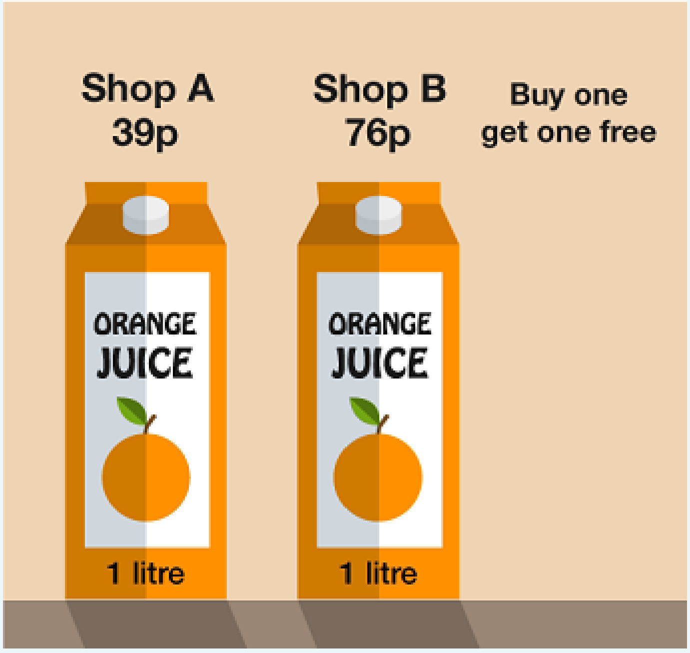


Figure 25 Orange juice options

Answer

2.

b. Shop A: 1 litre costs 39p

Shop B: 2 litres cost 76p (we get 1 litre free), so 1 litre costs:
76p ÷ 2 = 38p.

Comparing the cost of 1 litre of orange juice in each case, we see that Shop B offers the better deal.

3. A supermarket sells bread rolls in 3 different size packs. Which size offers the best value for money?

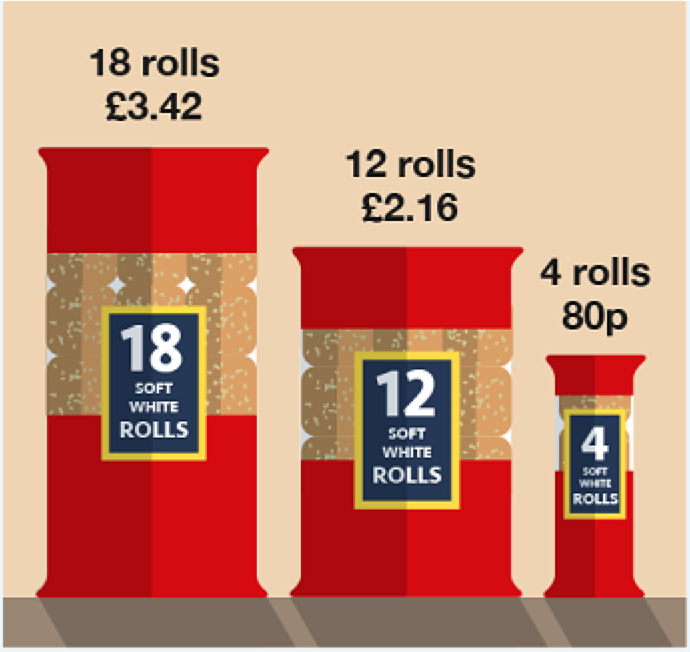


Figure 26 Bread rolls options

Answer

3. Calculate the cost of 1 roll in each pack:

   80p ÷ 4 = 20p

   £2.16 ÷ 12 = £0.18 or 18p

   £3.42 ÷ 18 = £0.19 or 19p

   The pack of 12 is best value.

You have now completed all elements of the ratio section and hopefully are feeling confident with each topic.

The next section of the course deals with formulas. This might sound daunting but you have actually already used a formula. Remember when you learned about how to work out the percentage change of an item? To do that you used a simple formula and you will now take a closer look at slightly more complex formulas.

Summary

In this section you have:

- learned about the three different types of ratio problems and that the aim of any ratio problem is to find out how much one part is worth
- practised solving each type of ratio problem:
  - where the total amount is given
  - where you are given the total of only one part
  - where only the difference in amounts is given

- learned about other useful applications of ratio, such as changing the proportions of a recipe.

11 Formulas

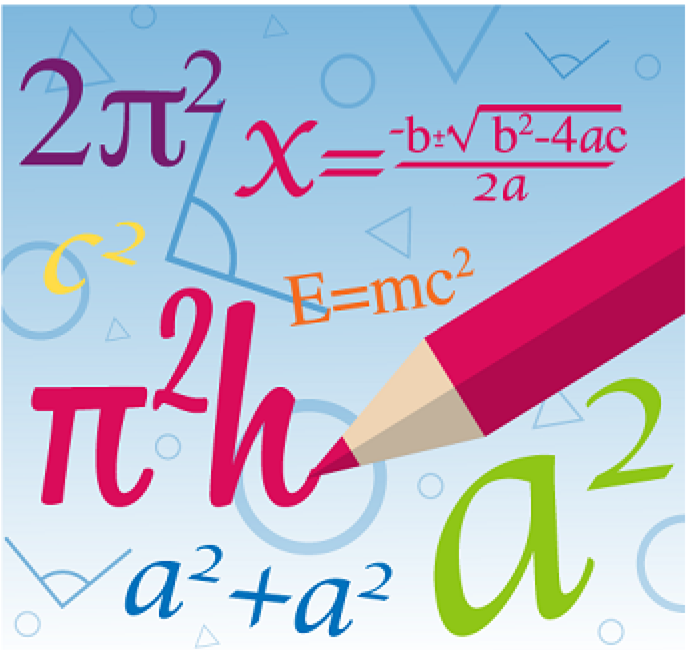


Figure 27 Formulas

Before diving in to this topic, you first need to learn about the order in which you need to carry out addition, subtraction, multiplication and division. Have you ever seen a question like the one below posted on social media?

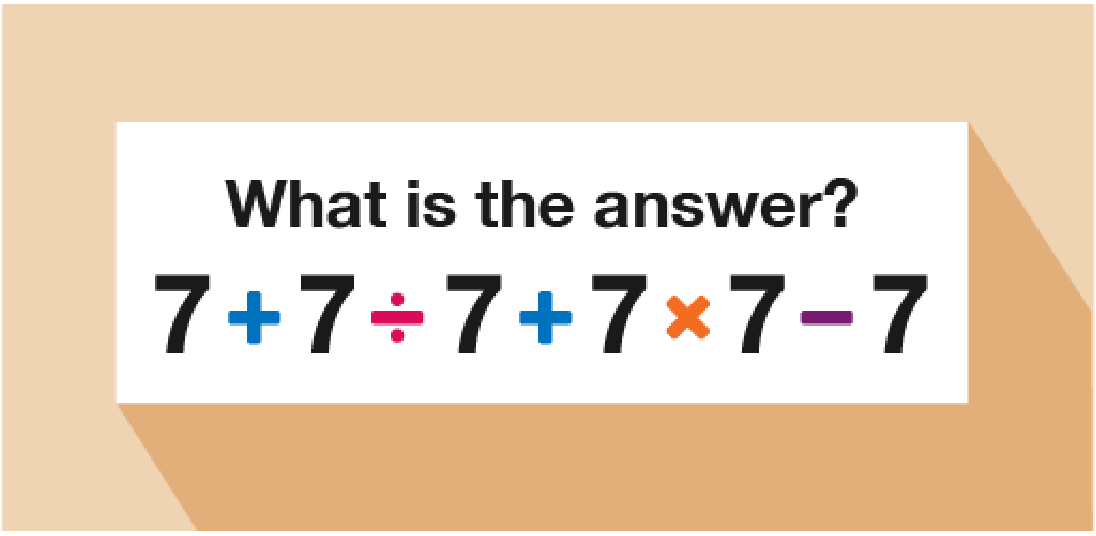


Figure 28 A calculation using the four operations

There are usually a wide variety of answers given by various people. But how is it possible that such a simple calculation could cause so much confusion? It's all to do with the order in which you carry out the calculations.

If you go from left to right:

7 + 7 = 14

14 ÷ 7 = 2

2 + 7 = 9

9 × 7 = 63

63 − 7 = 56

Check this on a calculator and you will see that the correct answer is actually 50. How do you arrive at this answer? You have to use the correct order of operations, sometimes called BIDMAS.

11.1 Order of operations

The order in which you carry out operations can make a big difference to the final answer. When doing any calculation that involves doing more than one operation, you must follow the rules of BIDMAS in order to arrive at the correct answer.

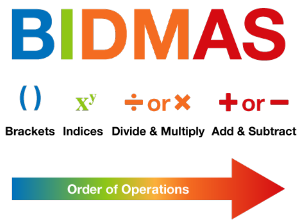


Figure 29 The BIDMAS order of operations

B: Brackets

Any calculation that is in brackets must be done first.

Example:

2 × (3 + 5)

2 × 8 = 16

Note that this could also be written as 2 (3 + 5) because if a number is next to a bracket, it means you need to multiply.

If there is more than 1 operation in the brackets, you must follow the rules of BIDMAS in the brackets.

I: Indices

After any calculations in brackets have been done, you must deal with any calculations involving indices or powers i.e.

$3^2 = 3 \times 3$

or

$4^3 = 4 \times 4 \times 4$

Example:

3×4^2

$3 \times (4 \times 4)$

$3 \times 16 = 48$

D: Divide

Next come any division or multiplication calculations. Of these two calculations, you should do them in the order that they appear in the sum from left to right.

Example:

$16 - 10 \div 5$

$16 - 2 = 14$

M: Multiply

Example:

$5 + 6 \times 2$

$5 + 12 = 17$

A: Add

Finally, any calculations involving addition or subtraction are done. Again, these should be done in the order that they appear from left to right.

S: Subtract

Example:

$24 + 10 - 2$

$34 - 2 = 32$

or

$24 + 8 = 32$

Activity 30: Using BIDMAS

Now have a go at carrying out the following calculations yourself. Make sure you apply BIDMAS!

1. 4 + 3 × 2
2. 5 (4 − 1)
3. 36 ÷ 3^2
4. 7 + 15 ÷ 3 − 4

Answer

1. 4 + 6 = 10
2. 5 × 3 = 15
3. 36 ÷ 9 = 4
4. 7 + 5 − 4 = 8

Now that you have learned the rules of BIDMAS you are ready to apply them when using formulas.

11.2 Formulas in practice

You will already have come across and used formulas in your everyday life. For example, if you are trying to work out the cost of a new carpet you will have used the formula:

area = *length* × *width*

to calculate how much carpet you would need.

Often division in a formula is shown as one number over another, for example:

6 ÷ 3 would be shown as $\frac{6}{3}$

Let's look at division in a formula:

$$speed = \frac{dis\,tan\,ce}{time}$$

A lorry driver travels 120 miles in 3 hours. What was the average speed during the journey?

Note: As the lorry driver was unlikely to have travelled at a constant speed for 120 miles we say we are calculating the average speed as this will give us the typical overall speed.

speed = $\frac{120}{3}$

speed = 40 miles per hour

Sometimes we use letters to represent the different elements used in a formula, e.g. the formula above might be shown as:

$s = \frac{d}{t}$

where:
'*s*' = speed in mph
'*d*' = distance in miles
'*t*' = time in hours

If you are trying to work out the time to cook a fresh chicken you may have used the formula:

Time (minutes) = 15 + $\frac{w}{500}$ × 25 where '*w*' is the weight of the chicken in grams.

For example, if you wanted to cook a chicken that weighs 2500 g you would do:

Time (minutes) = 15 + $\frac{2500}{500}$ × 25

Remembering to use BIDMAS you would then get:

Time (minutes) = 15 + 5 × 25
= 15 + 125
= 140 minutes

Let's look at another worked example before you try some on your own.

Example: Gas bill formula

The owner of a guesthouse receives a gas bill. It has been calculated using the formula:

Cost of gas (£) = $\frac{8d + u}{100}$

Note: 8*d* means you do 8 × *d*.

Where *d* = number of days and *u* = number of units used, if she used 3500 units of gas in 90 days, how much is the bill?

In this example, d = 90 and u = 3500 so you do:

$$\text{Cost of gas (£)} = \frac{8 \times 90 + 3500}{100}$$

$$= \frac{720 + 3500}{100}$$

$$= \frac{4220}{100}$$

= £42.20

Activity 31: Using formulas

1. Fuel consumption in Europe is calculated in litres per 100 kilometres. A formula to approximate converting from miles per gallon to litres per 100 kilometres is:

   $$L = \frac{280}{M}$$

   where L = number of litres per 100 kilometres and M = number of miles per gallon.
   A car travels 40 miles per gallon. What is this in litres per kilometres?

Answer

1.

   $L = \frac{280}{M}$ and in this case M = 40

   $$L = \frac{280}{40}$$

   L = **7 litres per 100 kilometres**

2. Using the formula $I = \frac{PRT}{100}$ where:

   I = interest
   P = principal amount of loan
   R = interest rate
   T = time in years

   calculate how much interest is due on a **loan of £5000** taken over **3 years** at an interest rate of **5.5%**.

Answer

2. $I = \frac{PRT}{100}$

In this case P = £5000, R = 5.5% and T = 3 years.

$I = \frac{5000 \times 5.5 \times 3}{100}$

$I = \frac{82500}{100}$

I = 825
So the interest paid would be **£825**.

3. The area of a trapezium can be calculated using the formula:

$A = \frac{h\ (a+b)}{2}$

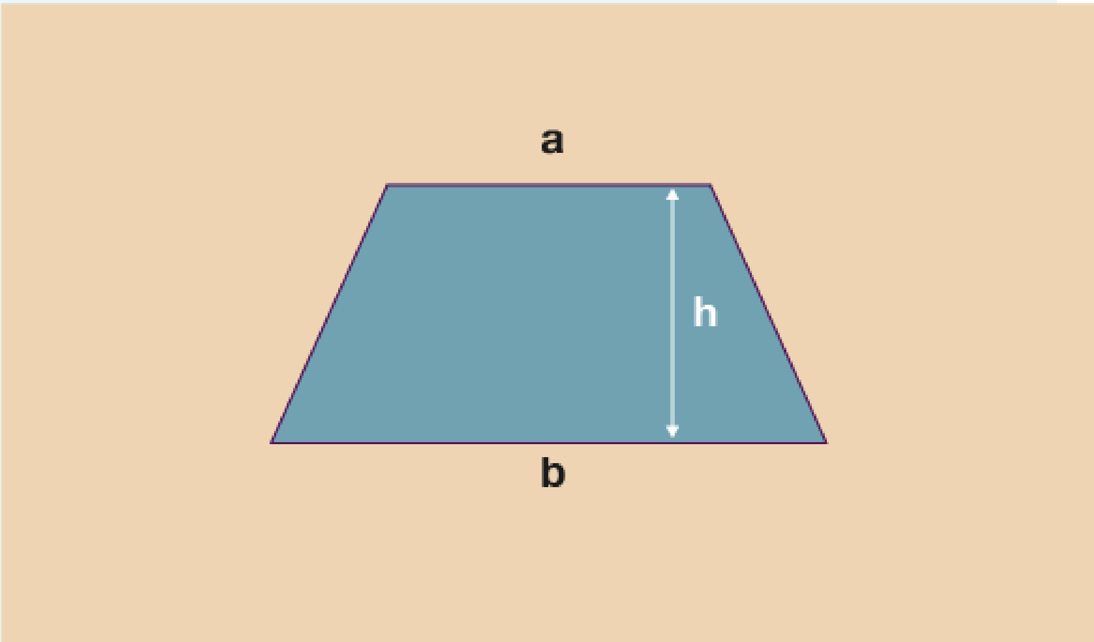


Figure 30 Dimensions of a trapezium

Find the area of trapeziums where:

iii. a = 5 cm, b = 9 cm and h = 7 cm
iv. a = 35 mm, b = 40 mm and h = 10 cm

Answer

3. $A = \frac{h\ (a+b)}{2}$

iii. $A = \frac{7\ (5+9)}{2}$

$A = \frac{7 \times 14}{2}$

$A = \frac{98}{2}$

A = **49 cm^2**

iv. In this question you must convert the units so that they are all the same. The units that you select will be the units that your answer will be given in, e.g. if you convert to mm your answer will be in mm^2 but if you convert to cm your answer will be in cm^2.

$A = \frac{h\ (a+b)}{2}$

Method 1 – converting to mm

Convert *h* measurement to mm:

10 × 10 = 100 mm

$A = \frac{100\ (35+40)}{2}$

$A = \frac{100 \times\ 75}{2}$

$A = \frac{7500}{2}$

A = **3750 mm^2**

Method 2 – converting to cm

Convert *a* and *b* measurements to cm:

a = 35 ÷ 10 = 3.5 cm

b = 40 ÷ 10 = 4 cm

$A = \frac{10\,(3.5+4.0)}{2}$

$A = \frac{10\times\ 7.5}{2}$

$A = \frac{75}{2}$

A = **37.5 cm^2**

Note: it is a good idea to show all the stages of the calculation to help you keep track of your workings.

4. A company uses the following formula to work out the total cost to the customer of hiring a bouncy castle:

$$T = hc + (0.45d) + 15$$

where:

T = total
h = number of days hire
c = cost of castle per day
d = delivery distance in miles.

Figure 31 Dues's Bouncy Fun – price list

Stuart lives 12 miles away and would like to hire a Supersonic Castle for 2 days. How much will it cost?

Answer

4. $T = hc + (0.45d) + 15$

In this case $h = 2$, $c = £42$, and $d = 12$, so:

$T = 2 \times 42 + (0.45 \times 12) + 15$
$T = 84 + 5.4 + 15$
$T = 104.4$

The total cost of hire would be **£104.40**.

Now that you have learned all the skills that relate to the number section of this course, there is just one final thing you need to be able to do before you will be ready to complete the end-of-session quiz for numbers.

You are now proficient at carrying out lots of different calculations including working out fractions and percentages of numbers, using ratio in different contexts and using formulas.

It is fantastic that you can now do all these things, but how do you check if an answer is correct? One way you can check would be to approximate an answer to the calculation (as you did in Section 3.2). Another way to check an answer is to use the inverse (opposite) operation.

Summary

In this section you have:

- learned about, and practised using BIDMAS – the order in which operations must be carried out
- seen examples of formulas used in everyday life and practised using formulas to solve a problem.

12 Checking your answers

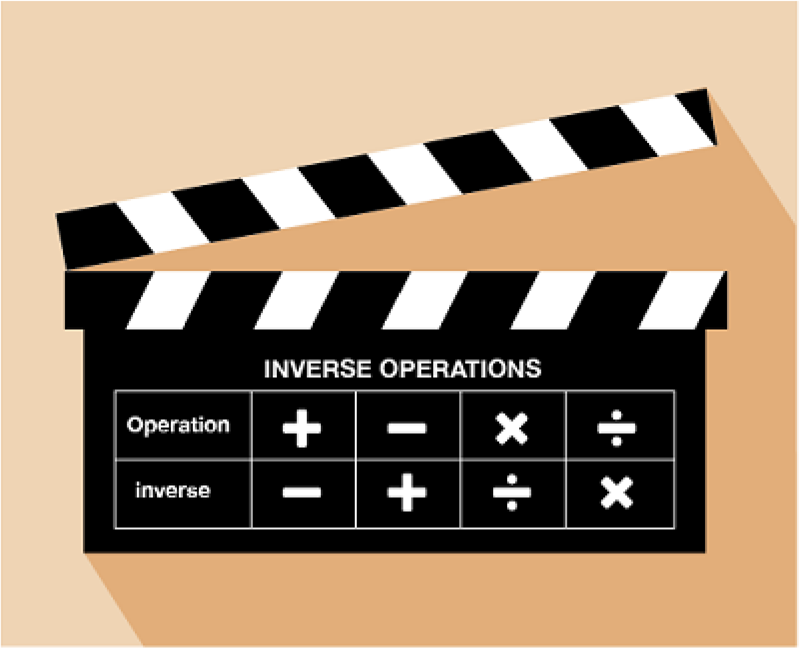


Figure 32 Inverse operations

An inverse operation is an opposite operation. In a sense, it 'undoes' the operation that has just been performed. Let's look at two simple examples to begin with.

Example: Check your working 1

6 + 10 = 16

Method
Since you have done an addition sum, the inverse operation is subtraction. To check this calculation, you can either do:

16 − 10 = 6

or

16 − 6 = 10

You will notice here that the same 3 numbers (6, 10 and 16) have been used in all the calculations.

Example: Check your working 2

5 × 3 = 15

Method
This time, since you have done a multiplication sum, the inverse operation is division. To check this calculation, you can either do:

15 ÷ 5 = 3

or

15 ÷ 3 = 5

Again, you will notice that the same 3 numbers (3, 5 and 15) have been used in all the calculations.

If you have done a more complicated calculation, involving more than one step, you simply ‘undo’ each step.

Example: Check your working 3

A coat costing £40 has a discount of 15%. How much do you pay?

Method

Firstly, we find out 15% of £40:

40 ÷ 100 × 15 = £6 discount

£40 − £6 = £34 to pay

To check this calculation, firstly you would check the subtraction sum by doing the addition:

£34 + £6 = £40

To check the percentage calculation you then do:

£6 ÷ 15 × 100 = £40

Don’t forget, sometimes it can also be helpful to use estimation to check your answers, particularly when using decimal or large numbers.

You have now completed the number section of the course. Before moving on to the next session, ‘Units of measure’, complete the quiz on the following page to check your knowledge and understanding.

Summary

In this section you have:

- learned that each of the four operations has an inverse operation (its opposite) and that these can be used to check your answers
- seen examples showing how to check answers using the inverse operation.

13 Session 1 quiz

Now it's time to review your learning in the end-of-session quiz.

Session 1 quiz.

Open the quiz in a new window or tab (by holding ctrl [or cmd on a Mac] when you click the link), then return here when you have done it.

Although the quizzes in this course do not require you to show your working to gain marks, real exams would do so. We therefore encourage you to practise this by using a paper and pen to clearly work out the answers to the questions. This will also help you to make sure you get the right answer.

14 Session 1 summary

You have now completed Session 1, 'Working with numbers'. If you have identified any areas that you need to work on, please ensure you refer back to this section of the course and retry the activities.

You should now be able to:

- use the four operations to solve problems in context
- understand rounding and look at different ways of doing this
- write large numbers in full and shortened forms
- carry out calculations with large numbers
- carry out multistage calculations
- solve problems involving negative numbers
- define some key mathematical terms (multiple, lowest common multiple, factor, common factor and prime number)
- identify lowest common multiples and factors
- use fractions, decimals and percentages and convert between them
- solve different types of ratio problems
- make substitutions within given formulas to solve problems
- use inverse operations and estimations to check your calculations.

All of the skills above will help you with tasks in everyday life. Whether you are at home or at work, number skills are an essential skill to have.

You are now ready to move on to Session 2, 'Units of measure'.

Session 2: Units of measure

Introduction

In this session you will be calculating using units of measure and focusing on length, time, weight, capacity and money. You will already be using these skills in everyday life when:

- working out which train you need to catch to get to your meeting on time
- weighing or measuring ingredients when cooking
- doing rough conversions if you are abroad to work out how much your meal costs in British pounds.

By the end of this session you will be able to:

- understand that there are different units used for measuring and how to choose the appropriate unit
- convert between measurements in the same system (e.g. grams and kilograms) and those in different systems (e.g. litres and gallons)
- use exchange rates to convert currencies
- work with time and timetables
- work out the average speed of a journey using a formula
- convert temperature measurements between Celsius (°C) and Fahrenheit (°F)
- read scales on measuring equipment.

First watch the video below which introduces units of measure.

Video content is not available in this format.

1 Units of measure

A unit of measure is simply what you measure something in and you will already be familiar with using centimetres, metres, kilograms, grams, litres and millilitres for measurement. When measuring something, you need to choose the appropriate unit of measure for the item you are measuring. You would not, for example, measure the length of a room in millimetres. Have a go at the short activity below and choose the most appropriate unit for each example.

Activity 1: Choosing the unit

Millilitres (ml)

Metres (m)

Kilograms (kg)

Grams (g)

Centimetres (cm)

Litres (l)

Match each of the items above to an item below.

Amount of liquid in a glass

Length of a garden fence

Weight of a dog

Weight of an egg

Length of a computer screen

Amount of water in a paddling pool

Hopefully you found that activity fairly straightforward. Next you'll take a closer look at some units of measure and how to convert between them.

1.1 Converting units of measure in the same system

Imagine you are catering for a party and go to the wholesale shop to buy flour. You need 200 g of flour for each batch of cookies you will be making and need to make 30 batches. You can buy a 5 kg bag of flour but aren't sure if that will be enough.

In order to work out if you have enough flour, you need both measurements to be in the same unit – kg or g – before you can do the calculation. For units of measure that are in the same system (so you are not dealing with converting cm and inches for example) it's a simple process to convert one measurement into another.

For now we will focus on what is known as the metric system of measurement (we will look at some units from the imperial system later on in the session). In most cases, if you want to convert between units in the metric system you will just need to multiply or divide by 10, 100 or 1000.

Take a look at the diagrams below which explain how to convert each type of measurement unit.

Length

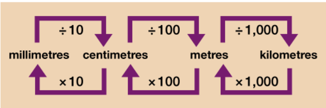


Figure 1 A conversion chart for length

Weight

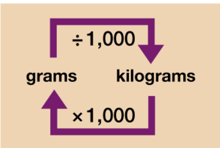


Figure 2 Converting between grams and kilograms

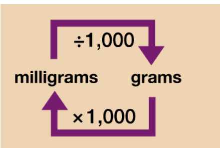


Figure 3 Converting between milligrams and grams

Money

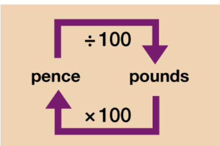


Figure 4 A conversion chart for money

Capacity

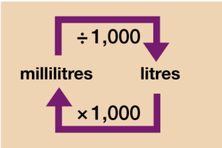


Figure 5 Converting between millilitres and litres

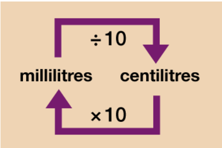


Figure 6 Converting between millilitres and centilitres

Note: Capacity may also be measured in centilitres (shortened to cl). On a standard bottle of water, for instance, it may give the capacity as 500 ml or 50 cl – there are 10 millilitres (ml) in 1 centilitre (cl).

When converting between units you may need to do the conversion in stages.

Example: Converting length

You measure the gap for a washing machine to be 0.62 m. You look on a retailer's website and it shows the width of a particular washing machine to be 600 mm. Will this fit in the gap?

Method

You can convert the gap for the washing machine in stages. Convert from metres to centimetres first by multiplying by 100:

0.62 m × 100 = 62 cm

Now convert the centimetres figure into millimetres by multiplying by 10:

62 cm × 10 = 620 mm

So the washing machine (600 mm wide) will fit in the gap of 620 mm.

You may have preferred to convert the washing machine width into metres instead and do the comparison that way:

600 mm ÷ 10 = 60 cm

60 cm ÷ 100 = 0.6 m

0.6 m is less than 0.62 m so the washing machine will fit the gap.

You'll learn how to convert between different currencies and units of measure in different systems (metric system kilograms (kg) to imperial system pounds (lb) for example) later in this session. For now, have a go at the activities below and test your metric conversion skills.

Activity 2: Converting units

Use your conversion skills to fill in the missing information in Table 1(a). You may want to carry out some of the calculations in stages.

Please carry out all of your calculations without using a calculator. However, double-check on a calculator if you need to.

Table 1(a)

Length	Weight	Capacity
55cm = ? mm	9.75 kg = ? g	235 ml = ? l
257 cm = ? m	652 g = ? kg	18.255 l = ? ml
28.7 km = ? m	5846 g = ? kg	16 ml = ? l
769 mm = ? cm	19.4 kg = ? g	7.88 l = ? ml
1.43 m = ? mm	44 g = ? mg	250 ml = ? cl
125 500 cm = ? km	750 000 mg = ? kg	7.4 l = ? cl

Answer

Table 1(b)

Length	Weight	Capacity
55cm × 10 = 550 mm	9.75 kg × 1000 = 9750 g	235 ml ÷ 1000 = 0.235 l
257 cm ÷ 100 = 2.57 m	652 g ÷ 1000 = 0.652 kg	18.255 l × 1000 = 18 225 ml
28.7 km × 1000 = 28 700 m	5846 g ÷ 1000 = 5.846 kg	16 ml ÷ 1000 = 0.016 l
769 mm ÷ 10 = 76.9 cm	19.4 kg × 1000 = 19 400 g	7.88 l × 1000 = 7880 ml
1.43 m × 100 = 143 cm × 10 = 1430 mm	44 g × 1000 = 44 000 mg	250 ml ÷ 10 = 25 cl
125 500 cm ÷ 100 = 1255 m ÷ 1000 = 1.255 km	750 000 mg ÷ 1000 = 750 g ÷ 1000 = 0.75 kg	7.4 l × 100 = 740 cl

Activity 3: Solving conversion problems

Solve the following problems. Please carry out all of your calculations without using a calculator. You may double check on a calculator if needed.

1. A sunflower is 1.8 m tall. Over the next month, it grows a further 34 cm. How tall is the sunflower at the end of the month?
2. Peter is a long distance runner. In a training session he runs around the 400 m track 23 times. He wanted to run a distance of 10 km. How many more times does he need to run around the track to achieve this?
3. A water cooler comes with water containers that hold 12 l of water. The cups provided for use hold 150 ml. A company estimates that each of its 20 employees will drink 2 cups of water a day. How many 12 l bottles will be needed for each working week (Monday to Friday)?
4. David is a farmer and has 52 goats. He is given a bottle of wormer that contains 0.3 litres. The bottle comes with the instructions:

   'Use 4 ml of wormer for each 15 kg of body weight.'

The average weight of David's goats is 21 kg and he wants to treat all of them. Does David have enough wormer?

Answer

1. 1 m = 100 cm so 1.8 × 100 = 180 cm

   180 cm + 34 cm = 214 cm or 2.14 m

2. 400 m × 23 = 9200 m that Peter has run.

   1 km = 1000 m so, 10 km = 10 × 1000 = 10 000 m

   The difference between what Peter wants to run and what he has already run is:

   10 000 − 9200 = 800 m
   800 ÷ 400 = 2

   Peter needs to run another 2 laps of the track.

3. 1 litre = 1000 ml so 12 l = 12 × 1000 = 12 000 ml

   Each employee will drink 2 × 150 ml = 300 ml a day

   There are 20 employees so 300 × 20 = 6000 ml for all employees per day

   A working week is 5 days:

   6000 × 5 = 30 000 ml for all employees for the week
   30 000 ÷ 12 000 = 2.5

   They will need 2 and a half containers per week.

4. 1 litre = 1000 ml so 0.3 litres = 0.3 × 1000 = 300 ml of wormer in the bottle

   52 goats at 21 kg each is 52 × 21 = 1092 kg total weight of goats

   To find the number of 4 ml doses required do:

   1092 ÷ 15 = 72.8
   72.8 × 4 ml = 291.2 ml of wormer needed.

   Yes, David has enough wormer.

Hopefully you will now be feeling confident with converting units of measure within the same system. You'll need to know how to do this and be able to remember how to convert from one to another by multiplying or dividing. If you need further practice converting between units, please revisit the 'Units of measure' session in the Everyday Maths 1 course.

If you are being asked to convert between units in different systems (e.g. between litres and gallons) or between currency (e.g. US dollars and British pounds), you won't be expected to know the conversion rate – you'll be given it in the question. The next section will show you how to use these conversion rates and you'll be able to practise solving problems that require you to do this.

Summary

In this section you have learned:

- that different units are used for measurement
- unit selection depends on the item or object being measured
- how to convert between units in the same system.

2 Converting currencies and measures between different systems

When working with measures you'll often need to be able to convert between units that are in different systems (kilograms to pounds for example) or currencies (British pounds to euros for example). Whilst this might sound tricky, it's really just using your multiplication and division skills. Let's begin by looking at converting currencies.

2.1 Converting currencies

Often when you are abroad, or buying something from the internet from another country, the price will be given in a different currency. You'll want to be able to work out how much the item costs in your own currency so that you can make a decision about whether to buy the item or not. When changing money for a holiday, you may also need to work out how much currency you will get for the amount of pounds sterling you are changing.

Figure 7 Different currency notes

Let's look at an example.

Example: Pounds and euros

To convert between pounds and euros, the first thing you need to know is the exchange rate. Exchange rates change daily. If you are given a currency conversion question, though, you will be given the exchange rate to use. In this example we will use the exchange rate of:

£1 = €1.15

So how do you convert?

Method

Look at the diagram below. In order to get from 1 to 1.15, you must multiply by 1.15. Following that, to get from 1.15 to 1, you must divide by 1.15.

In order to go from pounds to euros then, you multiply by 1.15 and to convert from euros to pounds, you divide by 1.15.

Figure 8 A conversion rate for pounds and euros

a. You decide you are going to change £300 into euros for your holiday. Using the exchange rate of £1 = €1.15, how many euros will you get?

   Referring back to the diagram above, you want to convert pounds to euros in this example so you must multiply by 1.15.

   However, you will not always have a diagram to refer to which means that you will need to be able to decide yourself whether you need to multiply by the conversion rate or divide by it. We will now look at the method to help you to do this.

   Method

   Write down your conversion rate with the single unit on the left-hand side:

   £1 = €1.15

   Next, you write down the figure you are converting under the correct side. In this case it is the £300 cash you are changing, so this figure needs to go under the pounds (£) side:

   £1 = €1.15
   £300 = €?

   This will help you to work out whether you need to go *forwards* (in which case you *multiply* by the conversion rate) or *backwards* (in which case you *divide* by the conversion rate).

   Here, you are converting from pounds to euros. This means you are going *forwards* so you need to *multiply* by the conversion rate:

   £1 = €1.15
   £300 = €?
   → *forwards* →

   Therefore, you must *multiply* by 1.15:

   £300 × 1.15 = €345

   Let's apply the method to another example.

b. You are in Spain and want to buy a souvenir. The price is €35 (35 euros). You know that £1 = €1.15.

   Method

Write the currency conversion rate down first with the single unit of measure (the 1) on the left hand-side.

£1 = €1.15

Next, write down the figure you are converting under the correct side. In this case it is the cost of the souvenir (€35), so this figure needs to go on the euros (€) side:

£1 = €1.15
£? = €35

Here, you are converting from euros to pounds. This means you are going *backwards* so you need to *divide* by the conversion rate:

£? = €35
← *backwards* ← therefore *divide*
€35 ÷ 1.15 = £30.43 (rounded to two decimal places)

Before you try some for yourself, let's look at one more example. This time involving pounds and dollars.

Example: Pounds and dollars

a. You want to buy an item from America. The cost is $120. You know that £1 = $1.30. How much does the item cost in pounds?

Method

Similarly to before, you can see from the diagram that in order to go from dollars to pounds you must divide by 1.30.

Figure 9 A conversion rate for pounds and dollars

To work out $120 in pounds then, you do:

$120 ÷ 1.30 = £92.31 (rounded to two decimal places)

To figure this out without the aid of a diagram, write down your conversion rate:

£1 = $1.30

Now write down the figure you want to convert (in this case the $120) under the correct side:

£1 = $1.30

£? = $120

As you are converting dollars to pounds, you are going *backwards* so need to *divide* by 1.30:

$120 ÷ 1.30 = £92.31 (rounded to two decimal places)

b. You are sending a relative in America £20 for their birthday. You need to send the money in dollars. How much should you send?

Method

To convert from pounds to dollars, you need to write down you conversion rate:

£1 = $1.30

Now write down the figure you want to convert (in this case the £20) under the correct side:

£1 = $1.30

£20 = $?

As you are converting pounds to dollars, you are going forwards so need to multiply by 1.30:

£20 × 1.30 = $26

You need to send $26.

Summary of method

Write your conversion rate down first, making sure you put the single unit (the 1) on the left-hand side. This will make it easier to decide whether you need to multiply by the conversion rate or divide by it.

- If you are going forwards, you *multiply* by the conversion rate.
- If you are going backwards, you *divide* by the conversion rate.

Activity 4: Currency conversions

Calculate the answers to the following questions without using a calculator. You may double-check your answers on a calculator if needed.

1. Sarah sees a handbag for sale on a cruise ship for €80. She sees the same handbag online for £65. She wants to pay the cheapest possible price for the bag. On that day the exchange rate is £1 = €1.25. Where should she buy the bag from?
2. Josh is going to South Africa and wants to change £250 into rand. He knows that £1 = 18.7 rand. How many rand will he get for his money?
3. Alice is returning from America and has $90 left over to change back into £s. Given the exchange rate of £1 = $1.27, how many £s will Alice receive? Give your answer rounded to two decimal places.

Answer

1. For this question you can either convert from £ to € or from € to £.

   Converting from £ to €:

   £1 = €1.25
   £65 = €?

   As you are converting pounds to euros, you are going forwards so need to multiply by 1.25:

   £65 × 1.25 = €81.25

   Sarah should buy the bag on the ship.

   Converting from € to £:

   £1 = €1.25
   £? = €80

   As you are converting euros to pounds you are going backwards so need to divide by 1.25:

   €80 ÷ 1.25 = £64

   Sarah should buy the bag on the ship.
2. You need to convert from £ to rand so you do:

   £1 = 18.7 rand
   £250 = ? rand

   As you are converting pounds to rand you are going forwards so need to multiply by 18.7:

   £250 × 18.7 = 4675 rand
3. You need to convert from $ to £ so you do:

   £1 = $1.27
   £? = $90

   As you are converting dollars to pounds, you are going backwards so need to divide by 1.27:

   $90 ÷ 1.27 = £70.87 (rounded to two d.p.)

Hopefully you are now feeling confident with converting between currencies. The next part of this section deals with converting other units of measure between different systems – centimetres to inches, pounds to kilograms etc. These conversions are incredibly similar to converting currency and the conversion rate will always be given to you so it's not something you have to remember.

2.2 Converting units of measure between different systems

This is a useful skill to have because you may often measure something in one unit, say your height in feet and inches, but then need to convert it to centimetres. You may, for example, be training to run a 10 km race and want to know how many miles that is. To do this all you need to know is the conversion rate and then you can use your multiplication and division skills to calculate the answer.

When performing these conversions, you are converting between metric and imperial measures. Metric measures are commonly used around the world but there are some countries (the USA for example) who still use imperial units. Whilst the UK uses metric units (g, kg, m, cm etc.) there are some instances where you may still need to convert a metric measure to an imperial one (inches, feet, gallons etc.).

In most cases, the conversion rate will be given in a similar format to the way money exchange rates are given, i.e. 1 inch = 2.5 cm or 1 kg = 2.2 lb. In these cases you can do exactly the same as you would with a currency conversion. The only exceptions here are where you don't have one of the units of measure as a single unit i.e. 5 miles = 8 km.

As this skill is very similar to the one you previously learnt with currency conversions, let's just look at two brief examples before you have a go at an activity for yourself.

Example: Centimetres and inches

You want to start your own business making accessories. One of the items you will be making is tote bags. The material you have bought is 156 cm wide. You need to cut pieces of material that are 15 inches wide.

How many pieces can you cut from the material you bought?

Use 1 inch = 2.54 cm.

Method

You need to start by converting either cm to inches or inches to cm so that you are working with units in the same system. Let's look at both ways so that you feel confident that you will get the same answer either way.

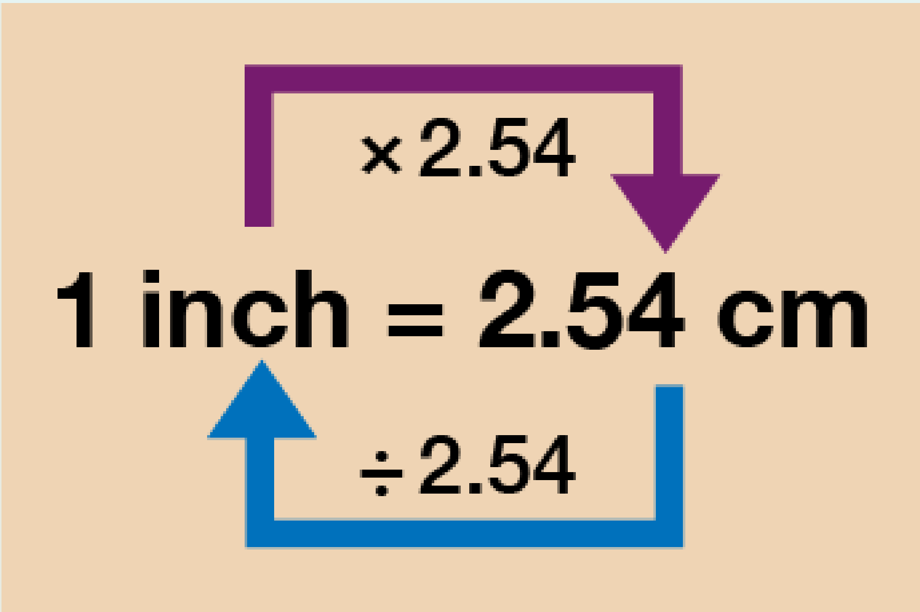


Figure 10 Converting between inches and centimetres

You can see from the diagram above that to convert from inches to cm you must multiply by 2.54. To convert from cm to inches you must divide by 2.54.

As with currency conversions, if you do not have a diagram to help you, you will have to work out yourself whether you need to multiply by the conversion rate or divide by it. To do this, write down your conversion rate:

1 inch = 2.54 cm (remember to put the single unit – the 1 – on the left-hand side)

Converting from inches to cm

Now write the figure you are converting under the correct side:

1 inch = 2.54 cm

15 inches = ? cm

Since you want to know what 15 inches is in centimetres, you are going forwards so you need to multiply by the conversion rate of 2.54:

15 inches × 2.54 = 38.1 cm. This is the length of the material needed for each bag.

To calculate how many pieces of material you can cut you then do:

156 ÷ 38.1 = 4.0944…. As you need whole pieces of material, you need to round this answer down to 4 pieces.

Converting from cm to inches

You may have decided to convert from centimetres to inches instead. You still need to write down your conversion rate first:

1 inch = 2.54 cm (remember to put the single unit – the 1 – on the left-hand side)

Now write the figure you are converting under the correct side (in this case you will convert the 156 centimetres into inches):

1 inch = 2.54 cm

? inches = 156 cm

Since you want to know what 156 centimetres is in inches, you are going backwards so you need to divide by the conversion rate of 2.54:

156 ÷ 2.54 = 61.417…, this is the length of the big piece of material.

To calculate how many pieces of material you can cut you then do:

61.417… ÷ 15 = 4.0944…. Again, as you need whole pieces of material, you need to round this answer down to 4 pieces.

You can see that no matter which way you choose to convert you will arrive at the same answer.

Note: Some numbers do not divide into each other exactly so you end up with lots of digits after the decimal point. Where this is the case, you need to round off the answer. You may be told what to round off to in the question (e.g. to the nearest whole number or to two decimal places) or you may have to use your own judgement.

For money you will normally round to two decimal places. In this case you are thinking about whole pieces of material – you only have four whole pieces so this is what you round to here.

Example: Kilometres and miles

You have signed up for a 60 km bike ride. There is a lake in a nearby park that you want to use for your training. You know that one lap of the lake is 2 miles. You want to cycle a distance of at least 40 km in your last training session before the race. How many full laps of the lake should you do?

You know that a rough conversion is 5 miles = 8 km.

There is more than one way to go about this calculation and, as ever, if you have a different method that works for you and you arrive at the same answer, feel free to use it!

Method

Since you already know how to convert if you are given a conversion that has a single unit of whichever measure you are using, it makes sense to just change the given conversion into one that you are used to dealing with.

Look at the diagram below – since you know that 5 miles is worth 8 km, you can find out what 1 mile is worth by simply dividing 5 by 5 so that you arrive at 1 mile. Whatever you do to one side, you must do to the other. Therefore, you also do 8 divided by 5. This then gives 1 mile = 1.6 km.

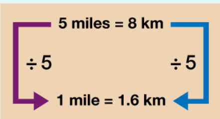


Figure 11 Calculating 1 mile in kilometres

Now that you know 1 mile = 1.6 km, you can solve this problem in its usual way:

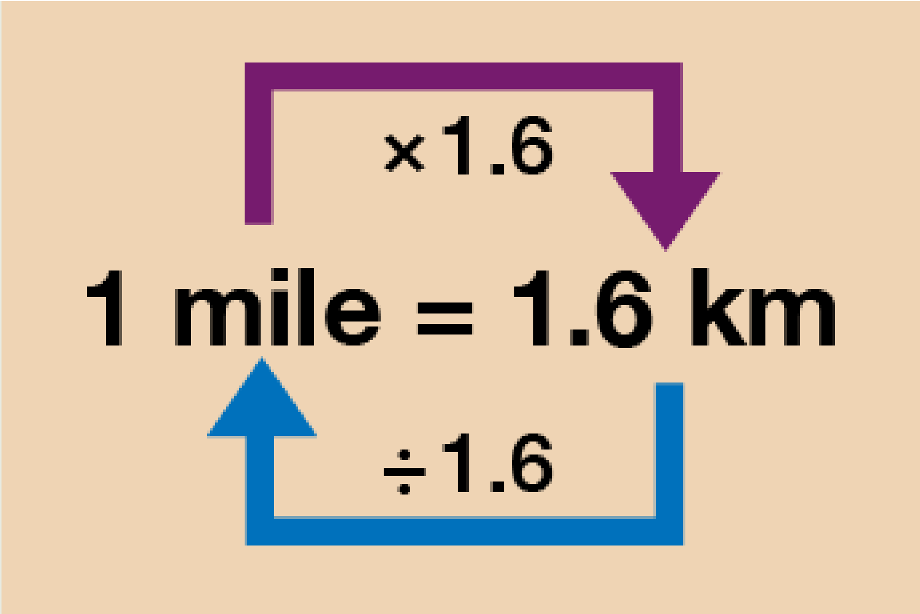


Figure 12 Converting between miles and kilometres

Write down your conversion rate:

1 mile = 1.6 km

Now write down the figure you want to convert under the correct side (in this case you want to convert 40 km into miles):

1 mile = 1.6 km

? miles = 40 km

Since you first want to know what 40 km is in miles, you are going backwards, so you need to divide by 1.6:

40 ÷ 1.6 = 25 miles

So, 40 km = 25 miles

You know that one lap of the lake is 2 miles so:

25 ÷ 2 = 12.5 laps

Therefore, you will need to cycle 12.5 laps around the lake in order to have cycled your target of 40 km.

Now that you have seen a couple of worked examples, have a go at the activity below to check your understanding.

Activity 5: Converting between systems

Wherever possible, please do the calculations first without a calculator. You may then double-check on a calculator if needed.

1. A Ford Fiesta car can hold 42 litres of petrol. Using the fact that 1 gallon = 4.54 litres, work out how many gallons of petrol the car can hold. Give your answer rounded to two decimal places.
2. The café you work at has run out of milk and you have been asked to go to the shop and buy 10 pints. When you arrive however, the milk is only available in 2 litre bottles. You know that 1 litre = 1.75 pints. How many 2 litre bottles should you buy?
3. You are packing to go on holiday and are allowed 25 kg of luggage on the flight. You've weighed your case on the bathroom scales and it weighs 3 stone and 3 pounds.

   You know that:

   1 stone = 14 pounds

   2.2 pounds = 1 kg

   Is your luggage over the weight limit?
4. Your son wants to go on a ride at a theme park that has a minimum height restriction of 122 cm. You know your son is 4 ft. 7 in. tall. You know that:

   1 foot (ft.) = 12 inches

   1 inch (in.) = 2.54 cm

   Can your child go on the ride?

Answer

1. You do:

   1 gallon = 4.54 litres

   ? gallons = 42 litres

As you are converting litres to gallons, you are going backwards so need to divide by 4.54.

   $42 \div 4.54 = 9.25$ gallons (to two d.p)

2. Firstly, you want to know how many litres there are in 10 pints, so you need to convert from pints to litres. You do:

   1 litre = 1.75 pints

   ? litres = 10 pints

   As you are converting pints to litres, you are going backwards so need to divide by 1.75:

   $10 \div 1.75 = 5.71\ldots$ litres. As you can only buy the milk in 2 litre bottles, you will have to buy 6 litres of milk in total. You therefore need:

   $6 \div 2 = 3$ bottles of milk
3. Firstly, you need to convert 3 stone and 3 pounds into just pounds. Since 1 stone = 14 pounds, to work out 3 stone you do:

   $3 \times 14 = 42$ pounds

   Then you need to add on the extra 3 pounds, so in total you have $42 + 3 = 45$ pounds.

   Now that you know this, you can convert from pounds to kg.

Since 2.2 pounds = 1 kg, you can also say that 1 kg = 2.2 pounds (this doesn't change anything, it just makes it a little easier as you can stick to your usual method of having the single unit on the left hand side):

1 kg = 2.2 pounds

? kg = 45 pounds

As you are converting from pounds to kg, you are going backwards so need to divide by 2.2:

45 ÷ 2.2 = 20.45 kg

Yes, your luggage is within the weight limit.

4. Firstly, you need to convert 4 ft. 7 in. into just feet. Since 1 foot = 12 inches, to work out 4 feet you do:

   4 × 12 = 48 inches

   Then you need to add on the extra 7 inches, so in total your son is 48 + 7 = 55 inches tall.

   Now that you know this, you can convert from inches to centimetres:

   1 inch = 2.54 cm

   55 inches = ? cm

   As you are converting from inches to centimetres, you are going forwards so need to multiply by 2.54:

   55 × 2.54 = 139.7 cm

   So your son is tall enough to go on the ride.

You should now be feeling confident with your conversion skills so it's time to move on to the next part of this session.

Summary

In this section you have learned:

- that different systems of measurement can be used to measure the same thing (e.g. a cake could be weighed in either grams or pounds)
- you can convert between these systems using your multiplication and division skills and the given conversion rate
- currencies can be converted in the same way as long as you know the exchange rate – this is particularly useful for holidays.

3 Time, timetables and average speed

Calculating with time is often seen as tricky, not surprising really considering how difficult it can be to learn how to tell the time. The reason many people find calculating with time tricky is because, unlike nearly every other mathematical concept, it does not work in 10s. Time works in 60s – 60 seconds in a minute, 60 minutes in an hour. You cannot therefore, simply use your calculator to add on or subtract time.

Figure 13 A radio alarm clock

Think about this simple example. If it's 9:50 and your bus takes 20 minutes to get to work, you cannot work out the time you will arrive by doing 950 + 20 on your calculator. This would give you an answer of 970 or 9:70 – there isn't such a time!

You will need to calculate with time and use timetables in daily life to complete basic tasks such as: getting to work on time, working out which bus or train to catch, picking your children up from school on time, cooking and so many other daily tasks.

3.1 Calculating with time and timetables

As previously discussed, calculators are not the most useful items when it comes to calculations involving time. A much better option is to use a number line to work out these calculations. Take a look at the examples below.

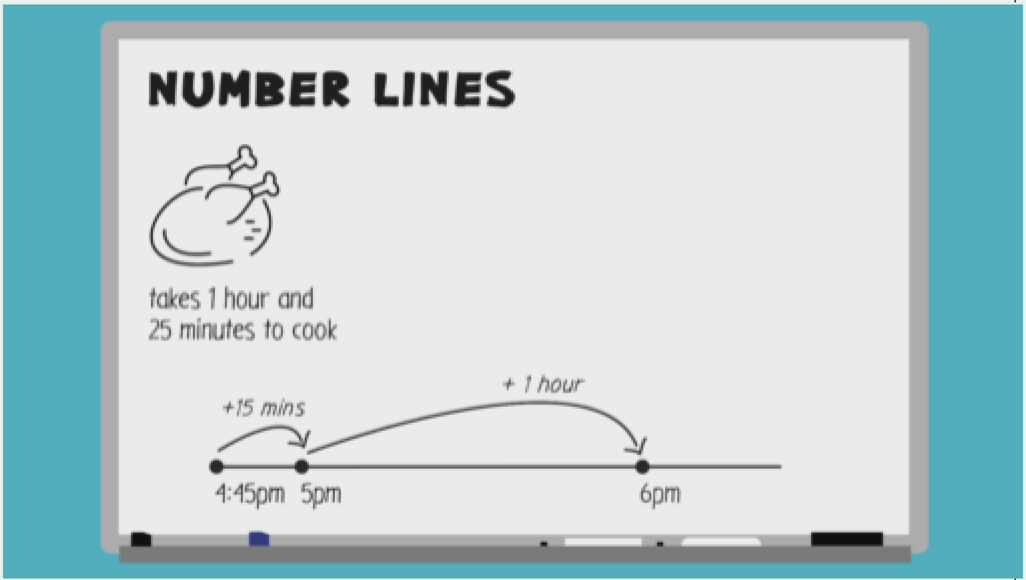
Example: Cooking

You put a chicken in the oven at 4:45 pm. You know it needs to cook for 1 hour and 25 minutes. What time should you take the chicken out?

Method

Watch the video below to see how the number line method works.

Video content is not available in this format.



Example: Time sheets

You work for a landscaping company and need to fill out your time sheet for your employer. You began working at 8:30 am and finished the job at 12:10 pm. How long did the job take?

Method

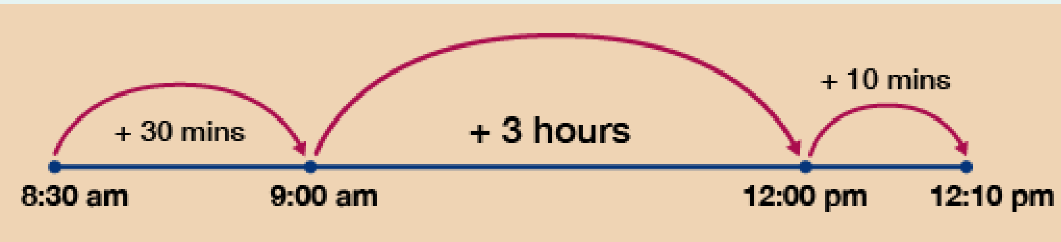


Figure 14 A number line for a time sheet

Again, for finding the time difference you want to work with easy ‘chunks’ of time. Firstly, you can move from 8:30 am to 9:00 am by adding 30 minutes. It is then simple to get to 12:00 pm by adding on 3 hours.

Finally, you just need another 10 minutes to take you to 12:10 pm. Looking at the total time added you have 3 hours and 40 minutes.

Another aspect of calculating with time comes in the form of timetables. You will be used to using these to work out which departure time you need to meet in order to get to a location on time or how long a journey will take. Once you can calculate with time, using timetables simply requires you to find the correct information before carrying out the calculation. Take a look at the example below.

Example: Timetables

Here is part of a train timetable from Swindon to London.

Table 2(a)

Swindon	06:10	06:27	06:41	06:58	07:01	07:17
Didcot	06:27	06:45	06:58	07:15	07:18	07:34
Reading	06:41	06:59	07:13	-	07:33	-
London	07:16	07:32	07:44	08:02	08:07	08:14

a. You need to travel from Didcot to London. You need to arrive in London by 8:00 am. What is the latest train you can catch from Didcot to arrive in London for 8:00 am?

Method

Table 2(b)

Swindon	06:10	06:27	06:41	06:58	07:01	07:17
Didcot	06:27	06:45	*06:58*	07:15	07:18	07:34
Reading	06:41	06:59	07:13	-	07:33	-
London	07:16	07:32	**07:44**	08:02	08:07	08:14

Looking at the arrival times in London, in order to get there for 8:00 am you will need to take the train that arrives in London at 07:44 (highlighted with bold). If you then move up this column of the timetable you can see that this train leaves Didcot at 06:58 (highlighted with italic). This is therefore the train you must catch.

b. How long does the 06:58 from Swindon take to travel to London?

Method

Table 2(c)

Swindon	06:10	06:27	06:41	*06:58*	07:01	07:17
Didcot	06:27	06:45	06:58	07:15	07:18	07:34
Reading	06:41	06:59	07:13	-	07:33	-
London	07:16	07:32	07:44	**08:02**	08:07	08:14

Firstly, find the correct train from Swindon (highlighted with italic). Follow this column of the timetable down until you reach London (highlighted with bold). You then need to find the difference in time between 06:58 and 08:02. Using the number line method from earlier in the section (or any other method you choose).

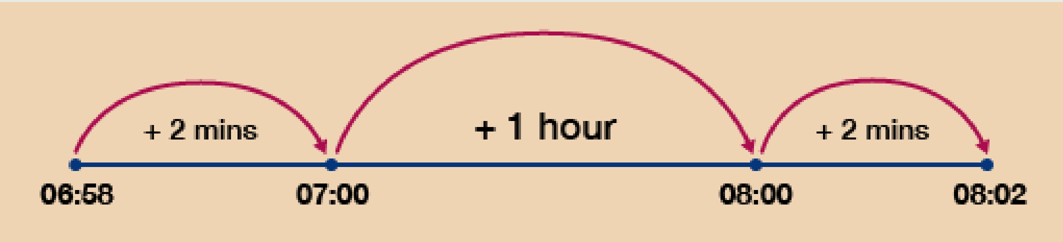


Figure 15 A number line for a timetable

You can then see that this train takes a total of 1 hour and 4 minutes to travel from Swindon to London.

Have a go at the activity below to practise calculating time and using timetables.

Activity 6: Timetables and calculating time

1. Kacper is a builder. He leaves home at 8:30 am and drives to the trade centre. He collects his items and loads them into his van. His visit takes 1 hour and 45 minutes. He then drives to work, which takes 50 minutes. What time does he arrive at work?
2. You have invited some friends round for dinner and find a recipe for roast lamb. The recipe requires:
   - 25 minutes preparation time
   - 1 hour cooking time
   - 20 minutes resting time

   You want to eat with your friends at 7:30 pm. What is the latest time you can start preparing the lamb?
3. Here is part of a train timetable from Manchester to Liverpool.

Table 3(a)

	Manchester to Liverpool					
Manchester	10:24	10:52	11:03	11:25	12:01	12:13
Warrington	10:38	11:06	11:20	11:45	12:15	12:28
Widnes	10:58	11:26	11:42	12:03	12:34	12:49
Liverpool Lime Street	11:09	11:38	11:53	12:14	12:46	13:02

You need to travel from Manchester to Liverpool Lime Street. You need to be in Liverpool by 12:30. Which train should you catch from Manchester and how long will your journey take?

Answer

1. Firstly, work out the total time that Kacper is out for:

   1 hour 45 minutes at the trade centre and another 50 minutes driving makes a total of 2 hours and 35 minutes.

   Then, using the number line, you have:

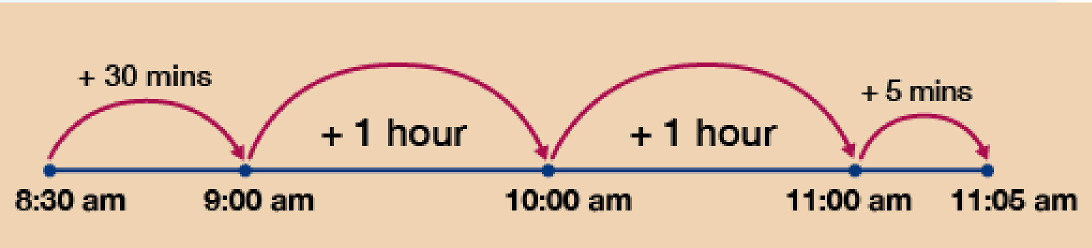


Figure 16 A number line for Question 1

   So Kacper arrives at work at 11:05 am.

   You could also do the calculation by adding on the 1 hour 45 minutes first:

   8:30 am + 1 hour = 9:30 am
   9:30 am + 45 minutes = 10:15 am

   Finally, you can add on the 50 minutes:

   10:15 am + 45 minutes = 11:00 am

   Then add on the remaining 5 minutes:

   11:00 am + 5 minutes = 11:05 am

2. Again, firstly work out the total time required:

   25 minutes + 1 hour + 20 minutes = 1 hour 45 minutes in total

   This time you need to work backwards on the number line so you begin at 7:30 and work backwards.

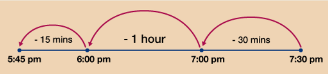


Figure 17 A number line for Question 2

You can now see that you must begin preparing the lamb at 5:45 pm at the latest.

As with the first question, you could have done this question by taking off each stage in the cooking process separately rather than finding the total time first:

7:30 pm − 20 minutes = 7:10 pm

7:10 pm − 1 hour = 6:10 pm

There are 25 minutes left so:

6:10 pm − 10 minutes = 6:00 pm

There are now 15 minutes left so:

6:00 pm − 15 minutes = 5:45 pm

3. Looking at the timetable for arrival at Liverpool, you can see that in order to arrive by 12:30 you need to catch the train that arrives at 12:14. This means that you need to catch the 11:25 from Manchester.

Table 3(b)

	Manchester to Liverpool					
Manchester	10:24	10:52	11:03	*11:25*	12:01	12:13
Warrington	10:38	11:06	11:20	11:45	12:15	12:28
Widnes	10:58	11:26	11:42	12:03	12:34	12:49
Liverpool Lime Street	11:09	11:38	11:53	**12:14**	12:46	13:02

You therefore need to work out the difference in time between 11:25 (italic) and 12:14 (bold).

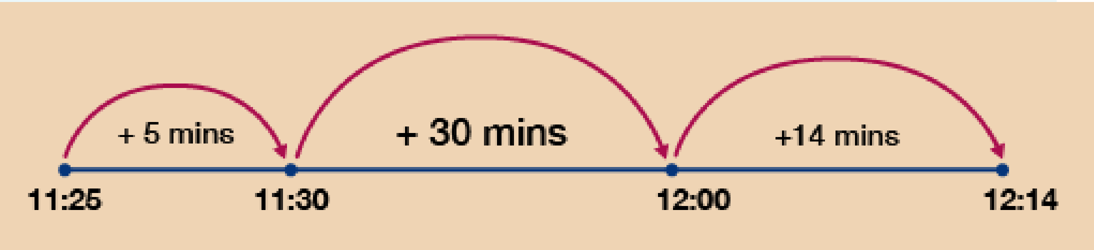


Figure 18 A number line for Question 3

Using the number line again, you can see that this is a total of 5 + 30 + 14 = 49 minutes.

You should now be feeling comfortable with calculations involving time and timetables. Before you move on to looking at problems that involve average speed, it is worth taking a brief look at time conversions. Since you are already confident with converting units of measure, this part will just consist of a brief activity so that you can practise converting units of time.

3.2 Converting units of time

You can see from the diagram below that to convert units of time you can use a very similar method to the one you used when converting other units of measure. There is one slight difference when working with time however.

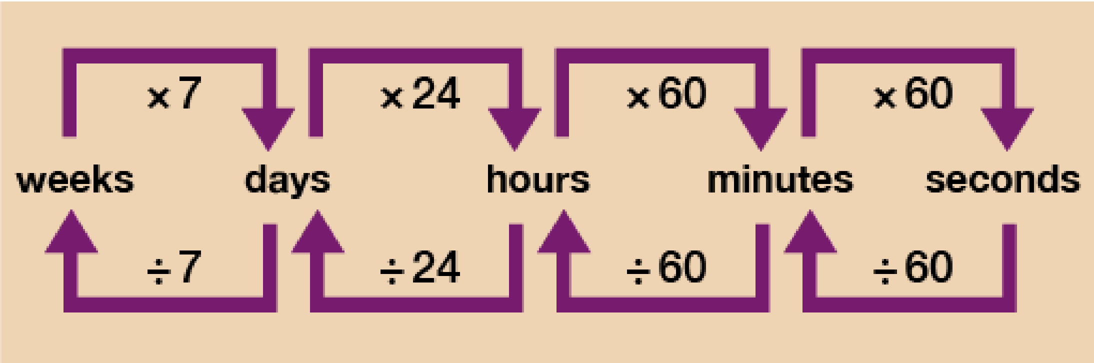


Figure 19 A conversion chart for time

Let's say you want to work out how long 245 minutes is in hours. The diagram above shows that you should do 245 ÷ 60 = 4.083. This is not a particularly helpful answer since you really want the answer in the format of: ___ hours __ minutes. Due to the fact that time does not work in 10s, you need to do a little more work once arriving at your answer of 4.083.

The answer is obviously 4 hours and an amount of minutes.

4 hours then is 4 × 60 = 240 minutes.

Since you wanted to know how long 245 minutes is you just do 245 – 240 = 5 minutes left over. So 245 minutes is 4 hours and 5 minutes.

It's a very similar process if you want to go from say minutes to seconds. Let's take it you want to know how long 5 minutes and 17 seconds is in seconds. 5 minutes would be 5 × 60 = 300 seconds. You then have a further 17 seconds to add on so you do 300 + 17 = 317 seconds.

Have a go at the activity below to make sure you feel confident with converting times.

Activity 7: Converting times

Convert the following times:

1. 6 hours and 35 minutes = ___ minutes.
2. 85 minutes = ____ hours and ____ minutes.
3. 153 seconds = ____ minutes and ___ seconds.
4. 46 days = ___ weeks and ____days.
5. 3 minutes and 40 seconds = ____ seconds.

Answer

1. 6 hours = 6 × 60 = 360 minutes
   360 minutes + 35 minutes = 395 minutes
2. 85 minutes ÷ 60 = 1.417 (rounded to three d.p)
   1 hour = 60 minutes.
   85 minutes − 60 minutes = 25 minutes remaining
   So 85 minutes = 1 hour and 25 minutes
3. 153 seconds ÷ 60 = 2.55
   2 minutes = 2 × 60 = 120 seconds
   153 seconds − 120 seconds = 33 seconds remaining
   So 153 seconds = 2 minutes and 33 seconds
4. 46 days ÷ 7 = 6.571 (rounded to three d.p)
   6 weeks = 6 × 7 = 42 days
   46 days − 42 days = 4 days remaining
   So 46 days = 6 weeks and 4 days
5. 3 minutes = 3 × 60 = 180 seconds
   180 seconds + 40 seconds = 220 seconds

Hopefully you found that activity fairly straightforward and are now feeling ready to move on to the next part of the 'Units of measure' session – Average speed.

3.3 Average speed

The sign below is commonly seen on motorways but it is not the only time when it is useful to know your average speed.

Figure 20 A speed camera sign

Being able to calculate and use average speed can help you to work out how long a journey is likely to take. The method for working out average speed involves using a simple formula.

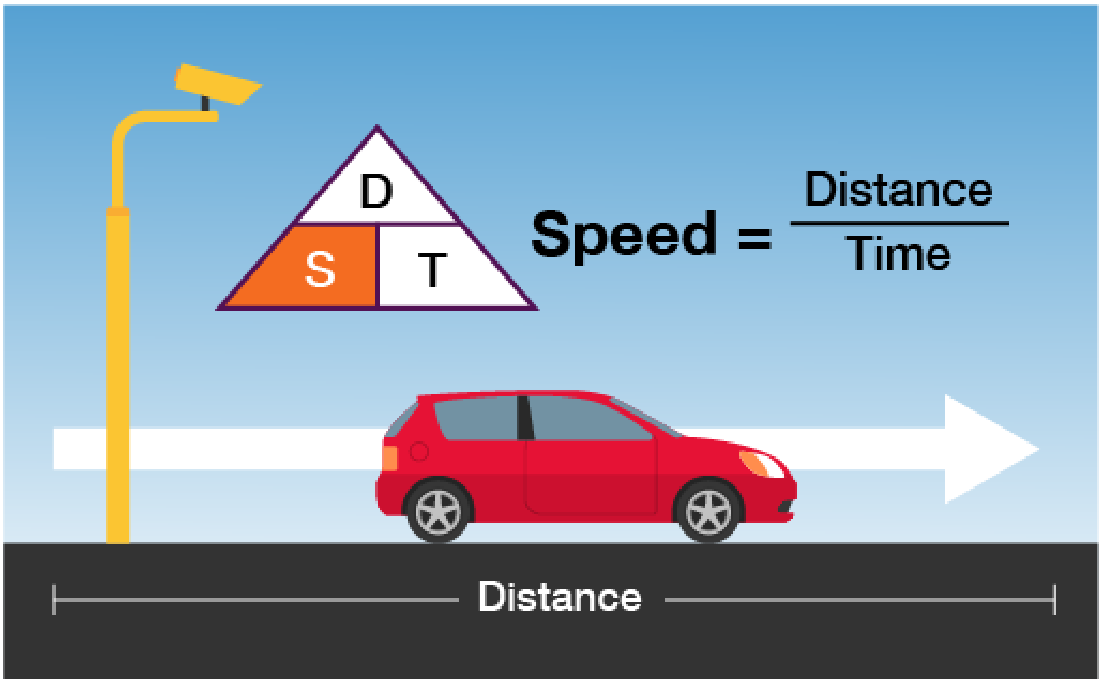


Figure 21 A formula for average speed

You can also use this formula to work out the distance travelled when given a time and the average speed, or the time taken for a journey when given the distance and average speed.

The formulas for this are shown in the diagram below. You can see that when given any two of the elements from distance, speed and time, you will be able to work out the third.

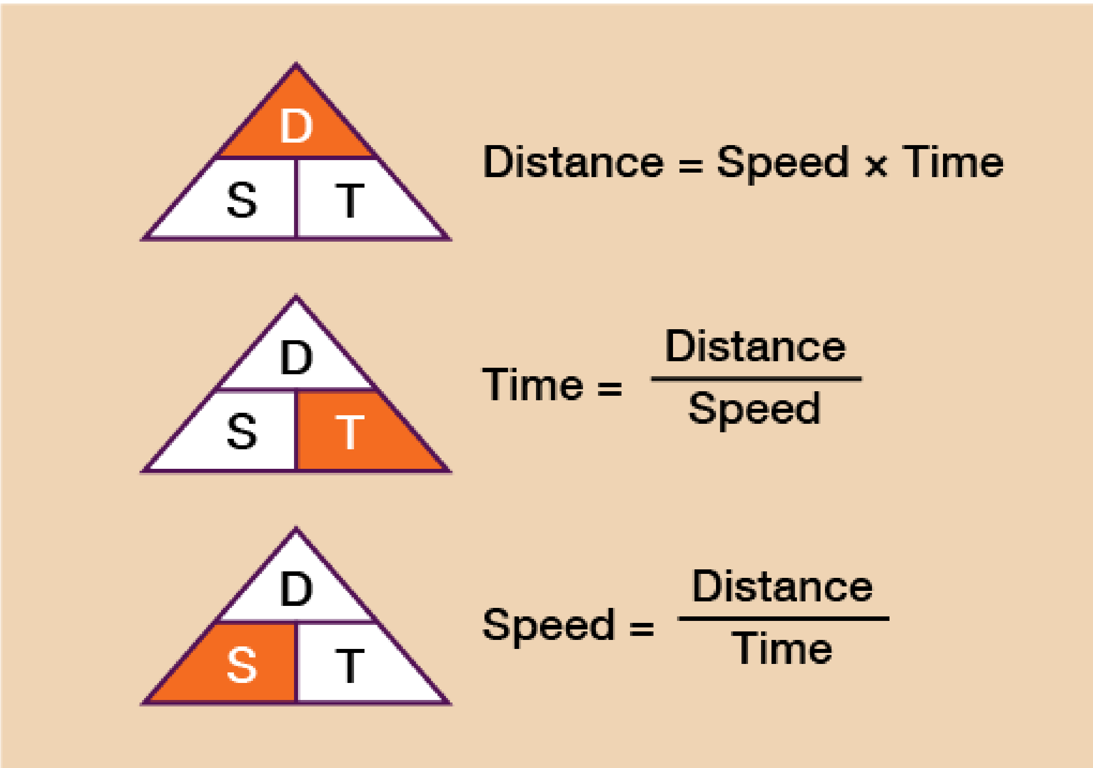


Figure 22 Distance, speed and time formulas

If you can learn this formula triangle, when you want to use it, you write it down and cover up what you want to work out (the segment in orange). This will tell you what calculation you need to do.

Let's look at an example of each so that you can familiarise yourself with it.

Example: Calculating distance

A car has travelled at an average speed of 52 mph over a journey that lasts 2 and a half hours. What is the total distance travelled?

Method

You can see that to work out the distance you need to do speed × time. In this example then we need to do 52 × 2.5. It is very important to note here that 2 and a half hours must be written as 2.5 (since 0.5 is the decimal equivalent of a half).

You cannot write 2.30 (for 2 hours and 30 mins). If you struggle to work out the decimal part of the number, convert the time into minutes (2 and a half hours = 150 minutes) and then divide by 60 (150 ÷ 60 = 2.5).

52 × 2.5 = 130 miles travelled

Example: Calculating time

A train will travel a distance of 288 miles at an average speed of 64 mph. How long will it take to complete the journey?

Method

You can see from the formula that to calculate time you need to do distance ÷ speed so you do:

288 ÷ 64 = 4.5 hours

Again, note that this is not 4 hours 50 minutes but 4 and a half hours.

If you are unsure of how to convert the decimal part of your answer, simply multiply the answer by 60, which will turn it into minutes and you can then convert from there.

In this case, 4.5 × 60 = 270 minutes. We already know from the answer of 4.5 hours that this is 4 whole hours and so many minutes, so we now need to work out how many minutes the .5 represents:

60 × 4 = 240 minutes

270 − 240 = 30 minutes

So 4.5 hours = 270 minutes = 4 hours, 30 minutes

Example: Calculating speed

A Formula One car covers a distance of 305 km during a race. The time taken to finish the race is 1 hour and 15 minutes. What is the car's average speed?

Method

The formula tells you that to calculate speed you must do distance ÷ time. Therefore, you do 305 ÷ 1.25 (since 15 minutes is a quarter of an hour and 0.25 is the decimal equivalent of a quarter):

$$305 \div 1.25 = 244 \text{ km/h}$$

In a similar way to example 1, if you are unsure of how to work out the decimal part of the time simply write the time (in this case 1 hour and 15 minutes) in minutes, (1 hour 15 minutes = 75 minutes) and then divide by 60:

$$75 \div 60 = 1.25$$

Now have a go at the following activity to check that you feel confident with finding speed, distance and time. Please do the calculations first without a calculator. You may then double-check on a calculator if needed.

Activity 8: Calculating speed, distance and time

1. Filip is driving a bus along a motorway. The speed limit is 70 mph. In 30 minutes, he travels a distance of 36 miles. Does his average speed exceed the speed limit?
2. A plane flies from Frankfurt to Hong Kong. The flight time was 10 hours and 45 minutes. The average speed was 185 km/h. What is the distance flown by the plane?
3. Malio needs to get to a meeting by 11:00 am. The time now is 9:45 am. The distance to the meeting is 50 miles and he will be travelling at an average speed of 37.5 mph. Will he be on time for the meeting?

Answer

1. You need to find the speed so you do: distance ÷ time.

   The distance is 36 miles. The time is 30 minutes but you need the time in hours:

   30 minutes ÷ 60 = 0.5 hours

   Now you do:

   36 ÷ 0.5 = 72 mph

   Yes, Filip's average speed did exceed the speed limit.
2. You need to find the distance so you do:

   speed × time

10 hours 45 minutes = 10.75 hours

If you are unsure how to express this in hours, convert 10 hours 45 minutes all into minutes:

10 × 60 = 600 + 45 = 645 minutes

Then divide by 60:

645 ÷ 60 = 10.75 hours

Now to work out the distance do:

speed × time = 185 × 10.75 = 1988.75 km from Frankfurt to Hong Kong

3. You need to find the time so you do:

distance ÷ speed

50 ÷ 37.5 = 1.33 hours (rounded to two d.p)

Note: The actual answer is 1.3333333 (the 3 is recurring or never-ending).

To convert this to minutes do:

1.33 × 60 = 79.8 minutes

round 79.8 minutes up to 80 minutes

80 minutes = 1 hour and 20 minutes

If the time now is 9:45 am and his meeting is at 11:00 am, then it is only 1 hour, 15 minutes until his meeting, so no, Malio will not make the meeting on time.

Hopefully you will now be feeling more confident with calculations involving speed, distance and time. You will now move on to temperature conversions.

Summary

In this section you have learned how to:

- use timetables to plan a journey and how to calculate time efficiently
- convert between units of time by using multiplication and division skills
- use the formula for calculating distance, speed and time.

4 Temperature

Temperature can be recorded in either degrees Celsius (°C) or degrees Fahrenheit (°F). In *Everyday Maths 1* you used conversion tables to help you to compare temperatures expressed in the different units. You will now look at how to convert between them using formulas.

4.1 Celsius and Fahrenheit formulas

The following formulas can be used to convert between Celsius and Fahrenheit.

To convert Celsius to Fahrenheit use the formula:

$$F = \frac{9}{5}C + 32$$

Method:

- divide the Celsius figure by 5
- multiply by 9
- add 32.

If you prefer, you can multiply the Celsius figure by 9 first and then divide by 5. You will still need to add on 32 at the end.

To convert Fahrenheit to Celsius use the formula:

$$C = \frac{5(F - 32)}{9}$$

Method:

- subtract 32 from the Fahrenheit figure
- multiply by 5
- divide by 9.

If you need a recap on the rules for using formulas, revisit Session 1 ‘Working with numbers’. We will now look at an example.

Example: Which city is warmer?

I look up the average temperature for New York on a particular day and it is 10°C. I know the average temperature in Swansea on the same day is 55°F. Which city is warmer?

You either need to convert New York’s temperature into °F or the Swansea temperature into °C.

Method 1 – Converting °C to °F

If we look back at the formulas above, the one we need to use to convert from °C to °F is:

$F = \frac{9}{5}\ C + 32$

We need to substitute the C with our °C figure of 10°C. We then need to follow the rules of BIDMAS to carry out the calculation in stages, as shown below:

$F = \frac{9}{5} \times 10 + 32$

Divide the celsius figure by 5:

10 ÷ 5 = 2

Multiply by 9:

2 × 9 = 18

Add 32:

18 + 32 = **50°F**

You may have done the calculation slightly differently by multiplying the Celsius figure by 9 first and then dividing by 5. The answer will work out the same:

$F = \frac{9}{5} \times 10 + 32$

Multiply by the Celsius figure by 9:

10 × 9 = 90

Divide by 5:

90 ÷ 5 = 18

Add 32:

18 + 32 = **50°F**

So which is warmer:

New York at 10°C (which we now know is 50°F) or Swansea at 55°F?
Swansea is warmer.

Method 2 – Converting °F to °C

The formula for converting from °F to °C:

$$C = \frac{5\,(F - 32)}{9}$$

We need to substitute the F with our °F figure of 55°F. We then need to follow the rules of BIDMAS to carry out the calculation in stages, as shown below:

Take 32 away from the Fahrenheit figure of 55:

$55 - 32 = 23$

Multiply by 5:

$23 \times 5 = 115$

Divide by 9:

$115 \div 9 =$ **12.8°C** (rounded to 1 decimal place)

So which is warmer:

New York at 10°C or Swansea at 55°F (which we now know is 12.8°C)? Swansea is warmer.

Hint: Google has its own unit converter (search for Google Unit Converter) which you can use to convert between various units of measure, including between °C and °F. You could try using it to double-check your answers to the questions below.

Activity 9: Temperature conversions

Work out the answers to the following without using a calculator. You may double-check your answers on a calculator or using the Google unit converter, if needed, and remember to check your answers with ours at the end. Round your answers off to one decimal place where needed.

1. Convert the following temperatures into degrees Fahrenheit (°F):
   a. 22°C
   b. 0°C
   c. −6°C

Answer

1. You need to use the following formula:

   $F = \frac{9}{5}\,C + 32$

   a. $F = \frac{9}{5} \times 22 + 32$

   $22 \div 5 = 4.4$

4.4 × 9 = 39.6

39.6 + 32 = **71.6°F**

b. $F = \frac{9}{5} \times 0 + 32$

0 ÷ 5 = 0

0 × 9 = 0

0 + 32 = **32°F**

c. $F = \frac{9}{5} \times -6 + 32$

−6 ÷ 5 = −1.2

−1.2 × 9 = −10.8

−10.8 + 32 = **21.2°F**

2. Convert the following temperatures into degrees Celsius (°C):
   b. 45°F
   c. 212°F
   d. 5°F

Answer

2. You need to use the following formula:

$C = \frac{5\,(F - 32)}{9}$

b. $C = \frac{5\,(45 - 32)}{9}$

45 − 32 = 13

13 × 5 = 65

65 ÷ 9 = **7.2°C** (to one d.p)

c. $C = \frac{5\,(212 - 32)}{9}$

212 − 32 = 180

180 × 5 = 900

900 ÷ 9 = **100°C**

d. $C = \frac{5\,(5 - 32)}{9}$

5 − 32 = −27

−27 × 5 = −135

−135 ÷ 9 = **−15°C**

3. I find a recipe which states that my oven needs to be set at a temperature of 400°F. My settings on my oven are in °C. What temperature should I set my oven to?

Answer

3. You need to convert 400°F to °C so use the formula:

$$C = \frac{5\,(F - 32)}{9}$$

$$C = \frac{5\,(400 - 32)}{9}$$

400 − 32 = 368

368 × 5 = 1840

1840 ÷ 9 = **204.4°C** (to one d.p).

As you would be unable to set an oven so accurately, you would set the temperature to 200°C.

4. I see Moscow's temperature is −4°C on a particular day in February, whilst the temperature in Toronto is 19°F. Which place is colder?

Answer

4. You either need to convert the Moscow temperature of −4°C to °F, or convert the Toronto temperature of 19°F to °C.

Method 1 – Converting °C to °F

If we look back at the formulas, the one we need to use to convert from °C to °F is:

$$F = \frac{9}{5}\,C + 32$$

We need to substitute the C with our °C figure of −4°C. We then need to follow the rules of BIDMAS to carry out the calculation in stages, as shown below:

$$F = \frac{9}{5} \times -4 + 32$$

−4 ÷ 5 = −0.8

Multiply by 9:

−0.8 × 9 = −7.2

Add 32:

−7.2 + 32 = **24.8°F**

So which is colder? Moscow at −4°C (which we now know is 24.8°F) or Toronto at 19°F? Toronto is colder.

Method 2 – Converting °F to °C

The formula for converting from °F to °C is:

$$C = \frac{5\,(F - 32)}{9}$$

We need to substitute the F with our °F figure of 19°F. We then need to follow the rules of BIDMAS to carry out the calculation in stages, as shown below:

$$C = \frac{5\,(19 - 32)}{9}$$

Take 32 away from the Fahrenheit figure of 19:

$19 - 32 = -13$

Multiply by 5:

$-13 \times 5 = -65$

Divide by 9:

$-65 \div 9 =$ **−7.2°C** (to one d.p.)

So which is colder: Moscow at −4°C or Toronto at 19°F (which we now know is −7.2°C)? Toronto is colder.

Hopefully you will be feeling more confident when solving problems relating to temperature. The next section will cover reading measurements on scales.

Summary

In this section you have:

- practised converting between degrees Celsius (°C) and degrees Fahrenheit (°F).

5 Reading scales

You may need to read a scale to measure out an amount of liquid, read a measurement on a ruler, weigh out ingredients for a recipe or to take someone's temperature.

Reading scales can be tricky because every scale is different.

To read a scale correctly, you need to ask yourself:

> What does the scale go up in? What steps or intervals does it use?

> **Note:** The marks on a scale may be referred to as any of the following: intervals, steps, increments or markers. These terms are often used interchangeably.

5.1 Scale examples

Take a look at the following examples.

Example 1: Reading scales

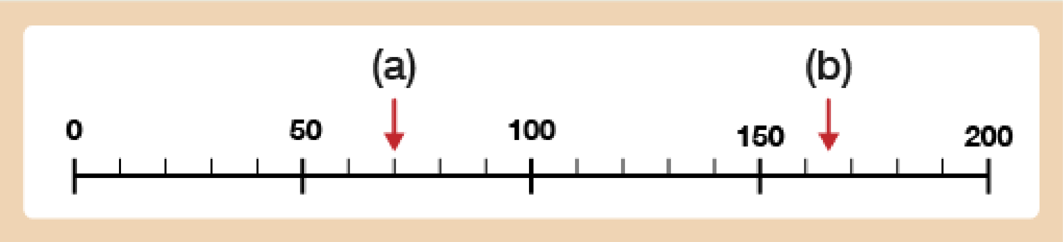


Figure 23 Scale with numbered intervals of 50

You can see that this scale is marked up in numbered intervals of 50. However, what does each line in between each numbered interval represent? You can use your judgement to help you to figure out what each small step represents.

Watch this video (https://corbettmaths.com/2013/04/27/reading-scales/) for further information about how to do this.

Alternatively, you can work this out using division. If you count on from 0 to 50 on this scale, there are 5 steps: 50 ÷ 5 = 10, so each step is 10.

a. The arrow is pointing to the 2nd mark after 50. As the steps are going up in 10s, the arrow is pointing to 70.
b. The arrow is halfway between the 1st and 2nd step after 150. The first step is 160 and the second is 170 so the arrow is pointing to 165.

Example 2: Reading scales

Sometimes you will need to read scales where the reading will be a decimal number.

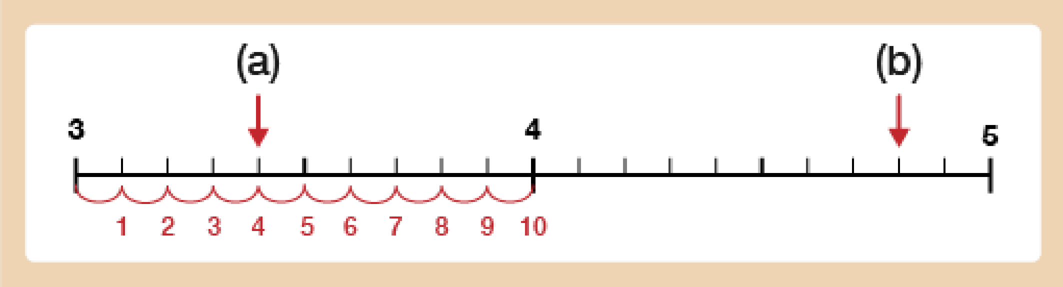


Figure 24 Scale with single numbered intervals going from 3–5

If you look at this scale, it goes up in numbered intervals of 1. From one whole number to the next whole number there are 10 small steps. 1 ÷ 10 = 0.1, so each step is 0.1.

Hint: Have a look at the image to show how to count the number of steps between the numbered markers.

a. The arrow is pointing to the fourth step after 3 so the arrow is pointing to 3.4.
b. The arrow is pointing to the eighth step after 4, so the arrow is pointing to 4.8.

5.2 Scales and measuring instruments

Now you've worked through some examples have a go at the following activities.

Activity 10: Reading scales

1. Read the scales below and find the values the arrows are pointing to for (a), (b) and (c).

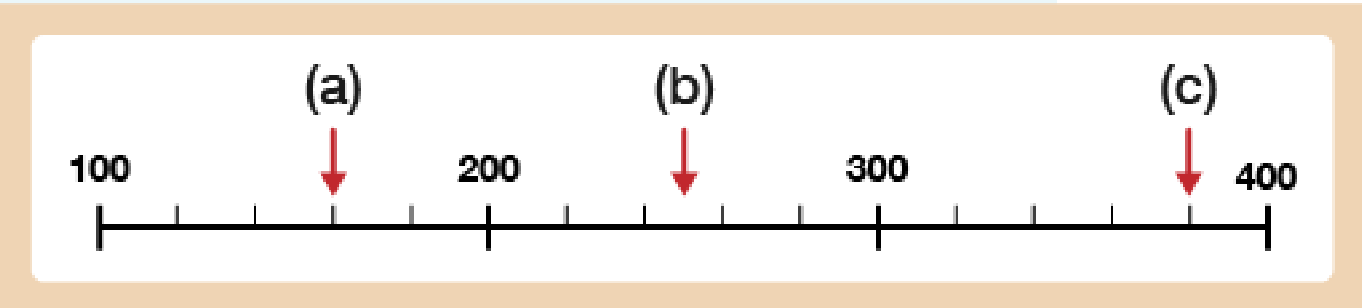


Figure 25 Scale with numbered intervals every hundred going from 100–400

Answer

1. The scale is going up in steps of 20 (the numbered markers are going up in intervals of 100 and there are 5 small steps between each numbered marker: 100 ÷ 5 = 20) so the answers are:
   a. 160
   b. This is halfway between 240 and 260 so the answer is 250

c. 380

2. Read the values for (a), (b) and (c) on the scales below.

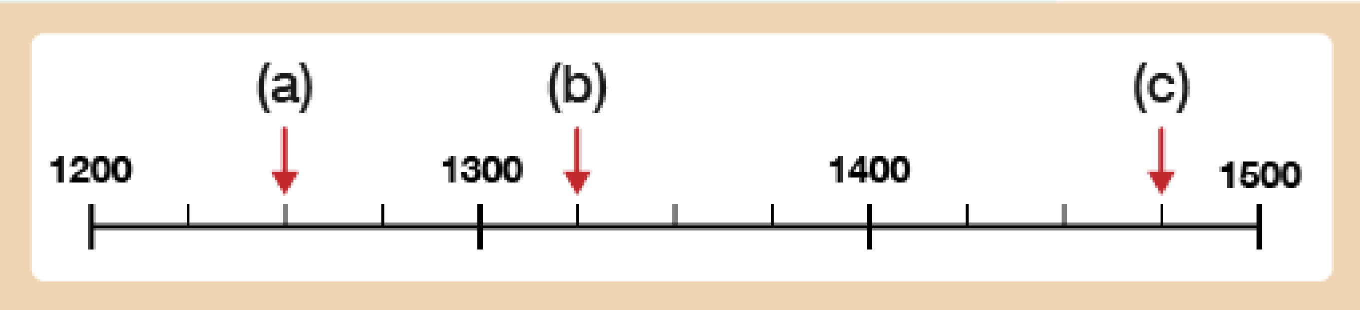


Figure 26 Scale with numbered intervals every hundred going from 1200–1500

Answer

2.

b. 1250
c. 1325
d. 1475

3. Read the values for (a), (b), (c) and (d) on the scales below.

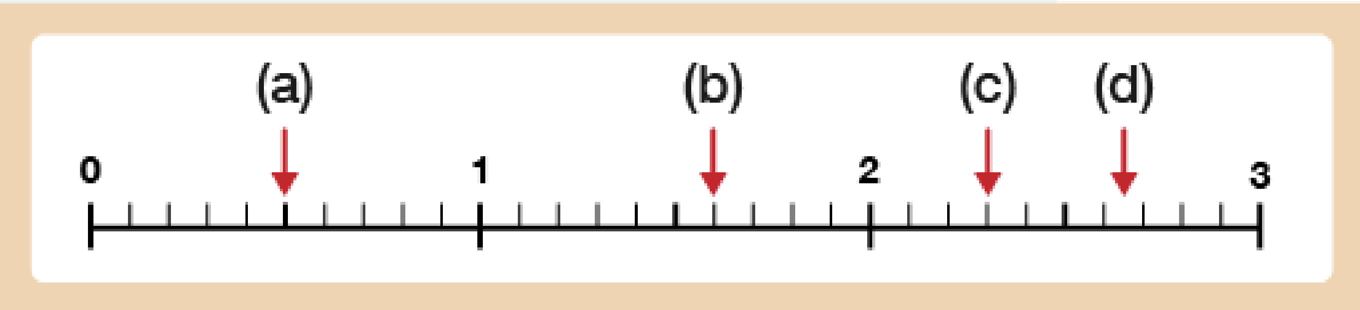


Figure 27 Scale with single number intervals from 0–3

Answer

3.

c. 0.5
d. 1.6
e. 2.3
f. The arrow is pointing to halfway between 2.6 and 2.7 so the reading is 2.65.

4. Read the values for (a), (b), and (c) on the scales below.

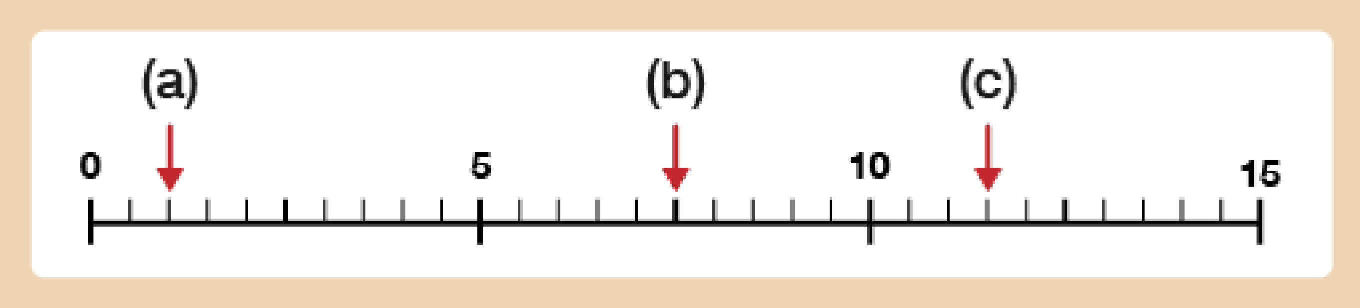


Figure 28 Scale with numbered intervals every 5 going from 0–15

Answer

4.

- d. 1.0 (or just 1)
- e. 7.5
- f. 11.5

Now have a go at reading the scales on the different instruments of measure.

Activity 11: Measuring instruments

1. How much water is left in the bottle, to the nearest 50 millilitres (ml)?

Figure 29 A water bottle with a scale on the side and water inside

Answer

1. The bottle holds 1 litre of liquid in total. There are 10 large steps marked on the bottle so each one marks 100 ml (1 litre = 1000 ml and 1000 ÷ 10 = 100).

   Halfway between each large step there is a small step so each of these marks off 50 ml.

   This means there is 250 ml of water left in the bottle to the nearest 50 ml.

2. Sara weighs her case using a set of luggage scales. She has a weight limit of 21 kg. How much more can she pack to the nearest 100 grams?

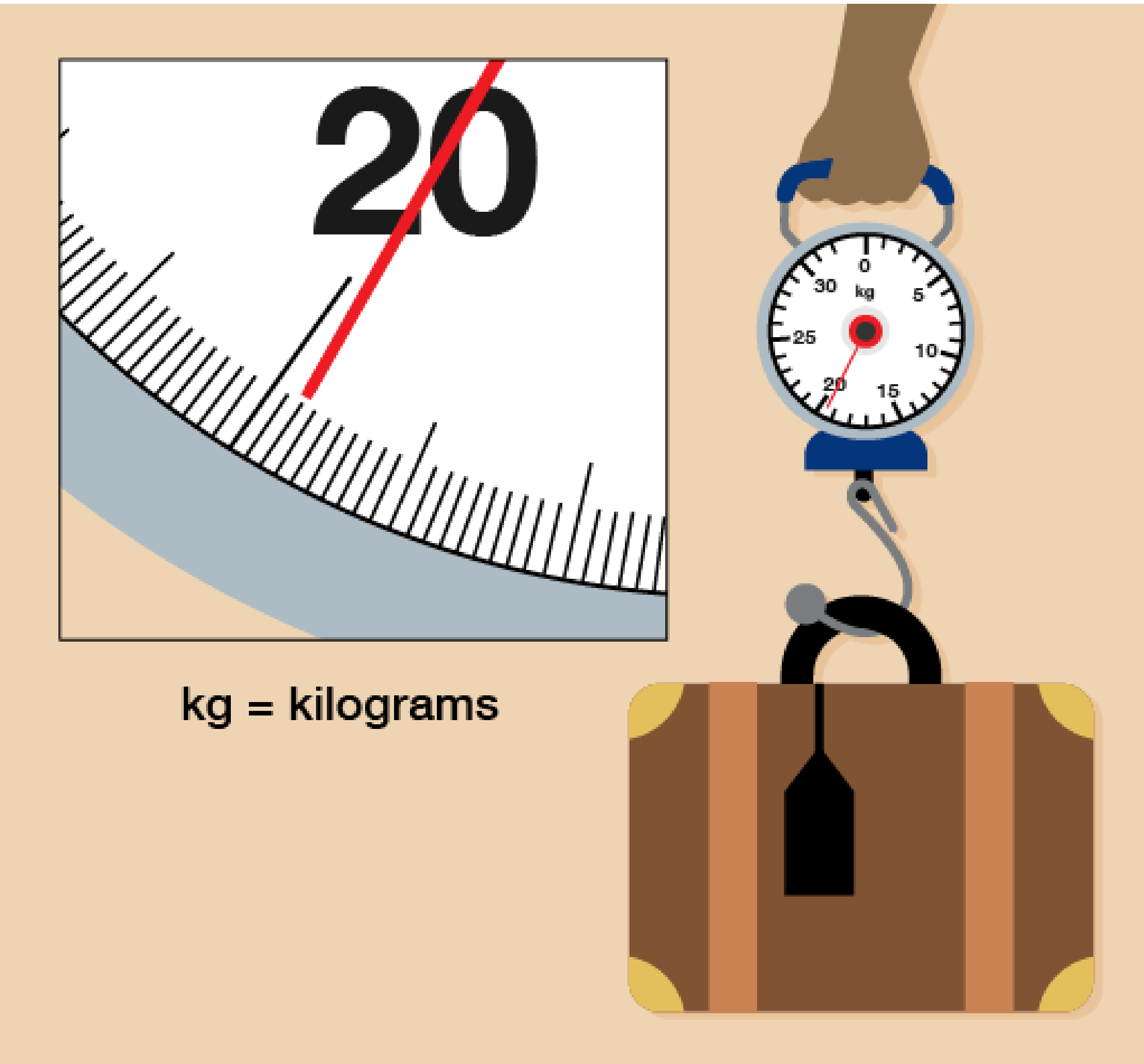


Figure 30 Luggage scales weighing luggage

Answer

2. To answer this question you need to remember that 1 kg = 1000 g.

   The scale is numbered at every 1 kg interval and there are 10 steps between each numbered interval, so each step marks 0.1 kg (1 ÷ 10 = 0.1). You could also think of each marker being 100 g (0.1 kg = 100 g).

   The arrow is almost at 19.8 kg (19 800 g). If Sara has a weight limit of 21 kg then:

   21 kg − 19.8 kg = 1.2 kg (21 000 g − 19 800 g = 1200 g)

   Sara can pack another 1.2 kg (or 1200 g) worth of luggage.

3. Simon needs to weigh out 4 kg of potatoes. Looking at the reading on the scale, how many more grams of potatoes does he need to add to make 4 kg?

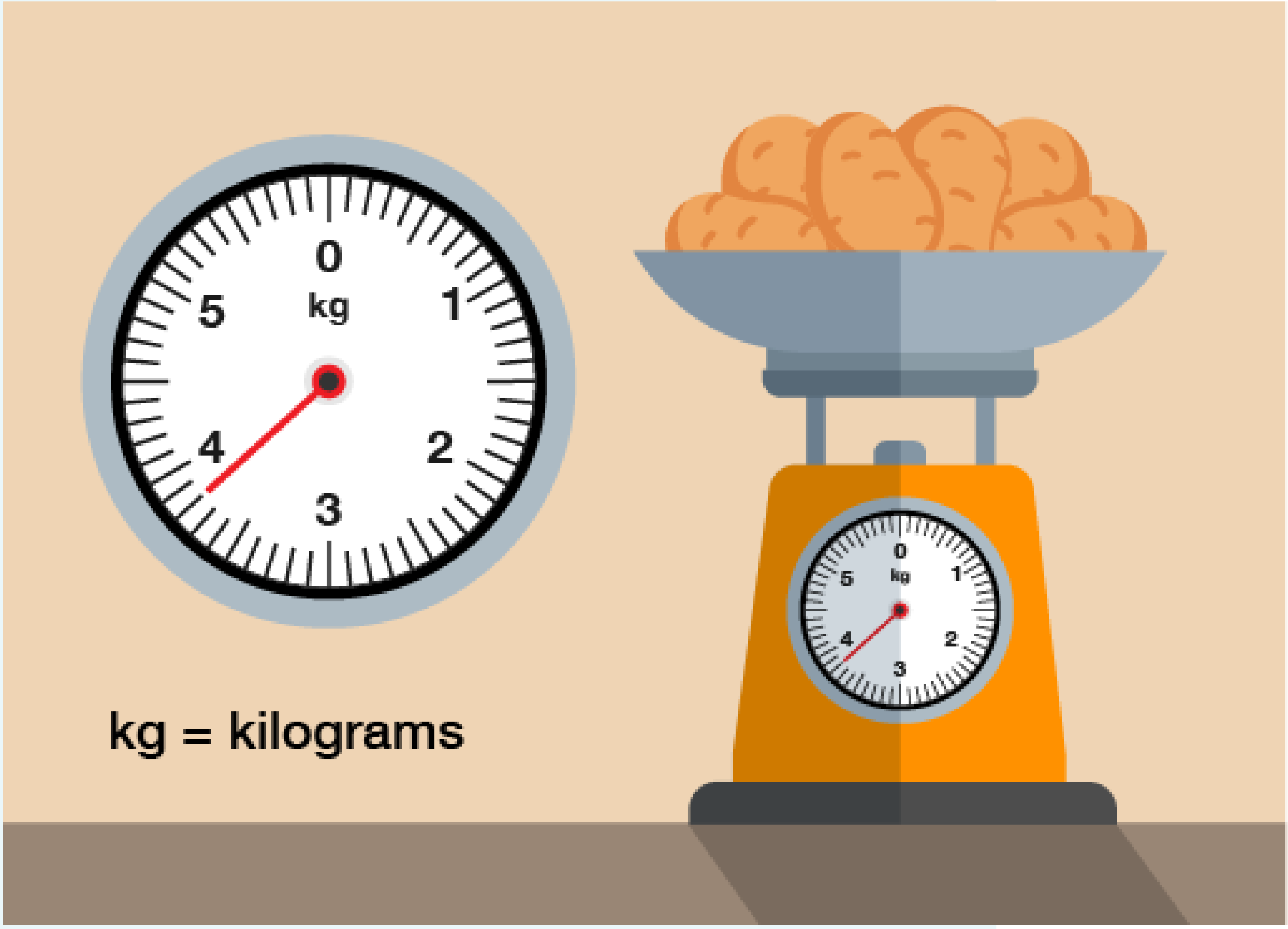


Figure 31 Food scales weighing potatoes

Answer

3. As with Question 2, you need to remember that 1 kg = 1000 g.

   The scale is numbered at every 1 kg interval and there are 10 steps between each numbered marker so each step marks 0.1 kg (1 ÷ 10 = 0.1). You could also think of each step being 100 g (0.1 kg = 100 g).

   The arrow is pointing to 3.8 kg (or 3800 g).

   If Simon needs 4 kg (4000 g) of potatoes then he needs to weigh out another 200 g.

Hopefully you will be feeling confident at reading scales on measuring devices now which leads you nicely onto the next section which looks at conversion scales.

5.3 Using conversion scales

Earlier on in the session you looked at converting between units of measure in different systems by carrying out calculations.

Many measuring instruments (e.g. thermometers, rulers, measuring jugs) have scales which show two or more different units of measure. This means that there may be times where you can compare the scales on the measuring instrument to make a conversion rather than carry out a calculation.

Look at the following example.

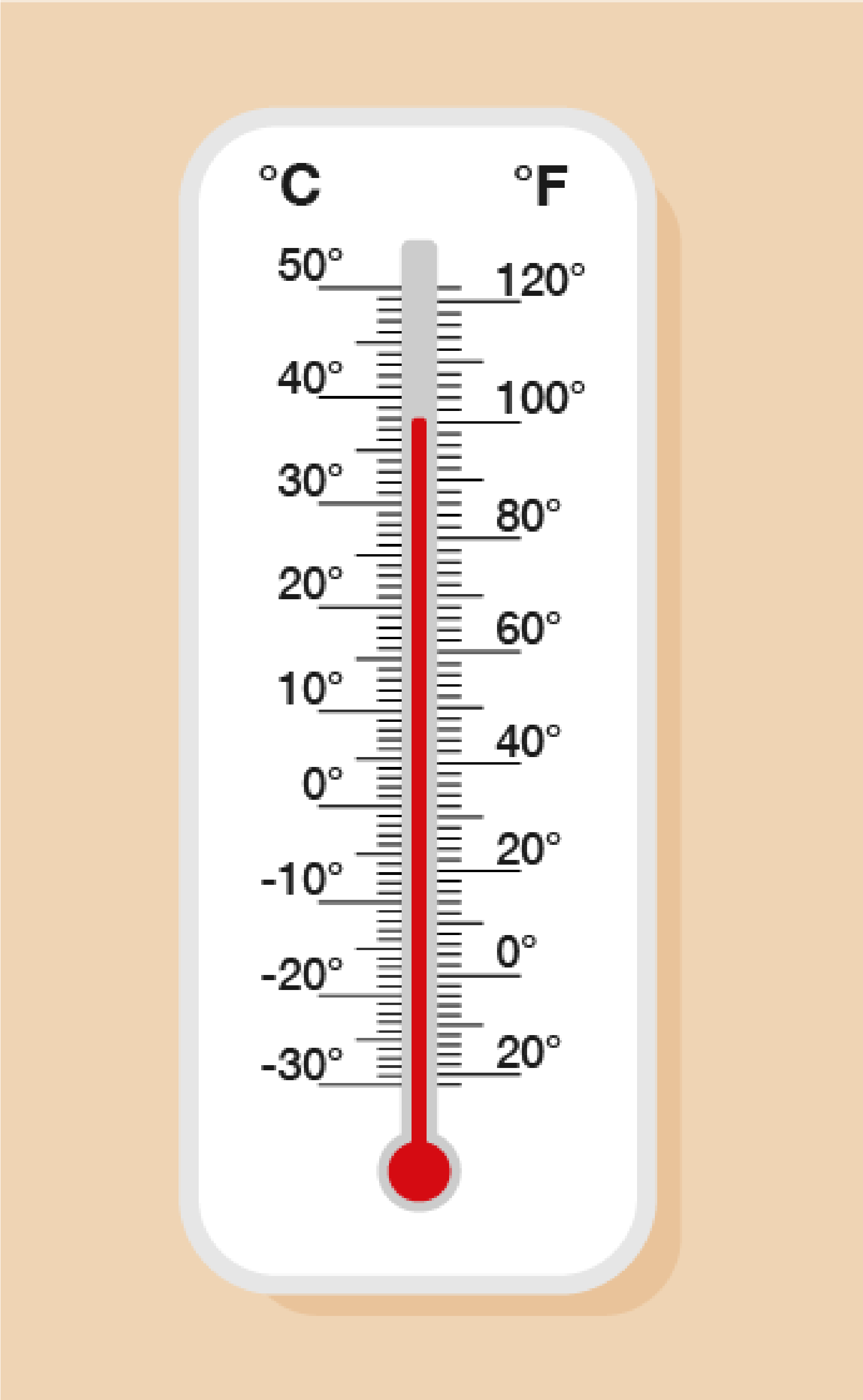


Figure 32 Example – Reading a thermometer

The thermometer above has a scale down the left-hand side which shows degrees Celsius (°C) and a scale on the right-hand side which shows degrees Fahrenheit (°F). This means that you can take a reading on this thermometer in both units of measure, depending on which is needed or which you are more familiar with. It can also help you to look up conversions between units.

You need to be careful with each scale, though – as they are showing different units, they are marked differently and go up in different steps.

On this thermometer, the degrees Celsius scale is going up in steps of 1°C, so the temperature shown is 38°C. If you want to take the reading in degrees Fahrenheit, you can see that it is 100°F (the scale is going up in steps of 2°F). It can be difficult to get a precise comparison between units, but using this thermometer, we can say that 38°C is roughly 100°F.

Now have a go at the following activity.

Activity 12: Using conversion scales

Look at the weighing scales below and answer the questions that follow.

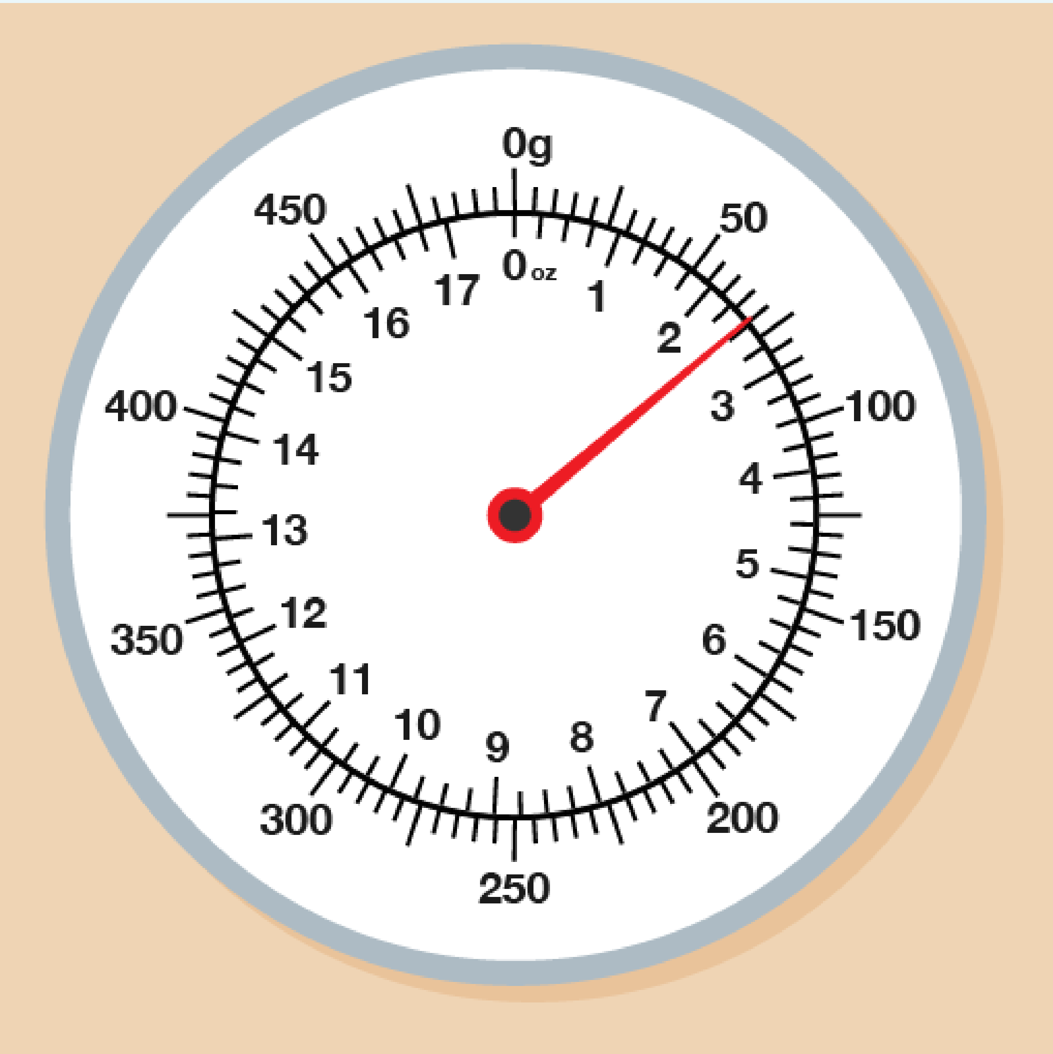


Figure 33 Weighing scales showing two units of measure

1. What is the reading shown by the arrow in grams?
2. How many ounces (oz) is 200 g, to the nearest whole oz?
3. Roughly how many grams is 1 oz, to the nearest 10 g?

4. I see a recipe which states that I need 6 oz of flour. Roughly, how many grams of flour is this?

Answer

Grams (g) are shown on the outside of this scale and ounces (oz) are shown on the inside.

1. The arrow is pointing to 70 g (the scale is going up in steps of 5).
2. You need to look on the outside of the scale to find 200 g and then look on the inside to see how many whole ounces it is nearest to.

   The nearest whole ounce is 7 oz.
3. Find 1 oz on the inside of the scale. Now look on the outside to take this reading in grams. The nearest marker is 30 grams (the grams scale goes up in steps of 5) so 1 oz is approximately 30 g.
4. Look on the inside of the scale for 6 oz. Then take the equivalent gram reading from the outside of the scale. 6 oz is approximately 170 g.

You have now learned all you need to know about units of measures! If you feel unsure on any part of this section, feel free to refer back to the examples or activities again to ensure you feel secure in all areas. All that remains of this section is the end of session quiz. Good luck!

Summary

In this section you have learned to read:

- measuring scales using different intervals
- scales on different measuring instruments
- conversion scales.

6 Session 2 quiz

Now it's time to review your learning in the end-of-session quiz.

Session 2 quiz.

Open the quiz in a new window or tab (by holding ctrl [or cmd on a Mac] when you click the link), then return here when you have done it.

7 Session 2 summary

You have now completed Session 2, 'Units of measure'. If you have identified any areas that you need to work on, please ensure you refer to this section of the course and retry the activities.

You should now be able to:

- understand that there are different units used for measuring and how to choose the appropriate unit
- convert between measurements in the same system (e.g. grams and kilograms) and those in different systems (e.g. litres and gallons)
- use exchange rates to convert currencies
- work with time and timetables
- work out the average speed of a journey using a formula
- convert temperature measurements between Celsius (°C) and Fahrenheit (°F)
- read scales on measuring equipment.

All of the skills listed above will help you with tasks in everyday life, such as measuring for new furniture or redesigning a room or garden. These are essential skills that will help you progress through your employment and education.

You are now ready to move on to Session 3, 'Shape and space'.

Session 3: Shape and space

Introduction

Take a look at the picture below. In order to decorate the room, you would need to know the length of skirting required (perimeter), how much carpet to order and how many tins of paint to buy (area). You would also need to use your rounding skills (as items such as paint and skirting must be bought in full units) and your addition and multiplication knowledge – to work out the cost!

Figure 1 Floor plan of a house

This session of the course will draw upon the skills you learned in the ‘Working with numbers’ and ‘Units of measure’ sessions.

Throughout this session you will learn how to find the perimeter, area and volume of simple and more complex shapes – if you’ve ever decorated a room you will be familiar with these skills already. It’s important to be able to work out area and perimeter accurately to ensure you buy enough of each material (but not too much to avoid wastage).

Once your room is beautifully decorated, you’re going to need to plan the best layout for your furniture (and make sure it fits!). You may well use a scale drawing to achieve this.

By the end of this session you will be able to:

- understand the difference between perimeter, area and volume and be able to calculate these for both simple and more complex shapes
- know that volume is a measure of space inside a 3D object and calculate volumes of shapes in order to solve practical problems
- draw and use a scale drawing or plan.

Video content is not available in this format.

1 Perimeter

As you will see from the picture below, perimeter is simply the distance around the outside of a shape or space. The shape you need to find the perimeter of could be anything from working out how much fencing you need to go around the outside of your garden to how much ribbon you need to go around the outside of a cake. Shapes or spaces with straight edges (rather than curved) are easiest to calculate so let’s begin by looking at these.

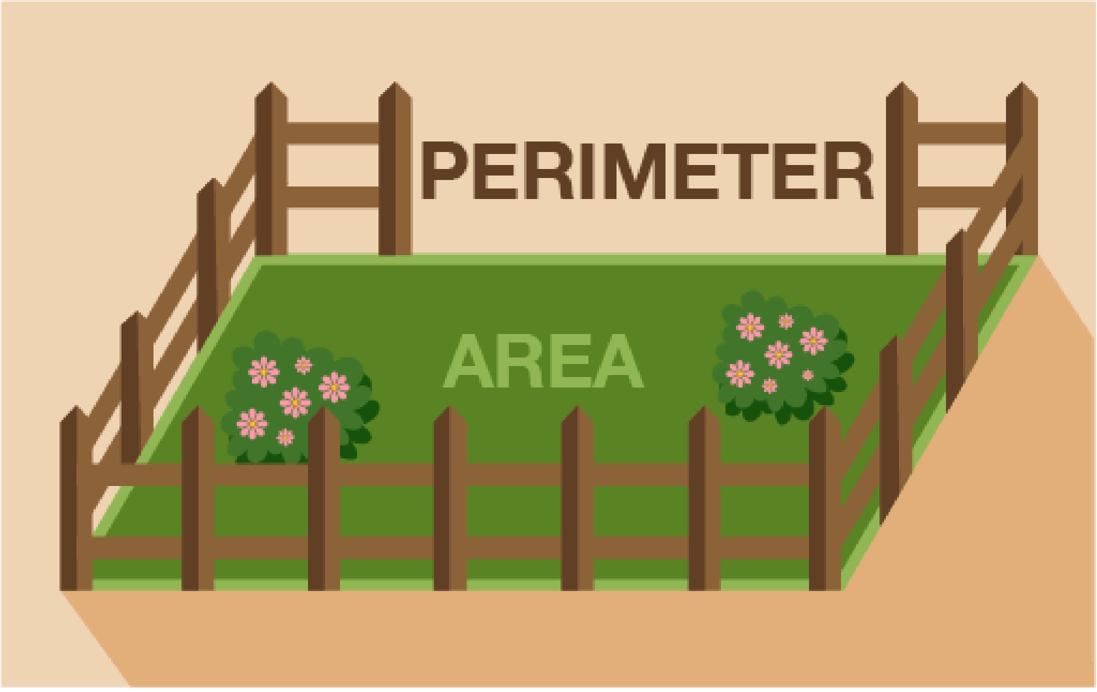


Figure 2 A perimeter fence

1.1 Perimeter of simple shapes

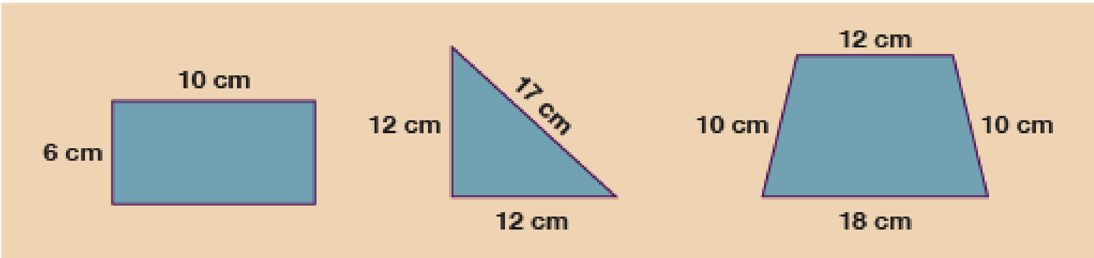


Figure 3 Finding the perimeter of simple shapes

In order to work out the perimeter of the shapes above, all you need to do is simply add up the total length of each of the sides.

- Rectangle: 10 + 10 + 6 + 6 = 32 cm.
- Triangle: 12 + 12 + 17 = 41 cm.
- Trapezium: 10 + 12 + 10 + 18 = 50 cm.

When you give your answer, make sure you write the units in cm, m, km etc. One other important thing to note is that before you work out the perimeter of any shape, you must make sure all the measurements are given in the same units. If, for example, two lengths are given in cm and one is given in mm, you must convert them all to the same unit before you work out the total. It doesn't usually matter which measurement you choose to convert but it's wise to check the question first because sometimes you might be asked to give your answer in a specific unit.

Activity 1: Finding the perimeter

Work out the perimeters of each of the shapes below.

Remember to give units in your answer and to check that all measurements are in the same units before you begin to add. Carry out your calculations without using a calculator. Remember to check your answers against ours.

1.

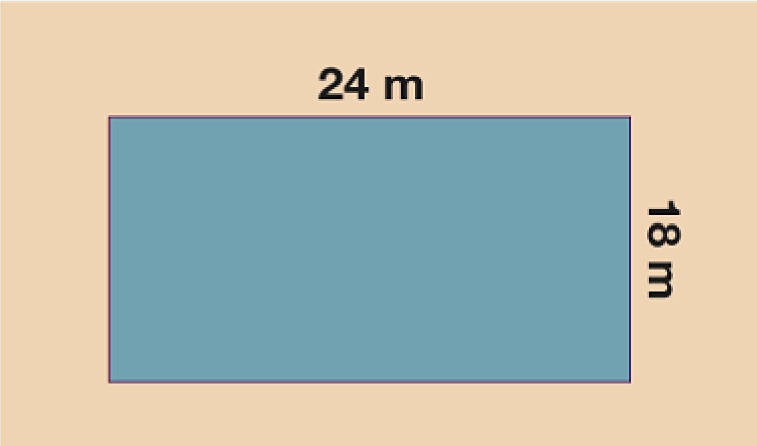


Figure 4 Calculating the perimeter – Question 1

2.

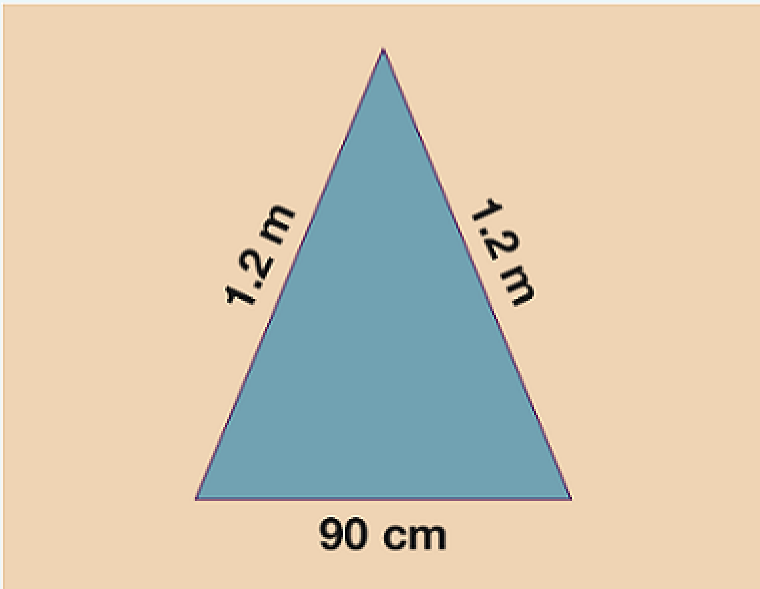


Figure 5 Calculating the perimeter – Question 2

3.

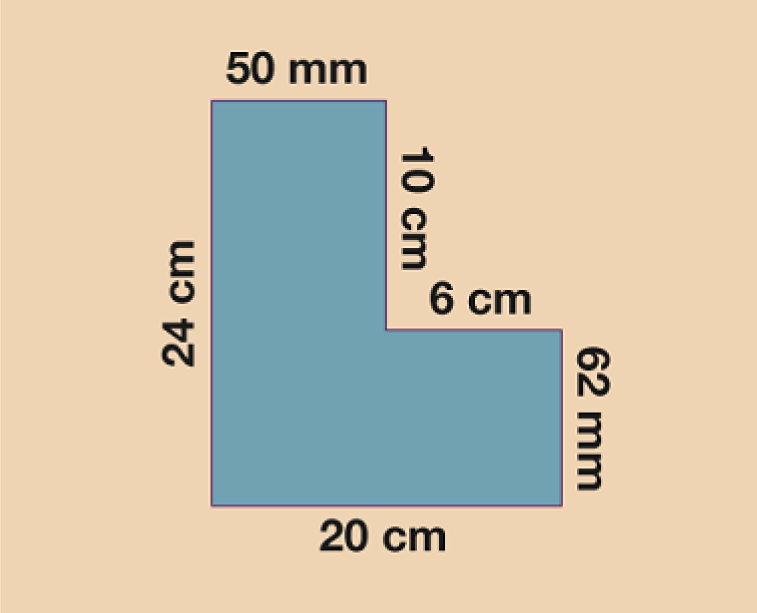


Figure 6 Calculating the perimeter – Question 3

Answer

1. 24 + 18 + 24 + 18 = 84 m.
2. If you worked in cm:

   1.2 m = 120 cm.

   Therefore, the perimeter is 120 + 120 + 90 = 330 cm.

   If you worked in m:

   90 cm = 0.9 m.

   Therefore, the perimeter is 0.9 + 1.2 + 1.2 = 3.3 m.
3. 50 mm = 5 cm, 62 mm = 6.2 cm.

   Therefore, the perimeter is 24 + 5 + 10 + 6 + 6.2 + 20 = 71.2 cm.

Hopefully, you found working out the perimeters of these shapes fairly straightforward.

The next step on from shapes like the ones you have just worked with, is finding the perimeter of shapes where not all the lengths are given to you. In shapes such as a rectangle or regular shapes like squares (where all sides are the same length) this is a simple process. However, in a shape where the sides are not the same as each other, you have a little more work to do.

1.2 Perimeters of shapes with missing lengths

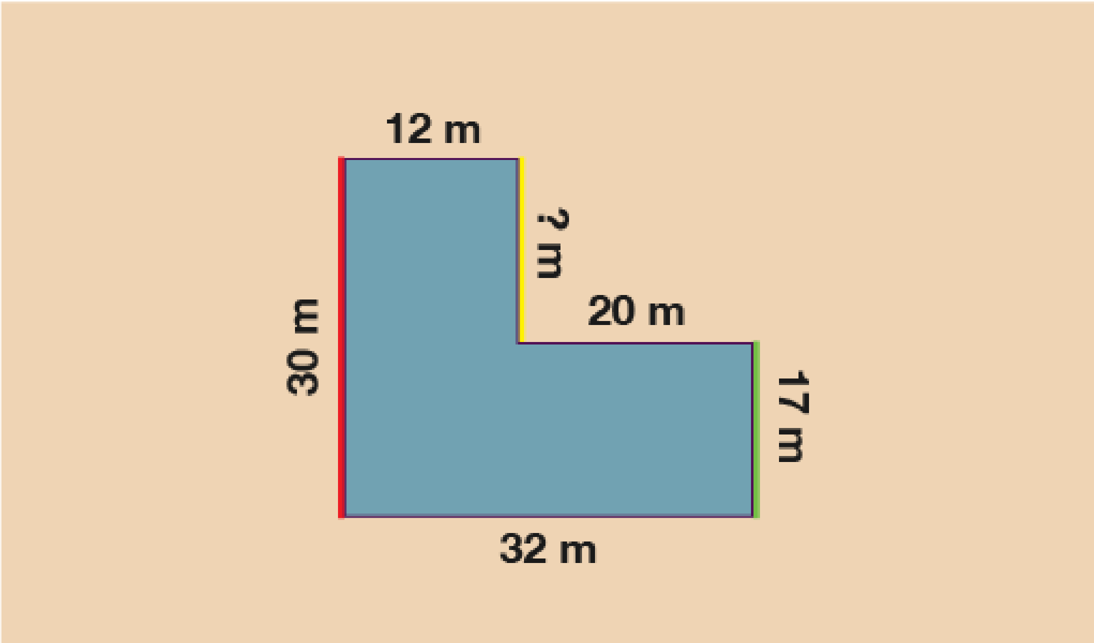


Figure 7 Finding the perimeter when measurements are missing

Look at the shape above. You can see that one of the lengths is missing from the shape. How do you find the perimeter when you don't have all the measurements? You cannot just assume that the missing length (yellow) is half of the red length, so how do you work it out? You'll need to use the information that is given in the rest of the shape.

If you look at all the vertical lengths (red, yellow and green) you can see that we know the length of two of the three. You can also see that green + yellow = red, since the two shorter lengths put together would equal the same as the longest vertical length.

If 17 + ? = 30, then in order to find the missing length you must do 30 − 17 = 13.

The missing length is therefore 13 m. Now that you know this, you can work out the perimeter of the shape in the normal way.

> 30 + 12 + 13 + 20 + 17 + 32 = 124
> So the perimeter of this shape is 124 m.

Let's look at one more example before you try some on your own.

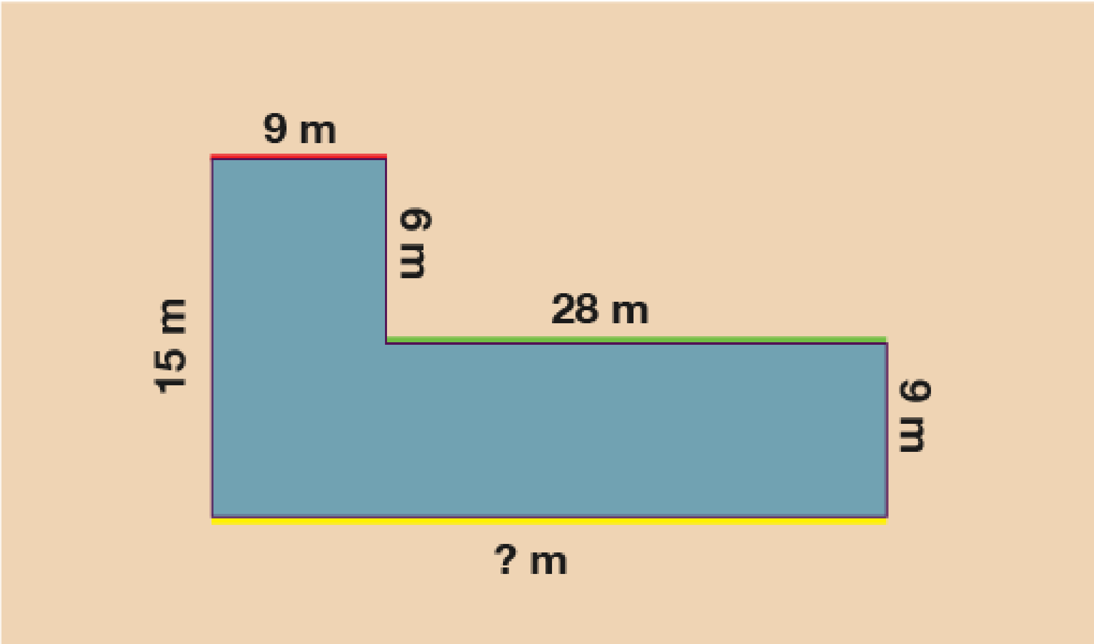


Figure 8 Finding the perimeter when a length is missing

In the example above, you will see that there is again a missing length. This time the missing length (yellow) is a horizontal one and so you need to look at the other two horizontal lengths (red and green) in order to work out the missing side.

You can see that this time that red + green = yellow, since the missing length is the sum of the two shorter lengths. You can now do 9 + 28 = 37, so the missing length is 37 m.

Now that you know all the side lengths, you can work out the perimeter by finding the total of all the lengths.

> 15 + 9 + 6 + 28 + 9 + 37 = 104
>
> So the perimeter of this shape is 104 m.

Activity 2: Perimeters and missing lengths

1. Work out the perimeter of the shape below.

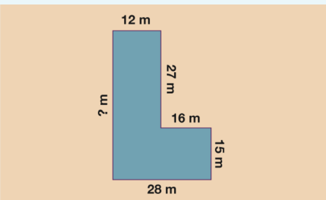


Figure 9 Perimeters and missing lengths – Question 1

2. You are redecorating your living room and need to replace the skirting board. The layout of the room is shown below. Skirting board can only be bought in lengths of 2 m.

   How many lengths should you buy?

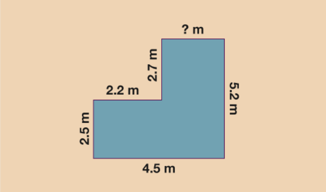


Figure 10 Perimeters and missing lengths – Question 2

Answer

1. Firstly, you need to work out the missing length:

   15 + 27 = 42 m

   So the perimeter is:

   42 + 12 + 27 + 16 + 15 + 28 = 140 m
2. Again, work out the missing length first:

   4.5 − 2.2 = 2.3 m

   Next, work out the perimeter of the room:

   2.5 + 2.2 + 2.7 + 2.3 + 5.2 + 4.5 = 19.4 m

   To see how many 2 m length of skirting you will need do:

   19.4 ÷ 2 = 9.7

   Since we can only buy whole lengths we need to round this up to 10 lengths of skirting.

Good work! You can now work out perimeters of simple and complex shapes, including shapes where there are missing lengths. There is just one more shape you need to consider – circles.

Whether it's ribbon around a cake or fencing around a pond, it's useful to be able to work out how much of a material you will need to go around the edge of a circular shape. The final part of your work on perimeter focusses on finding the distance around the outside of a circle.

1.3 Circumference of a circle

You may have noticed that a new term has slipped in to the title of this section. The term circumference refers to the distance around the outside of a circle – its perimeter. The perimeter and the circumference of a circle mean exactly the same, it's just that when referring to circles you would normally use the term circumference rather than perimeter.

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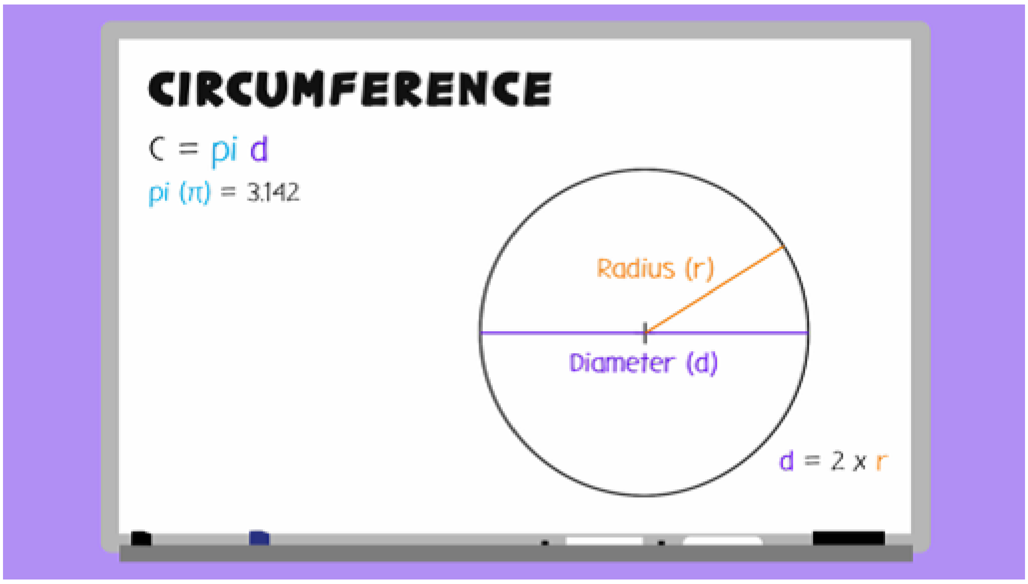

Activity 3: Finding the circumference

1. You have made a cake and want to decorate it with a ribbon.

   The diameter of the cake is 15 cm. You have a length of ribbon that is 0.5 m long. Will you have enough ribbon to go around the outside of the cake?

Figure 11 A round chocolate cake

2. You have recently put a pond in your garden and are thinking about putting a fence around it for safety. The radius of the pond is 7.4 m.

   What length of fencing would you require to fit around the full length of the pond? Round your answer up to the next full metre.

Figure 12 A round garden pond

Answer

1. d = 15 cm

   Using the formula $C = \pi d$

   $C = 3.142 \times 15$

C = 47.13 cm

Since you need 47.13 cm and have ribbon that is 0.5 m (50 cm) long, yes, you have enough ribbon to go around the cake.

2. Radius = 7.4 m so the diameter is 7.4 × 2 = 14.8 m

   Using the formula $C = \pi d$

   C = 3.142 × 14.8

   C = 46.5016 m which is 47 m to the next full metre.

You should now be feeling confident with finding the perimeter of all types of shapes, including circles. By completing Activity 3, you have also re-capped on using formulas and rounding.

The next part of this section looks at finding the area (space inside) a shape or space. As mentioned previously, this is incredibly useful in everyday situations such as working out how much carpet or turf to buy, how many rolls of wallpaper you need or how many tins of paint you need to give the wall two coats.

Summary

In this section you have learned:

- that perimeter is the distance around the outside of a space or shape
- how to find the perimeter of simple and more complex shapes
- how to use the formula for finding the circumference of a circle.

2 Area

The area of a shape is the amount of space inside it. This applies to only two dimensional (flat) shapes. If we are dealing with a three dimensional shape, the space inside this is called the volume. Since perimeter is the measure of a length or distance, the units it is measured in are cm, m, km etc.

As area is a measure of space rather than length or distance, it is measured in square units. This could be square metres, square centimetres, square feet and so on. You will often see these written as cm^2, m^2, ft^2, etc. Over the next few pages you will learn how to find the area of simple shapes, compound shapes and circles.

2.1 Area of simple shapes

The simplest shapes to begin with when looking at area are squares and rectangles. If you look at the rectangle in Figure 13 you can see that it's 6 cm long and 3 cm wide. If you count the squares, there are 18 of them. The area of the shape is 18 cm^2.

It is not always possible (or practical) to count the squares in a shape or space but it's a useful illustration to help you understand what area is.

More practically, to find the area (A) of a square or a rectangle, you would multiply the length (l) by the width (w), so the formula would be:

Area = *length* × *width* or:

$A = lw$ (remember the multiplication sign is not normally written in a formula)

In the example below $A = 6 \times 3 = 18$ cm^2.

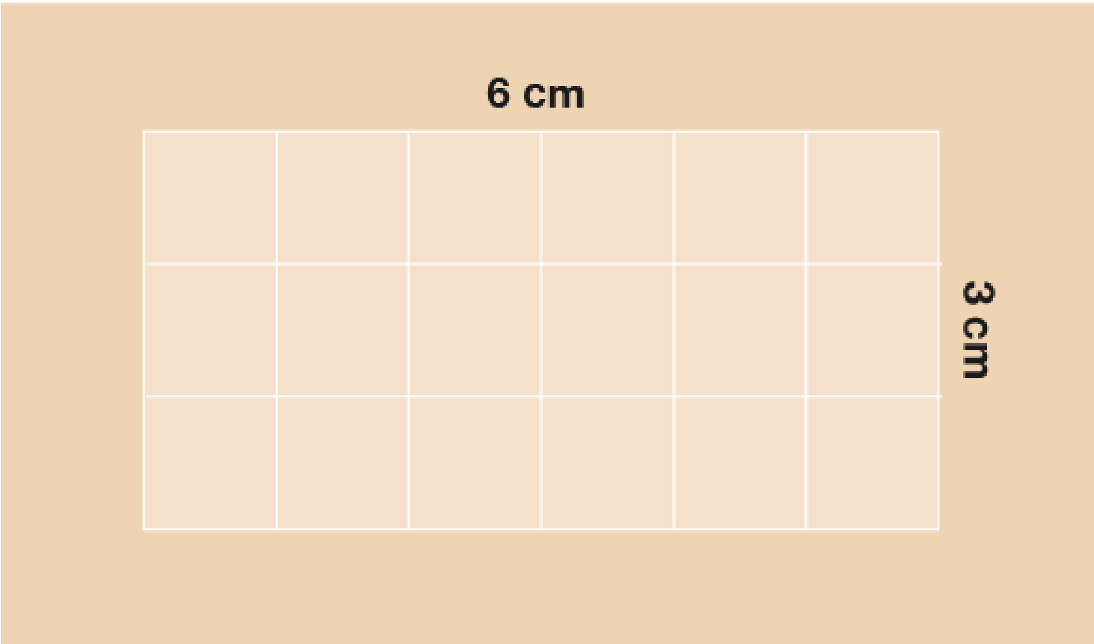

Figure 13 Finding the area of a rectangle

Triangles are another shape that you can find the area of relatively simply. If you think of a triangle, it is really just half of a rectangle. This is easiest to see with a right-angled triangle as shown below. You can see that the actual triangle (in yellow) is simply a rectangle that has been diagonally cut in half.

In order to find the area of the triangle then, you simply multiply the base by the height (as you would for a rectangle) and then halve the answer.

Sometimes this is shown as the formula:

$A = (b \times h) \div 2$

where b is the base of the triangle and h is the vertical height.

This formula may be written as:

$A = \frac{bh}{2}$

For the triangle below then, you would do:

$A = (5 \times 4) \div 2$
$A = 20 \div 2$
$A = 10\ \text{cm}^2$

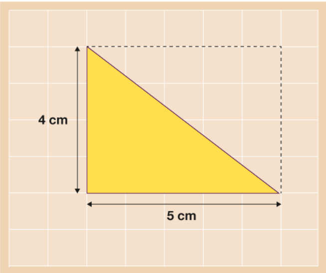

Figure 14 Finding the area of a right-angled triangle

This formula remains the same for any triangle. Take a look at the triangle below. It's a bit less obvious than with the example above but if you were to take the two yellow sections away and put them together, you would end up with a shape exactly the same size as the orange triangle.

The area of this triangle can be found in the same way as the previous one:

$A = (b \times h) \div 2$
$A = (4 \times 7) \div 2$
$A = 28 \div 2$
$A = 14\ \text{cm}^2$

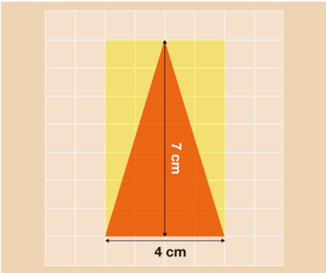

Figure 15 Finding the area of a triangle

Another shape you may need to find the area of is the trapezium. You will need to use a simple formula for this shape (don't panic when you see it, it looks scary but it really is quite easy to use!)

A trapezium looks like any of the shapes below.

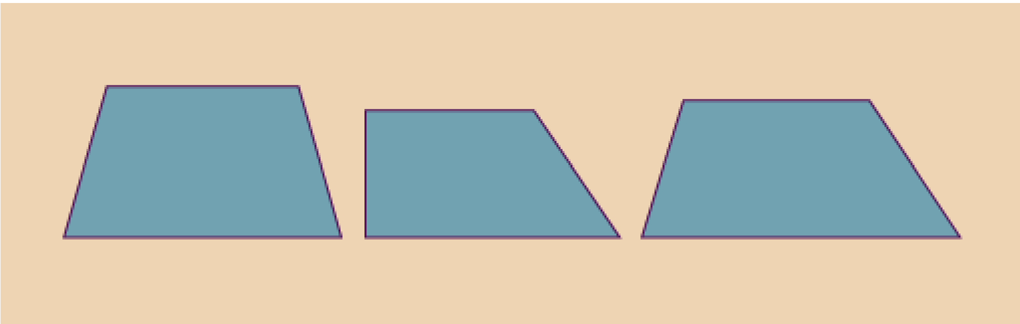

Figure 16 Examples of trapezium shapes

In order to work out the area of a trapezium you just need to know the vertical height and the length of the top and bottom sides. Traditionally, the length of the top is called *'a'*, the length on the bottom is *'b'* and the vertical height is *'h'*. Once this is clear you can then use the formula:

$$A = \frac{(a + b) \times h}{2}$$

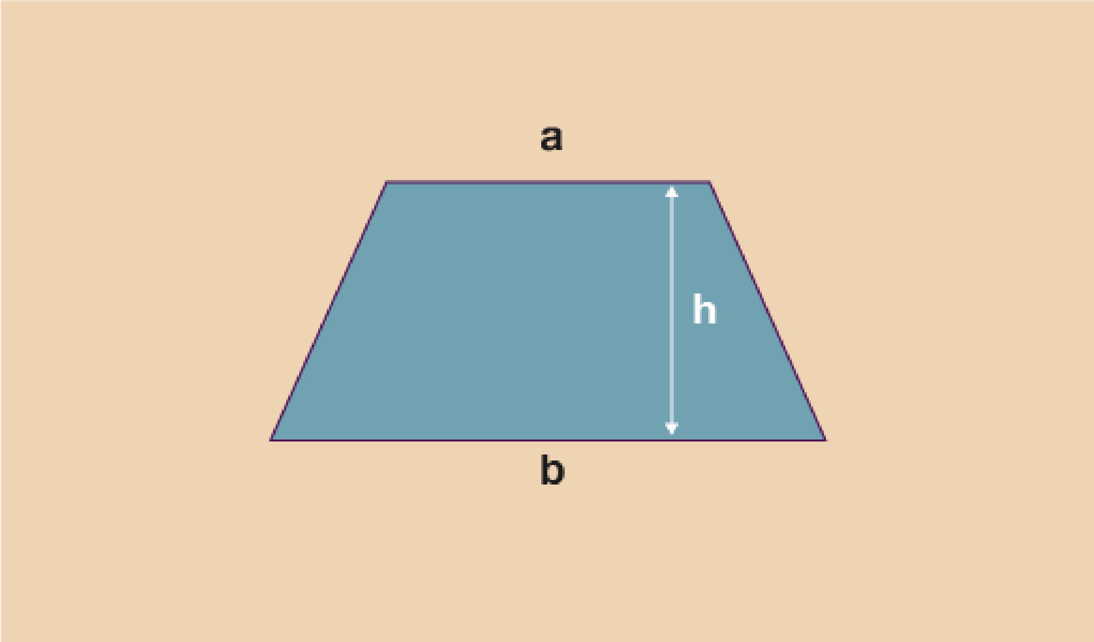

Figure 17 Dimensions of a trapezium

Let’s take a look at an example on how to work out the area of the trapezium below. We can see that the top length (*a*) = 12 cm. The bottom length (*b*) = 20 cm, and the height (*h*) = 13 cm.

Using these values and the formula:

$$A = \frac{(a + b) \times h}{2}$$

$$A = \frac{(12 + 20) \times 13}{2}$$

$$A = \frac{416}{2}$$

$$A = 208\text{ cm}^2$$

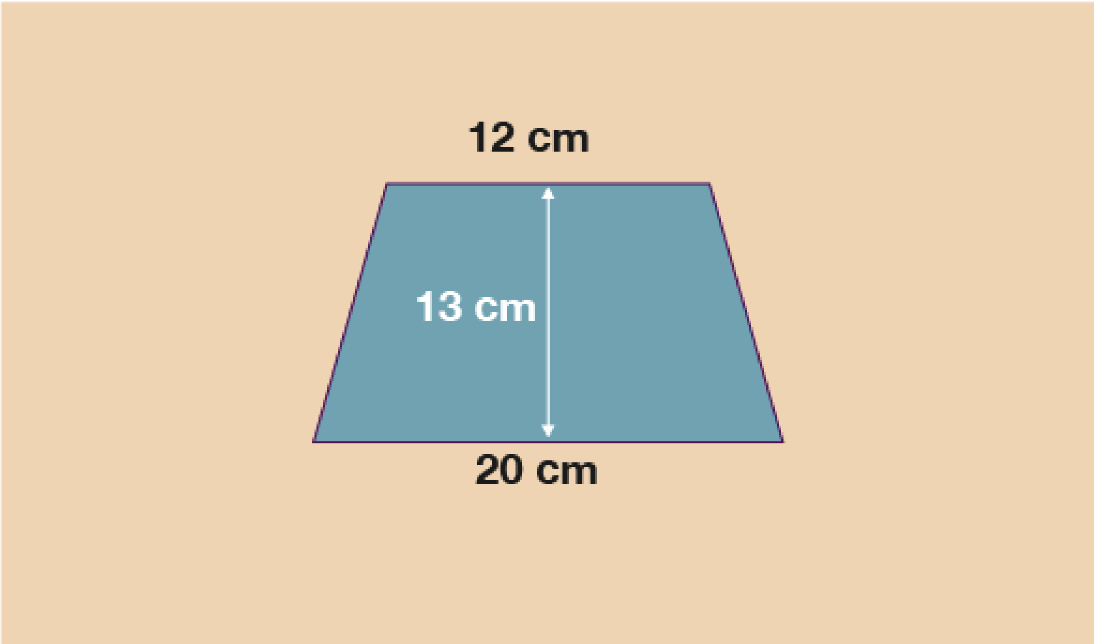

Figure 18 Finding the area of a trapezium

Now that you have seen how to work out the area of several basic shapes, it’s time to put your skills to the test. Have a go at the activity below. Don’t forget that, just as with perimeter, before you can begin any calculations you must make sure all measurements are in the same units.

Activity 4: Finding the area

Work out the area of each of the shapes below.

1.

Figure 19 Finding the area – Question 1

Answer

1. You need to convert the measurements into the same units before you can work out the area.

   If working in cm:

   1.6 m = 160 cm, so

   A = 160 × 95 = 15 200 cm^2

   If working in m:

   95 cm = 0.95 m, so

   A = 1.6 × 0.95 = 1.52 m^2

2.

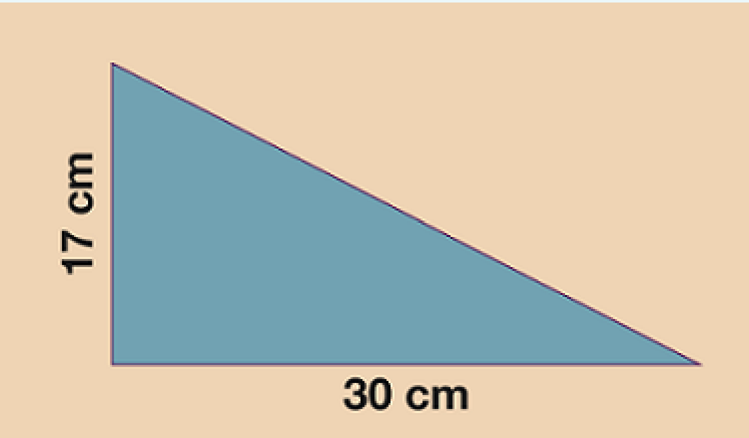

Figure 20 Finding the area – Question 2

Answer

2. The two measurements we need for the triangle are the base (30 cm) and the vertical height (17 cm). Don’t be fooled by the diagonal length of 46 cm, you do not need it for the area!

   $A = (b \times h) \div 2$

   $A = (30 \times 17) \div 2$

   $A = 510 \div 2$

   $A = 255\ cm^2$

3.

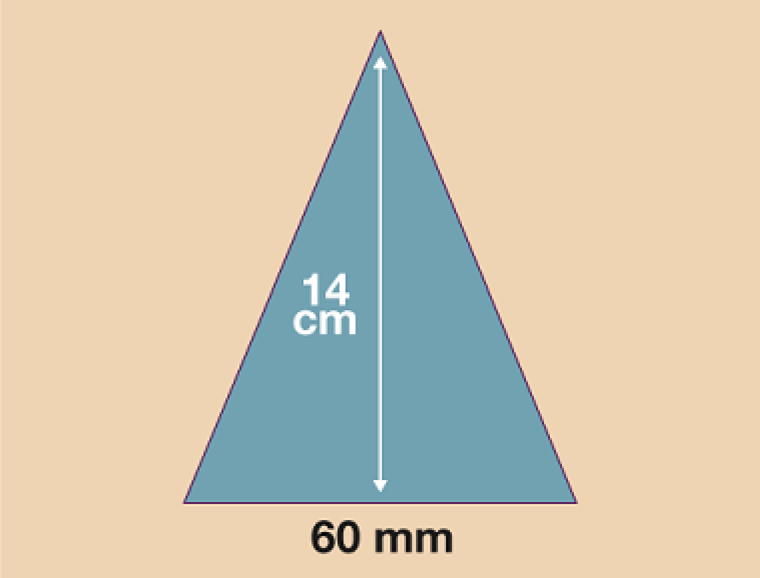

Figure 21 Finding the area – Question 3

Answer

3. You need to convert the measurements into the same units before you can work out the area.

   If working in mm: 14 cm = 140 mm

   $A = (b \times h) \div 2$
   $A = (60 \times 140) \div 2$
   $A = 8400 \div 2$
   $A = 4200\ \text{mm}^2$

   If working in cm: 60 mm = 6 cm

   $A = (b \times h) \div 2$
   $A = (6 \times 14) \div 2$
   $A = 84 \div 2$
   $A = 42\ \text{cm}^2$

4.

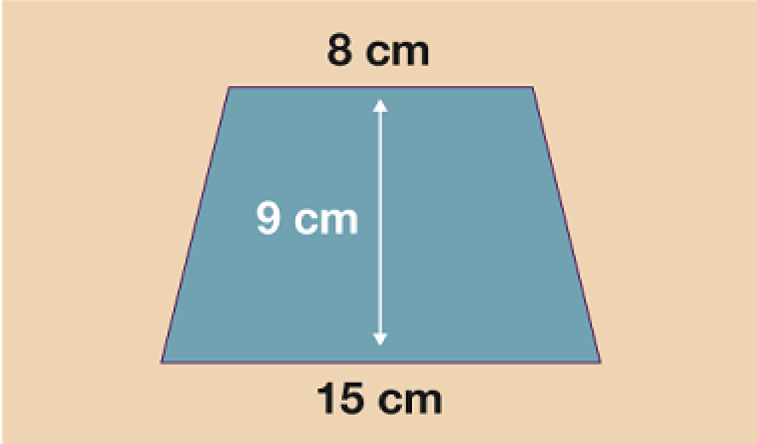

Figure 22 Finding the area – Question 4

Answer

4. Top length (a) = 8 cm

   Bottom length (b) = 15 cm
   Height (h) = 9 cm
   Using the formula for the area of a trapezium:

   $$A = \frac{(8 + 15) \times 9}{2}$$

   $$A = \frac{(23) \times 9}{2}$$

   $$A = \frac{207}{2}$$

   $A = 103.5\ \text{cm}^2$

Now that you've mastered finding the area of basic shapes, it's time to look at compound shapes.

A compound shape is just a shape that is made from more than one basic shape. Rarely will you find a floor space, garden area or wall that is perfectly rectangular. More often than not it will be a combination of shapes. The good news is that, to find the area of compound shapes, you simply split them up into their basic shapes, find the area of each of these, and then add them up at the end!

2.2 Area of compound shapes

Take a look at the shape below; this is an example of a compound shape. Whilst you cannot find the area of this shape as it is by using a formula as you have done previously, you can split it into two basic shapes (rectangles) and then use your existing knowledge to work out the area of each of these shapes.

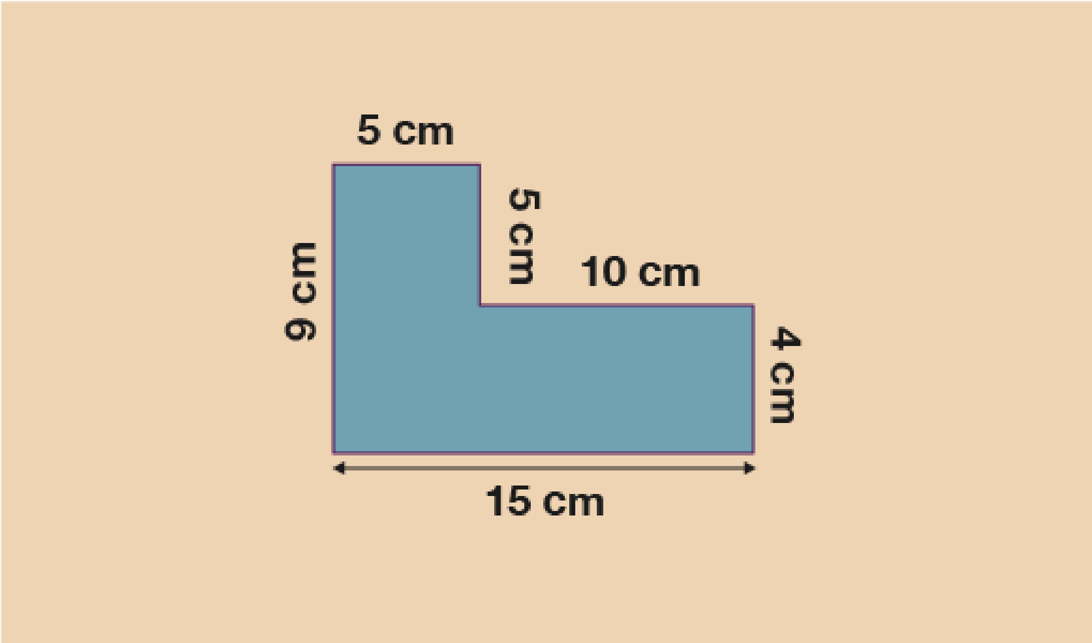

Figure 23 Finding the area of a compound shape

You should be able to see that you can split this shape into two rectangles. It does not matter which way you split it – you will get the same answer at the end.

You could split it like this:

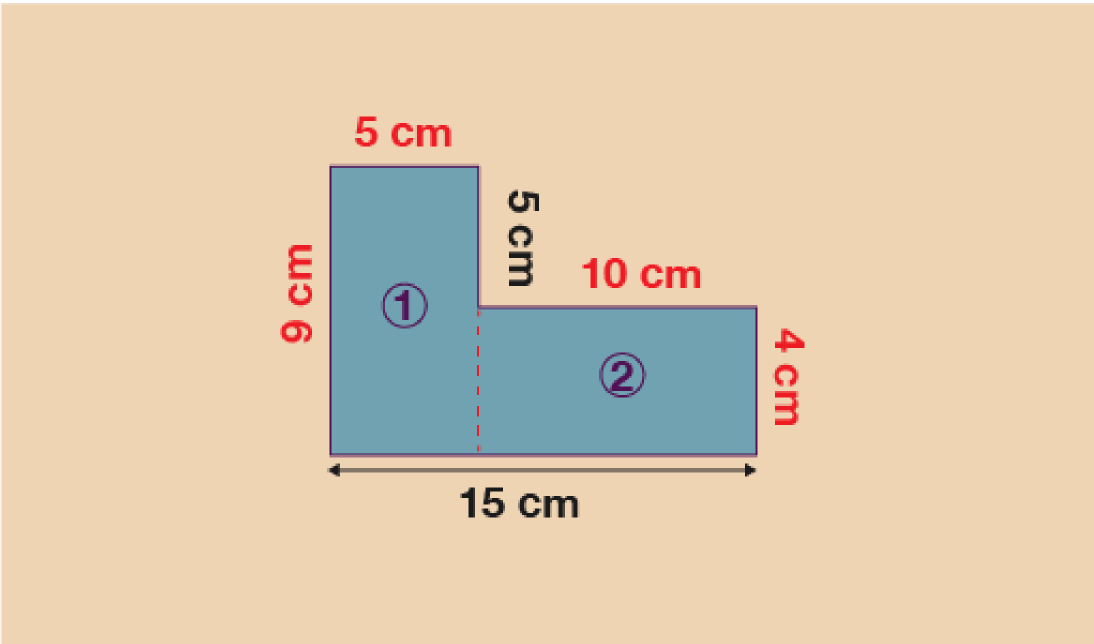

Figure 24 Splitting a compound shape horizontally to find the area

You now have two rectangles. To work out the area of rectangle ①, you do $A = 9 \times 5 = 45$ cm^2.

To work out the area of rectangle ②, you do $A = 10 \times 4 = 40$ cm^2.

Now that you have the area of both rectangles, simply add them together to find the area of the whole shape:

$$45 + 40 = 85 \text{ cm}^2$$

You need to be careful that you are using the correct measurements for the length and width of each rectangle (the measurements in red). In this example, the lengths of 15 cm and 5 cm (in black) are not required.

Alternatively, you could split the shape like this:

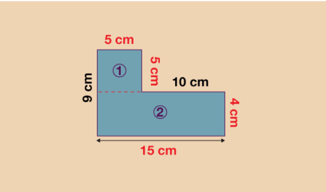

Figure 25 Splitting a compound shape vertically to find the area

Again, you now have two rectangles. To work out the area of rectangle ①, you do $A = 5 \times 5 = 25 \text{ cm}^2$.

To work out the area of rectangle ②, you do $A = 15 \times 4 = 60 \text{ cm}^2$.

Now that you have the area of both rectangles, simply add them together to find the area of the whole shape:

$$25 + 60 = 85 \text{ cm}^2$$

Again, you need to be careful that you are using the correct measurements for the length and width of each rectangle (the measurements in red). In this example, the lengths of 9 cm and 10 cm (in black) are not required.

You will notice that regardless of which way you choose to split the shape, you arrive at the same answer of 85 cm^2.

The best way for you to practise this skill is to try a few examples for yourself. Have a go at the activity below and then check your answers.

Activity 5: Finding the area of compound shapes

Work out the area of the shapes below.

1.

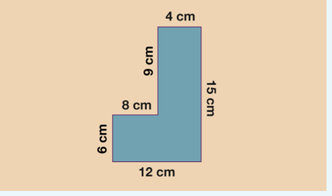

Figure 26 Finding the area of compound shapes – Question 1

Answer

1. The area of the whole shape is 108 cm^2

   Depending on how you split the shape you may have done:

   6 × 8 = 48 cm^2
   15 × 4 = 60 cm^2
   48 + 60 = 108 cm^2

   or:

   12 × 6 = 72 cm^2
   9 × 4 = 36 cm^2
   72 + 36 = 108 cm^2

2.

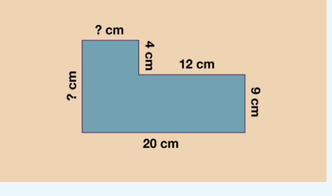

Figure 27 Finding the area of compound shapes – Question 2

Hint: You'll need to find some missing lengths on this shape before you can work out the area.

Answer

2. The missing vertical length is 13 cm (9 cm + 4 cm) and horizontal length is 8 cm (20 cm − 12 cm). The area of the whole shape is 212 cm^2.

   Depending on how you split the shape you may have done:

   13 × 8 = 104 cm^2
   12 × 9 = 108 cm^2
   104 + 108 = 212 cm^2

   or:

   20 × 9 = 180 cm^2
   4 × 8 = 32 cm^2

180 + 32 = 212 cm^2

Now that you can calculate the area of basic and compound shapes, there is just one other shape you will practise finding the area of: circles. Similarly to finding the perimeter of a circle, you'll need to use a formula involving the Greek letter π.

2.3 Area of a circle

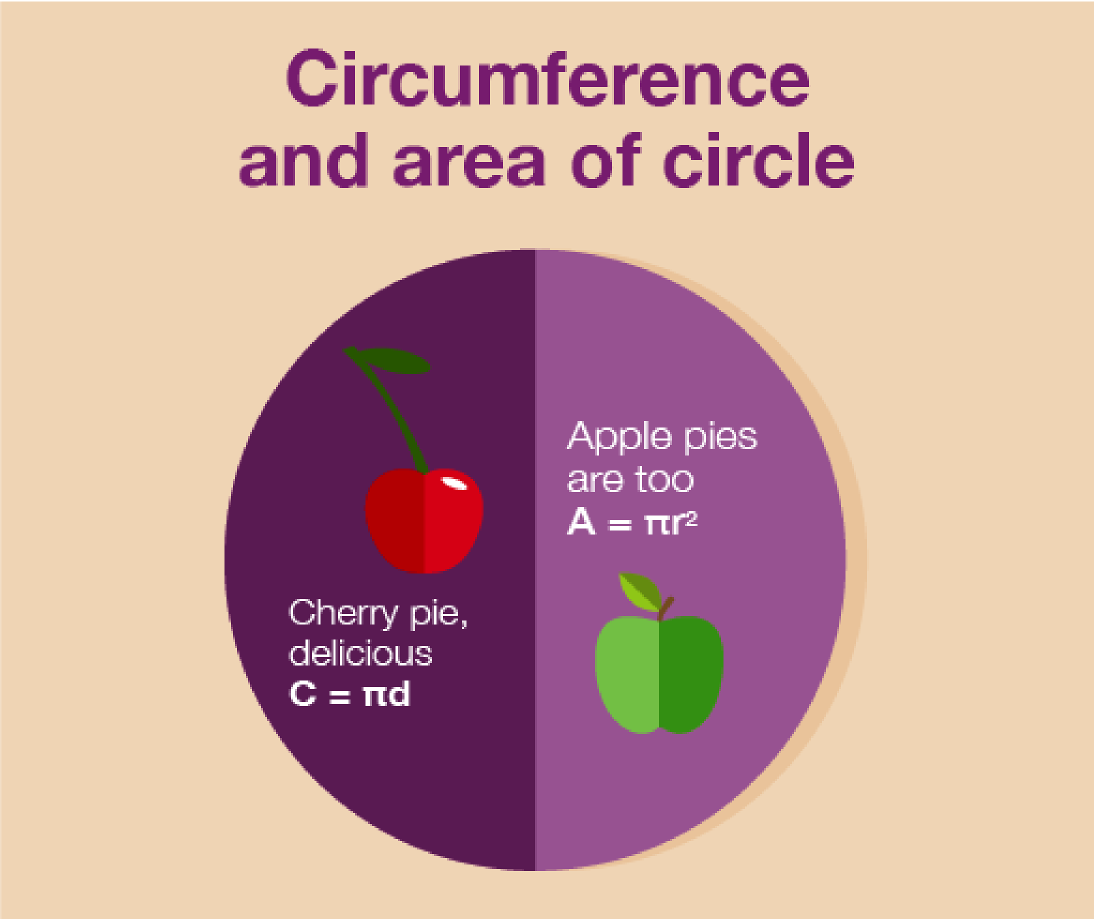

Figure 28 The circumference and area of a circle

You have already practised using the formula to find the circumference of a circle. You will now look at using the formula to find the area of one.

To find the area of a circle you need to use the formula:

Area of a circle = *pi* × *radius*^{2}

This can also be written as:

$A = \pi r^2$

where:

A = area

π = pi

r = radius
r^2!Warning! Cambria Math not supported means r squared.

Hint: Remember that when you square a number you simply multiply it by itself, *radius*^{2} is therefore simply *radius* × *radius*.

Let's look at an example.

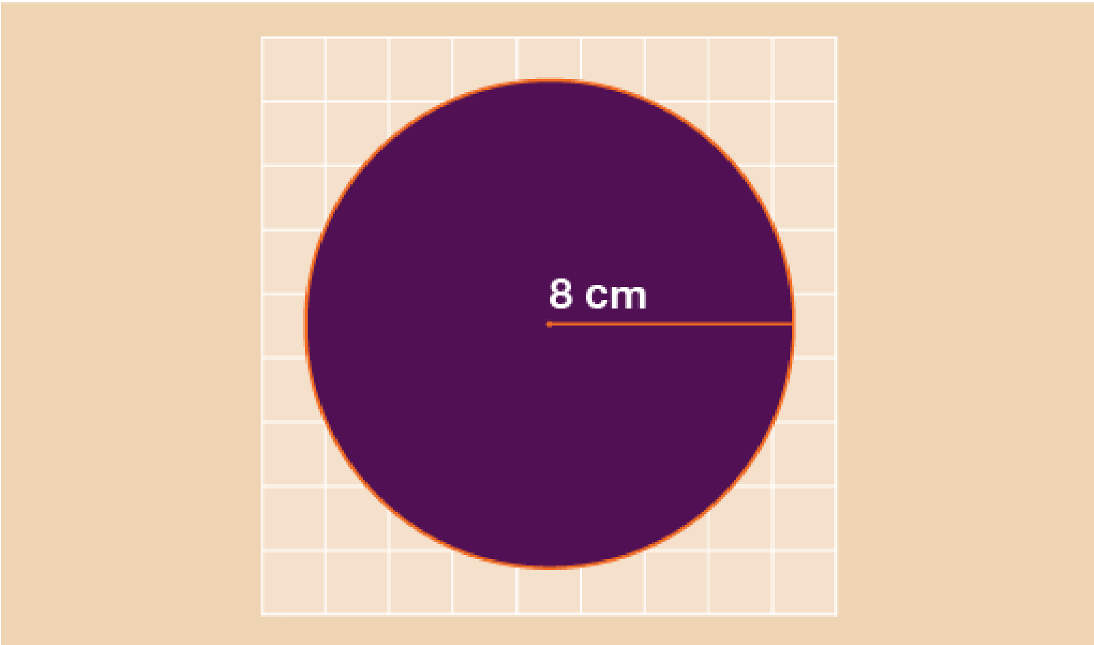

Figure 29 The radius of a circle

In the circle above you can see that the radius is 8 cm. For these tasks we will use the figure of 3.142 for π.

To find the area of the circle we need to do:

$A = \pi r^2$
$A = 3.142 \times 8 \times 8$
$A = 201.088\ \text{cm}^2$

Before you try some on your own, let's take a look at one further example.

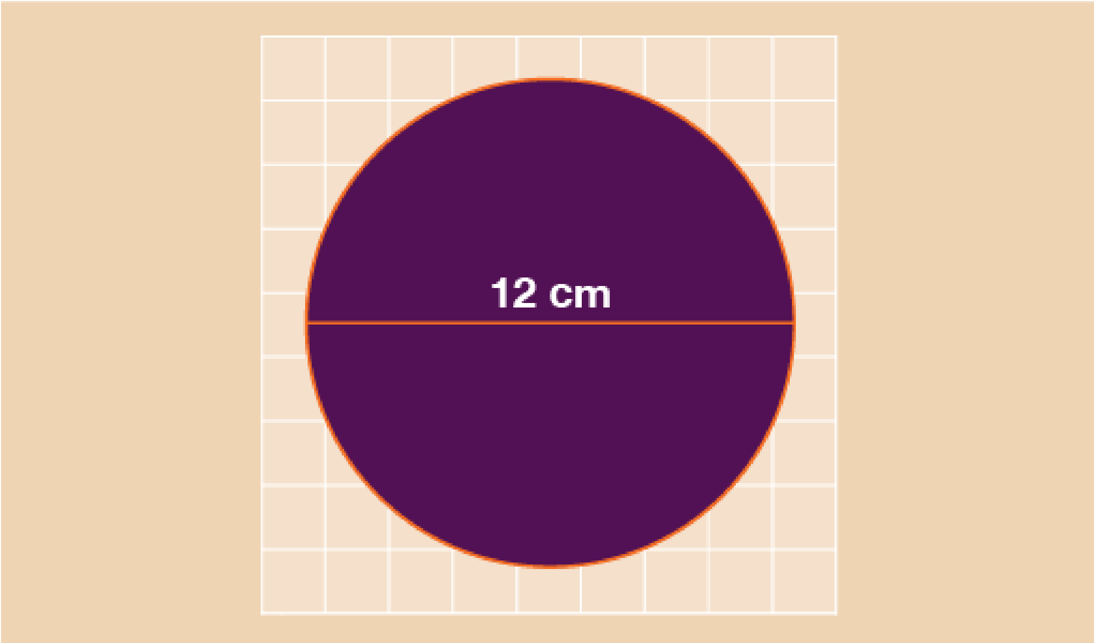

Figure 30 The diameter of a circle

This circle has a diameter of 12 cm. In order to find the area, you first need to find the radius. Remember that the radius is simply half of the diameter and so in this example radius = 12 ÷ 2 = 6 cm.

We can now use:

$A = \pi r^2$
$A = 3.142 \times 6 \times 6$
$A = 113.112 \text{ cm}^2$

Try a couple of examples for yourself before moving on to the next part of this section.

Activity 6: Finding the area of a circle

1. Find the area of the circle shown below. Give your answer to one decimal place.

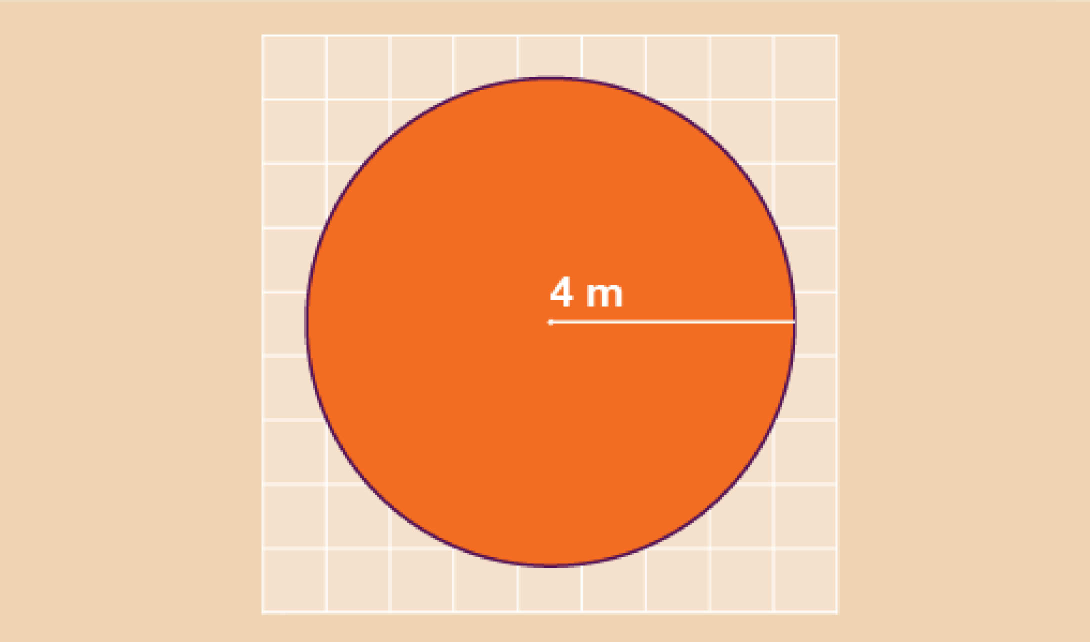

Figure 31 Finding the area of a circle – Question 1

2. Find the area of the circle shown below. Give your answer to 1 decimal place.

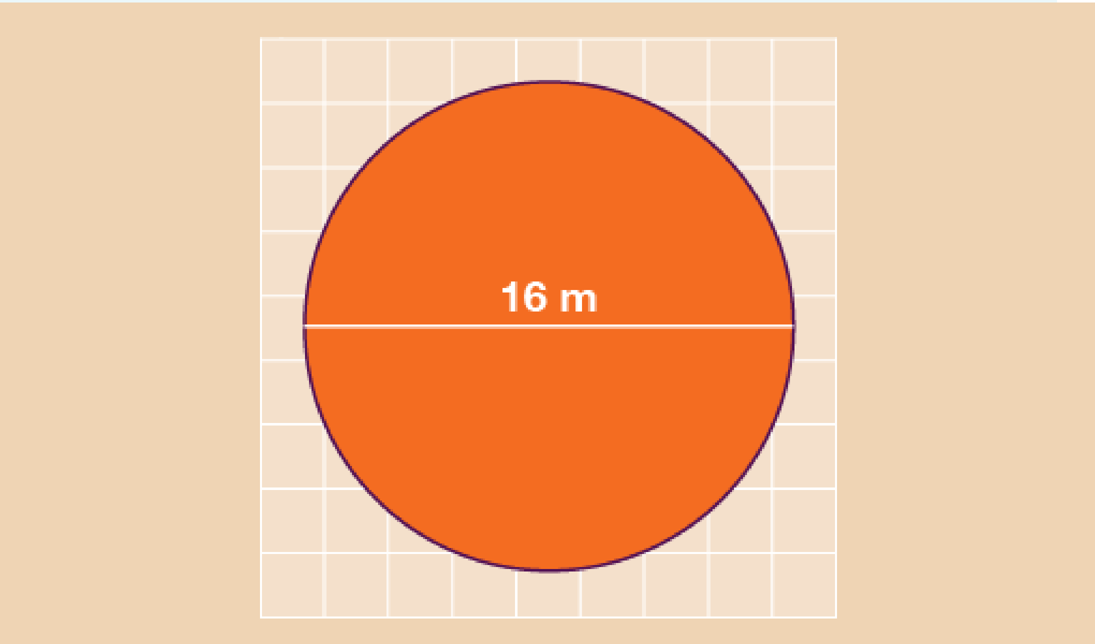

Figure 32 Finding the area of a circle – Question 2

3. You are designing a mural for a local school and need to decide how much paint you need. The main part of the mural is a circle with a diameter of 10 m as shown below. Each tin of paint will cover an area of 5 m^2. You will need to use two coats of paint. How many tins of paint should you buy?

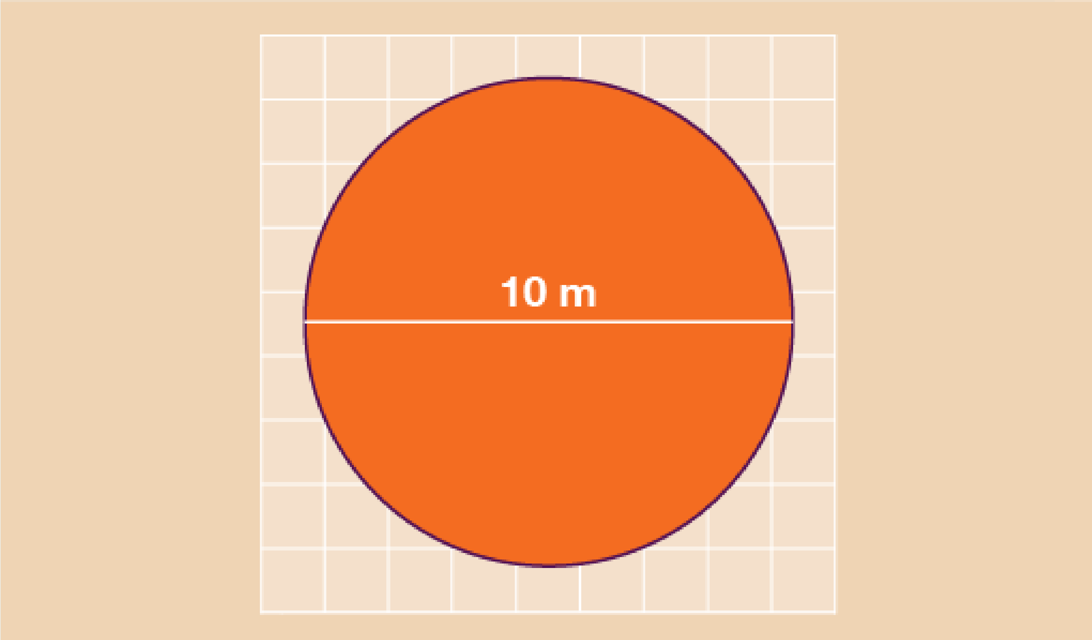

Figure 33 Finding the area of a circle – Question 3

Answer

1. To find the area of the circle you need to do:

   $A = \pi r^2$

   $A = 3.142 \times 4 \times 4$
   $A = 50.272$ m^2
   $A = 50.3$ m^2 to 1 d. p.
2. You need to find the radius of the circle first. Since the diameter of the circle is 16 cm, the radius is 16 cm ÷ 2 = 8 cm. Now you can find the area:

   $A = \pi r^2$
   $A = 3.142 \times 8 \times 8$
   $A = 201.088$ cm^2
   $A = 201.1$ cm^2 to one d.p.
3. You need to find the area of the circle first. Since the diameter of the circle is 10 m, the radius is 10 m ÷ 2 = 5 m.

   Now you can find the area of the circle:

   $A = \pi r^2$

   $A = 3.142 \times 5 \times 5$
   $A = 78.55$ m^2

   So the area of the circle you need to paint is 78.55 m^2

   Since you need to give 2 coats of paint, you need to double this number:

   $78.55 \times 2 = 157.1$ m^2

   You now need to work out how many tins of paint you need. As one tin of paint covers 5 m^2 you need to do:

   $157.1 \div 5 = 31.42$ tins

   Since you must buy whole tins of paint, you will need to buy 32 tins.

You have now learned all you need to know about finding the area of shapes! The last part of this section is on finding the volume of solid shapes – or three-dimensional (3D) shapes.

Summary

In this section you have learned:

- that area is the space inside a two-dimensional (2D) shape or space
- how to find the area of rectangles, triangles, trapeziums and compound shapes
- how to use the formula to find the area of a circle.

3 Volume

The volume of a shape is how much space it takes up. You might need to calculate the volume of a space or shape if, for example, you wanted to know how much soil to buy to fill a planting box or how much concrete you need to complete your patio.

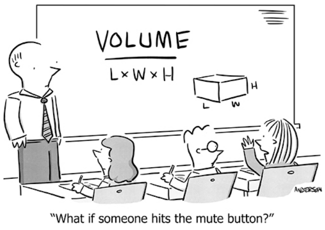

Figure 34 A volume cartoon

You'll need your area skills in order to calculate the volume of a shape. In fact, as you already know how to calculate the area of most shapes, you are one simple step away from being able to find the volume of most shapes too!

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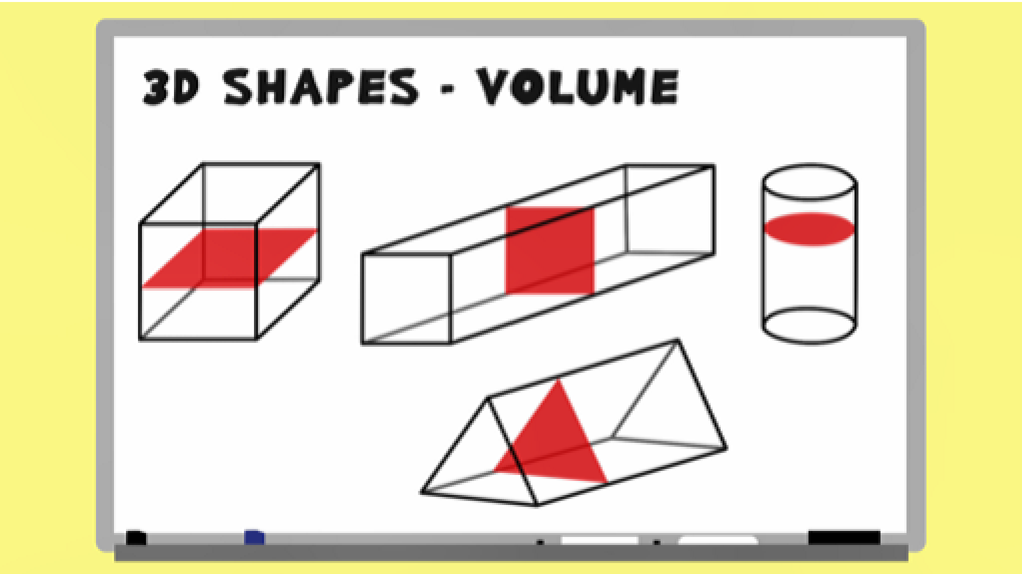

Example: Calculating volume

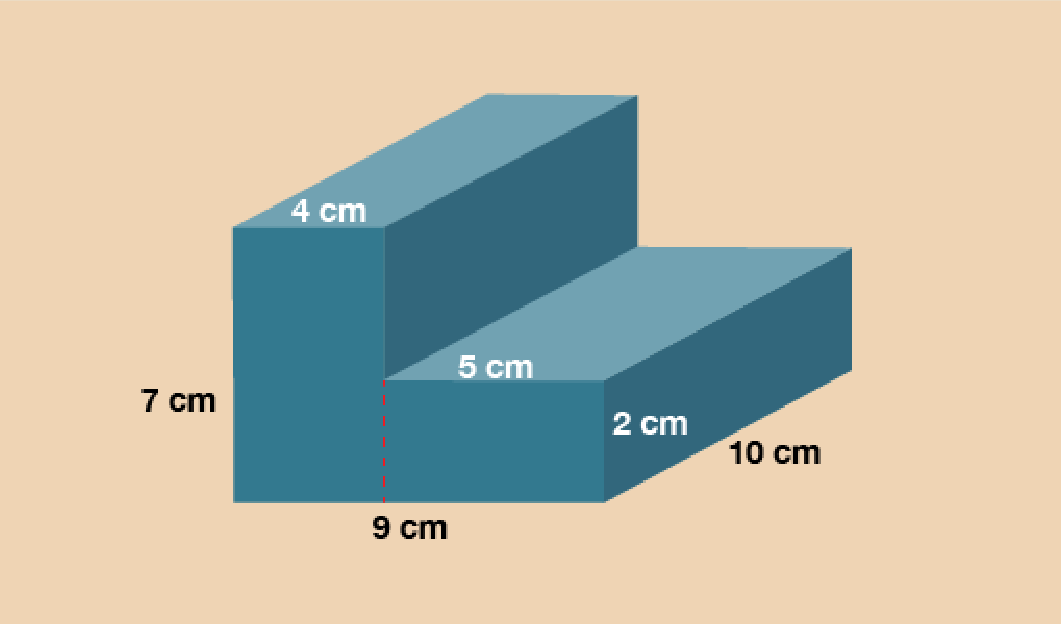

Figure 35 Calculating volume

The cross-section on this shape is the L shape on the front. In order to work out the area you'll need to split it up into two rectangles as you practised in the previous part of this section.

Rectangle 1 = 7 × 4 = 28 cm^2

Rectangle 2 = 5 × 2 = 10 cm^2

Area of cross-section = 28 + 10 = 38 cm^2

Now you have the area of the cross-section, multiply this by the length to calculate the volume.

$V = 38 \times 10 = \underline{380\ cm^3}$

Example: Calculating cylinder volume

The last example to look at is a cylinder. The cross-section of this shape is a circle. You'll need to use the formula to find the area of a circle in the same way you did in the previous part of this section.

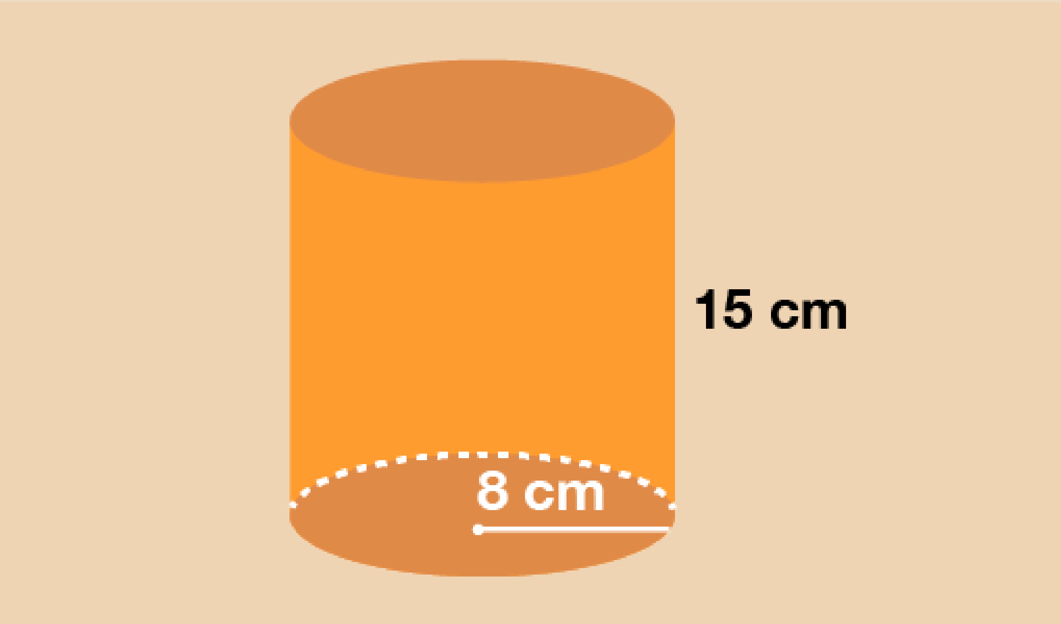

Figure 36 Calculating cylinder volume

You can see that the circular cross-section has a radius of 8 cm. To find the area of this circle, use the formula:

$A = \pi r^2$

$A = 3.142 \times 8 \times 8$

$A = 201.088 \text{ cm}^2$

Now you have the area, multiply this by the length of the cylinder to calculate the volume.

$V = 201.088 \times 15 = \underline{3016.32 \text{ cm}^3}$

Now try the following questions. Please carry out the calculations without a calculator. You can double check with a calculator if needed and remember to check your answers against ours.

Activity 7: Volume

Find the volume of the following shapes without using a calculator. The shapes are not drawn to scale.

1.

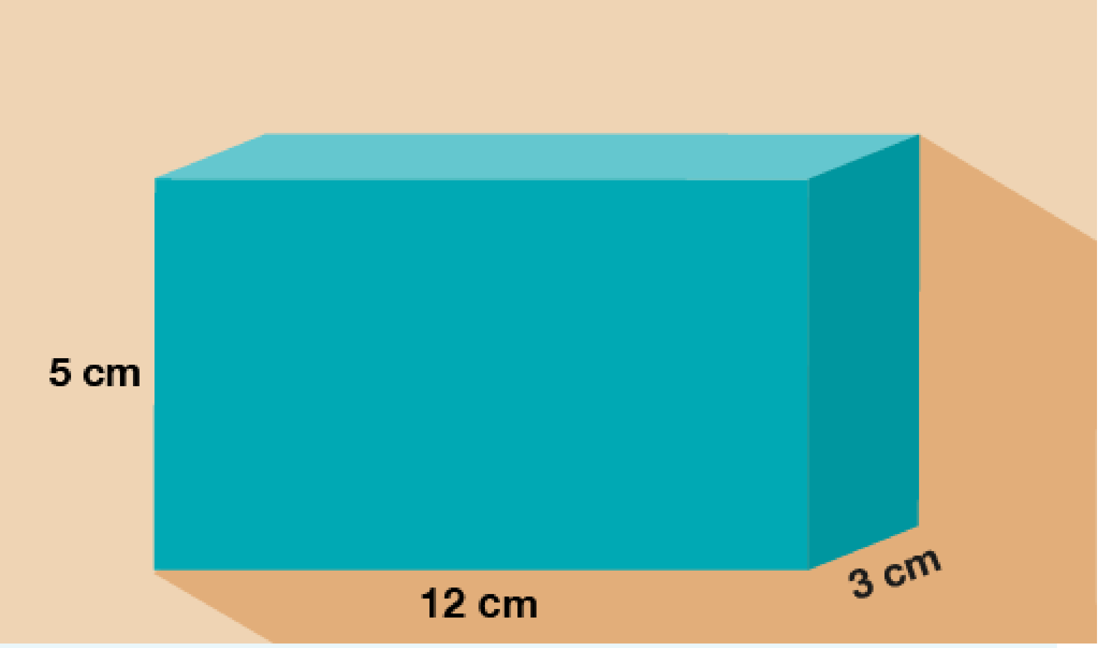

Figure 37 Rectangular prism (cuboid)

Answer

1. The area of the rectangular cross-section is:

   $5 \times 12 = 60\ \text{cm}^2$

   To get the volume, you now need to multiply the area of the cross-section by the length of the shape:

   $60 \times 3 = \underline{180\ \text{cm}^3}$

2.

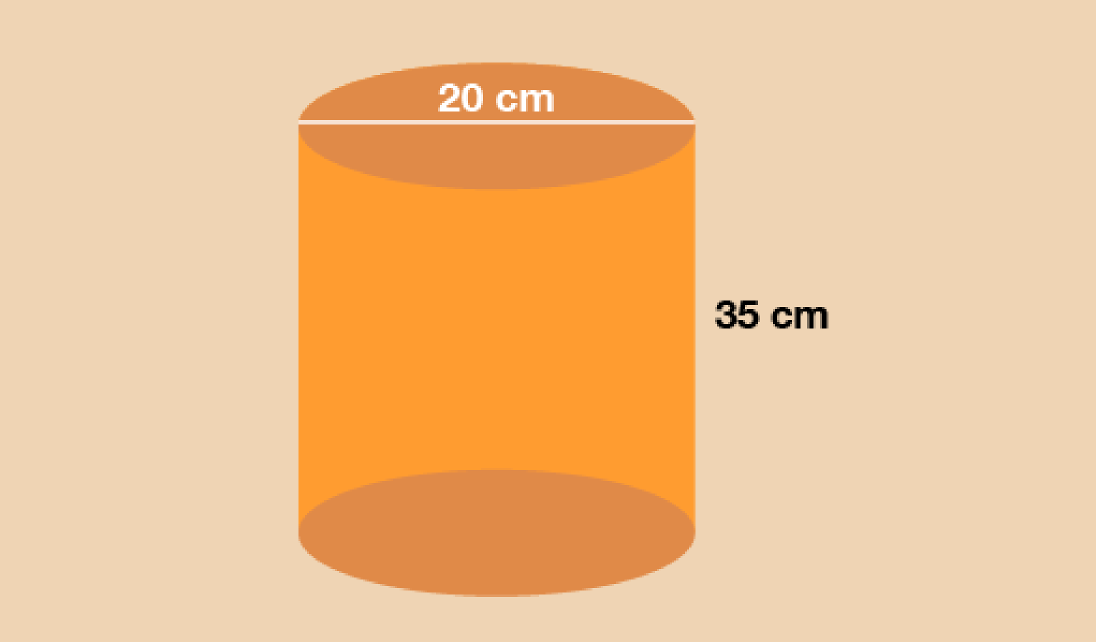

Figure 38 Circular prism (cylinder)

Answer

2. You can see that the circular cross-section has a diameter of 20 cm. To find the area of this circle, you need to find the radius first:

   radius = 20 ÷ 2 = 10 cm

   To find the area of the circular cross-section, use the formula:

   $A = \pi r^2$

   $A = 3.142 \times 10 \times 10$

   $A = 314.2\ \text{cm}^2$

   Now you have the area, multiply this by the length of the cylinder to calculate the volume:

   $V = 314.2 \times 35 = \underline{10\ 997\ \text{cm}^3}$

3.

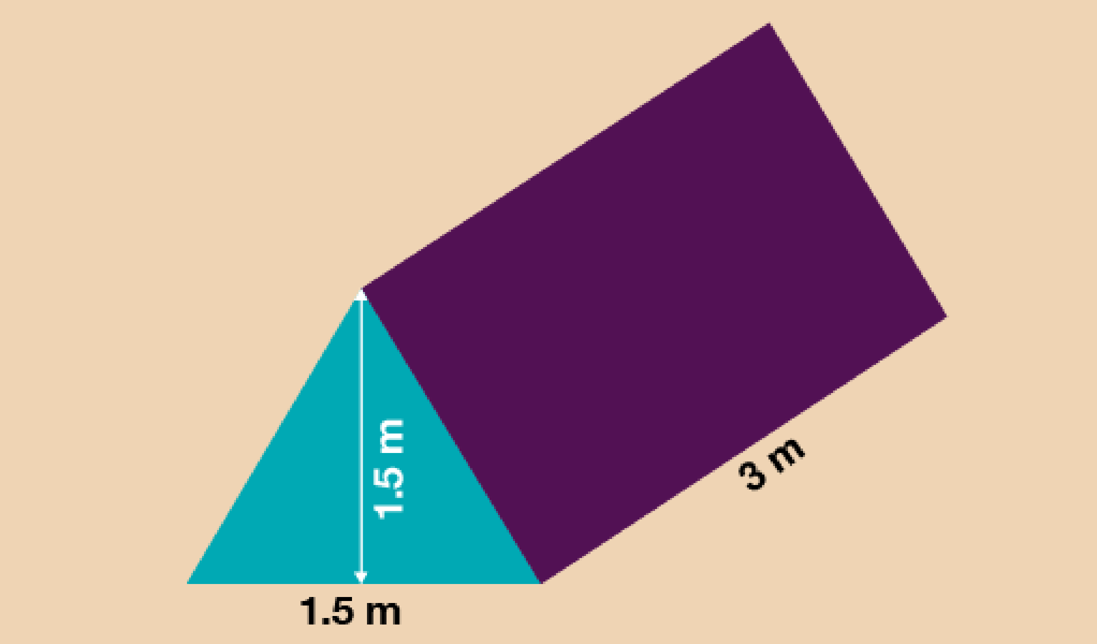

Figure 39 Triangular prism

Answer

3. The cross-section on this shape is the triangle at the front of the shape. To find the area of a triangle you use the formula:

   $A = (b \times h) \div 2$

   In this case then:

   $A = (1.5 \times 1.5) \div 2$

   $A = 2.25 \div 2$

   $A = 1.125\ \text{m}^2$

   Now you know the area of the cross-section, you multiply it by the length of the shape to find the volume (V):

   $V = 1.125 \times 3 = \underline{3.375\ \text{m}^3}$

4.

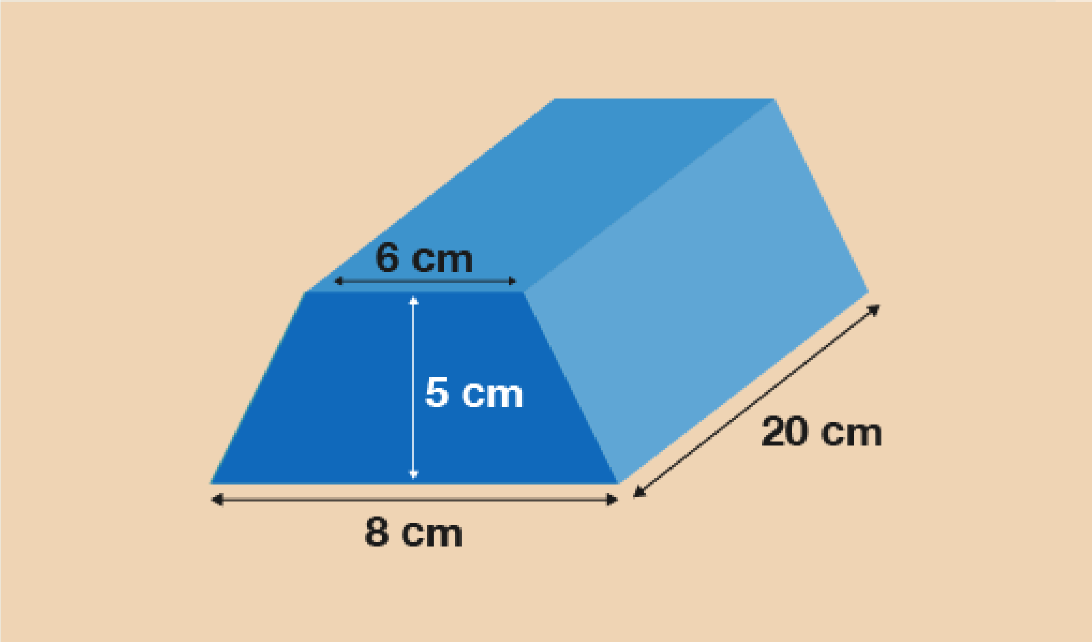

Figure 40 Trapezoidal prism

Hint: Go back to the section on area and remind yourself of the formula to find the area of a trapezium.

Answer

4. The cross-section is a trapezium so you will need the formula:

$$A = \frac{(a + b) \times h}{2}$$

$$A = \frac{(6 + 8) \times 5}{2}$$

$$A = \frac{14 \times 5}{2}$$

$$A = \frac{14}{2}$$

A = 35 cm^2

Now you have the area of the cross-section, multiply it by the length of the prism to calculate the volume:

V = 35 × 20 = <u>700 cm^3</u>

Summary

In this section you have learned:

- that volume is the space inside a 3D shape or space
- how to find the volume of prisms such as cuboids, cylinders and triangular prisms.

4 How many will fit?

You may need to work out how many tiles to buy to tile an area of wall or how many tins you can pack in a box. We will use the example below to illustrate.

Example 1: Tiling a wall

You are going to buy tiles measuring 0.75 m by 0.75 m to tile this area of wall:

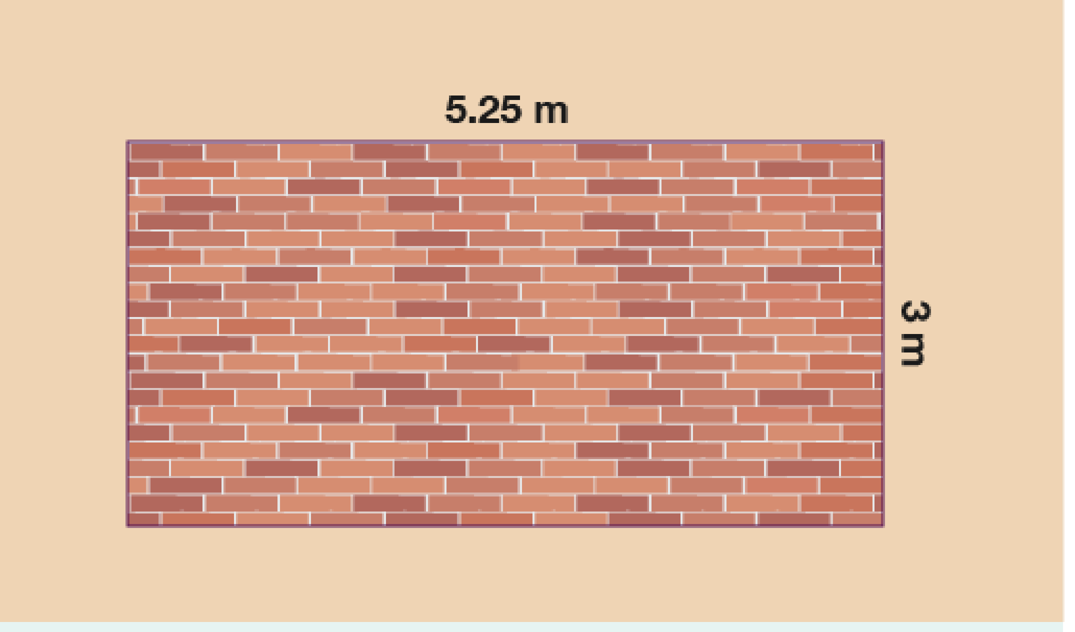

Figure 41 Wall measurements for tiling

How many tiles do you need to buy?

The easiest way to tackle a question such as this is to work out how many tiles will fit across the wall and how many will fit down the wall. You can then work out how many you need altogether.

The length across the wall is 5.25 m so we can divide by the length of the tile to work out how many will fit:

> 5.25 ÷ 0.75 = 7 tiles across

You may have preferred to convert the measurements into centimetres to avoid dividing by a decimal:

> 1 m = 100 cm, so
> 5.25 × 100 = 525 cm
> 0.75 × 100 = 75 cm
> 525 ÷ 75 = 7 tiles (you can see that the number of tiles is the same.)

Down the wall the measurement is 3 m, so down the wall you will fit:

> 3 ÷ 0.75 = 4 tiles (you may have converted to cm again and done
> 300 ÷ 75 = 4 tiles)

If 7 tiles will fit across the wall and 4 will fit down then altogether you will need:

7 × 4 = 28 tiles.

Example 2: Packing calculations

A shop is packing tins of varnish into boxes for delivery. The tins are cylindrical with a diameter of 7.5 cm and a height of 10 cm. The packing box is 60 cm long, 45 cm wide and 30 cm high.

How many tins can be packed into each box?

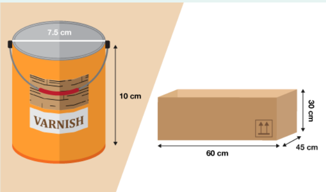

Figure 42 Dimensions of varnish tins and packing box

The easiest way to tackle this type of question is to work out how many tins you can fit in a single layer first and then to work out how many layers you can have. To work out how many tins will fit in a single layer use the same method you used above for working out how many tiles would fit.

The box is 60 cm long and tins are 7.5 cm wide:

60 ÷ 7.5 = 8 so 8 tins will fit across the box.

Hint: make sure you are working with the diameter and not the radius of the cylinder.

The box is 45 cm wide:

45 ÷ 7.5 = 6
so 6 tins would fit.

This would give 6 rows of 8 tins:

8 × 6 = 48
so 48 tins would fit in 1 layer.

Next you need to work out how many layers you could fit. The tins are 10 cm high and the box is 30 cm high:

30 ÷ 10 = 3
so the tins can be stacked 3 high:
3 × 48 = 144
The box would hold 144 tins.

Now try the following questions. Please carry out the calculations without a calculator. You can double check with a calculator if needed and remember to check your answers against ours.

Activity 8: How many will fit?

1. You want to re-carpet your floor using carpet tiles. Your floor measures 4.5 m by 6 m. The tiles you like measure 0.5 m by 0.5 m and cost £1.89 per tile.
   a. How many tiles will you need to buy?
   b. How much will they cost altogether?

Answer

1.
   a. You need to work out how many tiles will fit across the length of the floor and across the width of the floor.

      Length

      The floor is 6 m long. Each tile is 0.5 m by 0.5 m so to work out how many will fit do:

      6 ÷ 0.5 = 12 tiles

      You could also convert to centimetres:

      6 m = 6 × 100 = 600 cm

      0.5 m = 0.5 × 100 = 50 cm

      600 ÷ 50 = 12 tiles

      Width

      The floor is 4.5 m wide. Each tile is 0.5 m by 0.5 m so to work out how many will fit do:

      4.5 ÷ 0.5 = 9 tiles

      You could also convert to centimetres:

      4.5 m = 4.5 × 100 = 450 cm

      0.5 m = 0.5 × 100 = 50 cm

      450 ÷ 50 = 9 tiles

      So altogether you need:

      12 × 9 = <u>108 tiles</u>

   b. Each tile costs £1.89 and you need 108 tiles so the total cost of the tiles will be:

      1.89 × 108 = <u>£204.12</u>

2. How many 25 cm square tiles would you need to tile the floor area below?

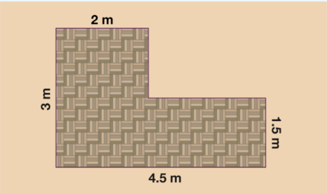

Figure 43 Area of floor for tiling

Answer

2. You need to split the floor up into 2 rectangles. You could do this in a couple of different ways.

   First method for splitting the floor as follows:

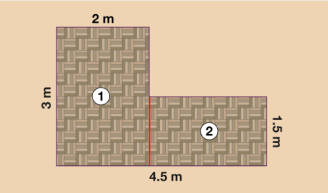

Figure 44 First method for splitting the floor

Rectangle 1

Rectangle ① is 3 m by 2 m. Convert the dimensions into centimetres:

3 m × 100 = 300 cm
2 m × 100 = 200 cm

The tiles are 25 cm square (25 cm by 25 cm) so you will need:

300 ÷ 25 = 12
200 ÷ 25 = 8
so for this part of the floor you will need:
12 × 8 = 96 tiles.

Rectangle 2

Rectangle ② is 1.5 m by 2.5 m (4.5 m − 2 m = 2.5 m). Convert the dimensions into centimetres:

1.5 m × 100 = 150 cm
2.5 m × 100 = 250 cm

The tiles are 25 cm square (25 cm by 25 cm) so you will need:

150 ÷ 25 = 6
250 ÷ 25 = 10
so for this part of the floor you will need:
6 × 10 = 60 tiles.

So altogether you will need:

60 + 96 = 156 tiles to tile the floor.

You may have split the floor area up differently.

Second method for splitting the floor as follows:

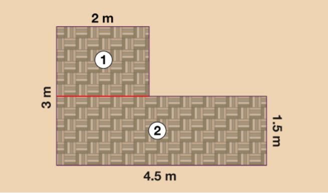

Figure 45 Second method for splitting the floor

Rectangle 1

Here rectangle ① is 1.5 m by 2 m (3 − 1.5 m = 2 m). Convert the dimensions into centimetres:

1.5 m × 100 = 150 cm

2 m × 100 = 200 cm

The tiles are 25 cm square (25 cm by 25 cm) so you will need:

150 ÷ 25 = 6

200 ÷ 25 = 8

so for this part of the floor you will need:

6 × 8 = 48 tiles

Rectangle 2

Here rectangle ② is 1.5 m by 4.5 m. Convert the dimensions into centimetres:

1.5 m × 100 = 150 cm

4.5 m × 100 = 450 cm

The tiles are 25 cm square (25 cm by 25 cm) so you will need:

150 ÷ 25 = 6

450 ÷ 25 = 18

so for this part of the floor you will need:

6 × 18 = 108 tiles

So altogether you will need:

48 + 108 = 156 tiles to tile the floor.

3. You are packing boxes of chocolates. Each chocolate box measures 23 cm long by 15 cm wide by 4 cm high and you are packing them into a storage box measuring 48 cm long by 30 cm wide by 34 cm high.

   What is the **maximum** number of chocolate boxes that can be packed in one storage box?

   Hint: The chocolate boxes need to be packed flat (horizontally) but you may want to try turning the chocolate box around.

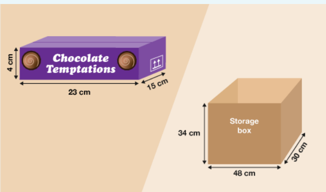

Figure 46 Chocolate boxes to fit into a storage box

Answer

3. The chocolate boxes need to be packed flat (horizontally). However, the boxes could be packed in 2 different ways:

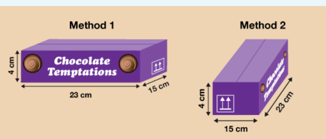

Figure 47 Method 1 and 2 for packing the chocolate boxes

Remember the dimensions of the storage box are as follows:

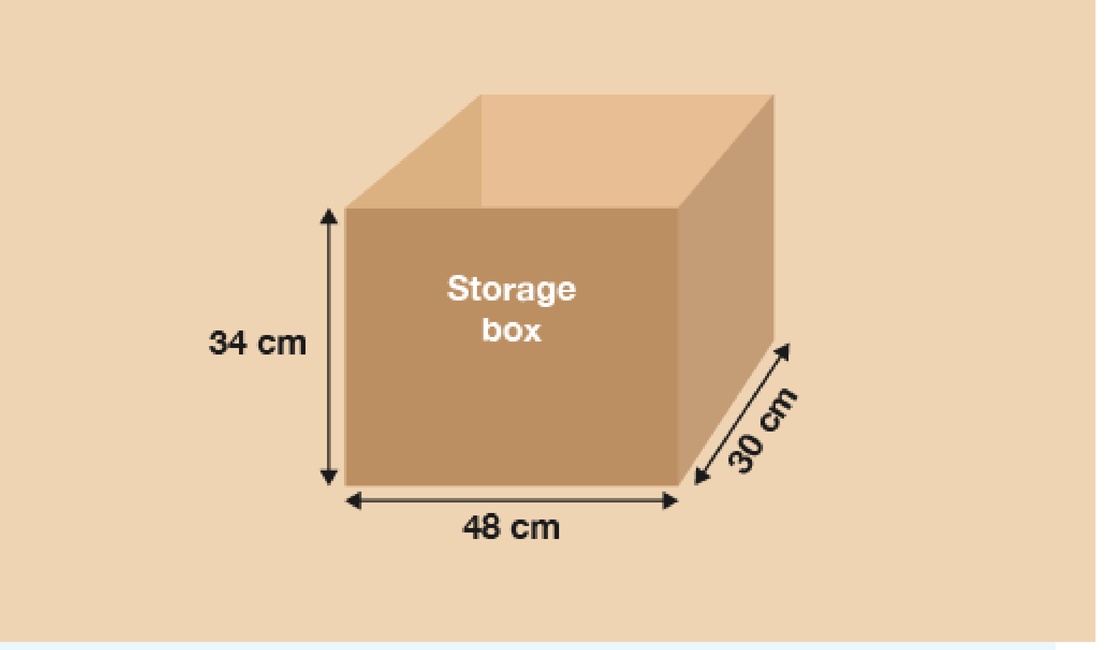

Figure 48 Storage box dimensions

Method 1

If you packed the chocolate boxes this way the storage box is 48 cm long and the chocolate boxes are 23 cm long so the calculation would be:

> 48 ÷ 23 = 2.1 (rounded to one d.p.)
>
> so only 2 chocolate boxes will fit.

The storage box is 30 cm wide and the chocolate boxes are 15 cm wide so:

> 30 ÷ 15 = 2
>
> so 2 chocolate boxes would fit.

This would give 2 rows of 2 chocolate boxes:

> 2 × 2 = 4
>
> so 4 chocolate boxes would fit in 1 layer.

Next you need to work out how many layers you could fit. The storage box is 34 cm high and the chocolate boxes are 4 cm high:

> 34 ÷ 4 = 8.5
>
> so 8 chocolate boxes will fit.

The chocolate boxes can be stacked 8 high so:

> 8 × 4 = 32

If using Method 1 the storage box would hold 32 chocolate boxes.

Method 2

If you packed the chocolate boxes this way the calculation would be as follows:

> 48 ÷ 15 = 3.2 so 3 boxes will fit
>
> 30 ÷ 23 = 1.3 (rounded to one d.p.) so only 1 will fit

In one layer, you could fit 3 × 1 = 3 chocolate boxes.

Next you need to work out how many layers you could fit. The storage box is 34 cm high and the chocolate boxes are 4 cm high:

> 34 ÷ 4 = 8.5
>
> so only 8 chocolate boxes will fit.

The chocolate boxes can be stacked 8 high:

$8 \times 3 = 24$

If using Method 2 the storage box would hold 24 chocolate boxes.

Therefore, the maximum number of chocolate boxes that could fit into the storage box is 32 and you would have to pack them the first way (Method 1) to achieve this.

4. A factory producing jars of jam packs the jars into boxes that measure 42 cm long, 42 cm wide and 45 cm high. The jars of jam have a radius of 3.5 cm and a height of 11 cm.

   How many jars of jam can be packed into a box? The jars must be packed upright.

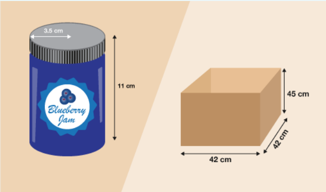

Figure 49 Jam jars (left) to fit into a packaging box (right)

Answer

4. First you need to calculate the diameter (width) of the jars. The radius of the jars is 3.5 cm:

   $3.5 \times 2 = 7$

   so the diameter of the jars is 7 cm.

   The box is 42 cm wide and jars are 7 cm wide:

   $42 \div 7 = 6$

   so 6 jars will fit across the box.

   The length of the box is also 42 cm so you know that there will be 6 rows of 6 jars on the bottom layer.

   $6 \times 6 = 36$

   so 36 jars will fit in 1 layer.

   Next you need to work out how many layers you could fit. The jars are 11 cm high, and the box is 45 cm high:

   $45 \div 11 = 4.09$ to two d.p.

   so the tins can be stacked 4 high.

   $4 \times 36 = 144$

   So 144 jars of jam would fit in each box.

The final part of his section will look at scale drawings and plans. It will require your previous knowledge of ratio as scale drawings are really just another application of ratio. The good news then, is that you already know how to do it!

Summary

In this section you have:

- learned to calculate how many smaller shapes or items will fit into larger areas or spaces.

5 Scale drawings and plans

If you completed *Everyday maths 1*, you will be familiar with the idea of scale. Scales are found on drawings, plans and maps and they are often written with the units indicated. Let's look at an example.

A football pitch is drawn to the scale of 1 cm to 5 m.

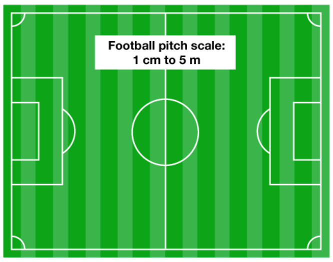

Figure 50 A football pitch drawn at 1 cm to 5 m

This means that every 1 cm measured on the plan is 5 m in real life.

If the plan is drawn with the length being 18 cm and the width being 9 cm, what are the dimensions of the football pitch in real life?

Write down the scale first:

1 cm to 5 m

You know the drawing dimensions so you need to work with these one at a time. Let's start with the length of 18 cm:

If the scale is 1 cm to 5 m then
18 cm = ? m.
If you have been given the drawing measurement and need to know the real life measurement, you multiply:
$18 \times 5 = 90$ m
so the length is 90 m.

Note: If you have been given the actual measurement and need to find the drawing measurement you would need to divide.

Now you can work out the width measurement:

If the scale is 1 cm to 5 m then
9 cm = ? m.
Again, you need to multiply:
$9 \times 5 = 45$ m
so the width is 45 m.

A lot of scales are written differently, without the units indicated. The scale of 1 cm to 5 m could also be written as 1:500.

This is the scale expressed as a ratio and it is independent of any units. A scale of 1:500 means that the actual real-life measurements are 500 times greater than those on the plan or map. This means that it does not matter whether you take the measurements on the plan in millimetres (mm), centimetres (cm) or metres (m) – the measurements will be 500 times as much in real life.

To write a scale as a ratio, you often have to convert. Let's look at the football pitch example again:

1 cm to 5 m

At the moment, the units of the scale are different. The plan side is given in centimetres (cm) and the real-life side is given in metres (m).

To express this as a ratio, you need to convert both sides to the same units. It is usually easiest to convert the real-life side of the scale into the same unit as the drawing side, so in this case it is easiest to convert 5 m into cm:

$5 \times 100 = 500$ cm

So you can now write the scale as a ratio:

1:500

It is standard to try to write the ratio in the simplest form possible, ideally with a single unit (a '1') on the drawing side of the ratio. This will make any calculations you do using the scale easier.

Now have a go a converting scales to ratios.

Activity 9: Writing a scale as a ratio

Rewriting these scales as a ratio in their simplest form:

1. 1 cm to 2 m
2. 2 cm to 5 m
3. 10 mm to 20 m
4. 1 cm to 1 km
5. 5 cm to 2 km

Answer

1. It is easiest to change the 2 m into cm:

   $2 \times 100 = 200$ cm so the scale expressed as a ratio would be **1:200**.
2. It is easiest to change the 5 m into cm:

   $5 \times 100 = 500$ cm so the scale expressed as a ratio could be written as:

   2:500

   However, we usually try to get the drawing side of the ratio down to a single unit (1) to make calculations easier. Therefore, you need to simplify the ratio. To do this here, divide both sides by 2:

   $2 \div 2 = 1$

500 ÷ 2 = 250 so the scale can be written as:

1:250

3. It is easiest to change the 20 m into mm. It might be easiest to do this in stages:

   Convert to cm first –

   1 m = 100 cm so 20 × 100 = 2000 cm

   Now convert to mm –

   1 cm = 10 mm so 2000 × 10 = 20 000 mm

   This makes the scale:

   10:20 000

   This can be simplified by dividing both sides by 10 to get:

   1:2000

4. Change the 1 km into cm. Again, this will be easiest to do in stages:

   Convert to m first –

   1 km = 1000 m so 1 × 1000 = 1000 m

   Now convert to cm –

   1 m = 100 cm so 1000 × 100 = 100 000 cm

   This means the scale should be written as:

   1:100 000

5. Change the 2 km into cm. In stages this can be done as follows:

   Convert to m first –

   1 km = 1000 m so 2 × 1000 = 2000 m

   Now convert to cm –

   1 m = 100 cm so 2000 × 100 = 200 000 cm

   This makes the scale:

   5:200 000

   This can be simplified by dividing both sides by 5 to get:

   1:40 000

Now you will look at using ratio scales to work out measurements.

5.1 Scale drawing method and problems

First watch the scale drawings example video.

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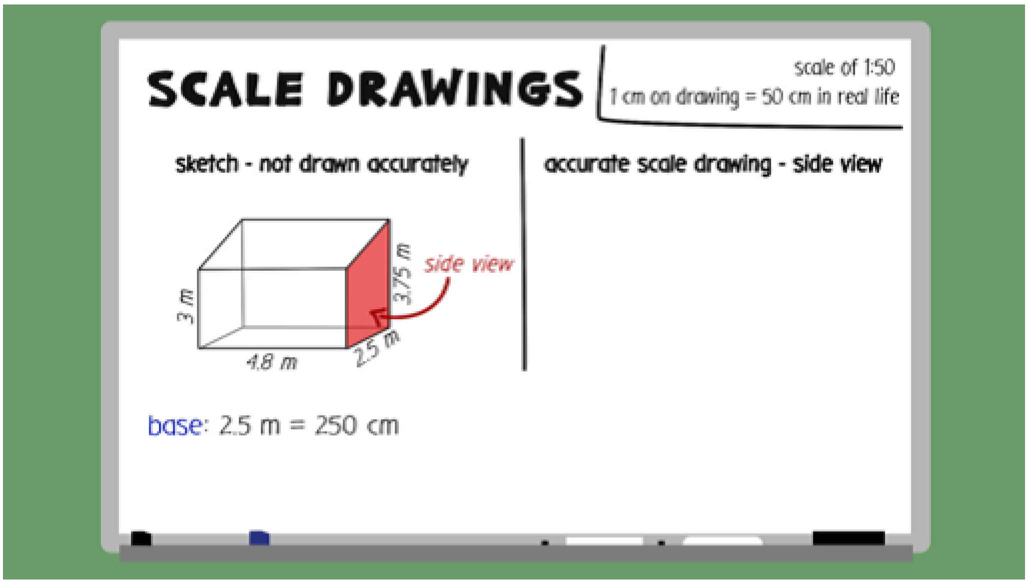

Summary of method

Remember if you are given the scale drawing measurement and asked to work out the real-life size, you need to *multiply*.

If you are given the real-life size and asked to work out the drawing measurement, you need to *divide*.

Example 1: Plan of a room

Rhodri has drawn a scale diagram of his living room. The scale he has used is 1:20.

On his diagram his living room is 30 cm long and 20 cm wide. What are the actual dimensions of his living room?

You need to work out the length and width separately.

Length

The scale he has used is 1:20, which means that everything is 20 times bigger in real life than on his diagram. You know the drawing measurement (30 cm) so you need to multiply by 20 to find the actual length of the room:

30 cm × 20 = 600 cm

It is a good idea to express the actual measurement in metres:

600 cm = 6 m

Width

You need to use the same scale of 1:20. If the width of the living room on the drawing is 20 cm then it will be 20 times as great in real life:

20 cm × 20 = 400 cm (400 cm = 4 m)

So Rhodri's actual living room measures 6 m by 4 m.

Now have a go at solving these problems involving scale. Do the calculations without a calculator. You may double-check on a calculator if you need to and make sure you check your answers against ours.

Activity 10: Scale problems

1. A scale drawing has been drawn below of a shed that a garden planner wants to build. The scale used for the drawing is 1:25.

   The area that the shed will be built on is a rectangle which measures 5.1 m by 4 m. Will the shed fit into the space allocated?

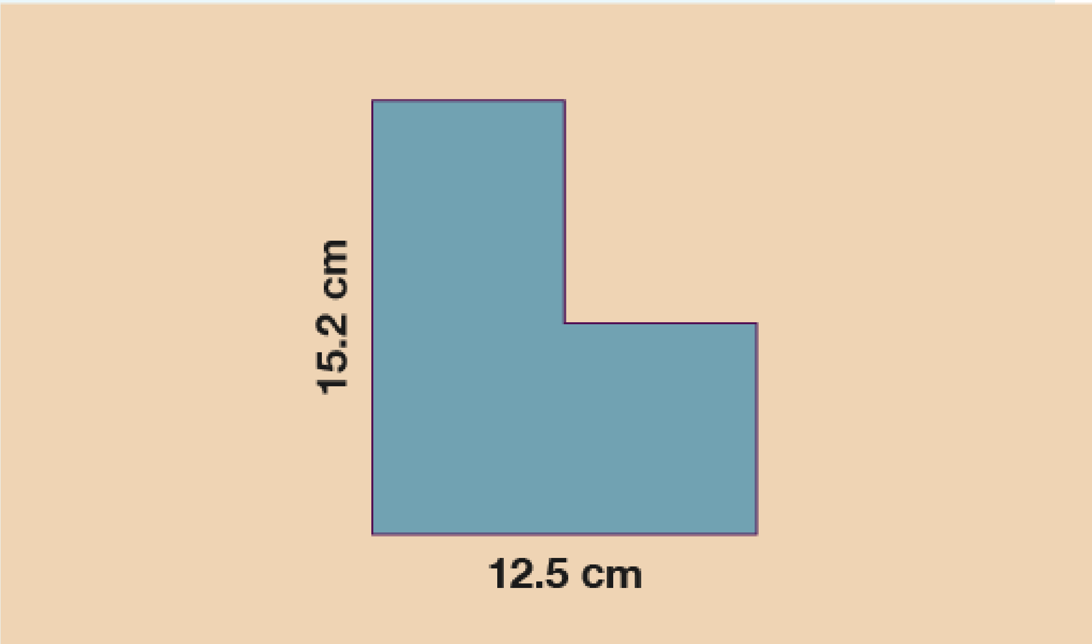

Figure 51 Calculating scale 1

Answer

1. You know the scale is 1:25, so 1 cm on the diagram represents 25 cm in real life. You have been given the measurements on the diagram and want to work out the actual measurements so you need to multiply.

   Horizontal length:

   12.5 × 25 = 312.5 cm = 3.125 m

   Vertical length:

   15.2 × 25 = 380 cm = 3.8 m

   Since both lengths for the shed are shorter than the lengths given for the area of land, you know the shed will fit.

2. A landscaper wants to put a wild area in your garden. She makes a scale plan of the wild area:

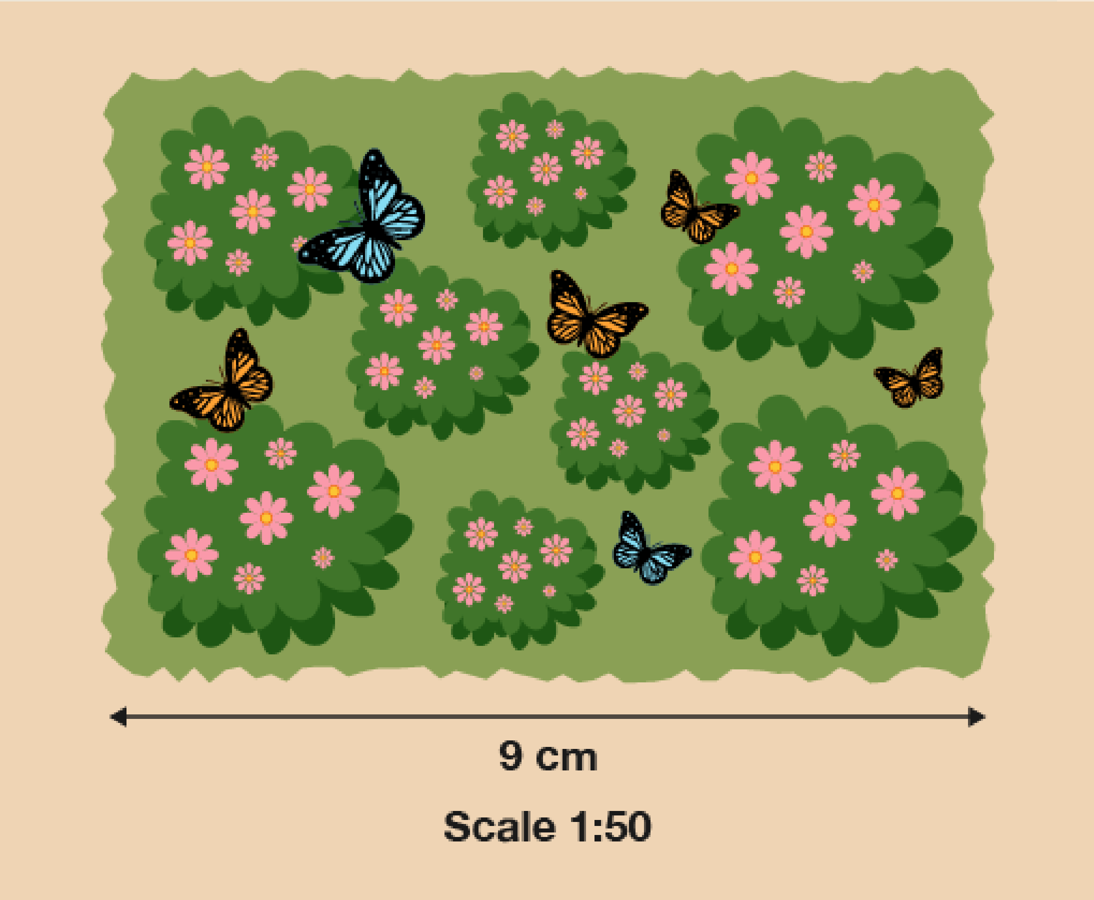

Figure 52 Calculating scale 2

What is the length of the longest side of the actual wild area in metres?

Answer

2. The length on the drawing is 9 cm, and the scale is 1:50. This means that 1 cm on the drawing is equal to 50 cm in real life. So to find out what 9 cm is in real life, you need to multiply it by 50:

   9 × 50 = 450 cm

   The question asks for the length in metres, so you need to convert centimetres into metres:

   450 ÷ 100 = 4.5 m

   The actual length of the wild area will be 4.5 m.

3. Here is a scale drawing showing one disabled parking space in a supermarket car park. The supermarket plans to add two more disabled parking spaces either side of the existing one.

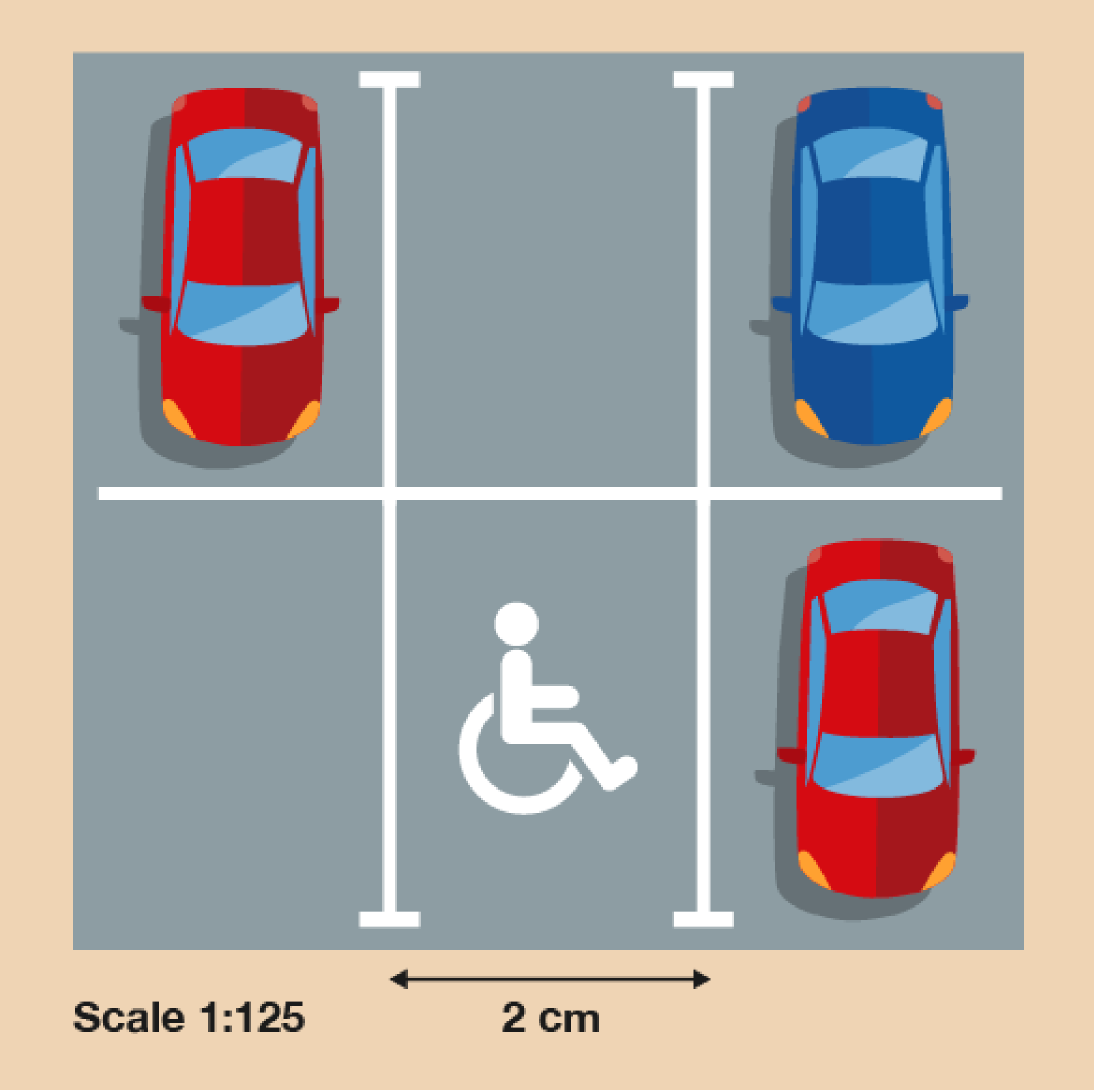

Figure 53 Calculating scale 3

What will be the total actual width of the three disabled parking spaces in metres?

Answer

3. You need to find out the width of three disabled parking spaces. The width of one parking space on the scale drawing is 2 cm, so first you need to multiply this by 3:

   2 × 3 = 6 cm

   The scale is 1:125. This means that 1 cm on the drawing is equal to 125 cm in real life. So to find out what 6 cm is in real life, you need to multiply it by 125:

   6 × 125 = 750 cm

   The question asks for the length in metres, so you need to convert centimetres into metres:

   750 ÷ 100 = 7.5 m

   The actual width of all three parking bays will be 7.5 m.

4. For this question you will need a pen or pencil, paper and a ruler.

Jane has a raised vegetable patch. She plans to build a slope leading up to the vegetable patch. Jane will cover the slope with grass turf.

She draws this sketch of the cross-section of the slope. The measurements indicated are the actual measurements.

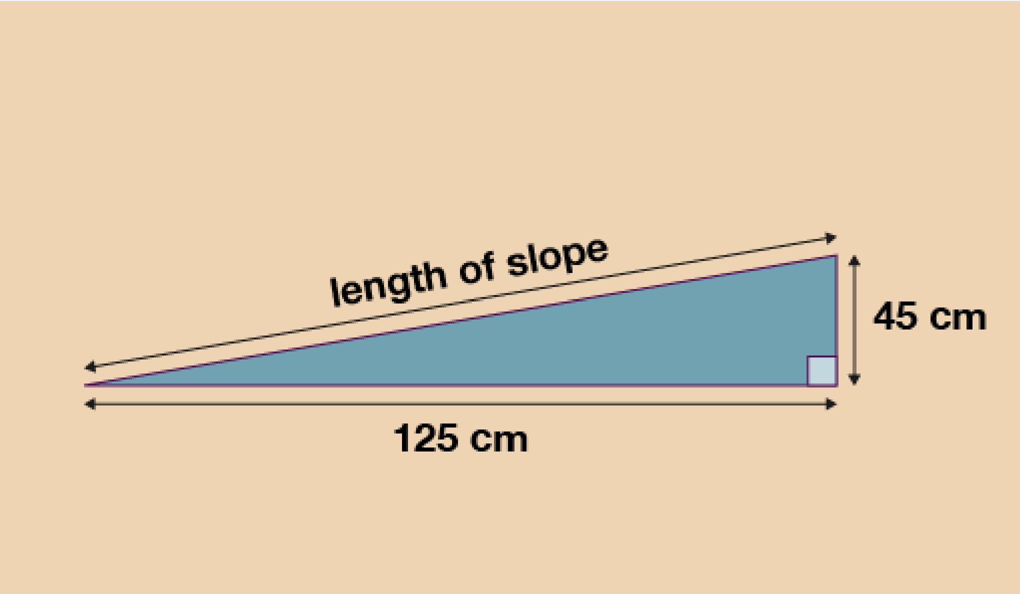

Figure 54 Finding the length of a slope

Jane will use a scale diagram to work out the length of the slope. She wants to use a scale of 1:10.

d. What will the measurements of the base and height of the slope be on her diagram?

e. Draw her scale diagram using the measurements you calculated in part (a).

   (i) What will the length of the slope be on the diagram?

   (ii) What will it be in real life?

Answer

4.

d. The scale is 1:10 and you want to work out the measurements for the scale diagram, so you need to divide the real-life measurements by 10:

   Base of patch = 125 ÷ 10 = 12.5 cm

   Height of patch = 45 ÷ 10 = 4.5 cm

e. (i) Draw the scale diagram with the base measuring 12.5 cm and the height measuring 4.5 cm.

   Now draw in your slope linking the end points of the base and height together. If you measure the length of the slope with a ruler it should measure around 13.2 cm on the diagram.

   (ii) To work out the length of the actual slope, you need to use the scale of 1:10 again, but this time you need to multiply by 10 to work out the actual slope length:

13.2 cm × 10 = 132 cm in real life.

Your answer may vary slightly from ours but should be within a reasonable range.

Summary

In this section you have:

- applied your ratio skills to the concept of scale plans and drawings
- interpreted scale plans.

Well done! You have now completed this section of your course. You are now ready to test the knowledge and skills you've learned in the end of session quiz. Good luck!

6 Session 3 quiz

Now it’s time to review your learning in the end-of-session quiz.

Session 3 quiz.

Open the quiz in a new window or tab (by holding ctrl [or cmd on a Mac] when you click the link), then return here when you have done it.

7 Session 3 summary

Well done! You have now completed Session 3 ‘Shape and space’. If you have identified any areas that you need to work on, please ensure you refer back to this section of the course.

You should now be able to:

- understand the difference between perimeter, area and volume and be able to calculate these for both simple and more complex shapes
- know that volume is a measure of space inside a 3D object and calculate volumes of shapes in order to solve practical problems
- draw and use a scale drawing or plan.

All of the skills above will help you with tasks in everyday life. Whether you are at home or at work, number skills are essential skills to have.

You are now ready to move on to Session 4, ‘Handling data’.

Session 4: Handling data

Introduction

Data is a big part of our lives and can be represented in many different ways. This session will take you through a number of these representations and show you how to interpret data to find specific information. Before delving into the world of charts, graphs and averages though, it is important to make the distinction between the two different types of data:

> **Qualitative data** – this is data that is not numerical e.g. eye – colours, favourite sports, models of cars.
>
> **Quantitative data** – this is numerical data related to things that can be counted or measured e.g. temperatures, house prices, GCSE grades. Quantitative data can be *discrete* or *continuous*.

By the end of this session you will be able to:

- identify different types of data
- create and use tally charts, frequency tables and data collection sheets to record information
- draw and interpret bar charts, pie charts and line graphs
- understand there are different types of averages and be able to calculate each type
- understand that probability is about how likely an event is to happen and the different ways that it can be expressed.

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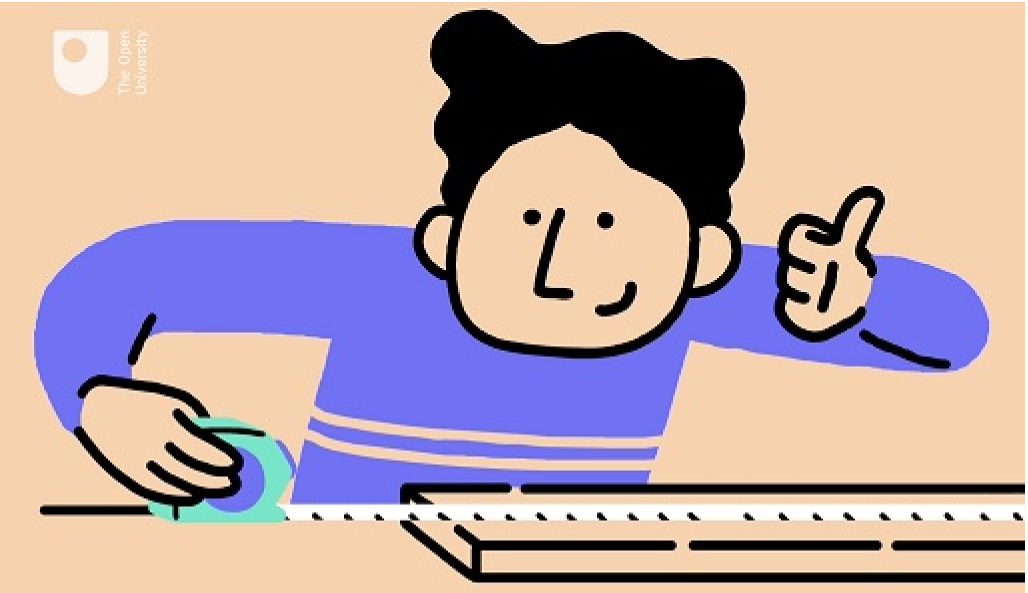

1 Discrete and continuous data

Discrete data is information that can only take certain values. These values don't have to be whole numbers (a child might have a shoe size of 3.5 or a company may make a profit of £3456.25 for example) but they are fixed values – a child cannot have a shoe size of 3.72!

The number of each type of treatment a salon needs to schedule for the week, the number of children attending a nursery each day or the profit a business makes each month are all examples of discrete data. This type of data is often represented using tally charts, bar charts or pie charts.

Continuous data is data that can take any value. Height, weight, temperature and length are all examples of continuous data. Some continuous data will change over time; the weight of a baby in its first year or the temperature in a room throughout the day. This data is best shown on a line graph as this type of graph can show how the data changes over a given period of time. Other continuous data, such as the heights of a group of children on one particular day, is often grouped into categories to make it easier to interpret.

You will have looked at different ways of presenting data in *Everyday maths 1*. Have a go at the next activity to see what you can remember.

Activity 1: Presenting discrete and continuous data

Match the best choice of graph for the data below.

1. Chart to show the heights of children in a class.
2. Chart to show favourite drink chosen by customers in a shopping centre.
3. Chart to show the temperature on each day of the week.
4. Chart to show percentage of each sale of ticket type at a concert.

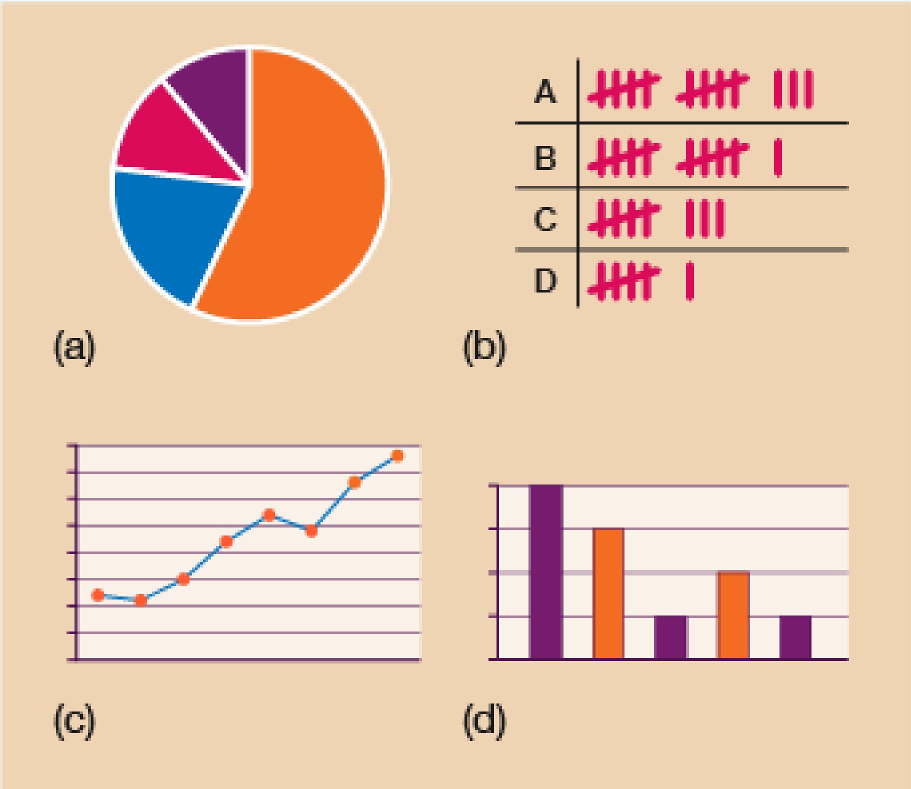

Figure 1 Different types of charts and graphs

Answer

1. Chart to show the heights of children in a class.

   The best choice here is **(d)** the bar chart as it can show each child's height clearly.
2. Chart to show favourite drink chosen by customers in a shopping centre.

   The best choice here is **(b)** the tally chart since you can add to this data as each customer makes their choice. A bar or pie chart would also be suitable.
3. Chart to show the temperature on each day of the week.

   The only choice here is **(c)** the line graph as it shows how the temperature changes over time.
4. Chart to show percentage of each sale of ticket type at a concert.

   The best choice here is probably **(a)** the pie chart since it shows clearly the breakdown of each type of ticket sale. A bar chart would also represent the data suitably.

Now that you are familiar with the two different types of data let's look in more detail at the different types of chart and graph; how to draw them accurately and how to interpret them.

Summary

In this section you have:

- learned about the two different types of data, discrete and continuous, and when and why they are used.

2 Tally charts, frequency tables and data collection sheets

Have you ever been stopped in the street or whilst out shopping and asked about your choice of mobile phone company or to sample some food or drink and give your preference on which is your favourite? If you have, chances are, the person who was conducting the survey was using a tally chart to collect the data.

Figure 2 Favourite fruit tally chart

Tally charts are convenient for this type of survey because you can note down the data as you go. Once all the data has been collected it can be counted up easily because, as shown in the picture above, every fifth piece of data for a choice is marked as a diagonal line. This allows you to count up quickly in fives to get the total. A frequency or total column can then be filled out to make the data easier to work with.

Take a look at the example below:

Table 1

Method of Travel	Tally	Frequency
Walk	卌 ||||	9

Bike	III	3
Car	卌 I	6
Bus	卌 卌 II	12
	TOTAL	30

You can see each category and its total, or frequency, clearly. Whilst this is a very simple example it demonstrates the purpose of a tally chart well. A tally chart may often be turned into a bar chart for a more visual representation of the data but they are useful for the actual data collection.

You can also use a tally chart for collecting grouped data. If for example, you want to survey the ages of clients or customers, you would not ask for each person's individual age, you would ask them to record which age group they came within. If you want to set yourself up a tally chart for this data it might look similar to the below:

Table 2

Age	Tally	Frequency
0–9		
10–19		
20–29		
30–39		
40–49		
50–59		
60–69		
70+		

Note that the age groups do not overlap; a common mistake would be to make the groups 0–10, 10–20 and so on. This is incorrect because if you were aged 10, you would not know which group you should place yourself in.

A more complex example of a tally chart can be seen below. In this example you can see that the information that has been collected is split into more than one category. These are sometimes called data summary sheets or data collection tables. It does not matter where each category is placed on the chart as long as all aspects are included.

Table 3

	fewer than 6 trips		6 trips or more	
	under 26 years	26 years and over	under 26 years	26 years and over
male	I (1)	(0)	III (3)	~~IIII~~ I (6)
female	III (3)	(0)	I (1)	IIII (4)

If you want to design a data collection or data summary sheet, you first need to know which categories of information you are looking for. Let's take a look at an example of how you might do this.

Imagine you work in a hotel and want to gather some data on your guests. You want to know the following information:

- Rating given by the guest: excellent, good, or poor.
- Length of stay: under 5 days, or 5 days or more.
- Location: from the UK, or from abroad.

A data summary sheet for this information could look like this:

Table 4

	Stayed for under 5 days			Stayed for 5 days or more		
	Excellent	Good	Poor	Excellent	Good	Poor
From the UK						
From abroad						

Have a go at the activity below and try creating a data collection sheet for yourself.

Activity 2: Data collection

You work at a community centre and want to gather some data on the people who use your services. You would like to know the following information:

- whether they are male or female
- if they use the centre during the day or during the evening
- which age category they are in: 0–25, 26–50, 51+.

Design a suitable data collection sheet to gather the information.

Answer

Your chart should look something like the example below. You may have chosen to put the categories in different rows and columns, which is perfectly fine. As long as all the options are covered your data collection sheet will be correct.

Table 5

	Daytime Visitor			Evening Visitor		
	0–25	26–50	51 +	0–25	26–50	51 +
Male						
Female						

Now that you know the different ways in which you can collect your data, it's time to look at how you can put this information into various charts and graphs.

Summary

In this section you have:

- explored the differences between tally charts, frequency tables and data collection sheets and understood their usefulness in data collection.

3 Bar charts

There are two main types of bar charts (also known as bar graphs) – single and dual. The most common type is a single bar chart. These bar charts use vertical or horizontal bars to display data as seen below.

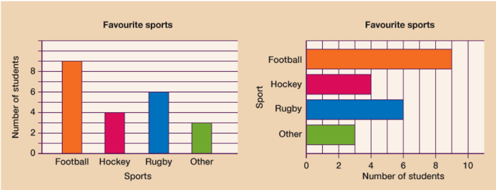

Figure 3 Vertical and horizontal single bar charts

A comparative bar chart (also known as a dual bar chart) shows a comparison between two or more sets of data. Whilst the chart below shows the favourite sports of a group of students, it further breaks down the data and gives a comparison between boys and girls; there are two bars for each sport. Depending on the type of data you are collecting, you could have more than two bars under each category.

Looking at the example comparative bar chart below, we can see that 6 girls said their favourite sport was football compared to only 3 boys.

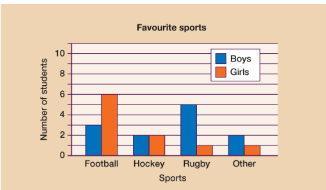

Figure 4 Dual bar chart

A third way of presenting data is a component bar chart. Instead of using separate bars, all the data is represented in a single bar. Each component, or part, of the bar is coloured differently. A key is again needed to show what each colour represents. Look at the example below:

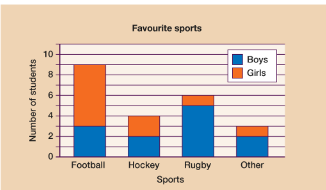

Figure 5 Single (component) bar chart

A component bar chart is useful for displaying data where we want to see the overall total together with a breakdown. In the chart above, we can easily see that in total 9 students said that football was their favourite sport. However, we can also see the data broken down to show us that 3 boys and 6 girls said that football was their favourite sport.

3.1 Features of a bar chart

Regardless of which type of bar chart you are drawing, there are certain features that all charts should have. Look at the diagram below to learn what they are.

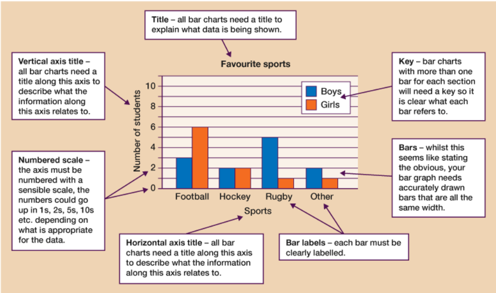

Figure 6 Features of a bar chart

If you are drawing a graph or chart by hand, you will need to use graph paper. However, creating a graph on a computer is much quicker and more convenient than doing it by hand. The computer will select the scale for the graph and it will plot it accurately. All you need to do is make sure you choose the best type of graph for your data and remember to label it all clearly and appropriately.

Activity 3: Drawing a bar chart

1. Draw a bar chart to represent the information shown in the table below. You may hand-draw your graph or create it using a computer package.

Table 6 Leisure centre users

	January	February	March	April
Male	175	154	120	165
Female	205	178	135	148

Remember to choose a suitable scale for your chart and ensure that it has a title and axis labels. It will also need a key.

Answer

1. Your bar chart should look something like the example below. You may have chosen to use a slightly different numbering scale but your chart should have the following features:
   - a title
   - labels on both the horizontal and vertical axes
   - bars labelled with the month
   - accurately drawn bars of the same width
   - a numbered scale on the vertical axis
   - a key to show which bars are for males and which are for females.

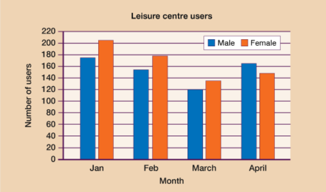

Figure 7 A bar chart showing numbers of leisure centre users

2. Draw a bar chart to represent the data shown in the table below. The numbers are much bigger than in the previous example so if you are drawing your chart by hand, you will need to consider your scale carefully.

Table 7 Food sales at a three-day summer music festival

Food types	Number of portions sold
Sandwiches and baguettes	37 000
Burgers	25 000
Fish and chips	16 000
Ice cream cones	12 000
Other	8 500

Answer

2. Your bar chart should look something like the example below.

   Note the use of gridlines to help with interpreting the data in the graph. Also note how the scale and label on the vertical axes (see the second example on the right-hand side of the graph) are slightly different. Both formats are shown here as different examples and either would be acceptable.

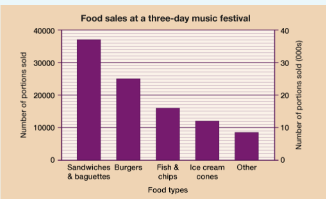

Figure 8 A bar chart showing food sales at three-day music festival

Your bar chart should be similar to the above and include:

- a title
- labels on both the horizontal and vertical axes
- bars labelled with the food types
- accurately drawn bars of the same width
- a numbered scale on the vertical axis, which goes up in equal intervals.

Now that you understand the features of bar charts and are able to draw them, let's look at how to interpret the information they show.

3.2 Interpreting bar charts

If you see a bar chart that is showing you information, it's useful to know how to interpret the information given. The most important thing you need to read to understand a bar chart correctly is the scale on the vertical axis.

As you know from drawing bar charts yourself, the numbers on the vertical axis can go up in steps of any size. You therefore need to work out what each of the smaller divisions is worth before you can use the information. The best way for you to learn is to practise looking at, and reading from, different bar charts.

Have a go at the activity below and see how you get on.

Activity 4: Interpreting a bar chart

The chart below shows the average daily viewing figures for Sky Cinema channels during 11–17 March 2019. Answer the questions that follow. Choose the correct answer in each case.

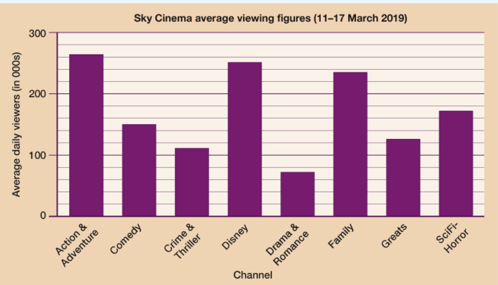

Figure 9 Sky Cinema average viewing figures (11–17 March 2019)

1. Which was the most popular channel?
    - a. Action & Adventure
    - b. Disney
    - c. Family

Answer

a. Action & Adventure

2. Approximately how many viewers tuned in to Sky Cinema Disney?
    - b. 250 viewers
    - c. 25 000 viewers
    - d. 250 000 viewers

Answer

c. 250 000 viewers – note how the vertical axis label states that the figures are in 000s (thousands) which means that the scale goes up in intervals of 20 000. This means the Disney bar goes up to 250 000.

3. Approximately how many more viewers watched Sky Cinema SciFi-Horror than Sky Cinema Drama & Romance?
   - c. 100 000
   - d. 80 000
   - e. 100

Answer

a. 100 000 – Sky Cinema SciFi-Horror had appoximately 175 000 viewers whilst Sky Cinema Drama & Romance had approximately 75 000 viewers, making a difference of 100 000 viewers.

Activity 5: Interpreting a comparative bar chart

The chart below shows the average house prices by local authority in Wales for 2017 and 2018. Answer the questions that follow.

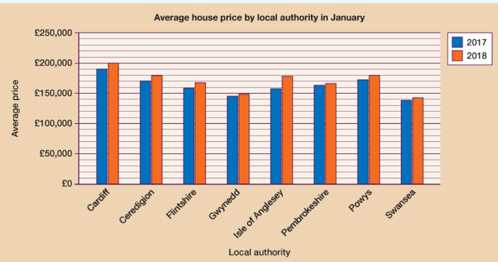

Figure 10 Average house price by local authority in January

1. Which local authority had the biggest average house price increase between 2017 and 2018?

Answer

Isle of Anglesey

2. Which of the following is the best estimate of the difference between Cardiff and Swansea's 2018 average house prices?
   - b. £50 000
   - c. £60 000
   - d. £120 000

Answer

b. £60 000 – note how the scale goes up in intervals of £10 000. In 2018 Cardiff's average house price was approximately £200 000 compared to Swansea's average house price of approximately £140 000, making a difference of £60 000.

Activity 6: Interpreting a component bar chart

The following chart shows the number of enrolments for different college courses. Answer the questions that follow.

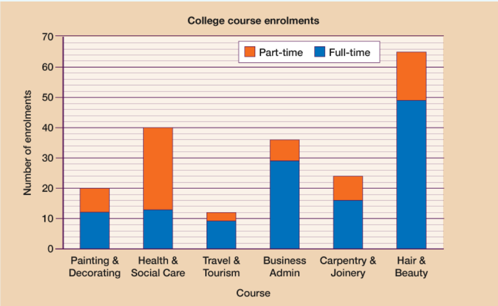

Figure 11 College course enrolments

1. Which course had the fewest number of part-time enrolments?

Answer

Travel & Tourism

2. Which course had more part-time than full-time enrolments?

Answer

Health & Social Care

3. How many full-time enrolments did Hair & Beauty have?

Answer

49.

Note how the scale goes up in 2s and the full-time component of the Hair & Beauty bar goes up to halfway between 48 and 50.

4. How many part-time enrolments did Business Admin have?

Answer

7

5. Approximately how many course enrolments were there in total?

Answer

197 – You need to count the total enrolments for all of the separate courses and then add them together:

- Painting & Decorating = 20
- Health & Social Care = 40
- Travel & Tourism = 12
- Business Admin = 36
- Carpentry & Joinery = 24
- Hair & Beauty = 65

20 + 40 + 12 + 36 + 24 + 65 = 197

You should now be feeling confident with both drawing and interpreting bar charts so it's time to move on to look at another type of chart – pie charts.

Summary

In this section you have learned:

- about the different features of a bar chart
- to use single and comparative bar charts to present data
- to interpret the information shown on different types of bar chart.

4 Pie charts

A pie chart is the best way of showing the proportion (or fraction) of the data that is in each category. It is very easy to visually see the largest and smallest sections of the chart and how they compare to the other sections.

This section will help you to understand when a pie chart should be used over other presentation methods. In the first part of this section you'll learn how to draw and then move on to interpreting pie charts.

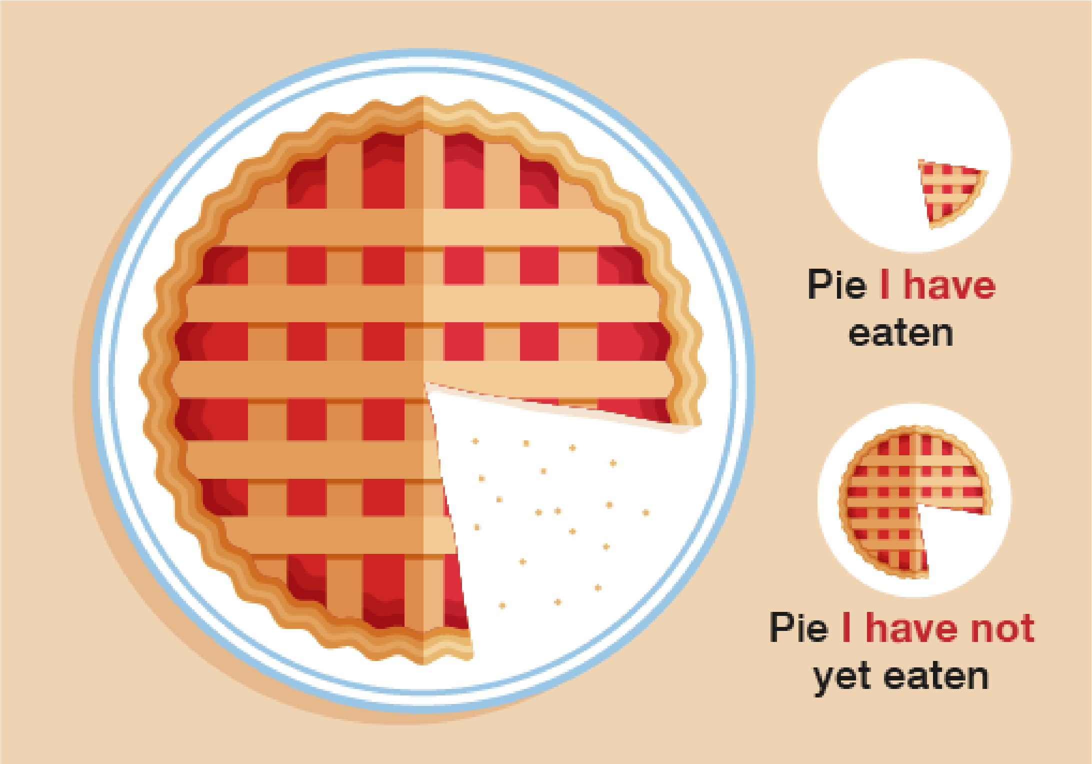

Figure 12 A partially eaten pie

In the first part of this section you'll learn how to draw and then move on to interpreting pie charts.

4.1 Drawing pie charts

The best way to understand the steps involved in drawing a pie chart is to watch the worked example in the video below.

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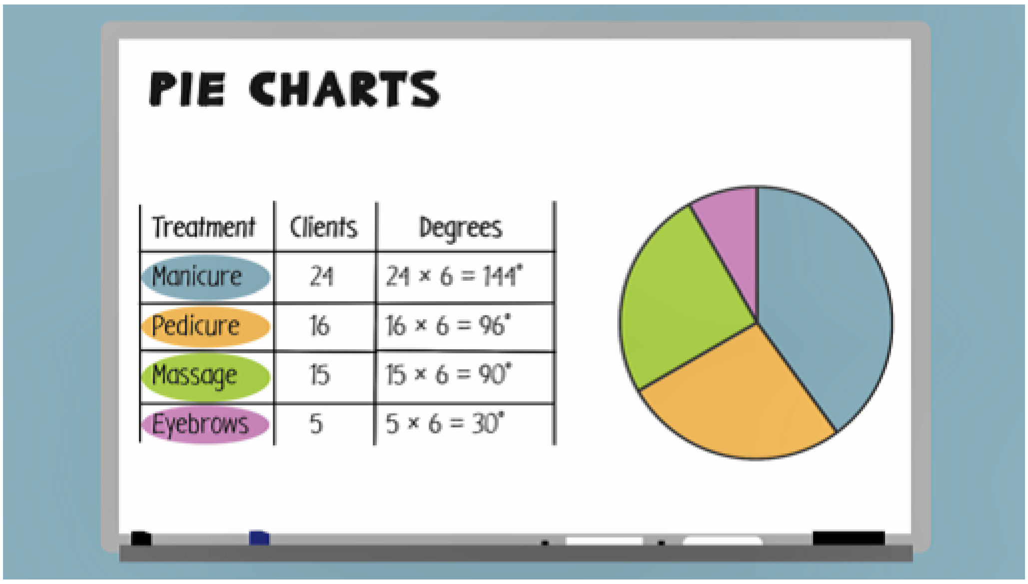

Now have a go at drawing a pie chart for yourself.

Activity 7: Drawing a pie chart

1. A leisure centre wants to compare which activities customers choose to do when they visit the centre. The information is shown in the table below. Draw an accurate pie chart to show this information. You may hand-draw your pie chart or create it using a computer package.

Table 8(a)

Activity	Number of customers
Swimming	26
Gym	17
Exercise class	20
Sauna	9

Answer

1. Firstly, work out the total number of customers:

   $26 + 17 + 20 + 9 = 72$

   Now work out the number of degrees that represents customer:

   $360^\circ \div 72 = 5^\circ$ per customer

Table 8(b)

Activity	Number of customers	Number of degrees
Swimming	26	26 × 5 = 130°
Gym	17	17 × 5 = 85°
Exercise class	20	20 × 5 = 100°
Sauna	9	9 × 5 = 45°

Now use this information to draw your pie chart. It should look something like this:

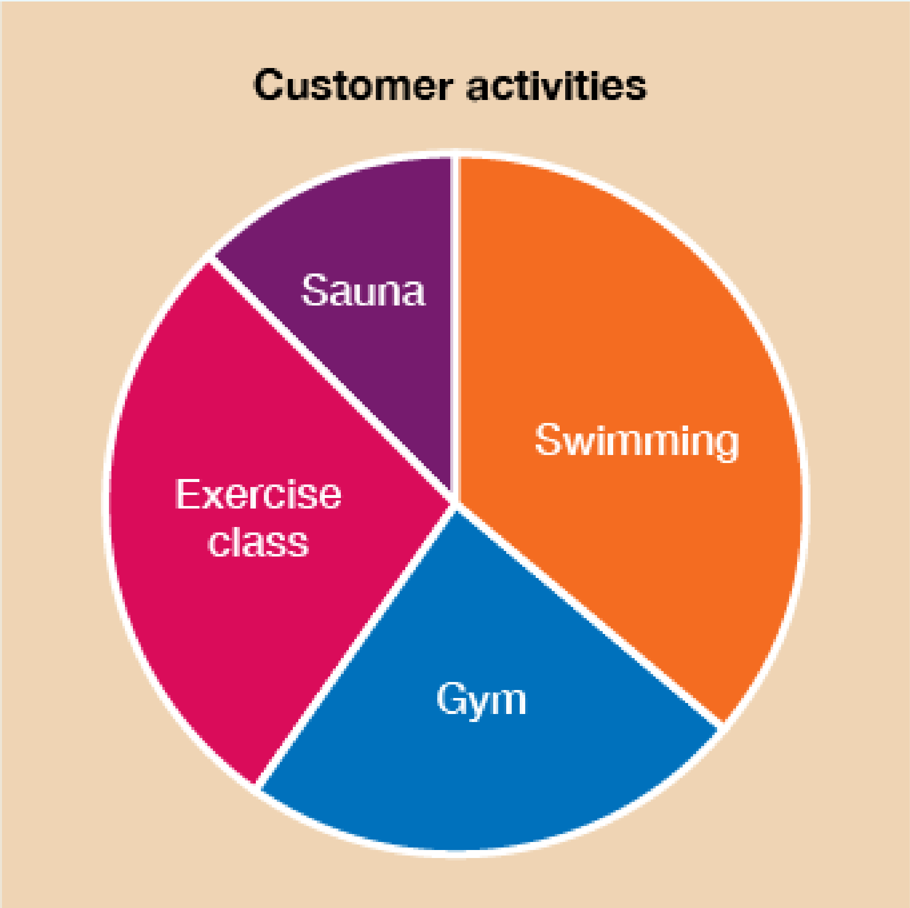

Figure 13 Pie chart for customer leisure centre activities

2. The table below shows the sandwich sales over one year for sandwich company, Belinda's Butties. Draw a pie chart to illustrate the data. You may hand-draw your pie chart or create it using a computer package.

Table 9(a)

Sandwich Type	Sales (000s)
Cheese and onion	20
Egg and cress	17
Prawn	11
Coronation chicken	12
Tuna mayonnaise	9
Ham	8
Beef and tomato	13

Answer

2. Firstly, work out the total number of sandwich sales:

   20 + 17 + 11 + 12 + 9 + 8 + 13 = 90 (000s)

   Now work out the number of degrees that represents each sale:

   360 ÷ 90 = 4˚ per sale

Table 9(b)

Sandwich Type	Sales (000s)	Number of Degrees
Cheese and onion	20	20 × 4 = 80°
Egg and cress	17	17 × 4 = 68°
Prawn	11	11 × 4 = 44°
Coronation chicken	12	12 × 4 = 48°
Tuna mayonnaise	9	9 × 4 = 36°
Ham	8	8 × 4 = 32°
Beef and tomato	13	13 × 4 = 52°

Now use this information to draw your pie chart. It should look something like the one below.

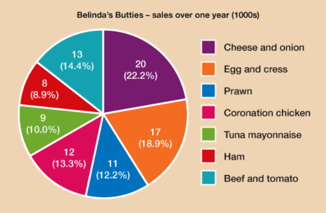

Figure 14 Belindas Butties – sales over one year (000s)

Now that you can accurately draw a pie chart, it's time to look at how to interpret them. You won't always be given the actual data, you may just be given the total number represented by the chart or a section of the chart and the angles on the pie chart itself. It's useful to know how to use your maths skills to work out the actual figures.

Here's a reminder of the degrees of a circle which will be useful when you come to read from pie charts.

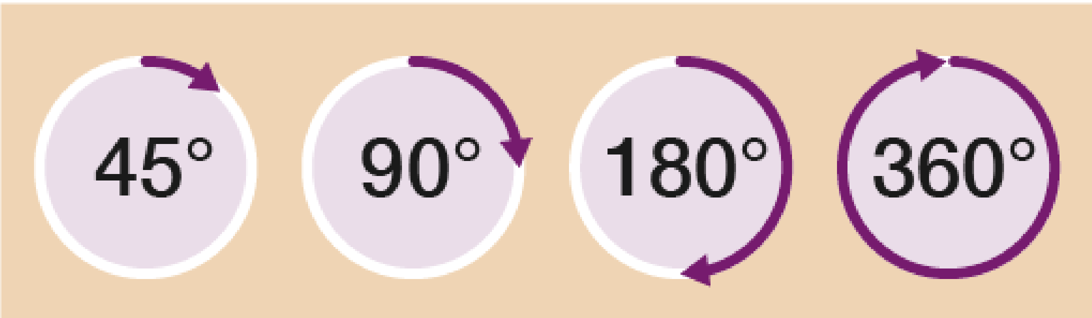

Figure 15 Degrees of a circle

4.2 Interpreting pie charts

Imagine you've been presented with the pie chart below. The chart shows the ages of students competing at an athletic event.

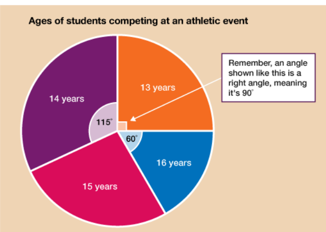

Figure 16 Ages of students competing at an athletic event

There are two possible pieces of information you could be given. You could be given the total number of students that were at the event or, you could be given the number of students in one of the age categories.

Example: Reading a pie chart 1

Let's say you were told that 72 people attended the competition. Since you know that 360˚ has been shared equally between all 72 people, you do 360 ÷ 72 = 5˚ per person.

Once you know this, if you wanted to know how many students that took part were 16 years old, you would look at the degrees on the chart for 16-year-olds which in this example is 60˚.

You then do 60 ÷ 5 = 12 students.

If you wanted to work out the number of 15-year-olds, you would first need to work out the missing angle; you know that all the angles will add up to 360˚ so just do:

360 – 115 – 90 – 60 = 95˚

And now do the same as before 95 ÷ 5 = 19 students who were 15 years old.

Example: Reading a pie chart 2

Using the same pie chart, let's say that all you were told was that 23 students took part who were 14 years old.

You can see that the angle for 14-year-olds is 115˚ and you've been told that this represents 23 students.

To find out how many degrees each student gets, you do 115 ÷ 23 = 5˚ per student.

Once you know this you can find out how many students are represented by each other section in the same way as we did in example 1. For example, the 13-year-olds have an angle of 90˚.

To find out how many there are you do 90 ÷ 5 = 18 students who were 13 years old.

Pie charts are very similar to ratio. In ratio questions you are always looking to find out how much 1 part is worth; in pie chart questions you are looking to find how many degrees represents 1 person (or whatever object the pie chart is representing).

As well as being closely linked with ratio, pie charts also involve the use of your fractions skills. If, for example, you were asked what fraction of the students were 16 years old, you can show this as $\frac{60}{360}$ since the 16-year-olds are 60 degrees out of the total 360 degrees.

Using your fractions skills however, the fraction $\frac{60}{360}$ can be simplified to $\frac{1}{6}$.

It's time for you to practise your skills at interpreting pie charts. Have a go at the activity below and then check your answers with the feedback given.

Activity 8: Interpreting pie charts

1. The pie chart below shows how long a gardener spent doing various activities over a month.

Figure 17 Time spent doing gardening activities

a. What fraction of the time was spent planting? Give your answer in its simplest form.
b. 5 hours were spent digging. How long was spent on cutting the grass?

Answer

1.

a. Planting = $\frac{80}{360}$ = $\frac{2}{9}$ in its simplest form.

b. Digging is 100° and you know that that was 5 hours.
100° ÷ 5 = 20° for each hour.
Since cutting the grass has an angle of 40°, you do 40 ÷ 20 = 2 hours cutting the grass.

2. 120 adults participating in an online course were asked if they felt there were enough activities for them to complete throughout the course.
The pie chart below shows the results.

Figure 18 Pie chart of opinions about an online course

b. What fraction of the adults thought there were too many activities? Give your answer in its simplest form.
c. How many adults thought there were enough activities?

Answer

2.

b. Too much = $\frac{105}{360}$ = $\frac{7}{24}$ in its simplest form.

c. You know that 120 adults took part in the survey. To find out how many degrees represents each adult, you do 360 ÷ 120 = 3 degrees per person.

Next you need to know the angle for those who said there were enough activities. For this, you do:

$360 - 105 - 60 - 45 = 150^\circ$

Now you know this, you can do:

150 ÷ 3 = 50 adults thought there were enough activities.

Well done! You can now draw and interpret bar charts and pie charts; both of which are good ways to represent discrete data. In the next part of this session, you will learn how to draw and interpret line graphs.

Summary

In this section you have learned:

- what types of information can be represented effectively on a pie chart
- how to use and interpret a pie chart
- how to draw an accurate pie chart when given a set of data.

5 Line graphs

Line graphs are a very useful way to spot patterns or trends over time. You will have looked at how to plot and interpret single line graphs from single data sources in *Everyday maths 1*. Now you will be looking at line graphs that show the results of two data sources.

The example below shows the monthly foreign exchange rate of £1 against the US dollar and the euro. You can clearly see how the pound was dropping in value against the euro and US dollar up to October 2016. Now that you understand how line graphs can be used and why they are useful, next you'll learn how to draw and interpret them.

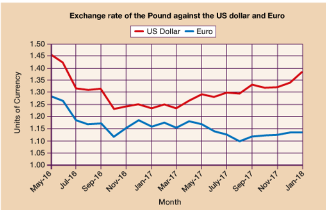

Figure 19 Monthly foreign exchange rate of £1 against the US dollar and the euro from May 2016 to January 2018.

Now that you understand how useful line graphs can be and how they can be used, next you'll learn how to draw and interpret them.

5.1 Drawing line graphs

Drawing a line graph is very similar to drawing a bar chart, and they have many of the same features.

Line graphs need:

- a title
- a label for the vertical axis (e.g. units of currency)
- a numbered scale on the vertical axis
- a label on the horizontal axis (e.g. month) so that it is clear to the reader what they are looking at.

The main difference when drawing a line graph rather than a bar chart, is that rather than a bar, you put a dot or a small cross to represent each piece of information. You then join each dot together with a line. There is significant debate over whether the dots should be joined with a curve or with straight line. Whilst the issue is (believe it or not!) hotly contested, the general consensus seems to be that dots should be joined with straight lines.

Activity 9: Drawing a line graph

1. Have a go at drawing a line graph to represent the data below.

   The table below shows the temperatures in Tenby during the first two weeks of July 2018.

Table 10

Date	Temperature High ˚C	Temperature Low ˚C
01/07/2018	25	15
02/07/2018	28	17
03/07/2018	29	12
04/07/2018	24	15
05/07/2018	26	13
06/07/2018	22	12
07/07/2018	23	12
08/07/2018	27	12
09/07/2018	26	14
10/07/2018	24	14
11/07/2018	22	14
12/07/2018	22	7
13/07/2018	24	12
14/07/2018	25	20

Answer

1. Your line graph should look similar to the one shown below with a title, key and axis labels.

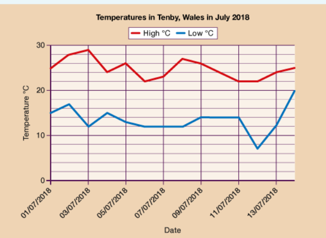

Figure 20 A line graph of temperatures in Tenby, Wales in July 2018

2. A clothing store has outlets in Llandudno and Aberystwyth. Use the data in the table below to draw a line graph comparing monthly sales between January and June.

Table 11

Month		Jan	Feb	Mar	Apr	May	Jun
Sales (£000s)	Llandudno	29	15	19	20	23	24
	Aberystwyth	25	10	16	16	21	26

Answer

2. Your line graph should look similar to the one shown below with a title, key and axis labels.

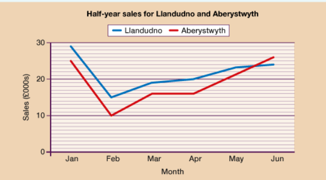

Figure 21 A line graph showing half-year sales for Llandudno and Aberystwyth

5.2 Interpreting line graphs

Interpreting line graphs is also very similar to the way you interpret bar charts; it's just about using the scales shown on the graph to find out information.

Given that you've already learned and practised interpreting bar charts, you can jump straight into an activity on interpreting line graphs.

Activity 10: A population line graph

1. The graph shows information about the population of a village in thousands over a period of time.

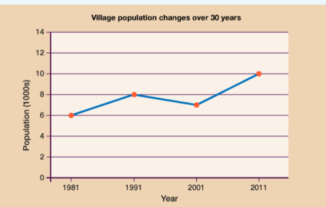

Figure 22 Line graph showing village population over time

   a. What was the population of the village in 1991?
   b. What was the increase in population from 1981 to 2011?
   c. By how much did the population drop between 1991 and 2001?

Answer

1.
   a. In 1991 the population was 8000.
   b. In 1981 the population was 6000, in 2011 the population was 10 000. This is an increase of 10 000 − 6000 = 4000.
   c. In 1991 the population was 8000 and by 2001 it was 7000. 8000 − 7000 = 1000. So the population dropped by 1000.

2. Jen's Gym kept a record of member numbers during 2018.

   The graph below shows information about the results.

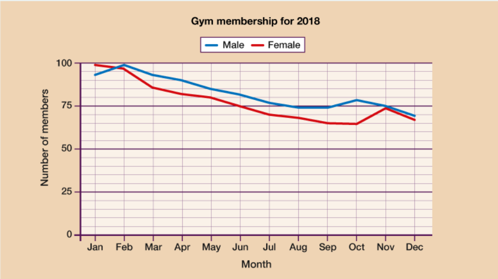

Figure 23 A line graph showing gym membership numbers for 2018

   b. Which was the only month when there were more female members than male members?
   c. Estimate the difference in membership numbers for October.
   d. In what month was the difference between male and female members smallest?

Answer

2.
   b. January
   c. 13 (+/− 1) allowed as this question is estimation
   d. November

How did you get on? Hopefully you were able to answer all the questions without too much difficulty. As long as you have worked out the scale correctly and read the question carefully, there's nothing too tricky involved.

You have now covered each drawing and interpreted each different type of chart and graph so it's time to move on to look at other uses for data: averages and range.

Summary

In this section you have learned:

- which types of data can be suitably represented by a line graph and which are best suited to other types of charts.

- how to interpret the information shown on a line graph
- how to draw an accurate line graph for a given set of data.

6 Mean, median, mode and range

The three types of averages that you will be focussing on in this part of the section are mean, median and mode. You will also be looking at range. For Level 2 Essential Skills Wales, you need to be able to calculate each of these without using a calculator.

6.1 Range

Figure 24 A mountain range of different sized peaks

Much like this stunning mountain range is made up of a variety of different sized mountains, a set of numerical data will include a range of values from smallest to biggest. The range is simply the difference between the biggest value and the smallest value. It shows how spread out a set of data is and can be useful to know because data sets with a big difference between the highest and lowest values can imply a certain amount of risk.

Let's say there are two basketball players and you are trying to choose which player to put on for the last quarter. If one player has a large range of points scored per game (sometimes they score a lot of points but other times they score very few – meaning their scoring is **variable**) and the other player has a smaller range (meaning they are more **consistent** with their point scoring) it might be safest to choose the more consistent player.

Take a look at the example below.

A farmer takes down information about the weight, in kg, of apples that one worker collected each day on his apple farm.

Table 12

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
56 kg	70 kg	45 kg	82 kg	67 kg	44 kg	72 kg

In order to find the range of this data, you simply find the biggest value (82 kg) and the smallest value (44 kg) and find the difference:

82 – 44 = 38 kg

The range is therefore 38 kg.

Now let's compare this worker to another worker whose information is shown in the table below.

Table 13

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
56 kg	60 kg	58 kg	62 kg	65 kg	49 kg	58 kg

This worker has a highest value of 65 kg and a lowest value of 49 kg. The range for this worker is therefore 65 – 49 = 16 kg. The second worker has a lower range than the first worker and is therefore a more consistent apple picker than the first worker, who is a more variable picker.

Now try one for yourself.

Activity 11: Finding the range

1. The table below shows the sales made by a café on each day of the week:

Table 14

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
£156.72	£230.54	£203.87	£179.43	£188.41	£254.70	£221.75

What is the range of sales for the café over the week?

Answer

1. Simply find the highest value: £254.70, and lowest value: £156.72, then find the difference:

   £254.70 − £156.72 = £97.98

2. A bowling team want to compare the scores for their players. The table below shows their results.

Table 15

Name	Andy	Bilal	Caz	Dom	Ede
Highest score	176	175	162	170	150
Lowest score	148	145	142	165	116

Which player is the most consistent? Give a reason for your answer.

Answer

2. You need to look at the range for each player:

   Andy: 176 − 148 = 28
   Bilal: 175 − 145 = 30
   Caz: 162 − 142 = 20
   Dom: 170 − 165 = 5
   Ede: 150 − 116 = 34

   The player with the smallest range is Dom and so Dom is the most consistent player.

3. Outside temperatures at a garden centre were taken daily over four weeks in January and displayed in the following table.

Table 16 Temperatures for January in ˚C

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Week 1	2	4	5	5	1	−1	−3
Week 2	−4	0	0	3	6	5	6
Week 3	3	2	−1	0	3	2	0
Week 4	0	4	7	8	3	−1	−2

   a. Which day of the week showed the most variable temperature range?
   b. Which day of the week showed the most consistent temperature range?
   c. Which days had the same range?

Answer

3.

   c. Sundays had a lowest temperature of −3˚C and a highest temperature of 6˚C so the difference was 9˚C, giving Sundays the highest range and making temperatures more variable.
   d. Tuesdays had a lowest temperature of 0˚C and a highest temperature of 4˚C so the difference was 4˚C, giving Tuesdays the lowest range and making temperatures more consistent.
   e. Wednesdays and Thursdays had the same range.

      Wednesdays had a lowest temperature of −1˚C and a highest temperature of 7˚C so the difference was 8˚C.

      Thursdays had a lowest temperature of 0˚C and a highest temperature of 8˚C so the difference was 8˚C.

As you have seen, finding the range of a set of data is very simple but it can give some useful insights into the data. The most commonly used average is the ‘mean average’ (or sometimes just the mean) and you’ll look at this next.

6.2 Mean average

You will now learn about:

- finding the mean when given a set of data
- finding the mean from a frequency table

Figure 25 Below average and mean average

The mean is a good method to use when you want to compare a large set of data, for example:

- the average amount spent by customers in a shop
- the average cost of a house in a certain area
- the average time taken for your chosen breakdown service to get to your car.

The mean average can help us make comparisons between sets of data which can then help you when making a decision.

6.3 Finding the mean from a set of data

To find the mean of a simple set of data, all you need to do is find the total, or sum, of all the items together and then divide this total by how many items of data there are.

Table 13 (repeated)

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
56 kg	60 kg	58 kg	62 kg	65 kg	49 kg	58 kg

Look again at the weight, in kg, of apples that one worker collected each day on an apple farm (shown above). If you want to calculate the mean average weight of apples collected, you first find the sum or total of the weight of apples collected in the week:

56 + 60 + 58 + 62 + 65 + 49 + 58 = 408 kg

Next, divide this total but the number of data items, in this case, 7:

408 ÷ 7 = 58.3 kg (rounded to one d.p.)

It is important to note that the mean may well be a decimal number even if the numbers you added together were whole numbers.
Another important thing to note here is that the two sums (the addition and then the division) are done as two separate sums. If you were to write:

56 + 60 + 58 + 62 + 65 + 49 + 58 ÷ 7

this would be incorrect (remember BIDMAS from Session 1?). Unless you are going to use brackets to show which sum needs to be done first
(56 + 60 + 58 + 62 + 65 + 49 + 58) ÷ 7, it is accurate to write two separate calculations.
Have a go at calculating the mean for yourself by completing the activity below.

Activity 12: Finding the mean

1. The table below shows the sale price of ten, 2 bedroom semi-detached houses in a town in Liverpool.

Table 16

1	2	3	4	5	6	7	8	9	10
£70 000	£65 950	£66 500	£71 200	£68 000	£62 995	£70 500	£68 750	£59 950	£67 900

What is the mean house price in this area?

Answer

1. First find the total of the house prices:
   £70 000 + £65 950 + £66 500 + £71 200 + £68 000 + £62 995 + £70 500 + £68 750 + £59 950 + £67 900 = £671 745
   Now divide this total by the number of houses (10):
   £671 745 ÷ 10 = **£67 174.50**

2. The table below shows the units of gas used by a household for the first 6 months of a year.

Table 17

January	February	March	April	May	June
1650	1875	1548	1206	654	234

Calculate the mean amount of gas units used per month.

Answer

2. Find the total number of units used:

   1650 + 1875 + 1548 + 1206 + 654 + 234 = 7167 units

   Now divide this total by the number of months (6):

   7167 ÷ 6 = **1194.5** units

This method of finding the mean is fine if you have a relatively small set of data. What about if the set of data you have is much larger? If this was the case, the data would probably not be presented as a list of numbers, it's much more likely to be presented in a frequency table.

In the next part of this section, you will learn how find the mean when data is presented in this way.

6.4 Finding the mean from a frequency table

Large groups of data will often be shown as a frequency table, rather than as a long list. This is a much more user friendly way to look at a large set of data. Look at the example below where there is data on how many times, over a year, customers used a gardening service.

Table 18

Number of visits in a year	Number of customers
1	6
2	10
3	11
4	16
5	4
6	3

We could write this as a list if we wanted to:

1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3 and so on

but it's much clearer to look at in the table format. But how would you find the mean of this data? Well, there were 6 customers who had 1 visit from the gardening company, that's a total of 1 × 6 = 6 visits. Then there were 10 customers who had 2 visits, that's a total of 2 × 10 = 20 visits. Do this for each row of the table, as shown below.

Table 19

Number of visits in a year	Number of customers	Total visits
1	6	1 × 6 = 6
2	10	2 × 10 = 20
3	11	3 × 11 = 33
4	16	4 × 16 = 64
5	4	5 × 4 = 20
6	3	6 × 3 = 18
	Total = 50 customers	Total = 161 visits

Finally, work out the totals for each column (highlighted in a lighter colour on the table above).

Now you have all the information you need to find the mean; the total number of visits is 161 and the total number of customers is 50 so you do 161 ÷ 50 = 3.22 visits per year as the mean average.

Warning! Many people trip up on these because they will find the total visits (161) but rather than divide by the total number of customers (50) they divide by the number of rows in the table (in this example, 6).

If you do 161 ÷ 6 = 26.83, logic tells you, that since the maximum number of visits any customer had was 6, the mean average cannot be 26.83. Always sense check your answer to make sure it is somewhere between the lowest and highest values of the table. For this example, anything below 1 or above 6 must be incorrect!

Have a go at a couple of these yourself so that you feel confident with this skill.

Activity 13: Finding the mean from frequency tables

1. The table below shows some data about the number of times children were absent from school over a term.

   Work out the mean average number of absences.

Table 20(a)

Number of absences	Number of children
1	26
2	13
3	0
4	35
5	6

Answer

1. First, work out the total number of absences by multiplying the absence column by the number of children. Next, work out the totals for each column.

Table 20(b)

Number of absences	Number of children	Total number of absences
1	26	1 × 26 = 26
2	13	2 × 13 = 26
3	0	3 × 0 = 0
4	35	4 × 35 = 140
5	6	5 × 6 = 30
	Total = 80	Total = 222

To find the mean do: 222 ÷ 80 = 2.775 mean average.

2. You run a youth club for 16–21-year-olds and are interested in the average age of young people who attend.

   You collect the information shown in the table below. Work out the mean age of those who attend. Give your answer rounded to one decimal place.

Table 21(a)

Age	Number of young people
16	3
17	8

18	5
19	12
20	6
21	1

Answer

2. Again, first work out the totals for each row and column.

Table 21(b)

Age	Number of young people	Total number
16	3	16 × 3 = 48
17	8	17 × 8 = 136
18	5	18 × 5 = 90
19	12	19 × 12 = 228
20	6	20 × 6 = 120
21	1	21 × 1 = 21
	Total = 35	Total = 643

To find the mean you then do:

- 643 ÷ 35 = 18.4 years (rounded to one d.p.)

If you are given all the data in a set and asked to find the mean, it's a relatively simple process.

For the Confirmatory Test element of Essential Skills Wales (ESW), you are expected to complete all of the necessary calculations without using a calculator, so remember to use alternative methods for checking e.g. inverse checks.

6.5 Calculating the median

The next type of average you look at briefly is called the median. Put very simply, the median is the middle number in a set of data and as it is in the middle, it is not affected by abnormally high or low data values. The important thing you need to remember is to put the numbers in size order, smallest to largest, before you begin. Firstly let's look at two simple examples.

Example: Finding the median 1

Find the median of this data set:

Method

5, 10, 8, 12, 4, 7, 10

Firstly, order the numbers from smallest to largest:

4, 5, 7, 8, 10, 10, 12

Now, find the number that is in the middle:

4, 5, 7, **8**, 10, 10, 12

8 is the number in the middle, so the median is 8.

Example: Finding the median 2

Find the median of this data set:

Method

24, 30, 28, 40, 35, 20, 49, 38

Again, you firstly need to order the numbers:

20, 24, 28, 30, 35, 38, 40, 49

And then find the one in the middle:

20, 24, 28, **30**, **35**, 38, 40, 49

In this example there are actually two numbers that are in the middle. You therefore find the middle of these two numbers by adding them together and then halving the answer:

(30 + 35) ÷ 2
75 ÷ 2 = 37.5

The median for this set of data is 37.5. As there are two numbers that are in the middle, your answer does not appear in the original set of data.

The example below is a little more complicated.

Example: Finding the median 3

Tracy did a survey of the number of cups of coffee her colleagues had drunk during one day. The frequency table shows her results.

Table 22

Number of cups of coffee	Frequency
2	1
3	5
4	3
5	4
6	6

First you need to calculate the number of colleagues by adding together the numbers in the frequency column:

1 + 5 + 3 + 4 + 6 = 19

You then need to work out the median or middle value in the number of cups of coffee. To do this you can list the number of cups of coffee in a line:

2 3 3 3 3 3 4 4 4 **5** 5 5 5 6 6 6 6 6 6

You know there are 19 colleagues, which is an odd number, so you will be able to find the exact midpoint:

9 + 9 = 18

so count in from either side until you reach the 10th number which can be seen as 5 in the list above.

Another way to do this is to calculate the number of colleagues, which you know is 19. You then find the midpoint by adding the numbers in the frequency table:

1 + 5 + 3 = 9

and the exact midpoint is the 10th colleague which the table shows as 4 in the frequency column.

If you look under the Number of cups of coffee column, you can see that the answer is 5 cups of coffee, so the median is 5.

Activity 14: Calculating the median

Now calculate the median of the following:

1. The ages of a group of students on a course are:
   16, 44, 32, 67, 25, 18, 22
2. The heights of a group of children in a gymnastics class are:
   1.24 m, 1.27 m, 1.20 m, 1.15 m, 1.26 m, 1.17 m
3. The frequency table below shows the number of televisions that a group of students on a media course have at home.

Table 23

Number of televisions	Number of students
0	1
1	4
2	8
3	9
4	3

Calculate the median number of televisions.

Answer

1. First you need to list the data in order of size so:

   16, 18, 22, 25, 32, 44, 67

   Now find the middle value. In this case there are 7 data values so you will be able to find the exact middle.

   16, 18, 22, **25**, 32, 44, 67

   The middle value is 25 so this is the median age of the group of students.
2. First you need to list the heights in order of size so:

   1.15 m, 1.17 m, 1.20 m, 1.24 m, 1.26 m, 1.27 m

   Now find the middle value. In this case there are 6 data values so first find the middle two values.

   1.15 m, 1.17 m, **1.20 m**, **1.24 m**, 1.26 m, 1.27 m

   Now add together the two middle values:

   1.20 m + 1.24 m = 2.44 m

   Then halve the answer:

   2.44 m ÷ 2 = 1.22 m

   So the median height of the students is 1.22 m.
3. First you need to calculate the number of students.

   To do this you add up each number in the frequency table:

   1 + 4 + 8 + 9 + 3 = 25 students

   You then need to work out the median or middle value in the number of televisions. To do this you can list the number of televisions in a line:

   0 1 1 1 1 2 2 2 2 2 2 2 **2** 3 3 3 3 3 3 3 3 3 4 4 4

   As you know there are 25 students, you need to find the midpoint which will be 13. Count in until you find the 13th number which is 2 in your list of televisions, so the median is 2.

   Another way to do this is to calculate the number of students, which you know is 25. You then find the midpoint as the 13th student using the frequency table. If you count up the ‘Number of students’ column in the frequency table, the 13th value is 2 televisions, so the median is 2.

If you want to see some more examples, or try some for yourself, use the link below:

https://www.mathsisfun.com/median.html

6.6 Calculating the mode

The final type of average to look at is the mode, which is the most common value in a data set. There can sometimes be more than one mode, which occurs when two or more values are equally common. Sometimes there will be no mode as each data value occurs only once. When you calculate the mode you will always find that it is one of the values in your original data set. The mode is also called the modal value.

Example: Finding the mode 1

What is the mode of the following numbers?

3, 7, 5, 6, 4, 5, 6, 5, 7, 5

First group the same numbers together by listing them in size order:

3, 4, 5, 5, 5, 5, 6, 6, 7, 7

It is then easy to identify the modal value as 5 as this number occurs the most.

Note: To remember how to calculate this average use *Mode = Most*.

Example: Finding the mode 2

What is the mode of the following amounts of money?

£8.99 £16.45 £17.50 £36.20 £6.75 £9.35 £12.99 £8.95

First list them in size order from lowest amount to highest:

£6.75 £8.95 £8.99 £9.35 £12.99 £16.45 £17.50 £36.20

You can now see that there is no mode (or modal value) as each amount of money occurs only once.

Example: Finding the mode 3

Below is a set of data showing the number of people attending a yoga class each week over a year. What is the mode?

17 17 13 16 18 15 12 16 16 17 18 13

First list them in size order from lowest amount to highest:

12 13 13 15 16 16 16 17 17 17 18 18

You can now see that two numbers occur the same number of times. 16 and 17 both occur three times, so in this set of data there are two modes or modal values – 16 and 17.

Example: Finding the mode 4

To find the mode from a frequency table you need to find the value with the highest frequency. The results of a survey on a block of flats are shown in the frequency table below. The highest frequency, which can be seen in the 'Number of flats' column, is 18. This means that the mode or modal number of occupants is 3.

Table 24

Number of occupants	Number of flats
0	2
1	9
2	13
3	**18**
4	6

Activity 15: Calculating the mode

Now calculate the mode of the following:

1. 3, 6, 5, 7, 3, 5, 6, 6, 3, 4, 9, 6
2. 13, 19, 11, 28, 17, 29, 16, 24, 15, 18
3. 81 cm, 53 cm, 74 cm, 62 cm, 53 cm, 70 cm, 81 cm, 74 cm, 42 cm, 90 cm
4. The table below shows the number of tries scored by a school rugby team during one month. What is the modal number of tries scored?

Table 25(a)

Number of tries	Frequency
0	5
1	8
2	6
3	3

Answer

1. First list them in size order from lowest amount to highest:

   3, 3, 3, 4, 5, 5, 6, 6, 6, 6, 7, 9

   It is then easy to identify the modal value as 6 as this number occurs the most.
2. First list them in size order from lowest amount to highest:

   11, 13, 15, 16, 17, 18, 19, 24, 28, 29

   You can now see that each value occurs only once so there is no mode/modal value.
3. First list them in size order from lowest amount to highest:

   42 cm, 53 cm, 53 cm, 62 cm, 70 cm, 74 cm, 74 cm, 81 cm, 81 cm, 90 cm

   You can now see that three numbers occur the same number of times. 53 cm, 74 cm and 81 cm occur twice, so in this set of data there are three modes or modal values.
4. To find the mode you need to look for the highest frequency in the table. In this case it is 8 which shows that the modal number of tries scored is 1.

Table 25(b)

Number of tries	Frequency
0	5
1	**8**
2	6
3	3

6.7 Choosing the best average

For some sets of data it may be better to use one type of average over another as it will be more representative of the data type.

Here are some of the advantages and disadvantages of each type of average.

Mean

Advantages

- Uses all the data values.

Disadvantages

- May not always be one of the values in the set of data or a value that does not make sense for the data, e.g. 1.6 people.

- The mean may not be useful for a set of data which has a value a lot higher or lower than the others, e.g. If you were to include the salary of the Managing Director with the wages of shop floor staff when calculating an average salary, it is likely that the mean would be distorted by the higher salary of the Managing Director, which means that it would not represent the data very well.

Median

Advantages

- It is the middle value so is not affected by very high or very low values. It is a useful type of average for data sets with such values.

Disadvantages

- Sometimes it will not be one of the values in the data set.

Mode

Advantages

- It will always be one of the values in the data set (if there is a mode).
- It is very good for certain types of data, e.g. finding the most common shoe size.

Disadvantages

- There may not be a mode.
- There may be several modes.
- It may be at one end of the data distribution.

Now have a go at finding the mean, median and mode.

Activity 16: Finding different averages

A bridal shop records the sizes of wedding dresses that it sells in one month. The table below shows the results.

1. Find the following:
   a. the mean
   b. the median
   c. the mode.
2. Which of the averages gives the most useful information for this data set?

Table 26(a)

Dress size	No. of dresses sold
8	2
10	8
12	11
14	12
16	5
18	2

Answer

1. (a) **The mean**
   - First, work out the total number of dresses sold by multiplying the dress size column by the number of dresses sold.
   - Add a frequency column to display your calculations.
   - Next, work out the totals for the number of dresses sold column and the frequency column.

Table 26(b)

Dress size	No. of dresses sold	Frequency
8	2	8 × 2 = 16
10	8	10 × 8 = 80
12	11	12 × 11 = 132
14	12	14 × 12 = 168
16	5	16 × 5 = 80
18	2	18 × 2 = 36
Totals	**40**	**512**

   - Finally calculate the mean by dividing the frequency by the number of dresses sold:
     512 ÷ 40 = 12.8
     Mean = 12.8

1. (b) **The median**

- First you need to calculate the total number of dresses sold. To do this add up each number in the frequency table:

  2 + 8 + 10 + 13 + 5 + 2 = 40 dresses sold.
- Now find the middle value by listing the quantity of each size of dress from smallest to biggest:

  8 8 10 10 10 10 10 10 10 10 12 12 12 12 12 12 12 12 12 **12 12** 14 14 14 14 14 14 14 14 14 14 14 14 16 16 16 16 16 18 18
- The midpoint of the 40 dresses sold is between values 20 and 21 so you need to find both of these values. In this example they are 12 and 12.

  You calculate the median by adding 12 + 12 and dividing by 2:

  12 + 12 = 24

  24 ÷ 2 = 12

  As you can see, in this example the median is size 12.
- A quicker way to do this is to calculate the number of dresses, which you know is 40.
- You then use the frequency table to find the midpoint which is between the 20th and the 21st dress size. If you count up the number of dresses sold in the frequency table:

  2 + 8 + 11 = 21

  you can see that the 20th and 21st values fall within size 12, so the median is size 12.

  <u>Median = 12</u>

1. (c) **The mode**
   - To find the mode you need to look for the highest number of dresses sold. In this case it is 12 which shows that the mode dress size is 14.

     <u>Mode = 14</u>

Table 26(c)

Dress size	No. of dresses sold
8	2
10	8
12	11
14	**12**
16	5
18	2

2. In this case the mean is size 12.8 which does not exist as a dress size so this is not useful.

The median result is size 12 which is a dress size, but it is still not the most commonly sold dress size.

The mode is dress size 14 which is the most commonly sold dress size and so this gives the most useful information.

Well done! You have now learned all you need to know about mean, median, mode and range. The final part of this section, before the end-of-course quiz, looks at probability.

Summary

In this section you have learned:

- that there are different types of averages that can be used when working with a set of data – mean, median and mode
- range is the difference between the largest data value and the smallest data value and is useful for comparing how consistently someone or something performs
- mean is what is commonly referred to when talking about the average of a data set
- how to find the mean from both a single data set and also a set of grouped data
- what the median of a data set is and how to find it for a given set of data
- what the mode of a data set is and how to find it for a given set of data
- how to choose the ‘best’ type of average for a given set of data.

7 Probability

You will use probability regularly in your day-to-day life:

- Should you take an umbrella out with you today?
- What are the chances of the bus being on time?
- How likely are you to meet your deadline?

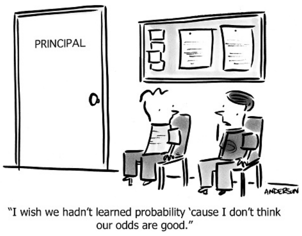

Figure 26 Probability – the odds are you are already using it

Probability is all about how likely, or unlikely, something is to happen. When you flip a coin for example, the chances of it landing on heads is $\frac{1}{2}$ or 50% or 0.5 (do you remember from your work in Session 1 how fractions, decimals and percentages can be converted into one another?).

The probability, or likelihood, of an event happening is perhaps most easily expressed as a fraction to begin with. Then, if you want to express it as a percentage or a decimal you can just convert it.

Let's look at an example.

Example: Chocolate probability

A box of chocolates contains 15 milk chocolates, 5 dark chocolates and 10 white chocolates. If the box is full and you choose a chocolate at random, what is the likelihood of choosing a dark chocolate?

Method

There are 5 dark chocolates in the box. There are 15 + 5 + 10 = 30 chocolates in the box altogether.

The probability of choosing a dark chocolate is therefore:

$$\frac{5}{30} = \frac{1}{6}$$

You could also be asked the probability of choosing either a dark or a white chocolate. For this you just need the total of dark and white chocolates:

5 + 10 = 15

The total number of chocolates in the box remains the same so the likelihood of choosing a dark or a white chocolate is:

$$\frac{15}{30} = \frac{1}{2}$$

You could even be asked what the likelihood is of an event not happening. For example, the likelihood that you will not choose a white chocolate. In this case, the total number of chocolates that are not white is 15 + 5 = 20.

Again, the total number of chocolates in the box remains the same and so the probability of not choosing a white chocolate is:

$$\frac{20}{30} = \frac{2}{3}$$

Now have a go yourself by completing the short activity below.

Activity 17: Calculating probability

1. You buy a packet of multi-coloured balloons for a children's party. You find that there are 26 red balloons, 34 green balloons, 32 yellow balloons and 28 blue balloons.

   You take a balloon out of the packet without looking. What is the likelihood of choosing a green balloon?

   Give your answer as a fraction in its simplest form.
2. At the village fete, 350 raffle tickets are sold. There are 20 winning tickets. What is the probability that you will not win the raffle?

   Give your answer as a percentage rounded to two decimal places.

Answer

1. There are 34 green balloons. The total number of balloons is 26 + 34 + 32 + 28 = 120.

   The likelihood of choosing a green balloon is therefore:

   $\frac{34}{120} = \frac{17}{60}$ in its simplest form.
2. If there are 20 tickets that are winning ones, there are 350 − 20 = 330 tickets that will not win a prize.

   As a fraction this is: $\frac{330}{350}$

   To convert to a percentage, you do 330 ÷ 350 × 100 = 94.29% rounded to two d.p.

You sometimes need to calculate the probability of more than one event occurring. In this case, the events might be:

- **independent** – which means that the outcome of one event does not affect the other
- **dependent** – which means that the outcome of one event does affect the other.

In either case, you can use **tree diagrams** or **tables** to help you to picture and solve these problems.

Example: Gender probability

If a couple has two children, the gender of the first child will not affect the gender of the second child. All possibilities can be shown in the form of a table:

B = boy

G = girl

Table 27 First and second child gender probability table

		First child	
		B	G
Second child	B	BB	GB
	G	BG	GG

Alternatively, it can be shown as a tree diagram:

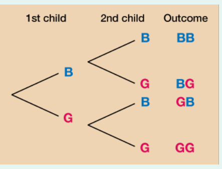

Figure 27 A tree diagram showing first and second child gender outcomes

Both the table and the tree diagram show that there are four possibilities:

1. **BB** – boy then another boy
2. **BG** – boy then a girl
3. **GB** – girl then a boy
4. **GG** – girl then another girl

Out of the possibilities there is 1 in 4 or $\frac{1}{4}$ (1 quarter/25%) chance of having 2 boys or 2 girls.

There is a 2 in 4 or $\frac{2}{4}$ ($\frac{1}{2}$/1 half/50%) chance of having one child of each gender.

Activity 18: Using diagrams and tables to calculate probability

1. Complete the missing details in the following tree diagram:

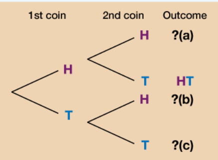

Figure 28 A tree diagram showing first and second coin toss outcomes

2. Draw a table showing all of the possibilities when 2 coins are tossed.
3. What is the probability of getting 2 tails?

Answer

1.
    a. HH
    b. TH
    c. TT
2. Your table should look like the one below.

Table 28 Coin toss probability table

		First coin	
		H	**T**
Second coin	**H**	**HH**	**TH**
	T	**HT**	**TT**

3. Out of the possibilities there is 1 in 4 or $\frac{1}{4}$ (1 quarter/25%) chance of getting 2 tails.

Remember that when you are checking your answers, you may have gone about the question in a different way. In a real exam it is always important to show your working as even if you don't arrive at the correct answer you can still gain marks.

You've now completed Session 4 of your course, congratulations!

Summary

In this section you have learned:

- that the probability of an event is how likely or unlikely that event is to happen and that this can be expressed as a fraction, decimal or percentage.

- how to use a table or tree diagram to show the different outcomes of two or more events.

8 End-of-course quiz

Now it's time to complete end-of-course quiz (Session 4 compulsory badge quiz). It's similar to previous quizzes, but in this one there will be 15 questions.

End-of-course quiz

Open the quiz in a new window or tab then come back here when you're done.

Remember, this quiz counts towards your badge. If you're not successful the first time, you can attempt the quiz again in 24 hours.

9 Session 4 summary

Well done! You have now completed ‘Handling data’, the fourth and final session of the course. If you have identified any areas that you need to work on, please ensure you refer back to this section of the course.

You should now be able to:

- identify different types of data
- create and use tally charts, frequency tables and data collection sheets to record information
- draw and interpret bar charts, pie charts and line graphs
- understand there are different types of averages and be able to calculate each type
- understand that probability is about how likely an event is to happen and the different ways that it can be expressed.

All of the skills listed above will help you when booking a holiday, budgeting, reading the news or analysing data at work.

You are now ready to test the knowledge and skills you’ve learned throughout each section in the end-of-course quiz (Session 4 compulsory badge quiz). Good luck!

10 Bringing it all together

Congratulations on completing *Everyday maths 2*. We hope you have enjoyed the experience and now feel inspired to develop your maths skills further.

Throughout this course you have developed your skills within the following areas:

- understanding and using whole numbers and decimal numbers, and understanding negative numbers in the context of money and temperature
- solving problems requiring the use of the four operations and rounding answers to a given degree of accuracy
- understanding and using equivalences between common fractions, decimals and percentages
- working out simple and more complex fractions and percentages of amounts
- calculating percentage change
- adding, subtracting, multiplying and dividing decimals up to three decimal places
- solving ratio problems where the information is presented in a variety of ways
- understanding the order of operations and using this to work with formulas
- solving problems requiring calculations with common measures, including money, time, length, weight, capacity and temperature
- converting units of measure in the same system and those in different systems
- extracting and interpreting information from tables, diagrams, charts and graphs
- collecting and recording discrete data, and organising and representing information in different ways
- finding the mean, median, mode and range for sets of data
- using data to assess the likelihood of an outcome and expressing this in different forms
- working with area, perimeter and volume, scale drawings and plans.

11 Next steps

If you would like to achieve a more formal qualification, please visit one of the centres listed below with your OpenLearn badge. They'll help you to find the best way to achieve the Level 2 Essential Skills Wales qualification in Application of Number, which will enhance your CV.

- **Coleg Cambria** • https://www.cambria.ac.uk/ • 0300 30 30 007
- **Addysg Oedolion Cymru | Adult Learning Wales** • https://www.adultlearning.wales/ • 03300 580845
- **Coleg Gwent** • https://www.coleggwent.ac.uk/ • 01495 333777
- **NPTC Group of Colleges** • https://www.nptcgroup.ac.uk/

Session 1: Working with numbers

Introduction

With an understanding of numbers you have the basic skills you need to deal with much of everyday life. In this session, you will learn about how to use each of the four different mathematical operations and how they apply to real life situations. Then you will encounter fractions and percentages which are incredibly useful when trying to work out the price of an item on special offer. Additionally, you will learn how to use ratio – handy if you are doing any baking – and how to work with formulas.

Throughout the course there will be various activities for you to complete in order to check your skills. All activities come with answers and show suggested working. It is important to know that there are often many ways of working out the same calculation. If your working is different to that shown but you arrive at the same answer, that's perfectly fine. If you need to refresh your written methods for addition, subtraction, multiplication or division please refer to the relevant number section in *Everyday maths 1*.

If you want more practice with your maths skills, please contact your local college to discuss the options available.

By the end of this session you will be able to:

- use the four operations to solve problems in context
- understand rounding and look at different ways of doing this
- write large numbers in full and shortened forms
- carry out calculations with large numbers
- carry out multistage calculations
- solve problems involving negative numbers
- define some key mathematical terms (multiple, lowest common multiple, factor, common factor and prime number)
- identify lowest common multiples and factors
- use fractions, decimals and percentages and convert between them
- solve different types of ratio problems
- make substitutions within given formulas to solve problems
- use inverse operations and estimations to check your calculations.

Video content is not available in this format.

1 Four operations

You will already be using the four operations in your daily life (whether you realise it or not). Everyday life requires you to carry out maths all the time; checking you've been given the correct change, working out how many packs of cakes you need for the children's birthday party and splitting the bill in a restaurant are all examples that come to mind.

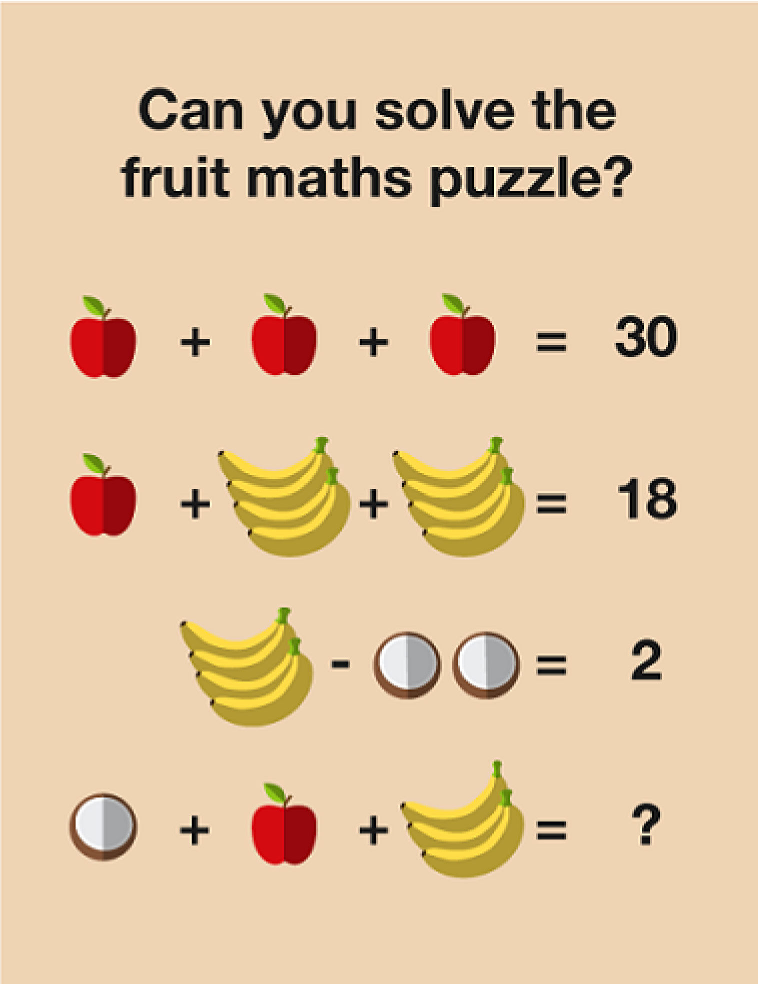

Figure 1 Fruit maths puzzle

The four operations are addition, subtraction, multiplication and division. You need to understand what each operation does and when to use it and at level 2, you will often be required to use more than one operation to answer a question.

- **Addition (+)**
  - This operation is used when you want to find the total, or sum, of two or more amounts.
- **Subtraction (−)**
  - This operation is used when you want to find the difference between two amounts or how much of something you have left after a quantity is used – for example, if you want to find the change owed after spending an amount of money.
- **Multiplication (×)**

- This operation is used to calculate multiple amounts of the same number. For example, if you wanted to know the cost of 16 plants costing £4.75 each you would multiply £4.75 by 16:

  £4.75 × 16 = £76

- **Division (÷)**
  - Division is used when sharing or grouping items. For example, if a group of 6 friends won £765 on the lottery and you wanted to know how much each person would get you would divide £765 by 6:

    £765 ÷ 6 = £127.50

If you are doing this course to prepare you for Essential Skills Wales Application of Number, remember the exam papers do not allow the use of a calculator so please attempt the calculations in this session by hand.

Activity 1: Operation choice

Each of the four questions below uses one of the four operations. Match the operation to the question.

Match each of the items above to an item below.

- You need to save £306 for a holiday. You have 18 months to save up that much money. How much do you need to save per month?
- Fourteen members of the same family go on holiday together. They each pay £155. What is the total cost of the holiday?
- You make an insurance claim worth £18 950. The insurance company pays you £12 648. What is the difference between what you claimed and what you actually received?
- You go to the local café and buy a coffee for £2.35, a tea for £1.40 and a croissant for £1.85. How much do you spend?

Try these questions to make sure that you are confident using each of the four operations.

Activity 2: Calculations with whole numbers

1. 1245 + 654
2. 187 + 65 401
3. 1060 − 264
4. 2000 − 173
5. 543 × 19
6. 1732 × 46
7. 1312 ÷ 8
8. 1044 ÷ 12
9. Stuart is saving £225 per month for a new car. How much would he have saved in:
   a. 1 year
   b. 2 years

10. A concert venue has 2190 seats arranged into 15 sections. How many seats are in each section?
11. A college sells 325 tickets to the charity ball at £12 each. How much money do they make from ticket sales?

Answer

1. 1245 + 654 = 1899
2. 187 + 65 401 = 65 588
3. 1060 − 264 = 796
4. 2000 − 173 = 1827
5. 543 × 19 = 10 317
6. 1732 × 46 = 79 672
7. 1312 ÷ 8 = 164
8. 1044 ÷ 12 = 87
9.
    a. Stuart is saving £225 per month for 1 year or 12 months so the calculation is 225 × 12 = £2700.
    b. Stuart saves £2700 in 1 year so to work out the amount saved in 2 years the calculation is 2 × £2700 = £5400. Alternatively, you could add £2700 + £2700 = £5400.
10. To calculate this you divide the number of seats by the number of sections. So, 2190 ÷ 15 = 146. So there are 146 seats in each section.
11. To calculate this you multiply the number of tickets by the cost per ticket. So, 325 × 12 = 3900. So the college will make £3900 from ticket sales.

1.1 Expressing a remainder as a decimal

To split a prize of £125 between 5 friends you would do this calculation:

£125 ÷ 5 and get the answer £25.

This is a convenient, exact amount of money. However, often when you perform calculations, especially those involving division, you do not always get an answer that is suitable for the question.

For example, if there were 4 friends who shared the same prize we would do the calculation £125 ÷ 4 and get the answer £31 remainder £1. If we did the same calculation on a calculator you would get the answer £31.25, the remainder has been converted into a decimal. Let’s look at how to express the remainder as a decimal.

You can write one hundred and twenty five pounds in two different ways: £125 or £125.00. Both ways show the same amount but the second way allows you to continue the calculation and express it as a decimal.

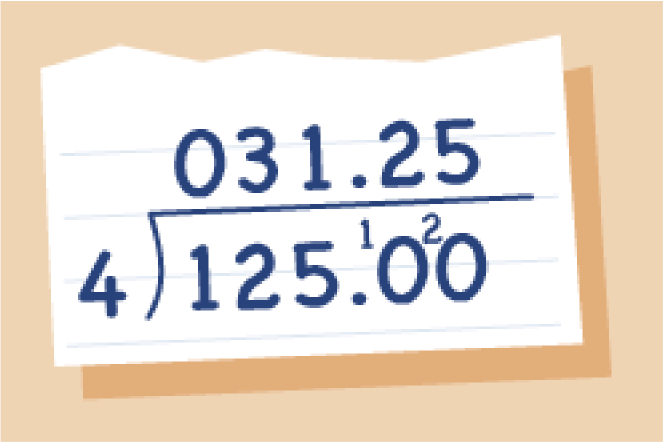

Figure 2 Expressed as a decimal: 125 ÷ 4

We can use the same principal with any whole number, adding as many zeros after the decimal point as required. Look at the following example.

A teacher wants to share 35 kg of clay between 8 groups of students. How much clay will each group get?

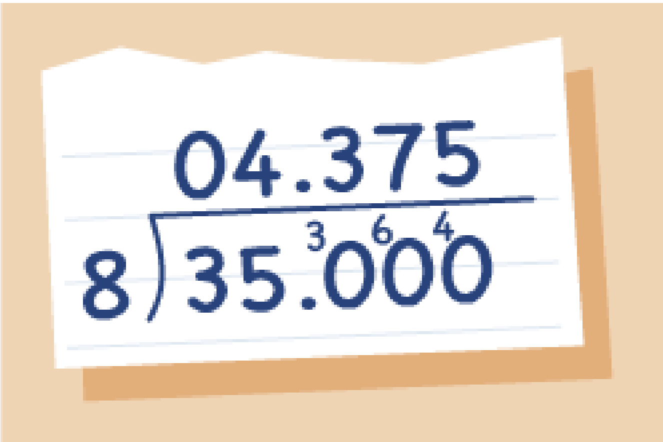

Figure 3 Expressed as a decimal: 35 ÷ 8

You can see that each group would get 4.375 kg of clay.

Activity 3: Expressing a remainder as a decimal

Work out the answers to the following without using a calculator.

1. 178 ÷ 4
2. 212 ÷ 5
3. 63 ÷ 8
4. 227 ÷ 4

Answer

1. 44.5
2. 42.4
3. 7.875

4. 56.75

1.2 Interpreting answers when dividing

After carrying out a division calculation you may not have an answer that is suitable.

For example, if you were at a restaurant and needed to split a bill of £126.49 between four people you would first calculate the division £126.49 ÷ 4 = £31.6225. Clearly you cannot pay this exact amount and so we would round it up to £31.63 to make sure the whole bill is covered.

In other situations, you may need to round an answer down. If you were cutting a length of wood that is 2 m (200 cm) long into smaller pieces of 35 cm you would initially do the calculation 200 ÷ 35. This would give an answer of 5.714…. As you will only actually be able to get 5 pieces of wood that are 35 cm long, you need to round your answer of 5.714 down to simply 5.

Note: The three full stops used in the answer above (5.714…) is a character called an ellipsis. In maths it is used to represent recurring decimal numbers so you don't have to display them all.

Activity 4: Interpreting answers

Calculate the answers to the following. Decide whether the answers need to be adjusted up or down after calculation of the division sum.

1. Apples are being packed into boxes of 52. There are 1500 apples that need packing. How many boxes are required?
2. A bag of flour contains 1000 g. Each batch of cakes requires 150 g of flour. How many batches can you make?
3. A child gets £2.50 pocket money each week. They want to buy a computer game that costs £39.99. How many weeks will they need to save up in order to buy the game?
4. A length of copper pipe measures 180 cm. How many smaller pieces that each measure 40 cm can be cut from the pipe?

Answer

1. 1500 ÷ 52 = 28.846 which must be adjusted up to 29 boxes.
2. 1000 ÷ 150 = 6.666 which must be adjusted down to 6 batches.
3. £39.99 ÷ £2.50 = 15.996 which must be adjusted up to 16 weeks.
4. 180 ÷ 40 = 4.5 which must be adjusted down to 4 pieces.

1.3 Dealing with decimals

We often deal with decimals in everyday life, for example, calculations involving money.

Try the following (without a calculator) to check your skills. For this activity, you will need to give your answers in full and only round or adjust your answer if required. If you need a reminder about how to carry out any of the calculations, please refer back to *Everyday maths 1*.

Activity 5: Calculations with decimals

1. 54.865 + 4.965 + 23.519
2. 6.938 − 5.517
3. 25 + 0.258
4. 54 − 0.65
5. 5.632 × 2.4
6. 1.542 × 1.9
7. 42.4 ÷ 4
8. 39.45 ÷ 1.5
9. 0.48 ÷ 0.025
10. A factory orders 425 gaskets that weigh 2.3 g each. What is the total weight of the gaskets?
11. A dispenser holds 15.5 litres of water. How many full 0.2 litre cups of water can you get from one dispenser?
12. Ahmed and Lea take part in the long jump. Their results are as follows:

    Ahmed 5.501 m

    Lea 5.398 m

    Who jumped furthest and by how much?

Answer

1. 83.349
2. 1.421
3. 25.258
4. 53.35
5. 13.5168
6. 2.9298
7. 10.6
8. 26.3
9. 19.2
10. 977.5 g
11. 77.5 (77 full cups)
12. Ahmed jumped the furthest by 0.103 m

Summary

In this section you have:

- recapped how to carry out calculations with whole numbers and decimals
- learned to adjust answers where needed
- learned how to express a remainder as a decimal.

2 Dealing with large numbers

It is important to be able to carry out calculations with numbers of any size. Large numbers can be written in different ways e.g.

1 200 000 (one million, two hundred thousand) or it can be written as 1.2 million.

Here is another example:

4 250 000 000 (four billion, two hundred and fifty million) is 4.25 billion.

It is often easier to deal with very large numbers when they are written as decimals.

Notice how the decimal is placed after the whole millions or billions.

Hint: A billion is a thousand million.

Using a place value grid can help you to read large numbers as it groups the digits for you, making the whole number easier to read.

Notice how the numbers above are written in this place value grid.

Table 1

Billion	**Million**			**Thousand**					
Billions	Hundreds of millions	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	**Hundreds**	**Tens**	**Units**
			1	2	0	0	0	0	0
4	2	5	0	0	0	0	0	0	0

Sometimes when dealing with large numbers it is sensible to round them, for example, the Office for National Statistics gives the number of people unemployed in the UK in February 2019 as 1.36 million. The number of people unemployed will not be exactly 1 360 000 but, by rounding the exact value and writing it as 1.36 million, it is easier to understand.

Activity 6: Rounding large numbers

The following table gives the population of countries.

Round each population to the nearest million and write the figure in shortened form, using decimals where needed.

Table 2(a)

Country	Population
UK	66 959 016
China	1 420 062 022

Answer

Table 2(b)

Country	Population	Population rounded	Shortened form
UK	66 959 016	67 000 000	67 million
China	1 420 062 022	1 420 000 000	1.42 billion

2.1 Calculations with large numbers

The best way to get used to these types of calculations is to go straight into an example.

Example: Calculations with large numbers

Calculate the total population of Malta (0.4 million) and Cyprus (1.2 million).

Method 1
Work in shortened form:

1.2 + 0.4 = 1.6 million

Method 2
Write the numbers in full:

1 200 000 + 400 000 = 1 600 000 (1.6 million)

Activity 7: Calculations with large numbers

1. Calculate the total turnover for Cambria Trading over the first quarter (3 months).

Table 3 Cambria Trading turnover

Month	Profit (£ million)
January	1.2
February	0.9
March	0.85
April	1.1
May	1.02
June	0.87
July	1.19
August	0.98
September	1.05
October	1.08
November	1.8
December	1.65

2. Calculate the turnover of the last quarter.
3. Calculate the difference in turnover between the first and last quarters.
4.
   b. Which month had the largest turnover?
   c. Which month had the smallest turnover?
   d. What is the difference between the largest and smallest turnovers?

Answer

1. Profit in 1st quarter 1.2 + 0.9 + 0.85 = £2.95 million
2. Profit in last quarter 1.08 + 1.8 + 1.65 = £4.53 million
3. The difference is 4.53 − 2.95 = £1.58 million
4.
   a. November had the largest turnover at £1.8 million.
   b. March had the smallest at £0.85 million.
   c. The difference is 1.8 − 0.85 = £0.95 million.

Summary

In this section you have learned how to:

- write large numbers in full and shortened forms
- round large numbers
- add and subtract large numbers.

3 Rounding

Why might you want to round numbers? You may wish to estimate the answer to a calculation or to use a guide rather than work out the exact total. Alternatively, you might wish to round an answer to an exact calculation so that it fits a given purpose, for example an answer involving money cannot have more than two digits after the decimal point.

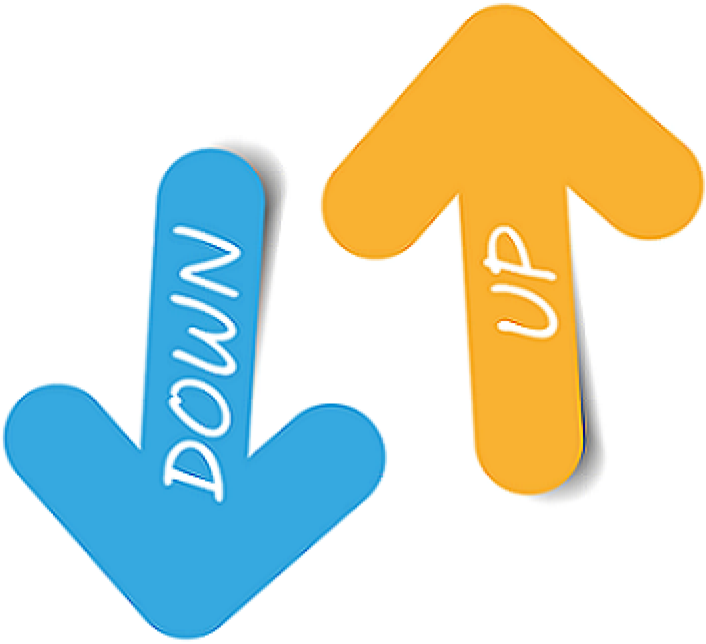

Figure 4 Rounding up and down

You will now explore each of these examples in more detail and practise your rounding skills in context.

3.1 Rounding to a degree of accuracy

Watch the short video below to see an example of how to round to 1, 2 and 3 decimal places.

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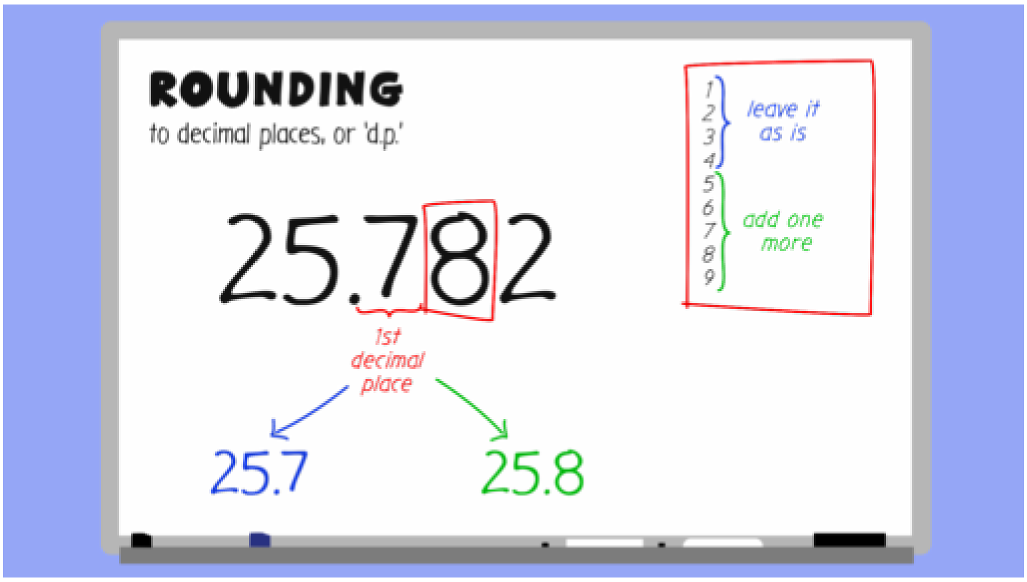

Remember this rounding rhyme to help you:

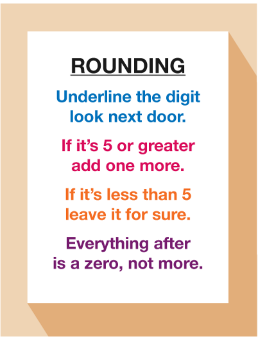

Figure 5 A rounding rhyme

Activity 8: Rounding skills

Practise your rounding skills by completing the below.

1. What is 24.638 rounded to one decimal place?
2. What is 13.4752 rounded to two decimal places?
3. What is 203.5832 rounded to two decimal places?
4. What is 345.6794 rounded to three decimal places?
5. What is 3.65 rounded to the nearest whole number?
6. What is £199.755 to the nearest penny?
7. What is £37.865 to the nearest pound?
8. What is 61.607 kg to the nearest kg?

Answer

1. 24.6
2. 13.48
3. 203.58
4. 345.679
5. 4
6. £199.76
7. £38
8. 62 kg

3.2 Rounding to approximate an answer

You might round in order to approximate an answer. At the coffee shop, you might want to buy a latte for £2.85, a cappuccino for £1.99 and a tea for £0.99. It is natural to round these amounts up to £3, £2 and £1 in order to arrive at an approximate cost of £6 for all three drinks. It is also very useful when checking calculations to make sure that your answer makes sense, especially when working with large numbers and decimals.

Activity 9: Approximation

Calculate the following using a calculator and use estimation to check your answers.

1. On 5th March 2019, 190 people matched 5 numbers and won £1650 each. What was the total prize fund?
2. Swansea AFC's home ground is the Liberty Stadium which holds 21 088 fans. Cardiff City FC plays at the Cardiff City Stadium which holds 33 316 fans.

   What is the difference in capacity at these grounds?

Answer

1. The actual answer is £313 500 (190 × 1650)

   Estimate 200 × 1700.

   2 × 17 = 34 so 200 × 1700 = 340 000 so the answer is sensible.
2. The actual answer is 12 228 (33 316 − 21 088)

   Estimate 33 000 − 21 000 = 12 000 so the answer is sensible.

Summary

In this section you have learned:

- how and when to use rounding to approximate an answer to a calculation

- how to round an answer to a given degree of accuracy – e.g. rounding to two decimal places.

4 Multistage calculations

Often in daily life you will come across problems that require more than one calculation to reach the final answer.

Example: Multistage calculations

Four friends are planning a holiday. The table below shows the costs:

Table 4

Item	Price
Flight (return)	£305 per person
Taxes	£60 per person
Hotel	£500 per room. 2 people per room
Taxi to airport	£45

The friends will be sharing the total cost equally between them. How much do they each pay?

Method

First we use multiplication to find the cost of items that we need more than one of:

Flights = £305 × 4 = £1220
Taxes = £60 × 4 = £240
Hotel = 2 rooms required for 4 people = £500 × 2 = £1000

Now we use addition to add these totals together along with the taxi fare:

£1220 + £240 + £1000 + £45 = £2505

Finally, we need to use division to find out how much each person pays:

£2505 ÷ 4 =£626.25 each

Activity 10: Multistage calculations

1. Your current mobile phone contract costs you £24.50 per month.

   You are considering changing to a new provider. This provider charges £19.80 per month along with an additional, one off connection fee of £30.

   How much will you save over the year by switching to the new provider?
2. You are going on holiday and you have decided to stay in a cottage in North Wales for 7 nights.

   There will be 12 of you staying and the total cost of the cottage you have chosen is £460 per night. If you split the cost equally, how much will each of you pay?

Answer

1. **First** calculate the cost with the current provider
   monthly cost × 12
   £24.50 × 12 = £294 (current provider)
   Second, calculate the cost of the new provider.
   To do this you need to calculate the total monthly costs:
   £19.80 × 12 = £237.60
   and then add on the one off connection fee of £30:
   £237.60 + £30 = £267.60 (new provider)
   Finally you can calculate the difference between the two providers:
   £294 − £267.60 = £26.40 saved
2. To do your calculation, you need to work out the cost for 7 nights. Once you have done this, you can divide the total by 12:
   460 × 7 = £3220
   £3220 ÷ 12 = £268.33 (rounded to two d.p.)
   Here we would probably round this amount up to £268.34 per person to make sure the full cost is covered.

All the examples we have looked at until now have used positive numbers. However, as anyone with an overdraft will know, numbers (or bank balances) are not always positive! Our next section therefore deals with negative numbers.

Summary

In this section you have:

- applied the four operations to solve multistage calculations.

5 Negative numbers

Negative numbers come into play in two main areas of life: money and temperature. Watch the animations below for some examples.

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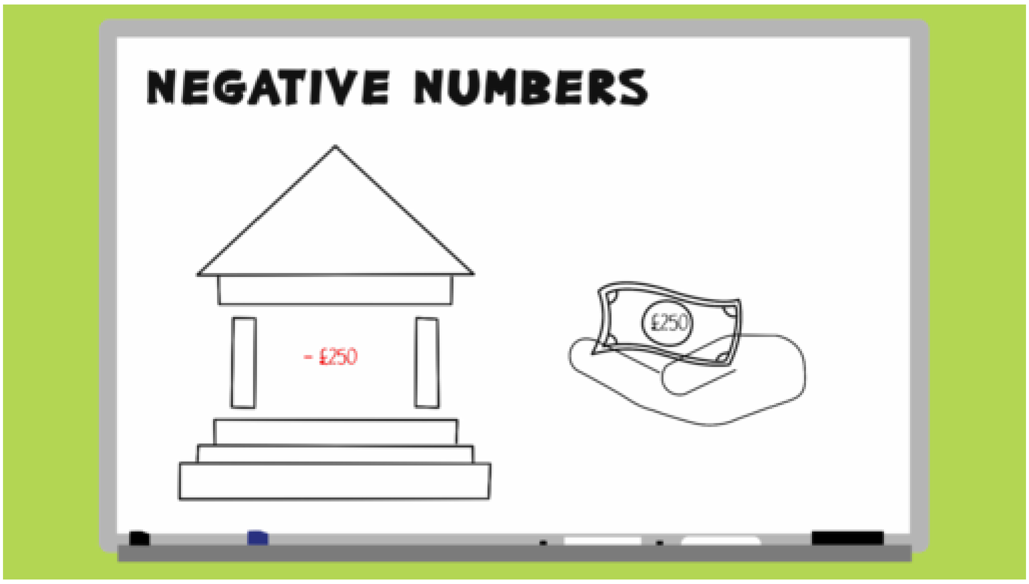

Activity 11: Negative and positive temperature

1. The table below shows the temperatures of cities around the world on a given day

Table 5

London	Oslo	New York	Kraków	Delhi
4˚C	−12C	7˚C	−3˚C	19˚C

   a. Which city was the warmest?
   b. Which city was the coldest?
   c. What is the difference in temperature between the warmest and coldest cities?

Answer

1.
   a. Delhi was the warmest city as it has the highest positive temperature.
   b. Oslo was the coldest city as it has the largest negative temperature.
   c. The difference between the temperatures in these cities is 31˚C.

From 19˚C down to 0˚C is 19˚C and then you need to go down a further 12˚C to get to −12˚C.

2. Look at this bank statement.

Bank statement

SBH Bank
Tadworth Road
Colesdale

Miss Sonia Cedar
Account no. 23395
Summary for 1-31 October

Date	Description	Amount	Balance
09 Oct	Bank Transfer		100.00
11 Oct	Direct Debit		-20.00
15 Oct	Automated Pay In		70.00
20 Oct	Bank Transfer		100.00
21 Oct	Direct Debit		-50.00
25 Oct	Bank Transfer		200.00

Figure 6 A bank statement

b. On which days was Sonia Cedar overdrawn, and by how much?
c. How much money was withdrawn between 9 and 11 of October?
d. How much was added to the account on 15 October?

Answer

2.

b. The minus sign (−) indicates that the customer is overdrawn, i.e. owes money to the bank.

The amount shows how much they owe. So Sonia Cedar was overdrawn on 11 October by £20 and by £50 on 21 October.

c. £120 was withdrawn on 11 October.

The customer had £100 in the account and must have withdrawn another £20 (i.e. £100 + £20 = £120 in total) in order to be £20 overdrawn.

d. The customer owed £20 and is now £70 in credit, so £90 must have been added to the account.

3. Look at the table below showing a company's profits over 6 months.

Hint: a negative profit means that the company made a loss.

Table 6

Month	Profit (£000)
January	166
February	182
March	−80
April	124
May	98
June	−46
Balance	

a. Which month had the greatest profit?
b. Which month has the greatest loss?
c. What was the overall balance for the six months?
Hint: start by calculating the total profits and the total losses.

Answer

3.

a. February had the largest profit with £182 000 (remember to look at the column heading which shows that the figures are in 000s – thousands).
b. March showed the greatest loss at £80 000.
c. To calculate the overall balance you need to first calculate the total profits and the total losses. To calculate the profits you need to do this calculation:
166 + 182 + 124 + 98 = 570

So the profit was £570 000.

Next you need to calculate the total losses; two months showed a loss so you need to add these values:

80 + 46 = 126 so the losses over the six months were £126 000.

Now you can calculate the overall balance by subtracting the losses from the profits:

£570 000 − £126 000 = £444 000

This is a positive value so it means the company made an overall profit of £444 000.

Summary

In this section you have:

- learned the two main contexts in which negative numbers arise in everyday life – money (or debt!) and temperature
- practised working with negative numbers in these contexts.

6 Mathematical terms

It is important to know the meaning of the following terms:

- multiples
- lowest common multiple
- factors
- common factors
- prime numbers

Multiples

A multiple of a number can be found by multiplying that number by any whole number e.g. multiples of 2 include 2, 4, 6, 8, 10 etc. (all are in the 2 times table).

Note: To check if a number is a multiple of another, see if it divides exactly into the multiple, e.g. to see if 3 is a multiple of 81 do 81 ÷ 3 = 27. It divides exactly so 81 *is*a multiple of 3.

Lowest common multiple

In maths, we sometimes need to find the lowest common multiple of numbers.

The lowest common multiple (LCM) is simply the smallest multiple that is common to more than one number.

Example: Lowest common multiple of 3 and 5

Hint: when looking for multiples, it is easiest to start by listing the multiples of the highest number first. This saves you going any further than you need to with the list.

The first few multiples of 5 are:

5, 10, **15**, 20, 25, 30 etc.

The first few multiples of 3 are:

3, 6, 9, 12, **15**, 18, 21 etc.

You can see that the lowest number that is a common multiple of 3 and 5 is 15.

Activity 12: Finding the lowest common multiple

Find the lowest common multiple of:

1. 6 and 12
2. 2 and 7

Answer

1. The lowest common multiple of 6 and 12 is 12:

   Multiples of 12:

   12, 24, 36, 48, 60 etc.

   Multiples of 6:

   6, **12**, 18, 24, 30 etc.

   You can see from the list that 24 is also a common multiple of 6 and 12, but 12 is the lowest common multiple.
2. The lowest common multiple of 2 and 7 is 14:

   Multiples of 7:

   7, **14**, 21, 28, 35, 42 etc.

   Multiples of 2:

   2, 4, 6, 8, 10, 12, **14** etc.

Factors, common factors and prime numbers

Factors of a number divide into it exactly. Factors of all numbers include 1 and the number itself. However, most numbers have other factors as well. If you think of all of the numbers that multiply together to make that number, you will find all of the factors of that number.

Example: What are the factors of 8?

$8 \times 1 = 8$

$2 \times 4 = 8$

So the factors of 8 are 1, 2, 4 and 8.

Activity 13: Finding factors

1. What are the factors of 54?
2. What are the factors of 165?

Answer

1. The factors of 54 are 1, 2, 3, 6, 9, 18, 27 and 54
2. The factors of 165 are 1, 3, 5, 11, 15, 33, 55 and 165

A common factor is a factor that goes into more than one number. For example, 4 is a common factor of 8 and 12 because it divides exactly into both numbers.

Prime Numbers

A prime number is a number which only has 2 factors: 1 and itself.

The prime numbers between 1 and 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

Note:

- 1 is *not* a prime number as it only has one factor.
- 2 is the only even prime number.

You have now learned how to use all four operations, how to work with negative numbers and learned some important mathematical terms. Every other mathematical concept hinges around what you have learned so far; so once you are confident with these, you'll be a success!

Summary

In this section you have:

- learned some key mathematical terms: multiple, lowest common multiple, factor, common factor and prime number
- identified the lowest common multiple
- identified factors.

7 Fractions

You will be used to seeing fractions in your everyday life, particularly when you are out shopping or scouring the internet for the best deals. It's really useful to be able to work out how much you'll pay if an item is on sale or if a supermarket deal really is a good deal!

Figure 7 A poster advertising a sale

There are several different elements to working with fractions. First you will look at simplifying fractions.

7.1 Simplifying fractions

Watch the video below which looks at how to simplify fractions before having a go yourself in Activity 14.

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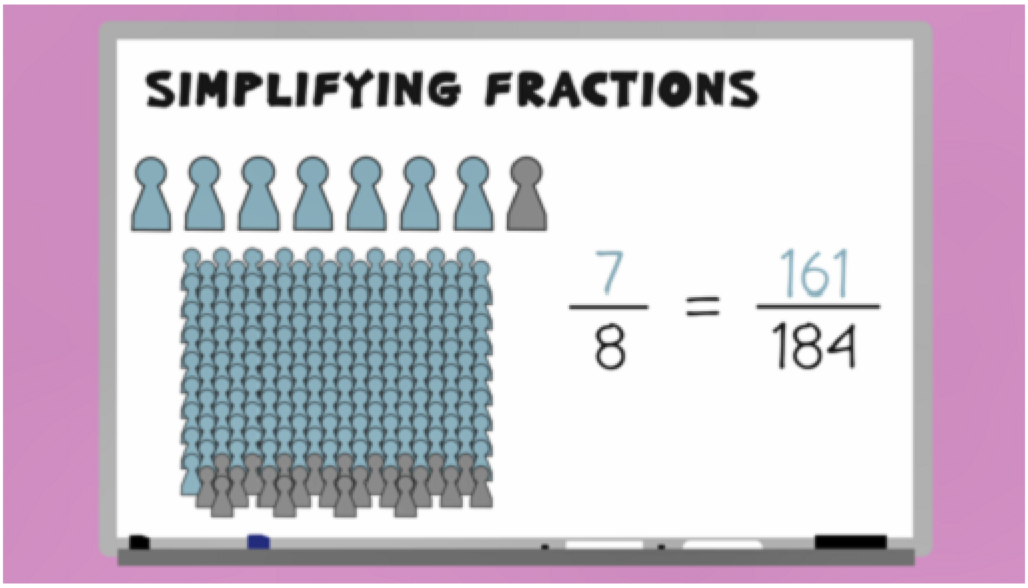

Activity 14: Simplifying fractions

Show the following fractions in simplest form, where possible:

a. $\frac{25}{75}$

b. $\frac{12}{36}$

c. $\frac{72}{96}$

d. $\frac{32}{48}$

e. $\frac{5}{126}$

f. $\frac{164}{256}$

Answer

a. $\frac{25}{75} = \frac{1}{3}$

b. $\frac{12}{36} = \frac{1}{3}$

c. $\frac{72}{96} = \frac{3}{4}$

d. $\frac{32}{48} = \frac{2}{3}$

e. $\frac{5}{126}$ can't be simplified

f. $\frac{164}{256} = \frac{41}{64}$

Next you'll look at expressing a quantity of an amount as a fraction.

7.2 Writing a quantity of an amount as a fraction

Sometimes you will need to show one amount as a fraction of another. This might sound complicated, but it's actually very logical. Look at the examples below.

Example 1: Fraction of an amount

In Figure 8, what fraction of Smarties are red?

Figure 8 Smarties in different colours

$$\frac{number\ of\ red\ smarties}{total\ number\ of\ smarties} = \frac{4}{30}$$

To express the fraction of Smarties that are red, you simply need to count the red Smarties (4) and the total number of Smarties (30). Since there are 4 red Smarties out of 30 altogether, the fraction is $\frac{4}{30}$. It is worth noting here that this could also be written as 4/30.

You may well be asked to give your answer as a fraction in its simplest form, so always check to see if you can simplify your answer. In this case $\frac{4}{30}$ will simplify to $\frac{2}{15}$.

Example 2: Fraction of an amount

250 g of flour is taken from a 1 kg bag. What fraction is this?

Hint: there are 1000 g in a kg.

To express quantities as fractions, the top and bottom numbers need to be in the same units, so here you need to make sure that you express both the top and bottom values in grams:

> The flour removed is already expressed in grams: 250 g
> The total amount is in kilograms so you need to convert to grams: 1 kg = 1000 g

Now write the amount taken over the total amount to express as a fraction:

$\frac{250}{1000}$ (250 g of flour out of the 1000 g bag has been taken or used)

Then cancel down (or simplify) if possible:

$$\frac{250}{1000} = \frac{1}{4}$$

So $\frac{1}{4}$ of the flour has been used.

Activity 15: Expressing one number as a fraction of another

1. What fraction of a kilogram is:
   a. 100 g
   b. 750 g
   c. 640 g
   d. 20 g

Answer

1. 100 g = 1000 g, so:
   a. 100 g $\frac{100}{1000}$ = $\frac{1}{10}$ of a kilogram

   b. 750 g $\frac{750}{1000}$ = $\frac{3}{4}$ of a kilogram

   c. 640 g $\frac{640}{1000}$ = $\frac{16}{25}$ of a kilogram

   d. 20 g = $\frac{20}{1000}$ = $\frac{1}{50}$ of a kilogram

2. What fraction of an hour is:
   b. 15 minutes
   c. 20 minutes
   d. 35 minutes
   e. 48 minutes

Answer

2. 1 hour = 60 minutes so:
   b. 15 minutes = $\frac{15}{60}$ = $\frac{1}{4}$ of an hour.

   c. 20 minutes = $\frac{20}{60}$ = $\frac{1}{3}$ of an hour.

   d. 35 minutes = $\frac{35}{60}$ = $\frac{7}{12}$ of an hour.

   e. 48 minutes = $\frac{48}{60}$ = $\frac{4}{5}$ of an hour.

3. A farmer takes 120 eggs to the local farmer's market. She has 24 eggs left at the end of the day. What fraction of the eggs are left?

Answer

3. $\frac{24}{120}$ are left. This cancels down to $\frac{1}{5}$, so $\frac{1}{5}$ of the eggs are left.

4. A class of students sit a test. 18 pass and 12 fail. What fraction passed the test?

Answer

4. Work out the total number of students by adding the number who passed to those who failed 18 + 12 = 30. Now work out the fraction that passed:

   $\frac{18}{30}$ (18 out of 30 students passed)

   Now cancel down:

   $\frac{18}{30} = \frac{3}{5}$.

   So $\frac{3}{5}$ passed the test.

5. Mary bought her car for £12 500. When she goes to trade it in she is offered £8750. What fraction of the original price is this?

Answer

5. $\frac{8750}{12500} = \frac{7}{10}$

6. 30 people entered a raffle. 6 of these people won a prize. What fraction of people did not win a prize? Give your answer as a fraction in its simplest form.

Answer

6. As this question wants the number of people who did not win a prize we must first do:

   30 − 6 = 24 people did not win a prize.

   As a fraction this becomes $\frac{24}{30}$ which simplifies to $\frac{4}{5}$.

Sometimes fractions will not cancel down easily. When this happens, you estimate the fraction by rounding the numbers to values that will cancel. Sometimes this means breaking the ‘rules’ of rounding.

Example: Estimating fractions

$\frac{1347}{2057}$ will not cancel.

By rounding 1347 up to 1400 and 2057 to 2100 we can cancel down the fraction to get:

$$\frac{1400}{2100} = \frac{2}{3}$$

Note: Round to numbers that are easy to cancel, but if you round off too much, you will lose the accuracy of your answer.

Now that you can express a quantity as a fraction, estimate and simplify fractions, the next step is to be able to work out fractions of amounts. For example, if you see a jacket that was priced at £80 originally but is in the sale with $\frac{2}{5}$ off, it's useful to be able to work out how much you will be paying.

7.3 Fractions of amounts

Fractions of amounts can be found by using your division and multiplication skills. To work out a fraction of any amount you first divide your amount by the number on the bottom of the fraction – the denominator. This gives you 1 part.

You then multiply that answer by the number on the top of the fraction – the numerator.

It is worth noting here that if the number on the top of the fraction is 1, multiplying the answer will not change it so there is no need for this step. Take a look at the examples below.

Example: Divide by the denominator

Method

To find $\frac{1}{5}$ of 90 we do 90 ÷ 5 = 18.

Since the number on the top of our fraction is 1, we do not need to multiply 18 by 1 as it will not change the answer.

So $\frac{1}{5}$ of 90 = 18.

Example: Multiply by the numerator

Method

To find $\frac{4}{7}$ of 42 we do 42 ÷ 7 = 6.

This means that $\frac{1}{7}$ of 42 = 6.

Since you want $\frac{4}{7}$ of 42, we then do 6 × 4 = 24.

So $\frac{4}{7}$ of 42 = 24.

Let's go back to the jacket that used to cost £80 but is now in the sale with $\frac{2}{5}$ off. How do you find out how much it costs? Firstly, you need to find $\frac{2}{5}$ of 80. To calculate this you do:

£80 ÷ 5 = £16 and then £16 × 2 = £32

This means that you save £32 on the price of the jacket. To find out how much you pay you then need to do £80 − £32 = £48.

You will have practised finding fractions of amounts in *Everyday maths 1*, but have a go at the following activity to recap this important skill.

Activity 16: Finding fractions of amounts

Work out the following without using a calculator. You may double-check on a calculator if you need to and remember to check your answers against ours.

1. You are looking to buy house insurance and want to get the best deal. Put the following offers in order, from cheapest to most expensive, after the discount has been applied.

Table 7

Company A	Company B	Company C
£120 per year Special offer: $\frac{1}{3}$ off!	£147 per year Special offer: $\frac{2}{7}$ off!	£104 per year Special offer: $\frac{1}{4}$ off!

Answer

1. Company C is cheapest:

   $\frac{1}{4}$ of £104 = £104 ÷ 4 = £26 discount

   £104 − £26 = £78

   Company A is second cheapest:

   $\frac{1}{3}$ of £120 = £120 ÷ 3 = £40 discount

   £120 − £40 = £80

   Company B is most expensive:

$\frac{2}{7}$ of £147 = £147 ÷ 7 × 2 = £42 discount

£147 − £42 = £105

2. A cinema sells 2400 tickets over a weekend. They review their ticket sales and find that $\frac{2}{3}$ of the weekend ticket sales were to adults. How many adult tickets were sold?

Answer

2. 1600 tickets sold to adults:

   2400 ÷ 3 = 800 to give $\frac{1}{3}$

   2 × 800 = 1600 to give $\frac{2}{3}$

3. A college has raised $\frac{3}{5}$ of its £40 000 charity fundraising target. How much money does the college need to raise to meet its target?

Answer

3. £16 000 needed to meet target.

   40 000 ÷ 5 = 8000 to give $\frac{1}{5}$

   8000 × 3 = 24 000 to give $\frac{3}{5}$ (the amount raised)

   But the question asks how much is needed to meet its target so we need to subtract the amount raised from the target:

   40 000 − 24 000 = £16 000

Discounts and special offers are not always advertised using fractions. Sometimes, you will see adverts with 10% off or 15% off. Another common area where we see percentages in everyday life would be when companies apply VAT at 20% to items or when a restaurant adds a 12.5% service charge. The next section looks at what percentages are, and how to calculate them.

Summary

In this section you have:

- learned how to express a quantity of an amount in the form of a fraction
- learned how to, and practised, simplifying fractions
- revised your knowledge on finding fractions of amounts.

8 Percentages

There are different ways of working out percentages of amounts. We will take a look at the most common methods now.

Figure 9 Percentage discounts in a sale

Note: You may use a different method to these ones. You may even use different methods depending on the percentage you are calculating. Do whatever works for you.

8.1 Calculating a percentage of an amount

Method 1

Percentages are just fractions where the number on the bottom of the fraction must be 100. If you wanted to find out 15% of 80 for example, you work out $\frac{15}{100}$ of 80, which you already know how to do!

Working out the percentage of an amount requires a similar method to finding the fraction of an amount. Take a look at the examples below to increase your confidence.

Example 1: Finding 17% of 80

Method

17% of 80 = $\frac{17}{100}$ of 80, so here we do:

80 ÷ 100 = 0.8

0.8 × 17 = 13.6

Another way of thinking about this method is that you are dividing by 100 to find 1% first and then you are multiplying by whatever percentage you want to find.

Alternatively, you could multiply the value by the top number first and then divide by 100:

17 × 80 = 1360

1360 ÷ 100 = 13.6

The answer will be the same.

Example 2: Finding 3% of £52.24

Method

3% of 52.24 = $\frac{3}{100}$ of 52.24, so we do:

52.24 ÷ 100 = 0.5224.

0.5224 × 3 = £1.5672 (£ 1.57 to two d.p.)

Or

52.24 × 3 = 156.72

156.72 ÷ 100 = £1.5672 (£1.57 to two d.p.)

This is a good method if you want to be able to work out every percentage in the same way. It can be used with and without a calculator. Many calculators have a percentage key, but different calculators work in different ways so you need to familiarise yourself with how to use the % button on your calculator.

Method 2

To use this method you only need to be able to work out 10% and 1% of an amount. You can then work out any other percentage from these.

Let's just recap how to find 10% and 1%.

10%

To find 10% of an amount divide by 10:

10% of £765 = 765 ÷ 10 = £76.50

10% of £34.50 = 34.50 ÷ 10 = £3.45

Hint: remember to move the decimal point one place to the left to divide by 10.

1%

To find 1% of an amount divide by 100:

1% of £765 = 765 ÷ 100 = £7.65

1% of £34.50 = 34.50 ÷ 100 = £0.345 (£0.35 to two d.p.)

Hint: remember to move the decimal point two places to the left to divide by 100.

Once you know how to work out 10% and 1%, you can work out any other percentage.

Example 1: Finding 24% of 60

Find 10% first:

60 ÷ 10 = 6

10% = 6

20% is 2 lots of 10% so:

6 × 2 = 12

20% = 12

Now find 1%:

60 ÷ 100 = 0.6

4% is 4 lots of 1% so:

0.6 × 4 = 2.4

4% = 2.4

Now add the 20% and 4% together:

12 + 2.4 = 14.4

Example 2: Finding 17.5% of £328

17.5% can be broken up into 10% + 5% + 2.5%, so you need to work out each of these percentages and then add them together.

Find 10% first:

328 ÷ 10 = 32.8

10% = 32.8

5% is half of the 10% so:

32.8 ÷ 2 = 16.4

5% = 16.4

2.5% is half of the 5% so:

16.4 ÷ 2 = 8.2

2.5% = 8.2

Now add the 10%, 5% and 2.5% figures together:

32.8 + 16.4 + 8.2 = £57.40

This is a good method to do in stages when you do not have a calculator.

Note: There are some other quick ways of working out certain percentages:

50% – divide the amount by two.

25% – halve and halve again.

These quick facts can be used in combination with method 2 to make calculations, e.g. 60% could be worked out by finding 50%, 10% and then adding the 2 figures together. You just need to look for the easiest way to split up the percentage to make your calculation.

Activity 17: Finding percentages of amounts

Use whichever method/s you prefer to calculate the answers to the following:

1. Find:

- a. 45% of £125
- b. 15% of 455 m
- c. 52% of £677
- d. 16% of £24.50
- e. $2\frac{1}{2}$% of 4000 kg
- f. 82% of £7.25
- g. $37\frac{1}{2}$% of £95

2. The Cambria Bank pays interest at 3.5%. What is the interest on £3000?
3. Sure Insurance offer a 30% No Claims Bonus. How much would be saved on a premium of £345.50?
4. Sunshine Travel Agents charge 1.5% commission on foreign exchanges. What is the charge for changing £871?

Answer

1.
   - a. £56.25
   - b. 68.25 m
   - c. £352.04
   - d. £3.92
   - e. 100 kg
   - f. £5.945 (£5.95 to 2 d.p.)
   - g. £35.625 (£35.63 to two d.p.)
2. £105
3. £103.65
4. £13.065 (£13.07 to two d.p.)

Just as with fractions you will often need to be able to work out the price of an item after it has been increased or decreased by a given percentage. The process for this is the same as with fractions; you simply work out the percentage of the amount and then add it to, or subtract it from, the original amount.

Activity 18: Percentages increase and decrease

1. You earn £500 per month. You get a 5% pay rise.
   - a. How much does your pay increase by?
   - b. How much do you now earn per month?

Answer

1.
   - a. £25
   - b. £525 per month.

2. You buy a new car for £9500. By the end of the year its value has decreased by 20%.
   b. How much has the value of the car decreased by?
   c. How much is the car worth now?

Answer

2.
   b. The car has decreased by £1900.
   c. The car is now worth £7600.

3. You invest £800 in a building society account which offers fixed-rate interest at 4% per year.
   c. How much interest do you earn in one year?
   d. How much do you have in your account at the end of the year?

Answer

3.
   c. £32 interest earned.
   d. £832 in the account at the end of the year.

4. Last year Julie's car insurance was £356 per annum. This year she will pay 12% less. How much will she pay this year?

Answer

4. She will pay £42.72 less so her insurance will cost £313.28

5. A zoo membership is advertised for £135 per year. If Tracy pays for the membership in full rather than in monthly installments, she receives a 6% discount. How much will she pay if she pays in full?

Answer

5. She will save £8.10 so she will pay £126.90.

6. A museum had approximately 5.87 million visitors last year. Visitor numbers are expected to increase by 4% this year. How many visitors is the museum expecting this year?

Answer

6. 5.87 million = 5 870 000.
   4% of 5 870 000 = 234 800
   5 870 000 + 234 800 = 6 104 800 people

Next you'll look at how to express one number as a percentage of another.

8.2 Expressing one number as a percentage of another

Sometimes you need to write one number as a percentage of another. You have already practised writing one number as a fraction of another; this just takes it a bit further.

Example 1: What percentage are women?

A class is made up of 21 women and 14 men, what percentage of the class are women?

To work this out, you start by expressing the numbers as a fraction. You then multiply by 100 to express as a percentage.

The formula is:

$$\frac{amount\ we\ need}{total} \times 100$$

In this case, 21 out of a total of 35 people are women so the sum we do would be:

$$\frac{21}{35} \times 100$$

The fraction line is also a divide line, so if you were doing this on a calculator you would do:

21 ÷ 35 × 100 = 60%

How would you work this out without a calculator?

There are different ways you can make the calculation. Two methods are shown below.

Method 1

$$\frac{21 \times 100}{35}$$

You start by multiplying the top number in the fraction by 100. The bottom number will stay the same:

$$\frac{21 \times 100}{35} = \frac{2100}{35}$$

Now you need to cancel the fraction down as much as possible:

$\frac{2100}{35}$ ÷ top and bottom by 5 = $\frac{420}{7}$, then, ÷ top and bottom by 7 = $\frac{60}{1}$

Anything over 1 is a whole number so the answer is 60.

So 60% of the class are women.

Note: When using this method, if you cancel as far as possible and you do not end up with an answer over 1, you will need to divide the top number by the bottom number to work out the final answer, e.g. the fraction $\frac{15}{4}$ cannot cancel any further, so:

15 ÷ 4 = 3.75

Method 2

The other method involves expressing the fraction as a decimal first and then converting it to a percentage. This means that you multiply by 100 at the very end of the calculation.

A local attraction sold 150 tickets last bank holiday, 102 of which were full price. What percentage of the tickets sold were at the concessionary price?

Work out the number of concessionary tickets sold:

150 – 102 = 48

Write the number of concessionary tickets sold as a fraction of the total number sold:

$\frac{48}{150}$

Cancel down your fraction:

$\frac{48}{150}$ ÷ top and bottom by 6 = $\frac{8}{25}$

Once you cannot cancel any further, you need to divide the top number by the bottom number to express as a decimal:

8 ÷ 25 = 0.32

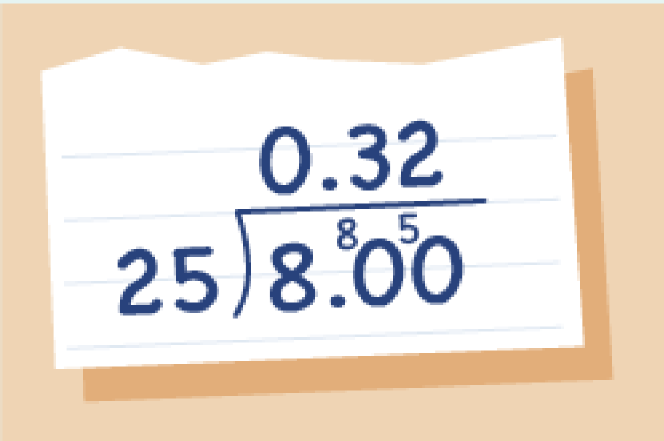

Figure 10 Expressed as a decimal: 8 divided by 25

Finally, multiply the decimal answer by 100 to express as a percentage:

$0.32 \times 100 = 32\%$

So 32% of the tickets were sold at the concessionary price.

Activity 19: Expressing one number as a percentage of another

Use whichever method you prefer to calculate the answers to the following. Give answers to two d.p. where appropriate.

Hint: make sure your units are the same first.

1. What percentage:
   a. of 1 kg is 200 g?
   b. of an hour is 48 minutes?
   c. of £6 is 30p?

Answer

1.
   a. 1 kg = 1000 g

   $\frac{200}{1000} \times 100 = 20\%$

   b. 1 hour = 60 minutes

   $\frac{48}{60} \times 100 = 80\%$

   c. £1 = 100 p

   $\frac{30}{600} \times 100 = 5\%$

2. Bea swam 50 laps of a 25 m swimming pool in a charity swim. A mile is almost 1600 m. What percentage of a mile did Bea swim?

Answer

2. $50 \times 25 = 1250$ m

   $\frac{1250}{1600} \times 100 = 78.13\%$ (to two d.p.)

3. A student gets the following results in the end of year tests:

Table 8

	Maths	English	Science	Art
Mark achieved	64	14	72	56
Possible total mark	80	20	120	70

Calculate her percentage mark for each subject.

Answer

Maths: $\frac{64}{80}$ × 100 = 80%

English: $\frac{14}{20}$ × 100 = 70%

Science: $\frac{72}{120}$ × 100 = 60%

Art: $\frac{56}{70}$ × 100 = 80%

4. Susan is planting her flower beds. She plants 13 yellow flowers, 18 white flowers and 9 red ones. What percentage of her flowers will not be white?

Answer

4. Number not white = 13 + 9 = 22

   Total number she is planting = 13 + 18 + 9 = 40

   $\frac{22}{40}$ × 100 = 55%

   55% of the flowers will not be white.

5. A building society charges £84 interest on a loan of £1200 over a year. What percentage interest is this?

Answer

5. $\frac{84}{1200}$ × 100 = 7%

   The interest rate is 7%.

Next you will look at percentage change. This can be useful for working out the percentage profit (or loss) or finding out by what percentage an item has increased or decreased in value.

8.3 Percentage change

Watch the video below on how to calculate percentage change, then complete Activity 20.

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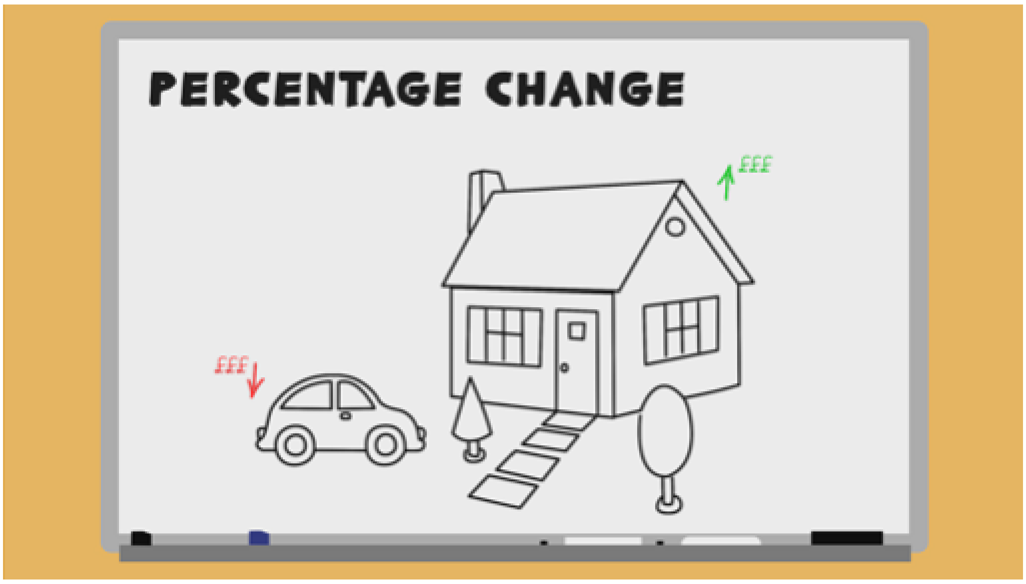

Activity 20: Percentage change formula

Practise using the percentage change formula which you learned about in the video above on the four questions below. Where rounding is required, give your answer to two decimal places.

1. Last year your season ticket for the train cost £1300. This year the cost has risen to £1450. What is the percentage increase?

Answer

1. Difference: £1450 − £1300 = £150

   Original: £1300

   Percentage change = $\frac{150}{1300}$ × 100

   Percentage change = 0.11538... × 100 = 11.54% increase (rounded to two d.p.)

2. You bought your house 10 years ago for £155 000. You are able to sell your house for £180 000. What is the percentage increase the house has made?

Answer

2. Difference: £180 000 − £155 000 = £25 000

   Original: £155 000

   Percentage change = $\frac{25000}{155000}$ × 100

   Percentage change = 0.16129... × 100 = 16.13% increase (rounded to two d.p.)

3. You purchased your car 3 years ago for £4200. You sell it to a buyer for £3600. What is the percentage decrease of the car?

Answer

3. Difference: £4200 − £3600 = £600

   Original: £4200

   Percentage change = $\frac{600}{4200}$ × 100

   Percentage change = 0.14285... × 100 = 14.29% decrease (rounded to two d.p.)

4. Stuart buys a new car for £24 650. He sells it 1 year later for £20 000. What is the percentage loss?

Answer

4. Difference: £24 650 − £20 000 = £4650

   Original: £24 650

   Percentage change = $\frac{4650}{24650}$ × 100

   4650 ÷ 24 650 × 100 = 18.86% loss (rounded to two d.p.)

Congratulations, you now know everything you need to know about percentages! As you have seen, percentages come up frequently in many different areas of life and having completed this section, you now have the skills to deal with all kinds of situations that involve them.

You saw at the beginning of the section that percentages are really just fractions. Decimals are also closely linked to both fractions and percentages. In the next section you will see just how closely related these three concepts are and also learn how to convert between each of them.

Summary

In this section you have:

- found percentages of amounts
- calculated percentage increase and decrease
- calculated percentage change using a formula
- expressed one number as a percentage of another.

9 Fractions, decimals and percentages

You have already worked with decimals in this course and many times throughout your life. Every time you calculate something to do with money, you are using decimal numbers. You have also learned how to round a number to a given number of decimal places.

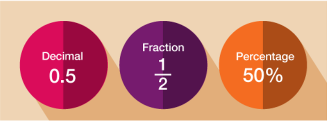

Figure 11 Equivalent decimals, fractions and percentages

9.1 Converting between percentages, decimals and fractions

Since fractions, decimals and percentages are all just different ways of representing the same thing, we can convert between them in order to compare. Take a look at the video below to see how to convert fractions, decimals and percentages.

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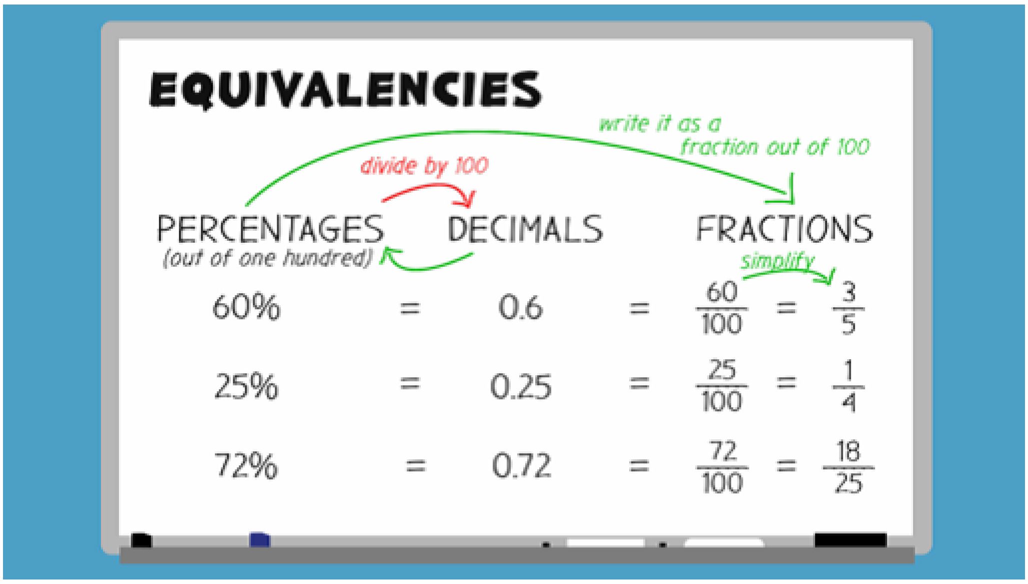

Lets look in more detail at changing a percentage to a fraction.

Example: 50% is $\frac{50}{100}$

As you can see this percentage is essentially a fraction of 100. However you can simplify it to $\frac{1}{2}$.

To change a percentage to a fraction, put the percentage over 100 and simplify if possible.

Sometimes we might see a percentage like this: 12.5%.

If we use the method above we get $\frac{12.5}{100}$ but we can't have a decimal in a fraction.

To get rid of the decimal in the fraction we must multiply the top and bottom of the fraction, the numerator and denominator, by any number that will give us whole numbers. In this case 10 or 2 both work well (12.5 × 10 = 125 and 12.5 × 2 = 25):

Method 1: × 10

$\frac{12.5}{100}$ × top and bottom by 10 = $\frac{125}{1000}$ = $\frac{1}{8}$

Method 2: × 2

$\frac{12.5}{100}$ × top and bottom by 2 = $\frac{25}{200}$ = $\frac{1}{8}$

Activity 21: Converting between percentages, decimals and fractions

1. Express these percentages as decimals:
   a. 62%
   b. 50%
   c. 5%
2. Express these decimals as percentages:
   a. 0.02
   b. 0.2
   c. 0.752
   d. 0.055
3. Express these percentages as fractions:
   a. 15%
   b. 2.5%
   c. 37.5%

Answer

1.
   a. 0.62
   b. 0.5
   c. 0.05
2.
   a. 2%
   b. 20%
   c. 75.2%
   d. 5.5%

3.

a. $\frac{15}{100} = \frac{3}{20}$

b. $\frac{2.5}{100}$ × top and bottom by 10 = $\frac{25}{1000} = \frac{1}{40}$

c. $\frac{37.5}{100}$ × top and bottom by 10 = $\frac{375}{1000} = \frac{3}{8}$

You may have multiplied by different numbers to get rid of the decimal in the last two questions. However, your final answers should still be the same as ours.

Now have a go at matching these fractions to decimals and percentages.

Activity 22: Matching fractions, decimals and percentages

Choose the correct fraction for each percentage and decimal.

$\frac{7}{20}$

$\frac{2}{5}$

$\frac{2}{25}$

$\frac{5}{8}$

Match each of the items above to an item below.

35% = 0.35 =

40% = 0.4 =

8% = 0.08 =

62.5% = 0.625 =

Next you'll look in more detail at how to change a fraction to a percentage.

9.2 Changing a fraction to a percentage

There are two ways you can do this.

Method 1

To change a fraction into a percentage, multiply it by $\frac{100}{1}$ (essentially, you are just multiplying the top number by 100 and the bottom number will stay the same).

Example: Change $\frac{3}{4}$ into a percentage

$\frac{3}{4} \times \frac{100}{1} = \frac{300}{4}$

This cancels to $\frac{75}{1}$ = 75%

Note: Remember anything over 1 is a whole number. If you do not end up with a 1 on the bottom, you will have to divide the top number by the bottom one to get your final answer.

Method 2

Divide the top of the fraction by the bottom (to express the fraction as a decimal) and then multiply the answer by 100.

Example: $\frac{3}{4}$ = 3 ÷ 4 = 0.75

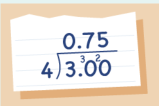

Figure 12 Expressed as a decimal: 3 ÷ 4

0.75 × 100 = 75%

Activity 22: Changing a fraction to a percentage

1. Express these fractions as percentages:
   a. $\frac{3}{8}$
   b. $\frac{9}{10}$
   c. $\frac{4}{5}$

Answer

1.
   a. 37.5%
   b. 90%
   c. 80%

Now you'll look at changing a fraction to a decimal.

9.3 Changing a fraction to a decimal

Again there are two ways to do this, both based on the two methods just shown for changing a fraction to a percentage.

Method 1

Example: Change the fraction into a percentage and divide by 100

$\frac{1}{4} \times \frac{100}{1} = \frac{100}{4}$ which cancels to $\frac{25}{1}$ = 25%

Now convert to a decimal by dividing by 100:

25 ÷ 100 = 0.25

Method 2

Example: Divide the top of the fraction by the bottom

$\frac{1}{4}$ = 1 ÷ 4 = 0.25.

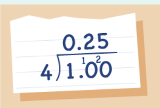

Figure 13 Expressed as a decimal: 1 ÷ 4

Activity 23: Changing a fraction to a decimal

Express these fractions as decimals:

1. $\frac{2}{5}$

2. $\frac{1}{8}$

3. $\frac{3}{10}$

Answer

1. 0.4
2. 0.125
3. 0.3

Fractions and percentages deal with splitting numbers into a given number of equal portions, or parts. When dealing with the next topic, ratio, you will still be splitting quantities into a given number of parts, but when sharing in a ratio, you do not share evenly. This might sound a little complicated but you'll have been doing it since you were a child.

Summary

In this section you have:

- learned about the relationship between fractions, decimals and percentages and are now able to convert between the three.

10 Ratio

As you can see from Figure 14, ratio is an important part of everyday life.

Figure 14 Day-to-day ratio

It is important to understand how to tell which part of the ratio is which. If for example, you have a group of men and women in the ratio of 5:4, as the men were mentioned first, they are the first part of the ratio.

The order of the ratio is very important. Consider the following:

> Julia attends a drama club where 100 members are men and 150 members are women. What is the ratio of women to men at the drama club?

Notice how the information that you need to answer the question is given in the opposite order to that required in the answer. It is very important that you give the parts of the ratio in the correct order.

The ratio of women to men is 150:100

If you were asked for the ratio of men to women it would be 100:150

10.1 Simplifying ratios

Sometimes you need to work out the ratio from the quantities you have.

If we refer back to the example we discussed earlier, we said that the ratio of women to men at the drama club is 150:100. However, you can simplify this ratio by dividing all parts by the same number. This is similar to simplifying fractions, which you have done.

With 150:100, we can divide each side of the ratio by 50 (you could also divide by 10 and then by 5), so the ratio will simplify to 3:2. Therefore, the ratio of women to men at the club is 3:2. Having it written in its simplest form makes it easier to think about and to use for other calculations. For every 2 men you have, there are 3 women.

Let’s look at another example.

Example: Recipes and ratio

Look at this recipe for a mocktail:

Sunset Smoothie

- 50 ml grenadine
- 100 ml orange juice
- 150 ml lemonade

The ratio of the ingredients is:

grenadine:orange juice:lemonade
50 : 100 : 150

To simplify this ratio you can divide all of the numbers by 50 (or by 10 and then 5).

This gives the ratio of grenadine to orange juice to lemonade as 1:2:3.

Activity 24: Simplifying ratios

Simplify the following ratios:

1. The ratio of women to men in a class is 15:20.
2. The ratio of management to staff in a warehouse is 10:250.
3. The ratio of home to away supporters is 24 000 to 8000.
4. The ratio of votes in a local election was candidate A 1600, candidate B 800, Candidate C 1200.
5. The ratio of fruit in a bag of mixed dried fruit is 150 g currants, 100 g raisins, 200 g sultanas and 50 g mixed peel.

Answer

1. Women to men is 3:4 (divide both sides by 5).
2. Management to staff is 1:25 (divide both sides by 10).
3. Home to away supporters is 3:1 (divide both sides by 8000 *or* by 1000 and then by 8).
4. A to B to C is 4:2:3 (divide each part of the ratio by 400 *or* by 100 and then by 4).
5. Currants to raisins to sultanas to mixed peel is 3:2:4:1 (divide by 50 *or* by 10 and then 5).

Ratio questions can be asked in different ways. There are three main ways of asking a ratio question. Take a look at an example of each below and see if you can identify the differences.

Type 1

A recipe for bread says that flour and water must be used in the ratio 5:3. If you wish to make 500 g of bread, how much flour should you use?

Type 2

You are growing tomatoes. The instructions on the tomato feed say ‘Use 1 part feed to 4 parts water’. If you use 600 ml of water, how much tomato feed should you use?

Type 3

Ishmal and Ailia have shared some money in the ratio 3:7. Ailia receives £20 more than Ishmal. How much does Ishmal receive?

In questions of type 1, you are given the total amount that both ingredients must add to, in this example, 500 g. In questions of type 2 however, you are not given the total amount but instead are given the amount of one part of the ratio. In this case you know that the 4 parts of water total 600 ml.

The final type of ratio question does not give us either the total amount or the amount of one part of the ratio. Instead, it gives us just the difference between the first and second part of the ratio. Whilst neither type of ratio question is more complicated than the others, it is useful to know which type you are dealing with as the approach for solving each type of problem is slightly different.

10.2 Solving ratio problems where the total is given

The best way for you to understand how to solve these problems is to look through the worked example in the video below.

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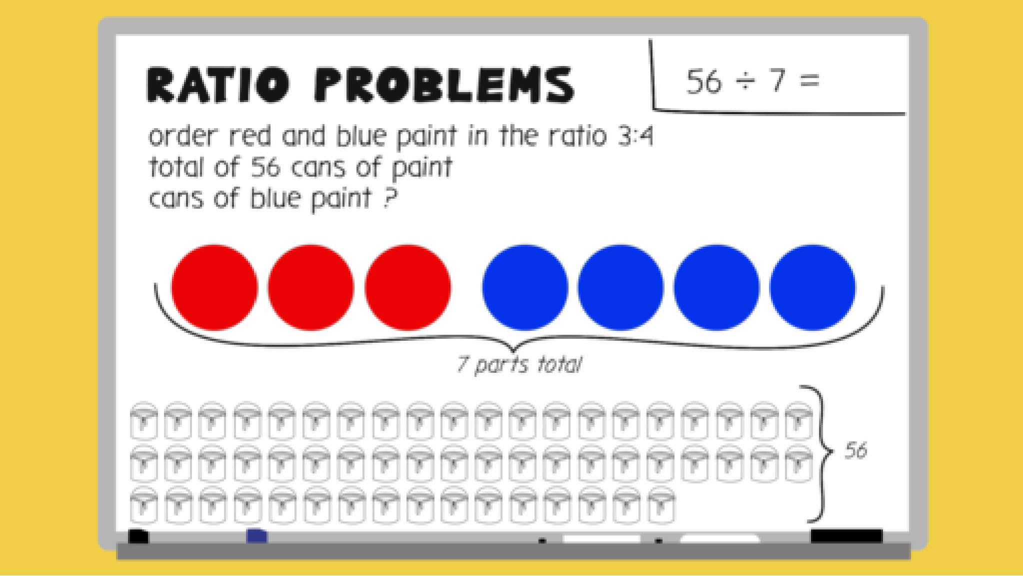


Activity 25: Ratio problems where the total is known

Try solving these ratio problems:

1. To make mortar you need to mix soft sand and cement in the ratio 4:1. You need to make a total of 1500 g of mortar.

   How much soft sand will you need?

Answer

1. Add the parts of the ratio:

   4 + 1 = 5

   Divide the total amount required by the sum of the parts of the ratio:

   1500 g ÷ 5 = 300 g

   Since soft sand is 4 parts, we do 300 g × 4 = 1200 g of soft sand.

   Check by working out how much cement you need. Cement is 1 part so you would need 300 g:

   1200 g + 300 g = 1500 g which is the correct total.

2. To make the mocktail 'Sea Breeze' you need to mix cranberry juice and grapefruit juice in the ratio 4:2.

   You want to make a total of 2700 ml of mocktail. How much grapefruit juice should you use?

Answer

2. Add the parts of the ratio:

   4 + 2 = 6

   Divide the total amount required by the sum of the parts of the ratio:

2700 ml ÷ 6 = 450 ml

Since grapefruit juice is 2 parts, we do 450 ml × 2 = 900 ml of grapefruit juice.

Check by working out how much cranberry juice you would use:

4 × 450 = 1800

1800 ml + 900 ml = 2700 ml

You may have simplified the ratio to 2:1 before doing the calculation, but you will see that your answers are the same as ours.

3. The instructions to mix Misty Morning paint are mix 150 ml of azure with 100 ml of light grey and 250 ml of white paint.

   How much light grey paint would you need to make 5 litres of Misty Morning?

Answer

3. Start by expressing and then simplifying the ratio:

   150:100:250 which simplifies to 3:2:5 = 10 parts

   5 litres = 5000 ml (converting to ml makes your calculation easier.)

   Divide the total amount required by the sum of the parts of the ratio:

   5000 ÷ 10 = 500 so 1 part = 500 ml

   Light grey is 2 parts:

   2 × 500 = 1000 ml or 1 litre

   Check:

   azure is 3 parts: 3 × 500 = 1500 ml or 1.5 litres

   white is 5 parts: 5 × 500 = 2500 ml or 2.5 litres

   1000 + 1500 + 2500 = 5000 ml or 5 litres

4. You want to make 14 litres of squash for a children's party. The concentrate label says mix with water in the ratio of 2:5.

   How much concentrate will you use?

Answer

4. Add the parts of the ratio:

   2 + 5 = 7

   Divide the total amount required by the sum of the parts of the ratio:

   14 litres ÷ 7 = 2 litres so 1 part = 2 litres

   (Note: this calculation was straightforward so there was no need to convert to ml.)

   Since the concentrate is 2 parts you will need 2 litres × 2 = 4 litres of concentrate.

   Check:

   Water is 5 parts:

   5 × 2 litres = 10 litres

   4 + 10 = 14 litres.

5. A man leaves £8400 in his will to be split between 3 charities:

   Dogs Trust, RNLI and MacMillan Research in the ratio 3:2:1.

   How much will each charity receive?

Answer

5. Add the parts of the ratio:

   3 + 2 + 1 = 6

   Divide the total amount required by the sum of the parts of the ratio:

   £8400 ÷ 6 = 1400

   – The Dogs Trust receives 3 parts: 3 × £1400 = £4200
   – The RNLI receives 2 parts: 2 × £1400 = £2800
   – MacMillan Research receives 1 part so: £1400

   Check:

   4200 + 2800 + 1400 = £8400

Next you'll look at ratio problems where the total of one part of the ratio is known.

10.3 Solving ratio problems where the total of one part of the ratio is given

Take a look at the worked example below:

You are growing tomatoes. The instructions on the tomato feed say:

Use 1 part feed to 4 parts water

If you use 600 ml of water, how much tomato feed should you use?

These questions make much more sense if you look at them visually:

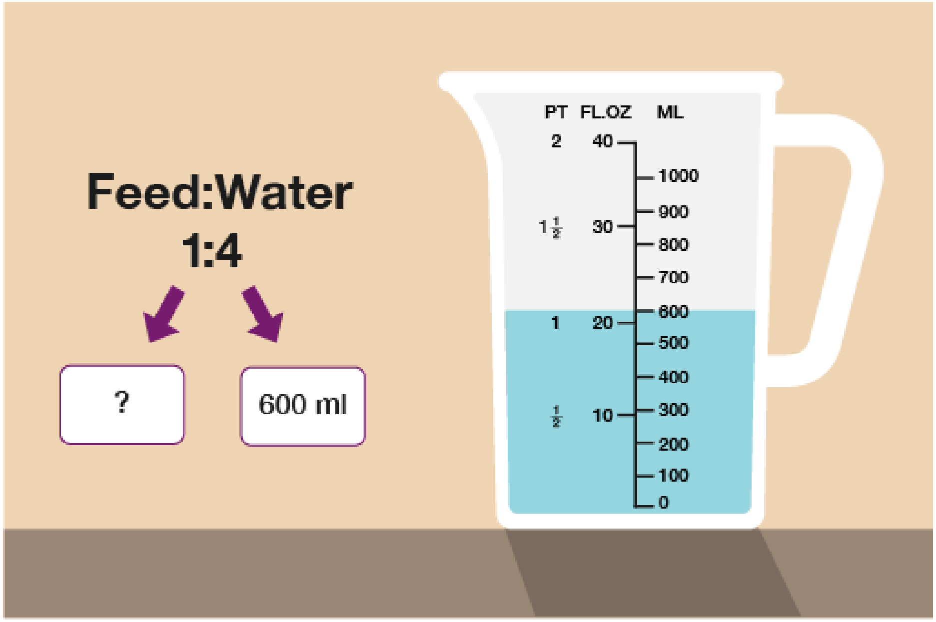


Figure 15 Solving ratio problems to grow tomatoes

You can now see clearly that 600 ml of water is worth 4 parts of the ratio. To find one part of the ratio you need to do:

600 ml ÷ 4 = 150 ml

Since the feed is only 1 part, feed must be 150 ml. If feed was more than one part you would multiply 150 ml by the number of parts.

!Warning! Calibri not supportedJust as with the previous type of question, you need to try to work out the value of 1 part. The value of any other number of parts can be worked out from this.

Activity 26: Ratio problems with one part given

Practise your skills by tackling the ratio problems below:

1. A recipe requires flour and butter to be used in the ratio 3:5. The amount of butter used is 700 g.
   How much flour will be needed?

Answer

1. **Flour:Butter**

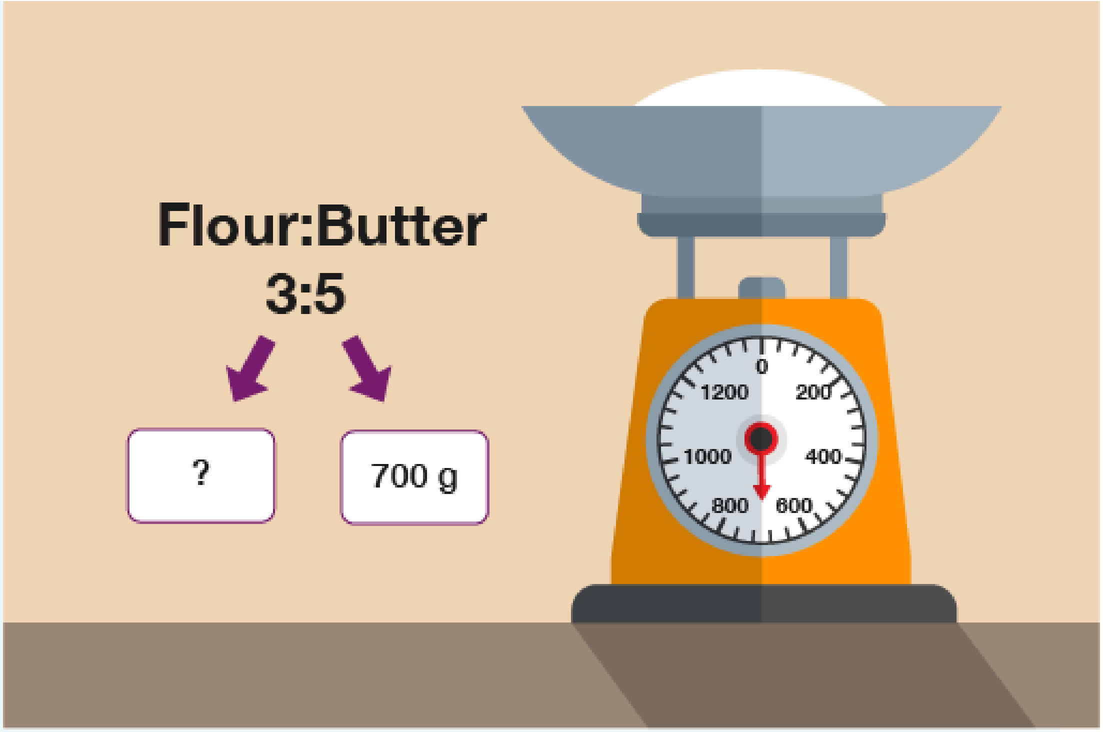


Figure 16 Using ratios in recipes

To find the amount of flour needed you then do 140 g × 3 = 420 g flour.

2. When looking after children aged between 7 and 10, the ratio of adults to children must be 1:8.
   - b. For a group of 32 children, how many adults must there be?
   - c. If there was one more child in the group, how would this affect the number of adults required?

Answer

2. **Adults:Children**

   b.

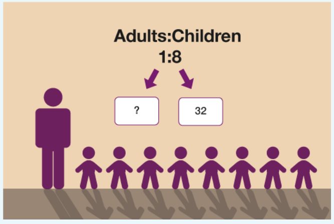


Figure 17 Working out the ratio of adults to children

To find one part you do 32 ÷ 8 = 4.

Since adults are only 1 part, you need 4 adults.

c. If there were 33 children, one part would be 33 ÷ 8 = 4.125.

Since you cannot have 4.125 adults, you need to round up to 5 adults so you would need one more adult for 33 children.

3. A shop mixes bags of muesli using oats, sultanas and nuts in the ratio 6:3:1. If the amount of sultanas used is 210 g, how heavy will the bag of muesli be?

Answer

3. **Oats:Sultanas:Nuts**

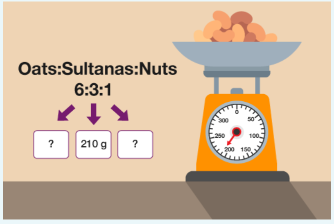


Figure 18 Working out the ratio of oats, sultanas and nuts

Sultanas are 3 parts so to find 1 part you do 210 g ÷ 3 = 70 g.
Oats are 6 parts so 6 × 70 = 420 g.
Nuts are only 1 part so they are 70 g.
The total weight of the bag would be 210 g + 420 g + 70 g = 700 g.

Next you'll look at ratio problems where only the difference in amounts is given.

10.4 Solving ratio problems where only the difference in amounts is given

Earlier in the section you came across the question below. Let's have a look at how we could solve this.

Example: Solving ratio amounts from the difference

Ishmal and Ailia have shared some money in the ratio 3:7.

Ailia receives £20 more than Ishmal. How much does Ishmal receive?

Ishmal:Ailia
3:7

You know that the difference between the amount received by Ishmal and the amount received by Ailia is £20. You can also see that Ailia gets 7 parts of the money whereas Ishmal only gets 3.

The difference in parts is therefore 7 − 3 = 4. So 4 parts = £20.

Now this is established, you can work out the value of one part by doing:

£20 ÷ 4 = £5

As you want to know how much Ishmal received you now do:

£5 × 3 = £15

As an extra check, you can work out Ailia’s by doing:

£5 × 7 = £35

This is indeed £20 more than Ishmal.

Activity 27: Ratio problems where difference given

Now try solving this type of problem for yourself.

1. The ratio of female to male engineers in a company is 2:9. At the same company, there are 42 more male engineers than females.

   How many females work for this company?
2. A garden patio uses grey and white slabs in the ratio 3:5. You order 30 fewer grey slabs than white slabs.

   How many slabs did you order in total?

Answer

1. The difference in parts between males and females is 9 − 2 = 7 parts.

   You know that these 7 parts = 42 people.

   To find 1 part you do:

   42 ÷ 7 = 6

   Now you know that 1 part is worth 6 people, you can find the number of females by doing

   6 × 2 = 12 females

   Check:

   The number of males is 6 × 9 = 54. The difference between 54 and 12 is 42.
2. The difference in parts between grey and white is 5 − 3 = 2 parts.

   These 2 parts are worth 30. To find 1 part you do:

   30 ÷ 2 = 15

   To find grey slabs do:

   15 × 3 = 45

   To find white slabs do:

   15 × 5 = 75

   Check:

   The difference between the number of grey and white slabs is 30 (75 − 45).

   Now you know both grey and white totals, you can find the total number of slabs by doing:

   45 + 75 = 120 slabs in total.

Even though there are different ways of asking ratio questions, the aim of any ratio question is to determine the value of one part. Once you know this, the answer is simple to find!

Ratio can also be used in less obvious ways. Imagine you are baking a batch of scones and the recipe makes 12 scones. However, you need to make 18 scones rather than 12. How do you work out how much of each ingredient you need? The final ratio section deals with other applications of ratio.

10.5 Other applications of ratio

A very common and practical use of ratio is when you want to change the proportions of a recipe. All recipes state the number of portions they will make, but this is not always the number that you wish to make.You may wish to make more or less than the actual recipe gives. If you wanted to make 18 scones but only have a recipe that makes 12, how do you know how much of each ingredient to use?

To make 12 scones

400 g self-raising flour

1 tablespoon caster sugar

80 g butter

250 ml milk

Figure 19 Scones on a plate

As you already know the ingredients to make 12 scones, you need to know how much of each ingredient to make an extra 6 scones. Since 6 is half of 12, if you halve each ingredient, you will have the ingredients for the extra 6 scones. To find the total for 18 scones you need to add together the ingredients for the 12 scones and the 6 scones.

Table 9

12 scones	6 scones	18 scones
400 g flour	400 g ÷ 2 = 200 g flour	400 g + 200 g = 600 g flour
1 tablespoon caster sugar	1 ÷ 2 = $\frac{1}{2}$ tablespoon caster sugar	1 + $\frac{1}{2}$ = 1 $\frac{1}{2}$ tablespoons caster sugar
80 g butter	80 g ÷ 2 = 40 g butter	80 g + 40 g = 120 g butter
250 ml milk	250 ml ÷ 2 = 125 ml milk	250 ml + 125 ml = 375 ml milk

Have a go at the activity below to check your skills.

Activity 28: Ratio and recipes

1. This recipe makes 18 biscuits:

220 g self-raising flour
150 g butter
100 g caster sugar
2 eggs

How much of each ingredient is needed for 9 biscuits?

Answer

1. Since 9 is half of 18, you need to halve each ingredient to find the amount required to make 9 biscuits.
   220g ÷ 2 = 110 g flour
   150g ÷ 2 = 75 g butter
   100g ÷ 2 = 50 g sugar
   2 ÷ 2 = 1 egg

2. To make strawberry milkshake you need:
   630 ml milk
   3 scoops of ice cream
   240 g of strawberries
   The recipe serves 3

   How much of each ingredient is needed for 9 people?

Answer

2. You know the ingredients for 3 but want to know the ingredients for 9. Since 9 is three times as big as 3, you need to multiply each ingredient by 3.
   630 ml × 3 = 1890 ml milk
   3 × 3 = 9 scoops of ice cream
   240 g × 3 = 720 g of strawberries

3. Angel Delight recipe:
   Add 60 g powder to 300 ml cold milk
   Serves 2 people

   How much of each ingredient is needed to serve 5 people?

Answer

3. You could work this out in 2 different ways.

   Method 1

   You know the ingredients for 2 people. You can find ingredients for 4 people by doubling the ingredients for 2. You then need ingredients for an extra 1 person. Since 1 is half of 2, you can halve the ingredients for 2 people.
   60 g + 60 g + 30 g = 150 g powder
   300 ml + 300 ml + 150 ml = 750 ml milk

   Method 2

You know the ingredients for 2 people so you can find the ingredients for 1 person by halving them. You can then multiply the ingredients for 1 person by 5.

60g ÷ 2 = 30 × 5 = 150 g powder

300ml ÷ 2 = 150 × 5 = 750 ml milk

The final practical application of ratio can be very useful when you are out shopping. Supermarkets often try and encourage us to buy in bulk by offering larger 'value' packs. But how can you work out if this is actually a good deal? Take a look at the example below.

Example: Ratio and shopping

Which of the boxes below offers the best value for money?

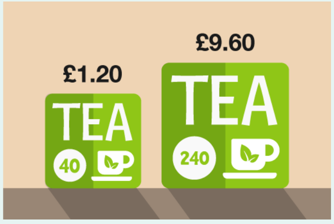


Figure 20 Shopping options: tea

There are various ways of comparing the prices.

Method 1

To work out which is the best value for money we need to find the price of 1 teabag.

If 40 teabags cost £1.20 then to find the cost of 1 teabag you do:

£1.20 ÷ 40 = £0.03, or 3p

If 240 teabags cost £9.60 then to find the cost of 1 teabag you do:

£9.60 ÷ 240 = £0.04, or 4p

The box containing 40 teabags is therefore better value than the larger box.

Method 2

The ratio of teabags is

40 : 240 which you can simplify to 1:6

If we use the price for the small box you can see that 1 part is £1.20

You can then use this value to calculate the price of the large box. At this price, the bigger box would be £1.20 × 6 = £7.20 so we can see that the small box is better value.

Activity 29: Practical applications of ratio

Use whichever method you prefer to work out the best deal in each case.

1. Work out which deal is the best value for money.
   a.

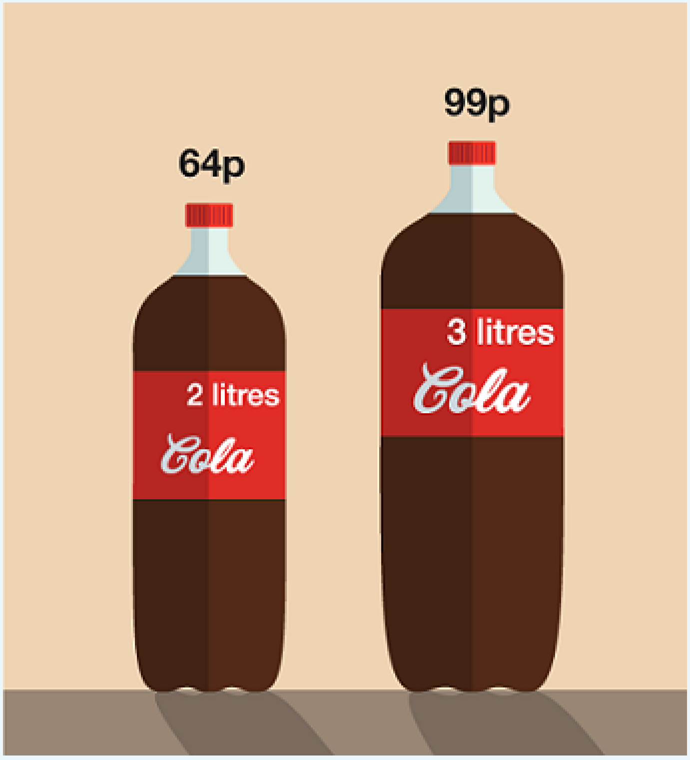


Figure 21 Cola options

Answer

1.
   a. 2 litres cost 64 p, so 1 litre costs 64 p ÷ 2 = 32p.
      3 litres cost 99p, so 1 litre costs 99p ÷ 3 = 33p.
      Comparing the cost of 1 litre in each case, we see that the 2-litre bottle is the best buy.

   b.

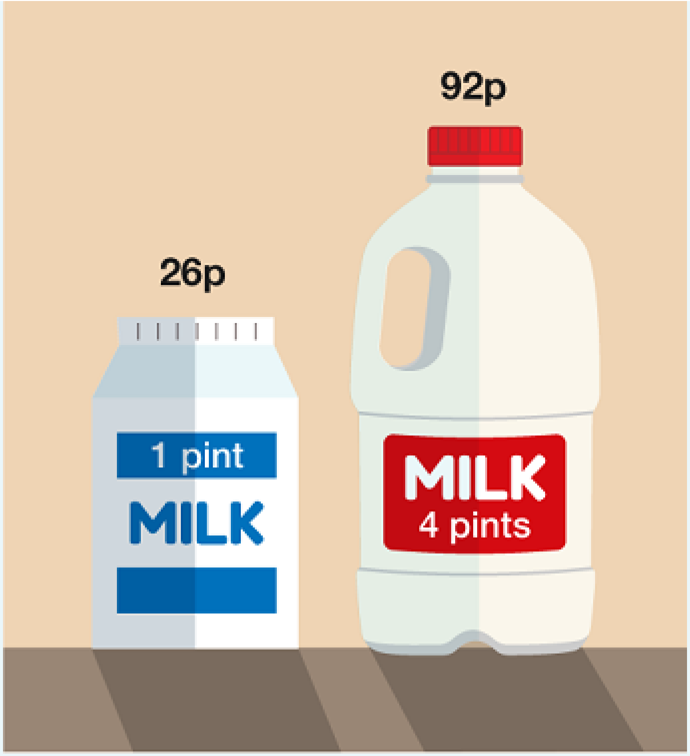


Figure 22 Milk options

Answer

1.

b. 1 pint costs 26p.

4-pint carton costs 92p, so 1 pint costs 92p ÷ 4 = 23p.

Comparing the cost of 1 pint of milk in each case, we see that the 4-pint carton is the best buy.

c.

Figure 23 Washing powder options

Answer

1.
   c. 5 kg costs £10, so 1 kg costs £10 ÷ 5 = £2.

      2 kg cost £3, so 1 kg costs £3 ÷ 2 = £1.50.

      Comparing the cost of 1 kg of powder in each case, we see that the 2 kg box is the best buy.

2. Two supermarkets sell the same brand of juice. Shop B is offering ‘buy one get second one half price’ for apple juice and ‘buy one get one free’ for orange juice.

   For each type of juice which shop is offering the best deal?

   b.

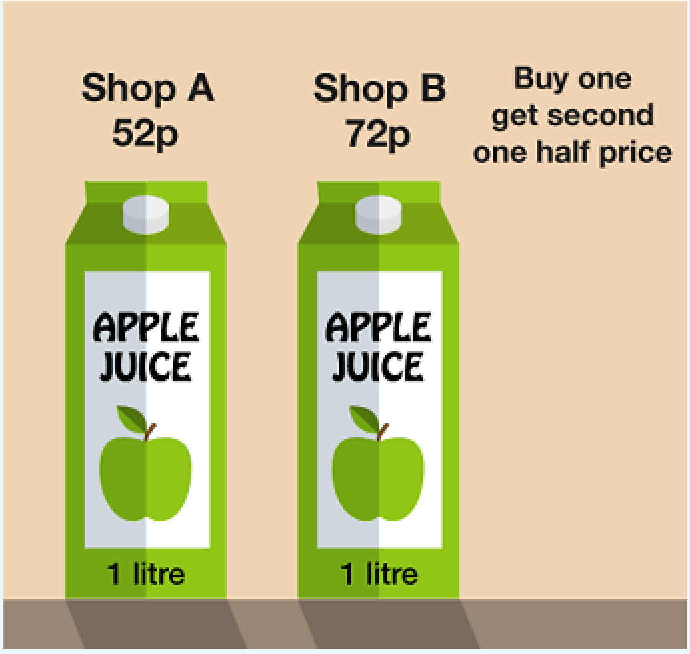


Figure 24 Apple juice options

Answer

2.

b. Shop A: 1 litre costs 52p

Shop B: 2 litres cost 72p + 36p = 108p (here we pay 72p for the first litre and 36p for second litre), so 1 litre costs 108p ÷ 2 = 54p.

Comparing the cost of 1 litre of apple juice in each case, we see that Shop A offers the better deal.

b.

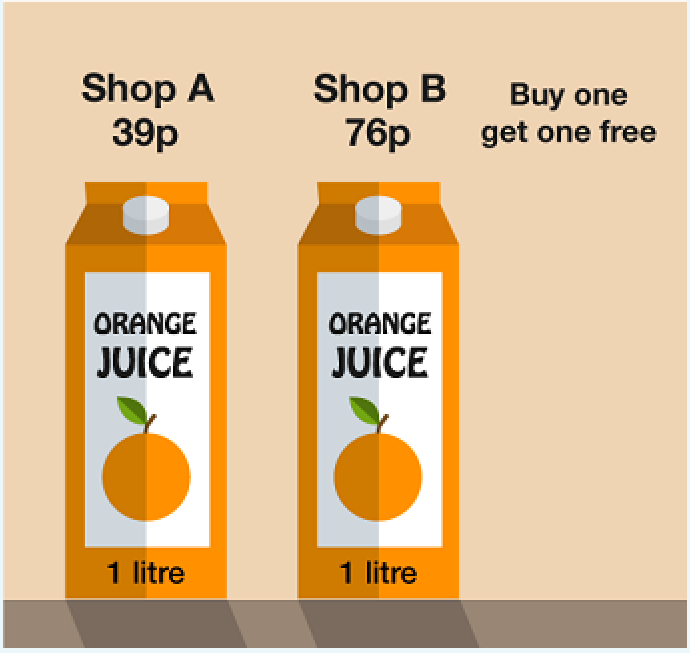


Figure 25 Orange juice options

Answer

2.

b. Shop A: 1 litre costs 39p

Shop B: 2 litres cost 76p (we get 1 litre free), so 1 litre costs:
76p ÷ 2 = 38p.

Comparing the cost of 1 litre of orange juice in each case, we see that Shop B offers the better deal.

3. A supermarket sells bread rolls in 3 different size packs. Which size offers the best value for money?

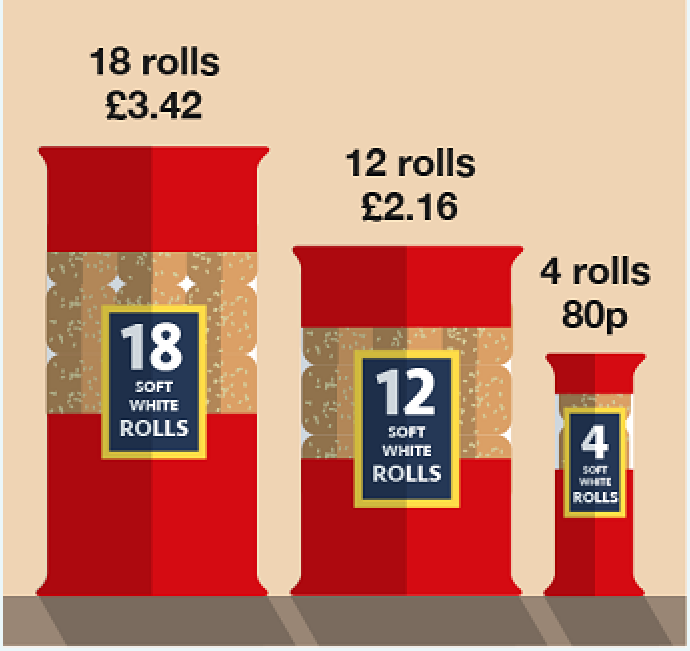


Figure 26 Bread rolls options

Answer

3. Calculate the cost of 1 roll in each pack:

    80p ÷ 4 = 20p

    £2.16 ÷ 12 = £0.18 or 18p

    £3.42 ÷ 18 = £0.19 or 19p

   The pack of 12 is best value.

You have now completed all elements of the ratio section and hopefully are feeling confident with each topic.

The next section of the course deals with formulas. This might sound daunting but you have actually already used a formula. Remember when you learned about how to work out the percentage change of an item? To do that you used a simple formula and you will now take a closer look at slightly more complex formulas.

Summary

In this section you have:

- learned about the three different types of ratio problems and that the aim of any ratio problem is to find out how much one part is worth
- practised solving each type of ratio problem:
    - where the total amount is given
    - where you are given the total of only one part
    - where only the difference in amounts is given

- learned about other useful applications of ratio, such as changing the proportions of a recipe.

11 Formulas

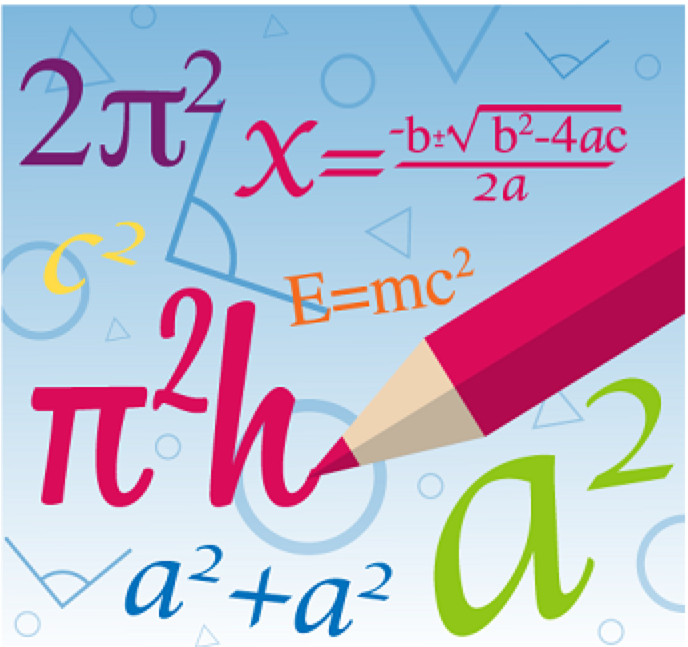


Figure 27 Formulas

Before diving in to this topic, you first need to learn about the order in which you need to carry out addition, subtraction, multiplication and division. Have you ever seen a question like the one below posted on social media?

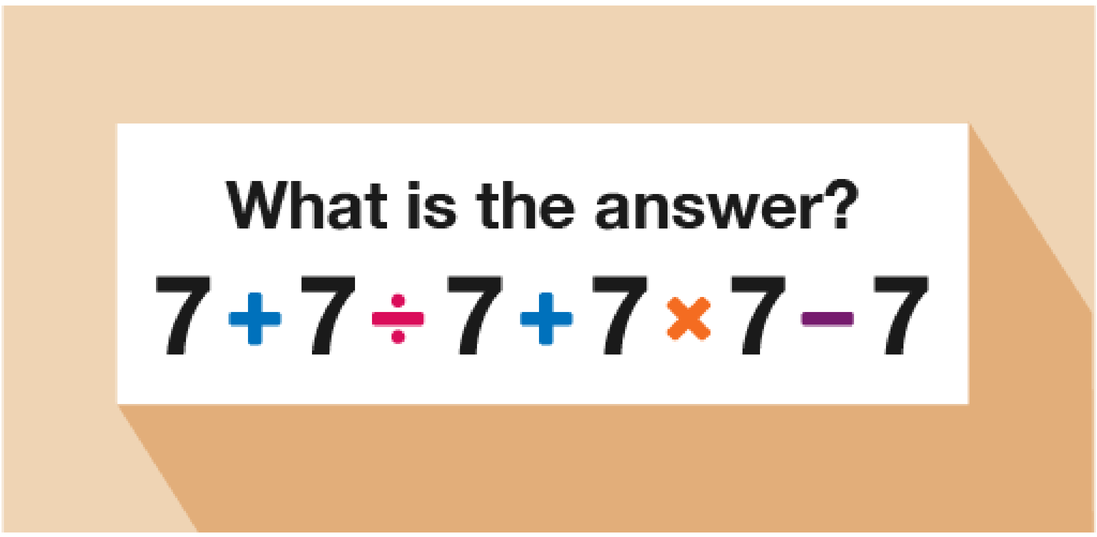


Figure 28 A calculation using the four operations

There are usually a wide variety of answers given by various people. But how is it possible that such a simple calculation could cause so much confusion? It’s all to do with the order in which you carry out the calculations.

If you go from left to right:

$7 + 7 = 14$

$14 \div 7 = 2$

$2 + 7 = 9$

$9 \times 7 = 63$

$63 - 7 = 56$

Check this on a calculator and you will see that the correct answer is actually 50. How do you arrive at this answer? You have to use the correct order of operations, sometimes called BIDMAS.

11.1 Order of operations

The order in which you carry out operations can make a big difference to the final answer. When doing any calculation that involves doing more than one operation, you must follow the rules of BIDMAS in order to arrive at the correct answer.

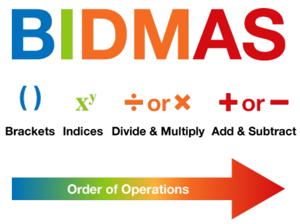


Figure 29 The BIDMAS order of operations

B: Brackets

Any calculation that is in brackets must be done first.

Example:

$2 \times (3 + 5)$

$2 \times 8 = 16$

Note that this could also be written as 2 (3 + 5) because if a number is next to a bracket, it means you need to multiply.

If there is more than 1 operation in the brackets, you must follow the rules of BIDMAS in the brackets.

I: Indices

After any calculations in brackets have been done, you must deal with any calculations involving indices or powers i.e.

$3^2 = 3 \times 3$

or

$4^3 = 4 \times 4 \times 4$

Example:

3×4^2

$3 \times (4 \times 4)$

$3 \times 16 = 48$

D: Divide

Next come any division or multiplication calculations. Of these two calculations, you should do them in the order that they appear in the sum from left to right.

Example:

$16 - 10 \div 5$

$16 - 2 = 14$

M: Multiply

Example:

$5 + 6 \times 2$

$5 + 12 = 17$

A: Add

Finally, any calculations involving addition or subtraction are done. Again, these should be done in the order that they appear from left to right.

S: Subtract

Example:

$24 + 10 - 2$

$34 - 2 = 32$

or

$24 + 8 = 32$

Activity 30: Using BIDMAS

Now have a go at carrying out the following calculations yourself. Make sure you apply BIDMAS!

1. 4 + 3 × 2
2. 5 (4 − 1)
3. $36 \div 3^2$
4. 7 + 15 ÷ 3 − 4

Answer

1. 4 + 6 = 10
2. 5 × 3 = 15
3. 36 ÷ 9 = 4
4. 7 + 5 − 4 = 8

Now that you have learned the rules of BIDMAS you are ready to apply them when using formulas.

11.2 Formulas in practice

You will already have come across and used formulas in your everyday life. For example, if you are trying to work out the cost of a new carpet you will have used the formula:

area = *length* × *width*

to calculate how much carpet you would need.

Often division in a formula is shown as one number over another, for example:

6 ÷ 3 would be shown as $\frac{6}{3}$

Let's look at division in a formula:

$$speed = \frac{dis\,tan\,ce}{time}$$

A lorry driver travels 120 miles in 3 hours. What was the average speed during the journey?

Note: As the lorry driver was unlikely to have travelled at a constant speed for 120 miles we say we are calculating the average speed as this will give us the typical overall speed.

speed = $\frac{120}{3}$

speed = 40 miles per hour

Sometimes we use letters to represent the different elements used in a formula, e.g. the formula above might be shown as:

$s = \frac{d}{t}$

where:

'*s*' = speed in mph
'*d*' = distance in miles
'*t*' = time in hours

If you are trying to work out the time to cook a fresh chicken you may have used the formula:

Time (minutes) = 15 + $\frac{w}{500}$ × 25 where '*w*' is the weight of the chicken in grams.

For example, if you wanted to cook a chicken that weighs 2500 g you would do:

Time (minutes) = 15 + $\frac{2500}{500}$ × 25

Remembering to use BIDMAS you would then get:

Time (minutes) = 15 + 5 × 25
= 15 + 125
= 140 minutes

Let's look at another worked example before you try some on your own.

Example: Gas bill formula

The owner of a guesthouse receives a gas bill. It has been calculated using the formula:

Cost of gas (£) = $\frac{8d + u}{100}$

Note: 8*d* means you do 8 × *d*.

Where *d* = number of days and *u* = number of units used, if she used 3500 units of gas in 90 days, how much is the bill?

In this example, *d* = 90 and *u* = 3500 so you do:

Cost of gas (£) = $\frac{8 \times 90 + 3500}{100}$

= $\frac{720 + 3500}{100}$

= $\frac{4220}{100}$

= £42.20

Activity 31: Using formulas

1. Fuel consumption in Europe is calculated in litres per 100 kilometres. A formula to approximate converting from miles per gallon to litres per 100 kilometres is:

   L = $\frac{280}{M}$

   where *L* = number of litres per 100 kilometres and *M* = number of miles per gallon.
   A car travels 40 miles per gallon. What is this in litres per kilometres?

Answer

1.

   L = $\frac{280}{M}$ and in this case M = 40

   L = $\frac{280}{40}$

   L = **7 litres per 100 kilometres**

2. Using the formula *I* = $\frac{PRT}{100}$ where:

   I = interest
   P = principal amount of loan
   R = interest rate
   T = time in years

   calculate how much interest is due on a **loan of £5000** taken over **3 years** at an interest rate of **5.5%**.

Answer

2. $I = \frac{PRT}{100}$

   In this case P = £5000, R = 5.5% and T = 3 years.

   $I = \frac{5000 \times 5.5 \times 3}{100}$

   $I = \frac{82500}{100}$

   I = 825
   So the interest paid would be **£825**.

3. The area of a trapezium can be calculated using the formula:

   $A = \frac{h\ (a+b)}{2}$

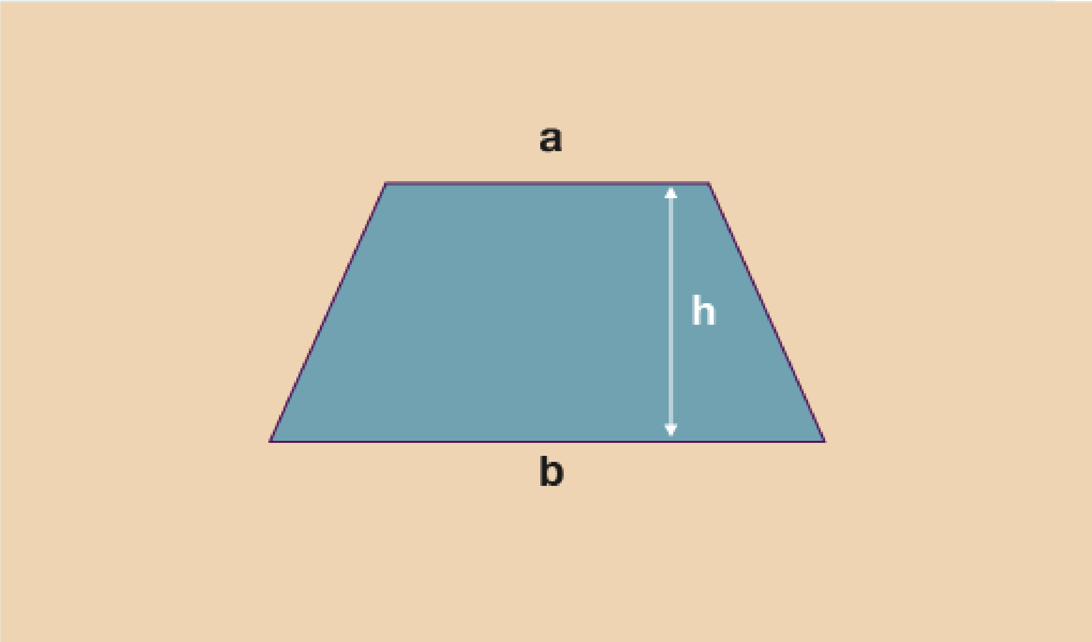


Figure 30 Dimensions of a trapezium

Find the area of trapeziums where:

iii. a = 5 cm, b = 9 cm and h = 7 cm
iv. a = 35 mm, b = 40 mm and h = 10 cm

Answer

3. $A = \frac{h\ (a+b)}{2}$

   iii. $A = \frac{7\ (5+9)}{2}$

$A = \frac{7 \times 14}{2}$

$A = \frac{98}{2}$

A = **49 cm²**

iv. In this question you must convert the units so that they are all the same. The units that you select will be the units that your answer will be given in, e.g. if you convert to mm your answer will be in mm^2 but if you convert to cm your answer will be in cm^2.

$A = \frac{h\ (a + b)}{2}$

Method 1 – converting to mm

Convert *h* measurement to mm:

10 × 10 = 100 mm

$A = \frac{100\ (35 + 40)}{2}$

$A = \frac{100 \times\ 75}{2}$

$A = \frac{7500}{2}$

A = **3750 mm^2**

Method 2 – converting to cm

Convert *a* and *b* measurements to cm:

a = 35 ÷ 10 = 3.5 cm

b = 40 ÷ 10 = 4 cm

$A = \frac{10\,(3.5 + 4.0)}{2}$

$A = \frac{10 \times\ 7.5}{2}$

$A = \frac{75}{2}$

A = **37.5 cm^2**

Note: it is a good idea to show all the stages of the calculation to help you keep track of your workings.

4. A company uses the following formula to work out the total cost to the customer of hiring a bouncy castle:

   $T = hc + (0.45d) + 15$

   where:

   T = total

   h = number of days hire

   c = cost of castle per day

   d = delivery distance in miles.

Figure 31 Dues's Bouncy Fun – price list

Stuart lives 12 miles away and would like to hire a Supersonic Castle for 2 days. How much will it cost?

Answer

4. $T = hc + (0.45d) + 15$

   In this case h = 2, c = £42, and d = 12, so:

   $T = 2 \times 42 + (0.45 \times 12) + 15$

   $T = 84 + 5.4 + 15$

   $T = 104.4$

   The total cost of hire would be **£104.40**.

Now that you have learned all the skills that relate to the number section of this course, there is just one final thing you need to be able to do before you will be ready to complete the end-of-session quiz for numbers.

You are now proficient at carrying out lots of different calculations including working out fractions and percentages of numbers, using ratio in different contexts and using formulas.

It is fantastic that you can now do all these things, but how do you check if an answer is correct? One way you can check would be to approximate an answer to the calculation (as you did in Section 3.2). Another way to check an answer is to use the inverse (opposite) operation.

Summary

In this section you have:

- learned about, and practised using BIDMAS – the order in which operations must be carried out
- seen examples of formulas used in everyday life and practised using formulas to solve a problem.

12 Checking your answers

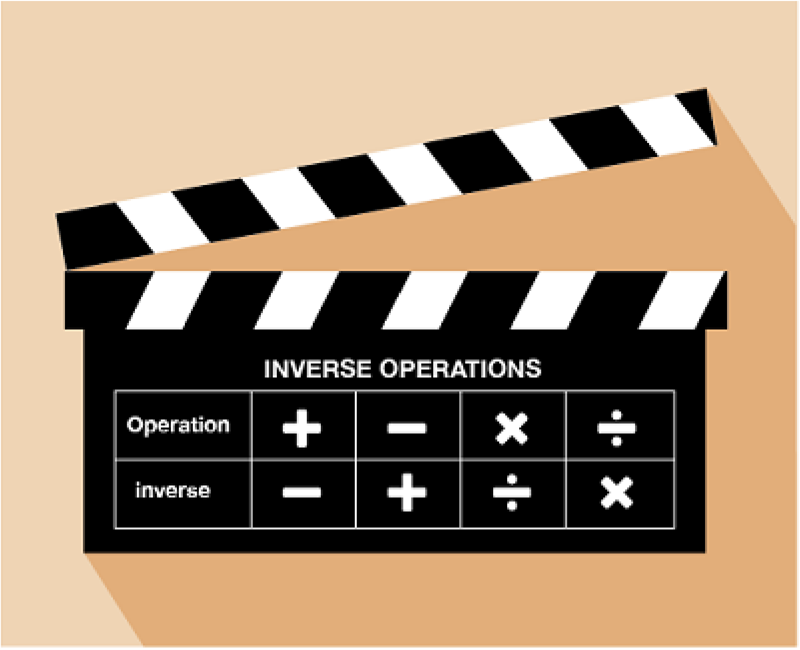


Figure 32 Inverse operations

An inverse operation is an opposite operation. In a sense, it 'undoes' the operation that has just been performed. Let's look at two simple examples to begin with.

Example: Check your working 1

6 + 10 = 16

Method

Since you have done an addition sum, the inverse operation is subtraction. To check this calculation, you can either do:

16 − 10 = 6

or

16 − 6 = 10

You will notice here that the same 3 numbers (6, 10 and 16) have been used in all the calculations.

Example: Check your working 2

5 × 3 = 15

Method

This time, since you have done a multiplication sum, the inverse operation is division. To check this calculation, you can either do:

15 ÷ 5 = 3

or

15 ÷ 3 = 5

Again, you will notice that the same 3 numbers (3, 5 and 15) have been used in all the calculations.

If you have done a more complicated calculation, involving more than one step, you simply ‘undo’ each step.

Example: Check your working 3

A coat costing £40 has a discount of 15%. How much do you pay?

Method

Firstly, we find out 15% of £40:

40 ÷ 100 × 15 = £6 discount

£40 − £6 = £34 to pay

To check this calculation, firstly you would check the subtraction sum by doing the addition:

£34 + £6 = £40

To check the percentage calculation you then do:

£6 ÷ 15 × 100 = £40

Don’t forget, sometimes it can also be helpful to use estimation to check your answers, particularly when using decimal or large numbers.

You have now completed the number section of the course. Before moving on to the next session, ‘Units of measure’, complete the quiz on the following page to check your knowledge and understanding.

Summary

In this section you have:

- learned that each of the four operations has an inverse operation (its opposite) and that these can be used to check your answers
- seen examples showing how to check answers using the inverse operation.

13 Session 1 quiz

Now it's time to review your learning in the end-of-session quiz.

Session 1 quiz.

Open the quiz in a new window or tab (by holding ctrl [or cmd on a Mac] when you click the link), then return here when you have done it.

Although the quizzes in this course do not require you to show your working to gain marks, real exams would do so. We therefore encourage you to practise this by using a paper and pen to clearly work out the answers to the questions. This will also help you to make sure you get the right answer.

Session 2: Units of measure

Introduction

In this session you will be calculating using units of measure and focusing on length, time, weight, capacity and money. You will already be using these skills in everyday life when:

- working out which train you need to catch to get to your meeting on time
- weighing or measuring ingredients when cooking
- doing rough conversions if you are abroad to work out how much your meal costs in British pounds.

By the end of this session you will be able to:

- understand that there are different units used for measuring and how to choose the appropriate unit
- convert between measurements in the same system (e.g. grams and kilograms) and those in different systems (e.g. litres and gallons)
- use exchange rates to convert currencies
- work with time and timetables
- work out the average speed of a journey using a formula
- convert temperature measurements between Celsius (°C) and Fahrenheit (°F)
- read scales on measuring equipment.

First watch the video below which introduces units of measure.

Video content is not available in this format.

1 Units of measure

A unit of measure is simply what you measure something in and you will already be familiar with using centimetres, metres, kilograms, grams, litres and millilitres for measurement. When measuring something, you need to choose the appropriate unit of measure for the item you are measuring. You would not, for example, measure the length of a room in millimetres. Have a go at the short activity below and choose the most appropriate unit for each example.

Activity 1: Choosing the unit

Millilitres (ml)
Metres (m)
Kilograms (kg)
Grams (g)
Centimetres (cm)
Litres (l)

Match each of the items above to an item below.

Amount of liquid in a glass
Length of a garden fence
Weight of a dog
Weight of an egg
Length of a computer screen
Amount of water in a paddling pool

Hopefully you found that activity fairly straightforward. Next you'll take a closer look at some units of measure and how to convert between them.

1.1 Converting units of measure in the same system

Imagine you are catering for a party and go to the wholesale shop to buy flour. You need 200 g of flour for each batch of cookies you will be making and need to make 30 batches. You can buy a 5 kg bag of flour but aren't sure if that will be enough.

In order to work out if you have enough flour, you need both measurements to be in the same unit – kg or g – before you can do the calculation. For units of measure that are in the same system (so you are not dealing with converting cm and inches for example) it's a simple process to convert one measurement into another.

For now we will focus on what is known as the metric system of measurement (we will look at some units from the imperial system later on in the session). In most cases, if you want to convert between units in the metric system you will just need to multiply or divide by 10, 100 or 1000.

Take a look at the diagrams below which explain how to convert each type of measurement unit.

Length

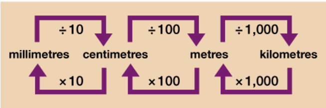


Figure 1 A conversion chart for length

Weight

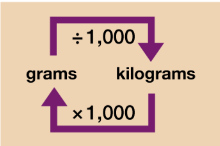


Figure 2 Converting between grams and kilograms

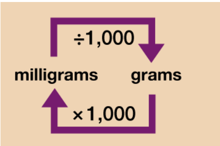


Figure 3 Converting between milligrams and grams

Money

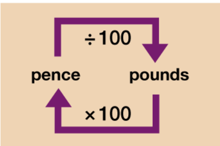


Figure 4 A conversion chart for money

Capacity

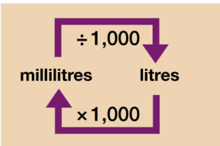


Figure 5 Converting between millilitres and litres

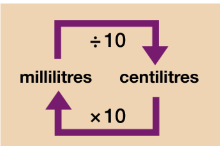


Figure 6 Converting between millilitres and centilitres

Note: Capacity may also be measured in centilitres (shortened to cl). On a standard bottle of water, for instance, it may give the capacity as 500 ml or 50 cl – there are 10 millilitres (ml) in 1 centilitre (cl).

When converting between units you may need to do the conversion in stages.

Example: Converting length

You measure the gap for a washing machine to be 0.62 m. You look on a retailer's website and it shows the width of a particular washing machine to be 600 mm. Will this fit in the gap?

Method

You can convert the gap for the washing machine in stages. Convert from metres to centimetres first by multiplying by 100:

0.62 m × 100 = 62 cm

Now convert the centimetres figure into millimetres by multiplying by 10:

62 cm × 10 = 620 mm

So the washing machine (600 mm wide) will fit in the gap of 620 mm.

You may have preferred to convert the washing machine width into metres instead and do the comparison that way:

600 mm ÷ 10 = 60 cm

60 cm ÷ 100 = 0.6 m

0.6 m is less than 0.62 m so the washing machine will fit the gap.

You'll learn how to convert between different currencies and units of measure in different systems (metric system kilograms (kg) to imperial system pounds (lb) for example) later in this session. For now, have a go at the activities below and test your metric conversion skills.

Activity 2: Converting units

Use your conversion skills to fill in the missing information in Table 1(a). You may want to carry out some of the calculations in stages.

Please carry out all of your calculations without using a calculator. However, double-check on a calculator if you need to.

Table 1(a)

Length	Weight	Capacity
55cm = ? mm	9.75 kg = ? g	235 ml = ? l
257 cm = ? m	652 g = ? kg	18.255 l = ? ml
28.7 km = ? m	5846 g = ? kg	16 ml = ? l
769 mm = ? cm	19.4 kg = ? g	7.88 l = ? ml
1.43 m = ? mm	44 g = ? mg	250 ml = ? cl
125 500 cm = ? km	750 000 mg = ? kg	7.4 l = ? cl

Answer

Table 1(b)

Length	Weight	Capacity
55cm × 10 = 550 mm	9.75 kg × 1000 = 9750 g	235 ml ÷ 1000 = 0.235 l
257 cm ÷ 100 = 2.57 m	652 g ÷ 1000 = 0.652 kg	18.255 l × 1000 = 18 225 ml
28.7 km × 1000 = 28 700 m	5846 g ÷ 1000 = 5.846 kg	16 ml ÷ 1000 = 0.016 l
769 mm ÷ 10 = 76.9 cm	19.4 kg × 1000 = 19 400 g	7.88 l × 1000 = 7880 ml
1.43 m × 100 = 143 cm × 10 = 1430 mm	44 g × 1000 = 44 000 mg	250 ml ÷ 10 = 25 cl
125 500 cm ÷ 100 = 1255 m ÷ 1000 = 1.255 km	750 000 mg ÷ 1000 = 750 g ÷ 1000 = 0.75 kg	7.4 l × 100 = 740 cl

Activity 3: Solving conversion problems

Solve the following problems. Please carry out all of your calculations without using a calculator. You may double check on a calculator if needed.

1. A sunflower is 1.8 m tall. Over the next month, it grows a further 34 cm. How tall is the sunflower at the end of the month?
2. Peter is a long distance runner. In a training session he runs around the 400 m track 23 times. He wanted to run a distance of 10 km. How many more times does he need to run around the track to achieve this?
3. A water cooler comes with water containers that hold 12 l of water. The cups provided for use hold 150 ml. A company estimates that each of its 20

employees will drink 2 cups of water a day. How many 12 l bottles will be needed for each working week (Monday to Friday)?

4. David is a farmer and has 52 goats. He is given a bottle of wormer that contains 0.3 litres. The bottle comes with the instructions:

   'Use 4 ml of wormer for each 15 kg of body weight.'

The average weight of David's goats is 21 kg and he wants to treat all of them. Does David have enough wormer?

Answer

1. 1 m = 100 cm so 1.8 × 100 = 180 cm

   180 cm + 34 cm = 214 cm or 2.14 m

2. 400 m × 23 = 9200 m that Peter has run.

   1 km = 1000 m so, 10 km = 10 × 1000 = 10 000 m

   The difference between what Peter wants to run and what he has already run is:

   10 000 − 9200 = 800 m
   800 ÷ 400 = 2

   Peter needs to run another 2 laps of the track.

3. 1 litre = 1000 ml so 12 l = 12 × 1000 = 12 000 ml

   Each employee will drink 2 × 150 ml = 300 ml a day

   There are 20 employees so 300 × 20 = 6000 ml for all employees per day

   A working week is 5 days:

   6000 × 5 = 30 000 ml for all employees for the week
   30 000 ÷ 12 000 = 2.5

   They will need 2 and a half containers per week.

4. 1 litre = 1000 ml so 0.3 litres = 0.3 × 1000 = 300 ml of wormer in the bottle

   52 goats at 21 kg each is 52 × 21 = 1092 kg total weight of goats

   To find the number of 4 ml doses required do:

   1092 ÷ 15 = 72.8
   72.8 × 4 ml = 291.2 ml of wormer needed.

   Yes, David has enough wormer.

Hopefully you will now be feeling confident with converting units of measure within the same system. You'll need to know how to do this and be able to remember how to convert from one to another by multiplying or dividing. If you need further practice converting between units, please revisit the 'Units of measure' session in the Everyday Maths 1 course.

If you are being asked to convert between units in different systems (e.g. between litres and gallons) or between currency (e.g. US dollars and British pounds), you won't be expected to know the conversion rate – you'll be given it in the question. The next section will show you how to use these conversion rates and you'll be able to practise solving problems that require you to do this.

Summary

In this section you have learned:

- that different units are used for measurement
- unit selection depends on the item or object being measured
- how to convert between units in the same system.

2 Converting currencies and measures between different systems

When working with measures you'll often need to be able to convert between units that are in different systems (kilograms to pounds for example) or currencies (British pounds to euros for example). Whilst this might sound tricky, it's really just using your multiplication and division skills. Let's begin by looking at converting currencies.

2.1 Converting currencies

Often when you are abroad, or buying something from the internet from another country, the price will be given in a different currency. You'll want to be able to work out how much the item costs in your own currency so that you can make a decision about whether to buy the item or not. When changing money for a holiday, you may also need to work out how much currency you will get for the amount of pounds sterling you are changing.

Figure 7 Different currency notes

Let's look at an example.

Example: Pounds and euros

To convert between pounds and euros, the first thing you need to know is the exchange rate. Exchange rates change daily. If you are given a currency conversion question, though, you will be given the exchange rate to use. In this example we will use the exchange rate of:

£1 = €1.15

So how do you convert?

Method

Look at the diagram below. In order to get from 1 to 1.15, you must multiply by 1.15. Following that, to get from 1.15 to 1, you must divide by 1.15.

In order to go from pounds to euros then, you multiply by 1.15 and to convert from euros to pounds, you divide by 1.15.

Figure 8 A conversion rate for pounds and euros

a. You decide you are going to change £300 into euros for your holiday. Using the exchange rate of £1 = €1.15, how many euros will you get?

   Referring back to the diagram above, you want to convert pounds to euros in this example so you must multiply by 1.15.

   However, you will not always have a diagram to refer to which means that you will need to be able to decide yourself whether you need to multiply by the conversion rate or divide by it. We will now look at the method to help you to do this.

   Method

   Write down your conversion rate with the single unit on the left-hand side:

   £1 = €1.15

   Next, you write down the figure you are converting under the correct side. In this case it is the £300 cash you are changing, so this figure needs to go under the pounds (£) side:

   £1 = €1.15
   £300 = €?

   This will help you to work out whether you need to go *forwards* (in which case you *multiply* by the conversion rate) or *backwards* (in which case you *divide* by the conversion rate).

   Here, you are converting from pounds to euros. This means you are going *forwards* so you need to *multiply* by the conversion rate:

   £1 = €1.15
   £300 = €?
   → *forwards* →

   Therefore, you must *multiply* by 1.15:

   £300 × 1.15 = €345

   Let's apply the method to another example.

b. You are in Spain and want to buy a souvenir. The price is €35 (35 euros). You know that £1 = €1.15.

   Method

Write the currency conversion rate down first with the single unit of measure (the 1) on the left hand-side.

£1 = €1.15

Next, write down the figure you are converting under the correct side. In this case it is the cost of the souvenir (€35), so this figure needs to go on the euros (€) side:

£1 = €1.15
£? = €35

Here, you are converting from euros to pounds. This means you are going *backwards* so you need to *divide* by the conversion rate:

£? = €35
← *backwards* ← therefore *divide*
€35 ÷ 1.15 = £30.43 (rounded to two decimal places)

Before you try some for yourself, let's look at one more example. This time involving pounds and dollars.

Example: Pounds and dollars

a. You want to buy an item from America. The cost is $120. You know that £1 = $1.30. How much does the item cost in pounds?

Method

Similarly to before, you can see from the diagram that in order to go from dollars to pounds you must divide by 1.30.

Figure 9 A conversion rate for pounds and dollars

To work out $120 in pounds then, you do:

$120 ÷ 1.30 = £92.31 (rounded to two decimal places)

To figure this out without the aid of a diagram, write down your conversion rate:

£1 = $1.30

Now write down the figure you want to convert (in this case the \$120) under the correct side:

£1 = \$1.30
£? = \$120

As you are converting dollars to pounds, you are going *backwards* so need to *divide* by 1.30:

\$120 ÷ 1.30 = £92.31 (rounded to two decimal places)

b. You are sending a relative in America £20 for their birthday. You need to send the money in dollars. How much should you send?

Method

To convert from pounds to dollars, you need to write down you conversion rate:

£1 = \$1.30

Now write down the figure you want to convert (in this case the £20) under the correct side:

£1 = \$1.30
£20 = \$?

As you are converting pounds to dollars, you are going forwards so need to multiply by 1.30:

£20 × 1.30 = \$26

You need to send \$26.

Summary of method

Write your conversion rate down first, making sure you put the single unit (the 1) on the left-hand side. This will make it easier to decide whether you need to multiply by the conversion rate or divide by it.

- If you are going forwards, you *multiply* by the conversion rate.
- If you are going backwards, you *divide* by the conversion rate.

Activity 4: Currency conversions

Calculate the answers to the following questions without using a calculator. You may double-check your answers on a calculator if needed.

1. Sarah sees a handbag for sale on a cruise ship for €80. She sees the same handbag online for £65. She wants to pay the cheapest possible price for the bag. On that day the exchange rate is £1 = €1.25. Where should she buy the bag from?
2. Josh is going to South Africa and wants to change £250 into rand. He knows that £1 = 18.7 rand. How many rand will he get for his money?
3. Alice is returning from America and has \$90 left over to change back into £s. Given the exchange rate of £1 = \$1.27, how many £s will Alice receive? Give your answer rounded to two decimal places.

Answer

1. For this question you can either convert from £ to € or from € to £.

   Converting from £ to €:

   £1 = €1.25
   £65 = €?

   As you are converting pounds to euros, you are going forwards so need to multiply by 1.25:

   £65 × 1.25 = €81.25

   Sarah should buy the bag on the ship.

   Converting from € to £:

   £1 = €1.25
   £? = €80

   As you are converting euros to pounds you are going backwards so need to divide by 1.25:

   €80 ÷ 1.25 = £64

   Sarah should buy the bag on the ship.
2. You need to convert from £ to rand so you do:

   £1 = 18.7 rand
   £250 = ? rand

   As you are converting pounds to rand you are going forwards so need to multiply by 18.7:

   £250 × 18.7 = 4675 rand
3. You need to convert from $ to £ so you do:

   £1 = $1.27
   £? = $90

   As you are converting dollars to pounds, you are going backwards so need to divide by 1.27:

   $90 ÷ 1.27 = £70.87 (rounded to two d.p.)

Hopefully you are now feeling confident with converting between currencies. The next part of this section deals with converting other units of measure between different systems – centimetres to inches, pounds to kilograms etc. These conversions are incredibly similar to converting currency and the conversion rate will always be given to you so it’s not something you have to remember.

2.2 Converting units of measure between different systems

This is a useful skill to have because you may often measure something in one unit, say your height in feet and inches, but then need to convert it to centimetres. You may, for example, be training to run a 10 km race and want to know how many miles that is. To do this all you need to know is the conversion rate and then you can use your multiplication and division skills to calculate the answer.

When performing these conversions, you are converting between metric and imperial measures. Metric measures are commonly used around the world but there are some countries (the USA for example) who still use imperial units. Whilst the UK uses metric units (g, kg, m, cm etc.) there are some instances where you may still need to convert a metric measure to an imperial one (inches, feet, gallons etc.).

In most cases, the conversion rate will be given in a similar format to the way money exchange rates are given, i.e. 1 inch = 2.5 cm or 1 kg = 2.2 lb. In these cases you can do exactly the same as you would with a currency conversion. The only exceptions here are where you don't have one of the units of measure as a single unit i.e. 5 miles = 8 km.

As this skill is very similar to the one you previously learnt with currency conversions, let's just look at two brief examples before you have a go at an activity for yourself.

Example: Centimetres and inches

You want to start your own business making accessories. One of the items you will be making is tote bags. The material you have bought is 156 cm wide. You need to cut pieces of material that are 15 inches wide.

How many pieces can you cut from the material you bought?

Use 1 inch = 2.54 cm.

Method

You need to start by converting either cm to inches or inches to cm so that you are working with units in the same system. Let's look at both ways so that you feel confident that you will get the same answer either way.

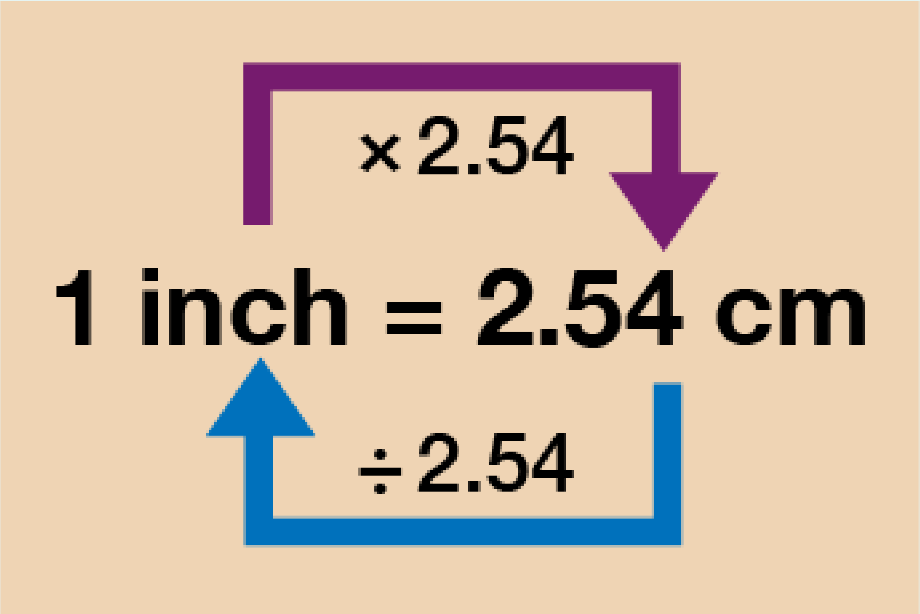


Figure 10 Converting between inches and centimetres

You can see from the diagram above that to convert from inches to cm you must multiply by 2.54. To convert from cm to inches you must divide by 2.54.

As with currency conversions, if you do not have a diagram to help you, you will have to work out yourself whether you need to multiply by the conversion rate or divide by it. To do this, write down your conversion rate:

1 inch = 2.54 cm (remember to put the single unit – the 1 – on the left-hand side)

Converting from inches to cm

Now write the figure you are converting under the correct side:

1 inch = 2.54 cm

15 inches = ? cm

Since you want to know what 15 inches is in centimetres, you are going forwards so you need to multiply by the conversion rate of 2.54:

15 inches × 2.54 = 38.1 cm. This is the length of the material needed for each bag.

To calculate how many pieces of material you can cut you then do:

156 ÷ 38.1 = 4.0944…. As you need whole pieces of material, you need to round this answer down to 4 pieces.

Converting from cm to inches

You may have decided to convert from centimetres to inches instead. You still need to write down your conversion rate first:

1 inch = 2.54 cm (remember to put the single unit – the 1 – on the left-hand side)

Now write the figure you are converting under the correct side (in this case you will convert the 156 centimetres into inches):

1 inch = 2.54 cm

? inches = 156 cm

Since you want to know what 156 centimetres is in inches, you are going backwards so you need to divide by the conversion rate of 2.54:

156 ÷ 2.54 = 61.417…, this is the length of the big piece of material.

To calculate how many pieces of material you can cut you then do:

61.417… ÷ 15 = 4.0944…. Again, as you need whole pieces of material, you need to round this answer down to 4 pieces.

You can see that no matter which way you choose to convert you will arrive at the same answer.

Note: Some numbers do not divide into each other exactly so you end up with lots of digits after the decimal point. Where this is the case, you need to round off the answer. You may be told what to round off to in the question (e.g. to the nearest whole number or to two decimal places) or you may have to use your own judgement.

For money you will normally round to two decimal places. In this case you are thinking about whole pieces of material – you only have four whole pieces so this is what you round to here.

Example: Kilometres and miles

You have signed up for a 60 km bike ride. There is a lake in a nearby park that you want to use for your training. You know that one lap of the lake is 2 miles. You want to cycle a distance of at least 40 km in your last training session before the race. How many full laps of the lake should you do?

You know that a rough conversion is 5 miles = 8 km.

There is more than one way to go about this calculation and, as ever, if you have a different method that works for you and you arrive at the same answer, feel free to use it!

Method

Since you already know how to convert if you are given a conversion that has a single unit of whichever measure you are using, it makes sense to just change the given conversion into one that you are used to dealing with.

Look at the diagram below – since you know that 5 miles is worth 8 km, you can find out what 1 mile is worth by simply dividing 5 by 5 so that you arrive at 1 mile. Whatever you do to one side, you must do to the other. Therefore, you also do 8 divided by 5. This then gives 1 mile = 1.6 km.

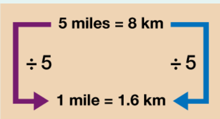


Figure 11 Calculating 1 mile in kilometres

Now that you know 1 mile = 1.6 km, you can solve this problem in its usual way:

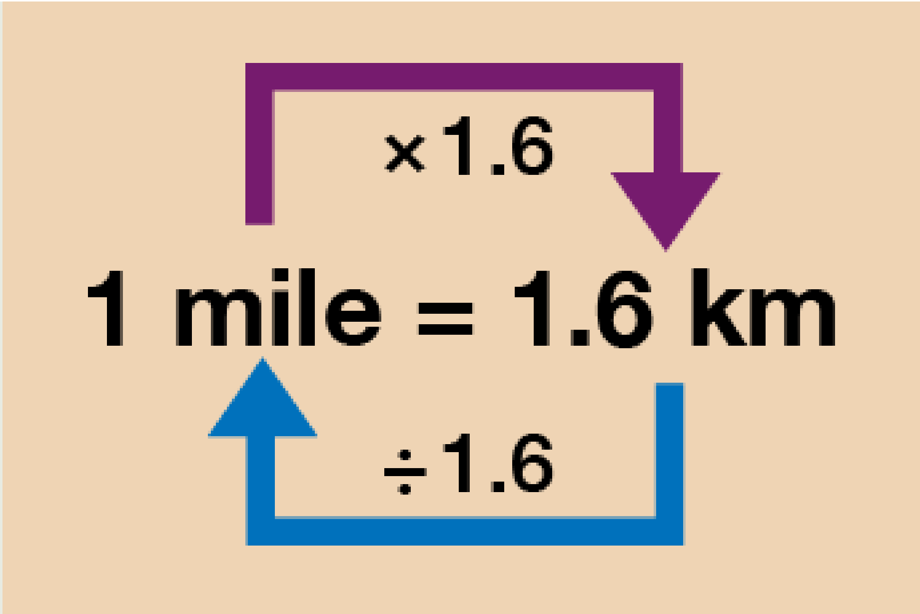


Figure 12 Converting between miles and kilometres

Write down your conversion rate:

1 mile = 1.6 km

Now write down the figure you want to convert under the correct side (in this case you want to convert 40 km into miles):

1 mile = 1.6 km

? miles = 40 km

Since you first want to know what 40 km is in miles, you are going backwards, so you need to divide by 1.6:

40 ÷ 1.6 = 25 miles

So, 40 km = 25 miles

You know that one lap of the lake is 2 miles so:

25 ÷ 2 = 12.5 laps

Therefore, you will need to cycle 12.5 laps around the lake in order to have cycled your target of 40 km.

Now that you have seen a couple of worked examples, have a go at the activity below to check your understanding.

Activity 5: Converting between systems

Wherever possible, please do the calculations first without a calculator. You may then double-check on a calculator if needed.

1. A Ford Fiesta car can hold 42 litres of petrol. Using the fact that 1 gallon = 4.54 litres, work out how many gallons of petrol the car can hold. Give your answer rounded to two decimal places.
2. The café you work at has run out of milk and you have been asked to go to the shop and buy 10 pints. When you arrive however, the milk is only available in 2 litre bottles. You know that 1 litre = 1.75 pints. How many 2 litre bottles should you buy?
3. You are packing to go on holiday and are allowed 25 kg of luggage on the flight. You've weighed your case on the bathroom scales and it weighs 3 stone and 3 pounds.

   You know that:

   1 stone = 14 pounds

   2.2 pounds = 1 kg

   Is your luggage over the weight limit?
4. Your son wants to go on a ride at a theme park that has a minimum height restriction of 122 cm. You know your son is 4 ft. 7 in. tall. You know that:

   1 foot (ft.) = 12 inches

   1 inch (in.) = 2.54 cm

   Can your child go on the ride?

Answer

1. You do:

   1 gallon = 4.54 litres

   ? gallons = 42 litres

As you are converting litres to gallons, you are going backwards so need to divide by 4.54.

   42 ÷ 4.54 = 9.25 gallons (to two d.p)

2. Firstly, you want to know how many litres there are in 10 pints, so you need to convert from pints to litres. You do:

   1 litre = 1.75 pints

   ? litres = 10 pints

   As you are converting pints to litres, you are going backwards so need to divide by 1.75:

   10 ÷ 1.75 = 5.71… litres. As you can only buy the milk in 2 litre bottles, you will have to buy 6 litres of milk in total. You therefore need:

   6 ÷ 2 = 3 bottles of milk
3. Firstly, you need to convert 3 stone and 3 pounds into just pounds. Since 1 stone = 14 pounds, to work out 3 stone you do:

   3 × 14 = 42 pounds

   Then you need to add on the extra 3 pounds, so in total you have 42 + 3 = 45 pounds.

   Now that you know this, you can convert from pounds to kg.

Since 2.2 pounds = 1 kg, you can also say that 1 kg = 2.2 pounds (this doesn't change anything, it just makes it a little easier as you can stick to your usual method of having the single unit on the left hand side):

1 kg = 2.2 pounds

? kg = 45 pounds

As you are converting from pounds to kg, you are going backwards so need to divide by 2.2:

45 ÷ 2.2 = 20.45 kg

Yes, your luggage is within the weight limit.

4. Firstly, you need to convert 4 ft. 7 in. into just feet. Since 1 foot = 12 inches, to work out 4 feet you do:

   4 × 12 = 48 inches

   Then you need to add on the extra 7 inches, so in total your son is 48 + 7 = 55 inches tall.

   Now that you know this, you can convert from inches to centimetres:

   1 inch = 2.54 cm

   55 inches = ? cm

   As you are converting from inches to centimetres, you are going forwards so need to multiply by 2.54:

   55 × 2.54 = 139.7 cm

   So your son is tall enough to go on the ride.

You should now be feeling confident with your conversion skills so it's time to move on to the next part of this session.

Summary

In this section you have learned:

- that different systems of measurement can be used to measure the same thing (e.g. a cake could be weighed in either grams or pounds)
- you can convert between these systems using your multiplication and division skills and the given conversion rate
- currencies can be converted in the same way as long as you know the exchange rate – this is particularly useful for holidays.

3 Time, timetables and average speed

Calculating with time is often seen as tricky, not surprising really considering how difficult it can be to learn how to tell the time. The reason many people find calculating with time tricky is because, unlike nearly every other mathematical concept, it does not work in 10s. Time works in 60s – 60 seconds in a minute, 60 minutes in an hour. You cannot therefore, simply use your calculator to add on or subtract time.

Figure 13 A radio alarm clock

Think about this simple example. If it's 9:50 and your bus takes 20 minutes to get to work, you cannot work out the time you will arrive by doing 950 + 20 on your calculator. This would give you an answer of 970 or 9:70 – there isn't such a time!

You will need to calculate with time and use timetables in daily life to complete basic tasks such as: getting to work on time, working out which bus or train to catch, picking your children up from school on time, cooking and so many other daily tasks.

3.1 Calculating with time and timetables

As previously discussed, calculators are not the most useful items when it comes to calculations involving time. A much better option is to use a number line to work out these calculations. Take a look at the examples below.

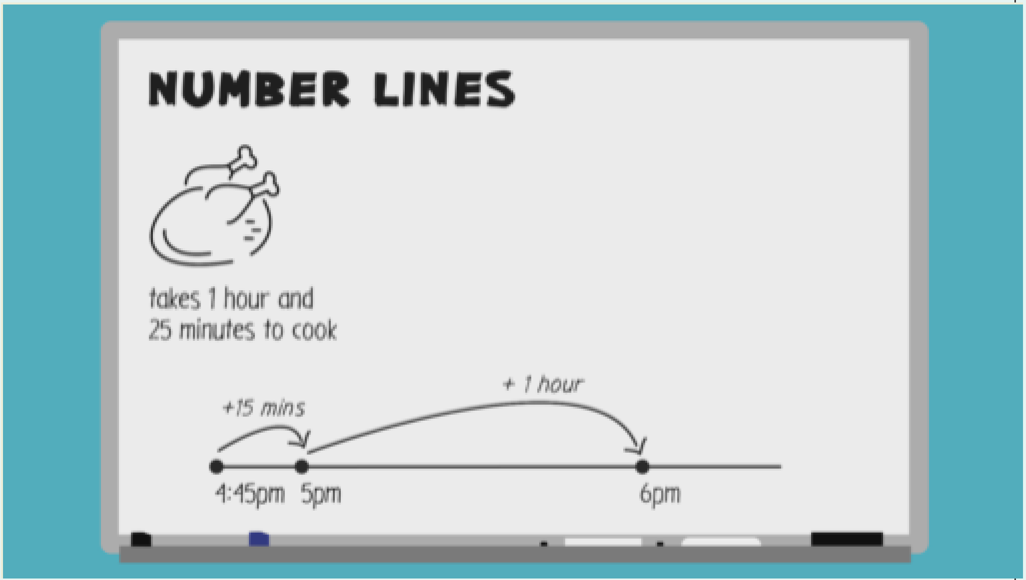
Example: Cooking

You put a chicken in the oven at 4:45 pm. You know it needs to cook for 1 hour and 25 minutes. What time should you take the chicken out?

Method

Watch the video below to see how the number line method works.

Video content is not available in this format.



Example: Time sheets

You work for a landscaping company and need to fill out your time sheet for your employer. You began working at 8:30 am and finished the job at 12:10 pm. How long did the job take?

Method

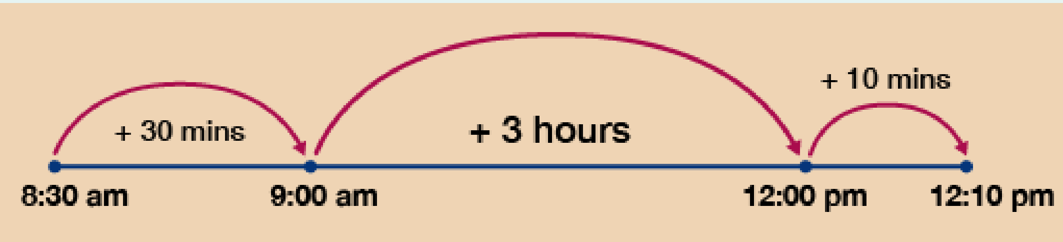


Figure 14 A number line for a time sheet

Again, for finding the time difference you want to work with easy ‘chunks’ of time. Firstly, you can move from 8:30 am to 9:00 am by adding 30 minutes. It is then simple to get to 12:00 pm by adding on 3 hours.

Finally, you just need another 10 minutes to take you to 12:10 pm. Looking at the total time added you have 3 hours and 40 minutes.

Another aspect of calculating with time comes in the form of timetables. You will be used to using these to work out which departure time you need to meet in order to get to a location on time or how long a journey will take. Once you can calculate with time, using timetables simply requires you to find the correct information before carrying out the calculation. Take a look at the example below.

Example: Timetables

Here is part of a train timetable from Swindon to London.

Table 2(a)

Swindon	06:10	06:27	06:41	06:58	07:01	07:17
Didcot	06:27	06:45	06:58	07:15	07:18	07:34
Reading	06:41	06:59	07:13	-	07:33	-
London	07:16	07:32	07:44	08:02	08:07	08:14

a. You need to travel from Didcot to London. You need to arrive in London by 8:00 am. What is the latest train you can catch from Didcot to arrive in London for 8:00 am?

Method

Table 2(b)

Swindon	06:10	06:27	06:41	06:58	07:01	07:17
Didcot	06:27	06:45	*06:58*	07:15	07:18	07:34
Reading	06:41	06:59	07:13	-	07:33	-
London	07:16	07:32	**07:44**	08:02	08:07	08:14

Looking at the arrival times in London, in order to get there for 8:00 am you will need to take the train that arrives in London at 07:44 (highlighted with bold). If you then move up this column of the timetable you can see that this train leaves Didcot at 06:58 (highlighted with italic). This is therefore the train you must catch.

b. How long does the 06:58 from Swindon take to travel to London?

Method

Table 2(c)

Swindon	06:10	06:27	06:41	*06:58*	07:01	07:17
Didcot	06:27	06:45	06:58	07:15	07:18	07:34
Reading	06:41	06:59	07:13	-	07:33	-
London	07:16	07:32	07:44	**08:02**	08:07	08:14

Firstly, find the correct train from Swindon (highlighted with italic). Follow this column of the timetable down until you reach London (highlighted with bold). You then need to find the difference in time between 06:58 and 08:02. Using the number line method from earlier in the section (or any other method you choose).

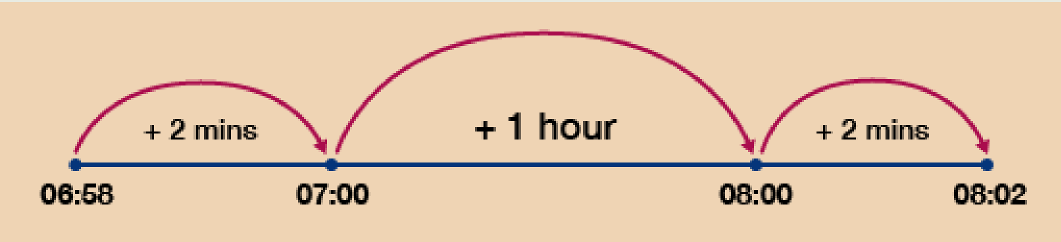


Figure 15 A number line for a timetable

You can then see that this train takes a total of 1 hour and 4 minutes to travel from Swindon to London.

Have a go at the activity below to practise calculating time and using timetables.

Activity 6: Timetables and calculating time

1. Kacper is a builder. He leaves home at 8:30 am and drives to the trade centre. He collects his items and loads them into his van. His visit takes 1 hour and 45 minutes. He then drives to work, which takes 50 minutes. What time does he arrive at work?
2. You have invited some friends round for dinner and find a recipe for roast lamb. The recipe requires:
   - 25 minutes preparation time
   - 1 hour cooking time
   - 20 minutes resting time

   You want to eat with your friends at 7:30 pm. What is the latest time you can start preparing the lamb?
3. Here is part of a train timetable from Manchester to Liverpool.

Table 3(a)

Manchester to Liverpool						
Manchester	10:24	10:52	11:03	11:25	12:01	12:13
Warrington	10:38	11:06	11:20	11:45	12:15	12:28
Widnes	10:58	11:26	11:42	12:03	12:34	12:49
Liverpool Lime Street	11:09	11:38	11:53	12:14	12:46	13:02

You need to travel from Manchester to Liverpool Lime Street. You need to be in Liverpool by 12:30. Which train should you catch from Manchester and how long will your journey take?

Answer

1. Firstly, work out the total time that Kacper is out for:

    1 hour 45 minutes at the trade centre and another 50 minutes driving makes a total of 2 hours and 35 minutes.

    Then, using the number line, you have:

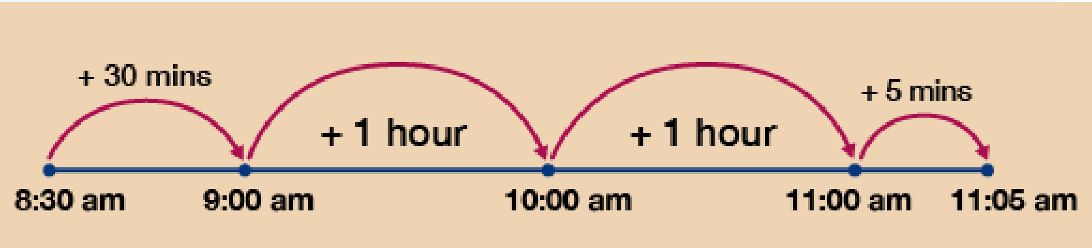


Figure 16 A number line for Question 1

So Kacper arrives at work at 11:05 am.

You could also do the calculation by adding on the 1 hour 45 minutes first:

8:30 am + 1 hour = 9:30 am
9:30 am + 45 minutes = 10:15 am

Finally, you can add on the 50 minutes:

10:15 am + 45 minutes = 11:00 am

Then add on the remaining 5 minutes:

11:00 am + 5 minutes = 11:05 am

2. Again, firstly work out the total time required:

    25 minutes + 1 hour + 20 minutes = 1 hour 45 minutes in total

    This time you need to work backwards on the number line so you begin at 7:30 and work backwards.

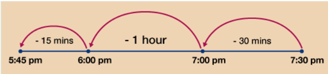


Figure 17 A number line for Question 2

You can now see that you must begin preparing the lamb at 5:45 pm at the latest.

As with the first question, you could have done this question by taking off each stage in the cooking process separately rather than finding the total time first:

7:30 pm − 20 minutes = 7:10 pm

7:10 pm − 1 hour = 6:10 pm

There are 25 minutes left so:

6:10 pm − 10 minutes = 6:00 pm

There are now 15 minutes left so:

6:00 pm − 15 minutes = 5:45 pm

3. Looking at the timetable for arrival at Liverpool, you can see that in order to arrive by 12:30 you need to catch the train that arrives at 12:14. This means that you need to catch the 11:25 from Manchester.

Table 3(b)

Manchester to Liverpool						
Manchester	10:24	10:52	11:03	*11:25*	12:01	12:13
Warrington	10:38	11:06	11:20	11:45	12:15	12:28
Widnes	10:58	11:26	11:42	12:03	12:34	12:49
Liverpool Lime Street	11:09	11:38	11:53	**12:14**	12:46	13:02

You therefore need to work out the difference in time between 11:25 (italic) and 12:14 (bold).

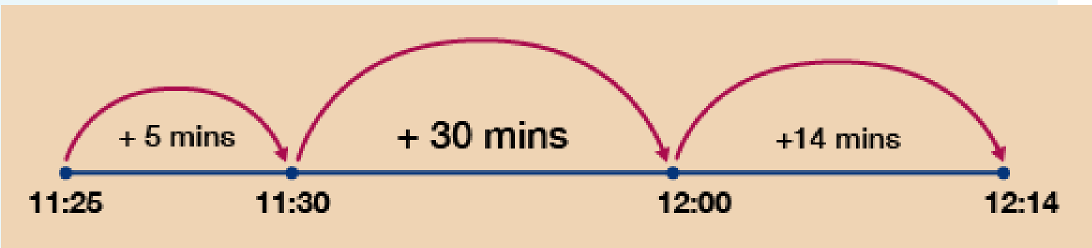


Figure 18 A number line for Question 3

Using the number line again, you can see that this is a total of 5 + 30 + 14 = 49 minutes.

You should now be feeling comfortable with calculations involving time and timetables. Before you move on to looking at problems that involve average speed, it is worth taking a brief look at time conversions. Since you are already confident with converting units of measure, this part will just consist of a brief activity so that you can practise converting units of time.

3.2 Converting units of time

You can see from the diagram below that to convert units of time you can use a very similar method to the one you used when converting other units of measure. There is one slight difference when working with time however.

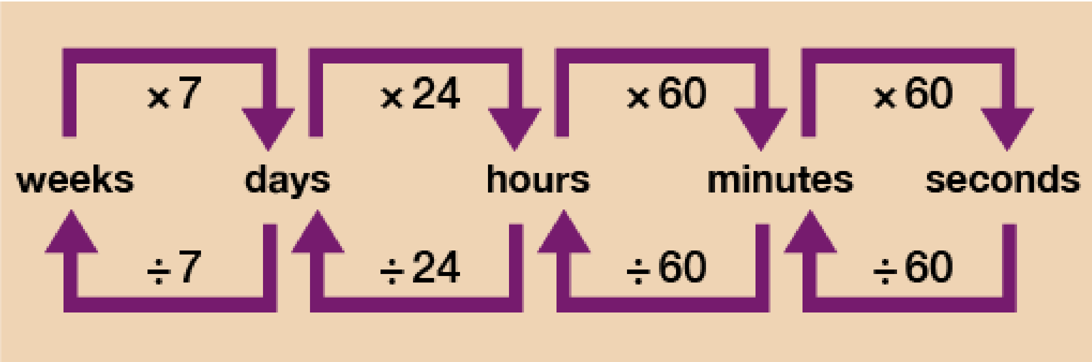


Figure 19 A conversion chart for time

Let's say you want to work out how long 245 minutes is in hours. The diagram above shows that you should do 245 ÷ 60 = 4.083. This is not a particularly helpful answer since you really want the answer in the format of: ___ hours __ minutes. Due to the fact that time does not work in 10s, you need to do a little more work once arriving at your answer of 4.083.

The answer is obviously 4 hours and an amount of minutes.

4 hours then is 4 × 60 = 240 minutes.

Since you wanted to know how long 245 minutes is you just do 245 – 240 = 5 minutes left over. So 245 minutes is 4 hours and 5 minutes.

It's a very similar process if you want to go from say minutes to seconds. Let's take it you want to know how long 5 minutes and 17 seconds is in seconds. 5 minutes would be 5 × 60 = 300 seconds. You then have a further 17 seconds to add on so you do 300 + 17 = 317 seconds.

Have a go at the activity below to make sure you feel confident with converting times.

Activity 7: Converting times

Convert the following times:

1. 6 hours and 35 minutes = ___ minutes.
2. 85 minutes = ____ hours and ____ minutes.
3. 153 seconds = ____ minutes and ___ seconds.
4. 46 days = ___ weeks and ____days.
5. 3 minutes and 40 seconds = ____ seconds.

Answer

1. 6 hours = 6 × 60 = 360 minutes

   360 minutes + 35 minutes = 395 minutes
2. 85 minutes ÷ 60 = 1.417 (rounded to three d.p)

   1 hour = 60 minutes.

   85 minutes − 60 minutes = 25 minutes remaining

   So 85 minutes = 1 hour and 25 minutes
3. 153 seconds ÷ 60 = 2.55

   2 minutes = 2 × 60 = 120 seconds

   153 seconds − 120 seconds = 33 seconds remaining

   So 153 seconds = 2 minutes and 33 seconds
4. 46 days ÷ 7 = 6.571 (rounded to three d.p)

   6 weeks = 6 × 7 = 42 days

   46 days − 42 days = 4 days remaining

   So 46 days = 6 weeks and 4 days
5. 3 minutes = 3 × 60 = 180 seconds

   180 seconds + 40 seconds = 220 seconds

Hopefully you found that activity fairly straightforward and are now feeling ready to move on to the next part of the 'Units of measure' session – Average speed.

3.3 Average speed

The sign below is commonly seen on motorways but it is not the only time when it is useful to know your average speed.

Figure 20 A speed camera sign

Being able to calculate and use average speed can help you to work out how long a journey is likely to take. The method for working out average speed involves using a simple formula.

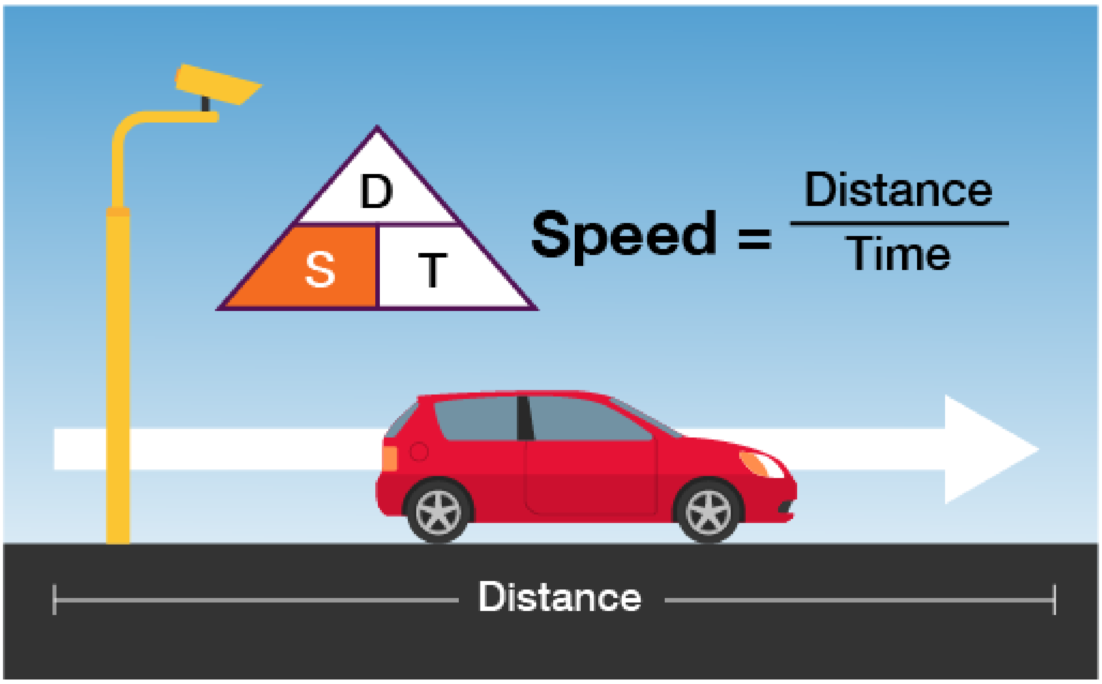


Figure 21 A formula for average speed

You can also use this formula to work out the distance travelled when given a time and the average speed, or the time taken for a journey when given the distance and average speed.

The formulas for this are shown in the diagram below. You can see that when given any two of the elements from distance, speed and time, you will be able to work out the third.

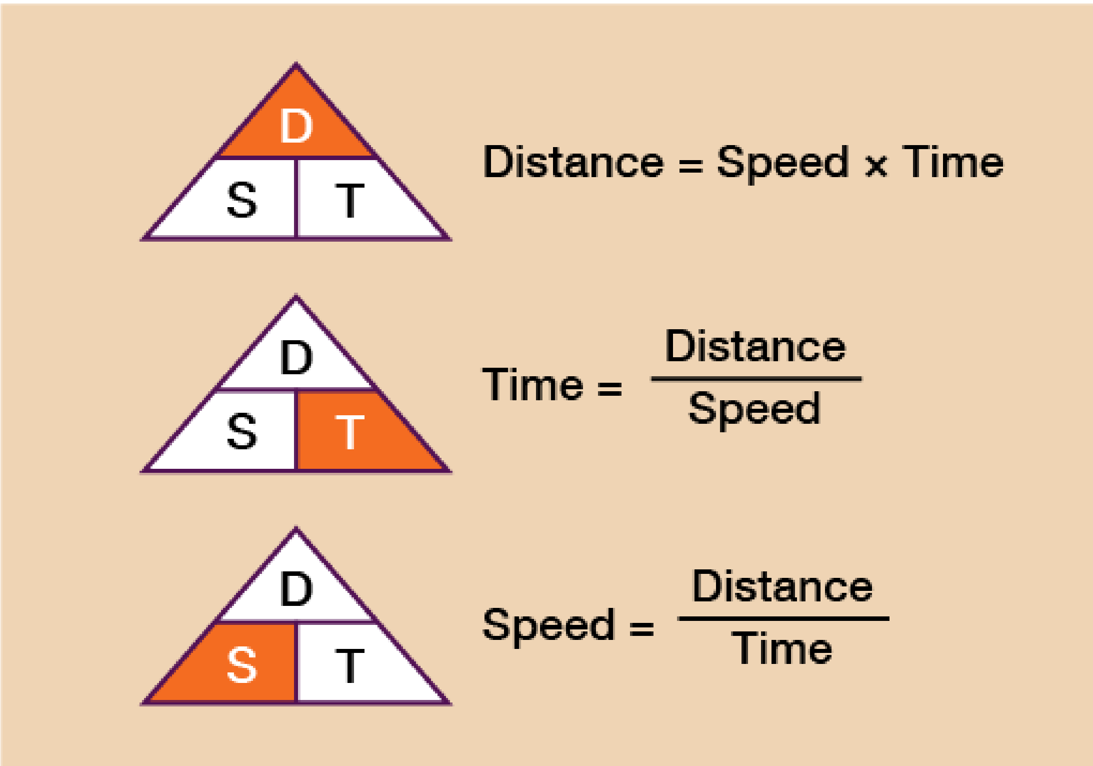


Figure 22 Distance, speed and time formulas

If you can learn this formula triangle, when you want to use it, you write it down and cover up what you want to work out (the segment in orange). This will tell you what calculation you need to do.

Let's look at an example of each so that you can familiarise yourself with it.

Example: Calculating distance

A car has travelled at an average speed of 52 mph over a journey that lasts 2 and a half hours. What is the total distance travelled?

Method

You can see that to work out the distance you need to do speed × time. In this example then we need to do 52 × 2.5. It is very important to note here that 2 and a half hours must be written as 2.5 (since 0.5 is the decimal equivalent of a half).

You cannot write 2.30 (for 2 hours and 30 mins). If you struggle to work out the decimal part of the number, convert the time into minutes (2 and a half hours = 150 minutes) and then divide by 60 (150 ÷ 60 = 2.5).

52 × 2.5 = 130 miles travelled

Example: Calculating time

A train will travel a distance of 288 miles at an average speed of 64 mph. How long will it take to complete the journey?

Method

You can see from the formula that to calculate time you need to do distance ÷ speed so you do:

288 ÷ 64 = 4.5 hours

Again, note that this is not 4 hours 50 minutes but 4 and a half hours.

If you are unsure of how to convert the decimal part of your answer, simply multiply the answer by 60, which will turn it into minutes and you can then convert from there.

In this case, 4.5 × 60 = 270 minutes. We already know from the answer of 4.5 hours that this is 4 whole hours and so many minutes, so we now need to work out how many minutes the .5 represents:

60 × 4 = 240 minutes

270 − 240 = 30 minutes

So 4.5 hours = 270 minutes = 4 hours, 30 minutes

Example: Calculating speed

A Formula One car covers a distance of 305 km during a race. The time taken to finish the race is 1 hour and 15 minutes. What is the car's average speed?

Method

The formula tells you that to calculate speed you must do distance ÷ time. Therefore, you do 305 ÷ 1.25 (since 15 minutes is a quarter of an hour and 0.25 is the decimal equivalent of a quarter):

$$305 \div 1.25 = 244 \text{ km/h}$$

In a similar way to example 1, if you are unsure of how to work out the decimal part of the time simply write the time (in this case 1 hour and 15 minutes) in minutes, (1 hour 15 minutes = 75 minutes) and then divide by 60:

$$75 \div 60 = 1.25$$

Now have a go at the following activity to check that you feel confident with finding speed, distance and time. Please do the calculations first without a calculator. You may then double-check on a calculator if needed.

Activity 8: Calculating speed, distance and time

1. Filip is driving a bus along a motorway. The speed limit is 70 mph. In 30 minutes, he travels a distance of 36 miles. Does his average speed exceed the speed limit?
2. A plane flies from Frankfurt to Hong Kong. The flight time was 10 hours and 45 minutes. The average speed was 185 km/h. What is the distance flown by the plane?
3. Malio needs to get to a meeting by 11:00 am. The time now is 9:45 am. The distance to the meeting is 50 miles and he will be travelling at an average speed of 37.5 mph. Will he be on time for the meeting?

Answer

1. You need to find the speed so you do: distance ÷ time.

   The distance is 36 miles. The time is 30 minutes but you need the time in hours:

   30 minutes ÷ 60 = 0.5 hours

   Now you do:

   36 ÷ 0.5 = 72 mph

   Yes, Filip's average speed did exceed the speed limit.
2. You need to find the distance so you do:

   speed × time

10 hours 45 minutes = 10.75 hours

If you are unsure how to express this in hours, convert 10 hours 45 minutes all into minutes:

10 × 60 = 600 + 45 = 645 minutes

Then divide by 60:

645 ÷ 60 = 10.75 hours

Now to work out the distance do:

speed × time = 185 × 10.75 = 1988.75 km from Frankfurt to Hong Kong

3. You need to find the time so you do:

distance ÷ speed

50 ÷ 37.5 = 1.33 hours (rounded to two d.p)

Note: The actual answer is 1.3333333 (the 3 is recurring or never-ending).

To convert this to minutes do:

1.33 × 60 = 79.8 minutes

round 79.8 minutes up to 80 minutes

80 minutes = 1 hour and 20 minutes

If the time now is 9:45 am and his meeting is at 11:00 am, then it is only 1 hour, 15 minutes until his meeting, so no, Malio will not make the meeting on time.

Hopefully you will now be feeling more confident with calculations involving speed, distance and time. You will now move on to temperature conversions.

Summary

In this section you have learned how to:

- use timetables to plan a journey and how to calculate time efficiently
- convert between units of time by using multiplication and division skills
- use the formula for calculating distance, speed and time.

4 Temperature

Temperature can be recorded in either degrees Celsius (°C) or degrees Fahrenheit (°F). In *Everyday Maths 1* you used conversion tables to help you to compare temperatures expressed in the different units. You will now look at how to convert between them using formulas.

4.1 Celsius and Fahrenheit formulas

The following formulas can be used to convert between Celsius and Fahrenheit.

To convert Celsius to Fahrenheit use the formula:

$$F = \frac{9}{5}C + 32$$

Method:

- divide the Celsius figure by 5
- multiply by 9
- add 32.

If you prefer, you can multiply the Celsius figure by 9 first and then divide by 5. You will still need to add on 32 at the end.

To convert Fahrenheit to Celsius use the formula:

$$C = \frac{5\,(F - 32)}{9}$$

Method:

- subtract 32 from the Fahrenheit figure
- multiply by 5
- divide by 9.

If you need a recap on the rules for using formulas, revisit Session 1 ‘Working with numbers’. We will now look at an example.

Example: Which city is warmer?

I look up the average temperature for New York on a particular day and it is 10°C. I know the average temperature in Swansea on the same day is 55°F. Which city is warmer?

You either need to convert New York’s temperature into °F or the Swansea temperature into °C.

Method 1 – Converting °C to °F

If we look back at the formulas above, the one we need to use to convert from °C to °F is:

$F = \frac{9}{5}\ C + 32$

We need to substitute the C with our °C figure of 10°C. We then need to follow the rules of BIDMAS to carry out the calculation in stages, as shown below:

$F = \frac{9}{5} \times 10 + 32$

Divide the celsius figure by 5:

$10 \div 5 = 2$

Multiply by 9:

$2 \times 9 = 18$

Add 32:

18 + 32 = **50°F**

You may have done the calculation slightly differently by multiplying the Celsius figure by 9 first and then dividing by 5. The answer will work out the same:

$F = \frac{9}{5} \times 10 + 32$

Multiply by the Celsius figure by 9:

$10 \times 9 = 90$

Divide by 5:

$90 \div 5 = 18$

Add 32:

18 + 32 = **50°F**

So which is warmer:

New York at 10°C (which we now know is 50°F) or Swansea at 55°F?
Swansea is warmer.

Method 2 – Converting °F to °C

The formula for converting from °F to °C:

$$C = \frac{5\,(F - 32)}{9}$$

We need to substitute the F with our °F figure of 55°F. We then need to follow the rules of BIDMAS to carry out the calculation in stages, as shown below:

Take 32 away from the Fahrenheit figure of 55:

$55 - 32 = 23$

Multiply by 5:

$23 \times 5 = 115$

Divide by 9:

$115 \div 9 =$ **12.8°C** (rounded to 1 decimal place)

So which is warmer:

New York at 10°C or Swansea at 55°F (which we now know is 12.8°C)? Swansea is warmer.

Hint: Google has its own unit converter (search for Google Unit Converter) which you can use to convert between various units of measure, including between °C and °F. You could try using it to double-check your answers to the questions below.

Activity 9: Temperature conversions

Work out the answers to the following without using a calculator. You may double-check your answers on a calculator or using the Google unit converter, if needed, and remember to check your answers with ours at the end. Round your answers off to one decimal place where needed.

1. Convert the following temperatures into degrees Fahrenheit (°F):
   a. 22°C
   b. 0°C
   c. −6°C

Answer

1. You need to use the following formula:

   $F = \frac{9}{5}C + 32$

   a. $F = \frac{9}{5} \times 22 + 32$

   $22 \div 5 = 4.4$

4.4 × 9 = 39.6

39.6 + 32 = **71.6°F**

b. $F = \frac{9}{5} \times 0 + 32$

0 ÷ 5 = 0

0 × 9 = 0

0 + 32 = **32°F**

c. $F = \frac{9}{5} \times -6 + 32$

−6 ÷ 5 = −1.2

−1.2 × 9 = −10.8

−10.8 + 32 = **21.2°F**

2. Convert the following temperatures into degrees Celsius (°C):
   b. 45°F
   c. 212°F
   d. 5°F

Answer

2. You need to use the following formula:

$$C = \frac{5\,(F - 32)}{9}$$

b. $C = \frac{5\,(45 - 32)}{9}$

45 − 32 = 13

13 × 5 = 65

65 ÷ 9 = **7.2°C** (to one d.p)

c. $C = \frac{5\,(212 - 32)}{9}$

212 − 32 = 180

180 × 5 = 900

900 ÷ 9 = **100°C**

d. $C = \frac{5\,(5 - 32)}{9}$

5 − 32 = −27

−27 × 5 = −135

−135 ÷ 9 = **−15°C**

3. I find a recipe which states that my oven needs to be set at a temperature of 400°F. My settings on my oven are in °C. What temperature should I set my oven to?

Answer

3. You need to convert 400°F to °C so use the formula:

$$C = \frac{5\,(F - 32)}{9}$$

$$C = \frac{5\,(400 - 32)}{9}$$

400 − 32 = 368

368 × 5 = 1840

1840 ÷ 9 = **204.4°C** (to one d.p).

As you would be unable to set an oven so accurately, you would set the temperature to 200°C.

4. I see Moscow's temperature is −4°C on a particular day in February, whilst the temperature in Toronto is 19°F. Which place is colder?

Answer

4. You either need to convert the Moscow temperature of −4°C to °F, or convert the Toronto temperature of 19°F to °C.

Method 1 – Converting °C to °F

If we look back at the formulas, the one we need to use to convert from °C to °F is:

$$F = \frac{9}{5}\,C + 32$$

We need to substitute the C with our °C figure of −4°C. We then need to follow the rules of BIDMAS to carry out the calculation in stages, as shown below:

$$F = \frac{9}{5} \times -4 + 32$$

−4 ÷ 5 = −0.8

Multiply by 9:

−0.8 × 9 = −7.2

Add 32:

−7.2 + 32 = **24.8°F**

So which is colder? Moscow at −4°C (which we now know is 24.8°F) or Toronto at 19°F? Toronto is colder.

Method 2 – Converting °F to °C

The formula for converting from °F to °C is:

$$C = \frac{5\,(F - 32)}{9}$$

We need to substitute the F with our °F figure of 19°F. We then need to follow the rules of BIDMAS to carry out the calculation in stages, as shown below:

$$C = \frac{5\,(19 - 32)}{9}$$

Take 32 away from the Fahrenheit figure of 19:

$19 - 32 = -13$

Multiply by 5:

$-13 \times 5 = -65$

Divide by 9:

$-65 \div 9 =$ **−7.2°C** (to one d.p.)

So which is colder: Moscow at −4°C or Toronto at 19°F (which we now know is −7.2°C)? Toronto is colder.

Hopefully you will be feeling more confident when solving problems relating to temperature. The next section will cover reading measurements on scales.

Summary

In this section you have:

- practised converting between degrees Celsius (°C) and degrees Fahrenheit (°F).

5 Reading scales

You may need to read a scale to measure out an amount of liquid, read a measurement on a ruler, weigh out ingredients for a recipe or to take someone's temperature.

Reading scales can be tricky because every scale is different.

To read a scale correctly, you need to ask yourself:

> What does the scale go up in? What steps or intervals does it use?

Note: The marks on a scale may be referred to as any of the following: intervals, steps, increments or markers. These terms are often used interchangeably.

5.1 Scale examples

Take a look at the following examples.

Example 1: Reading scales

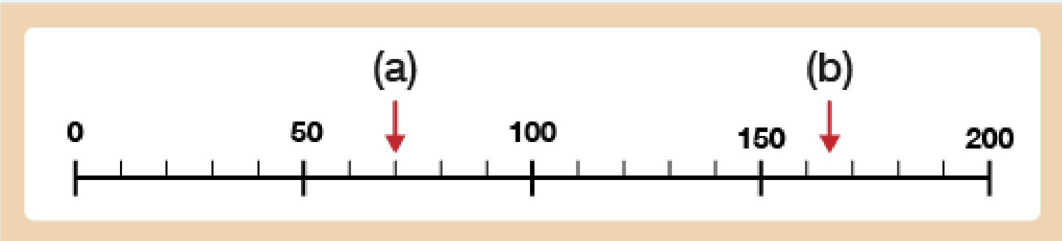


Figure 23 Scale with numbered intervals of 50

You can see that this scale is marked up in numbered intervals of 50. However, what does each line in between each numbered interval represent? You can use your judgement to help you to figure out what each small step represents.

Watch this video (https://corbettmaths.com/2013/04/27/reading-scales/) for further information about how to do this.

Alternatively, you can work this out using division. If you count on from 0 to 50 on this scale, there are 5 steps: 50 ÷ 5 = 10, so each step is 10.

a. The arrow is pointing to the 2nd mark after 50. As the steps are going up in 10s, the arrow is pointing to 70.
b. The arrow is halfway between the 1st and 2nd step after 150. The first step is 160 and the second is 170 so the arrow is pointing to 165.

Example 2: Reading scales

Sometimes you will need to read scales where the reading will be a decimal number.

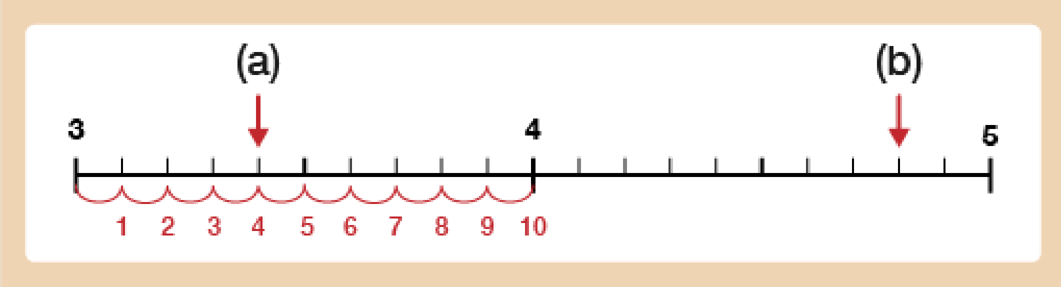


Figure 24 Scale with single numbered intervals going from 3–5

If you look at this scale, it goes up in numbered intervals of 1. From one whole number to the next whole number there are 10 small steps. 1 ÷ 10 = 0.1, so each step is 0.1.

Hint: Have a look at the image to show how to count the number of steps between the numbered markers.

a. The arrow is pointing to the fourth step after 3 so the arrow is pointing to 3.4.
b. The arrow is pointing to the eighth step after 4, so the arrow is pointing to 4.8.

5.2 Scales and measuring instruments

Now you've worked through some examples have a go at the following activities.

Activity 10: Reading scales

1. Read the scales below and find the values the arrows are pointing to for (a), (b) and (c).

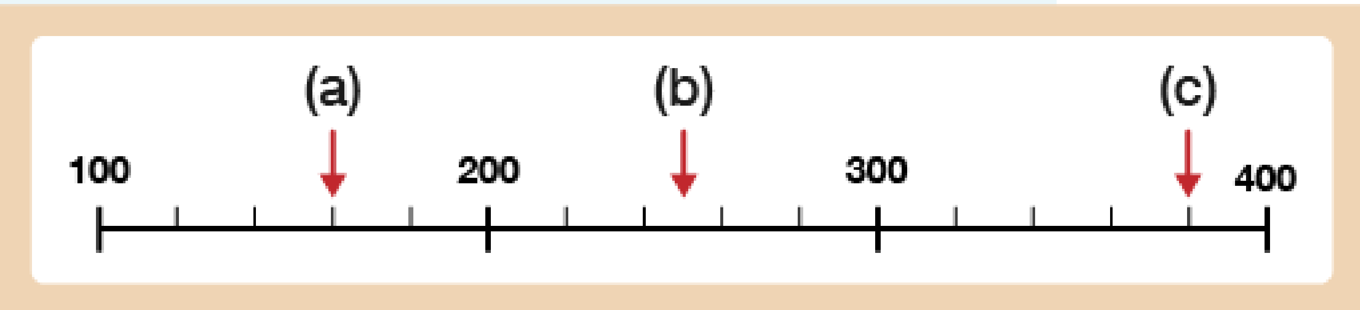


Figure 25 Scale with numbered intervals every hundred going from 100–400

Answer

1. The scale is going up in steps of 20 (the numbered markers are going up in intervals of 100 and there are 5 small steps between each numbered marker: 100 ÷ 5 = 20) so the answers are:
   a. 160
   b. This is halfway between 240 and 260 so the answer is 250

c. 380

2. Read the values for (a), (b) and (c) on the scales below.

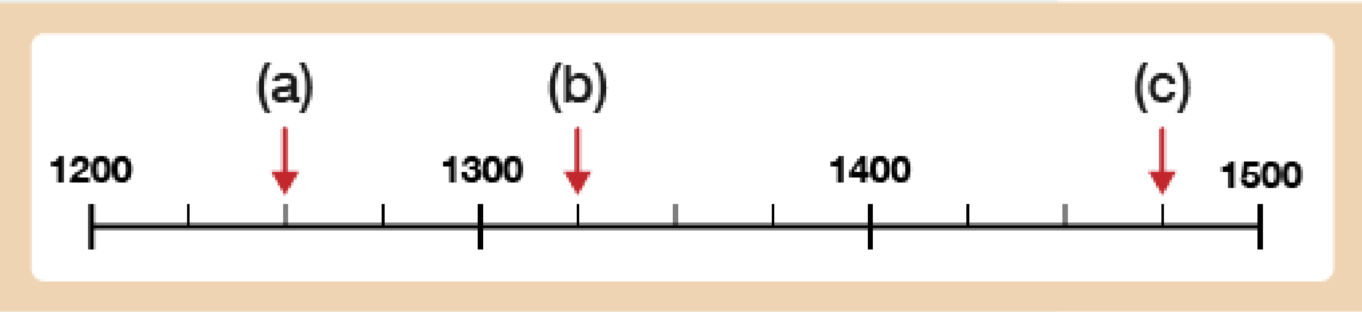


Figure 26 Scale with numbered intervals every hundred going from 1200–1500

Answer

2.

b. 1250
c. 1325
d. 1475

3. Read the values for (a), (b), (c) and (d) on the scales below.

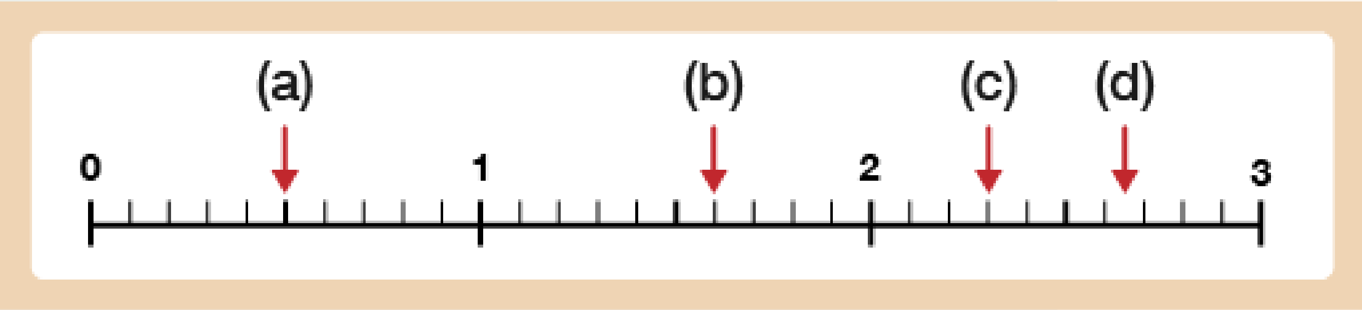


Figure 27 Scale with single number intervals from 0–3

Answer

3.

c. 0.5
d. 1.6
e. 2.3
f. The arrow is pointing to halfway between 2.6 and 2.7 so the reading is 2.65.

4. Read the values for (a), (b), and (c) on the scales below.

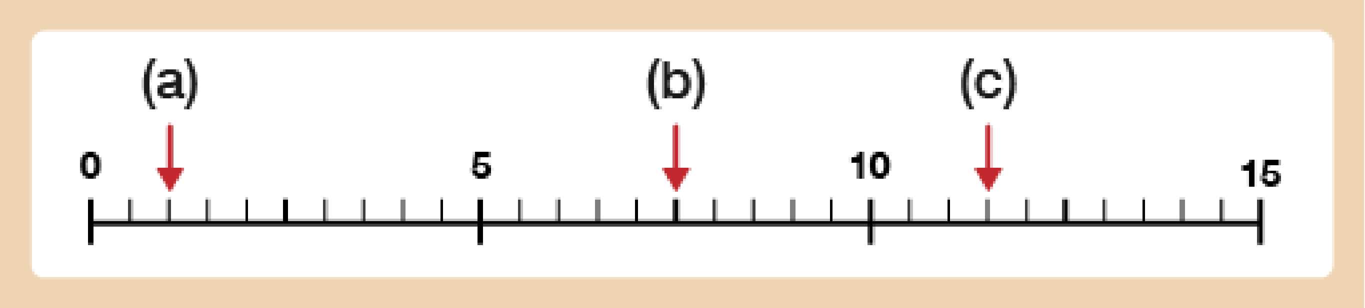


Figure 28 Scale with numbered intervals every 5 going from 0–15

Answer

4.

- d. 1.0 (or just 1)
- e. 7.5
- f. 11.5

Now have a go at reading the scales on the different instruments of measure.

Activity 11: Measuring instruments

1. How much water is left in the bottle, to the nearest 50 millilitres (ml)?

Figure 29 A water bottle with a scale on the side and water inside

Answer

1. The bottle holds 1 litre of liquid in total. There are 10 large steps marked on the bottle so each one marks 100 ml (1 litre = 1000 ml and 1000 ÷ 10 = 100).

   Halfway between each large step there is a small step so each of these marks off 50 ml.

   This means there is 250 ml of water left in the bottle to the nearest 50 ml.

2. Sara weighs her case using a set of luggage scales. She has a weight limit of 21 kg. How much more can she pack to the nearest 100 grams?

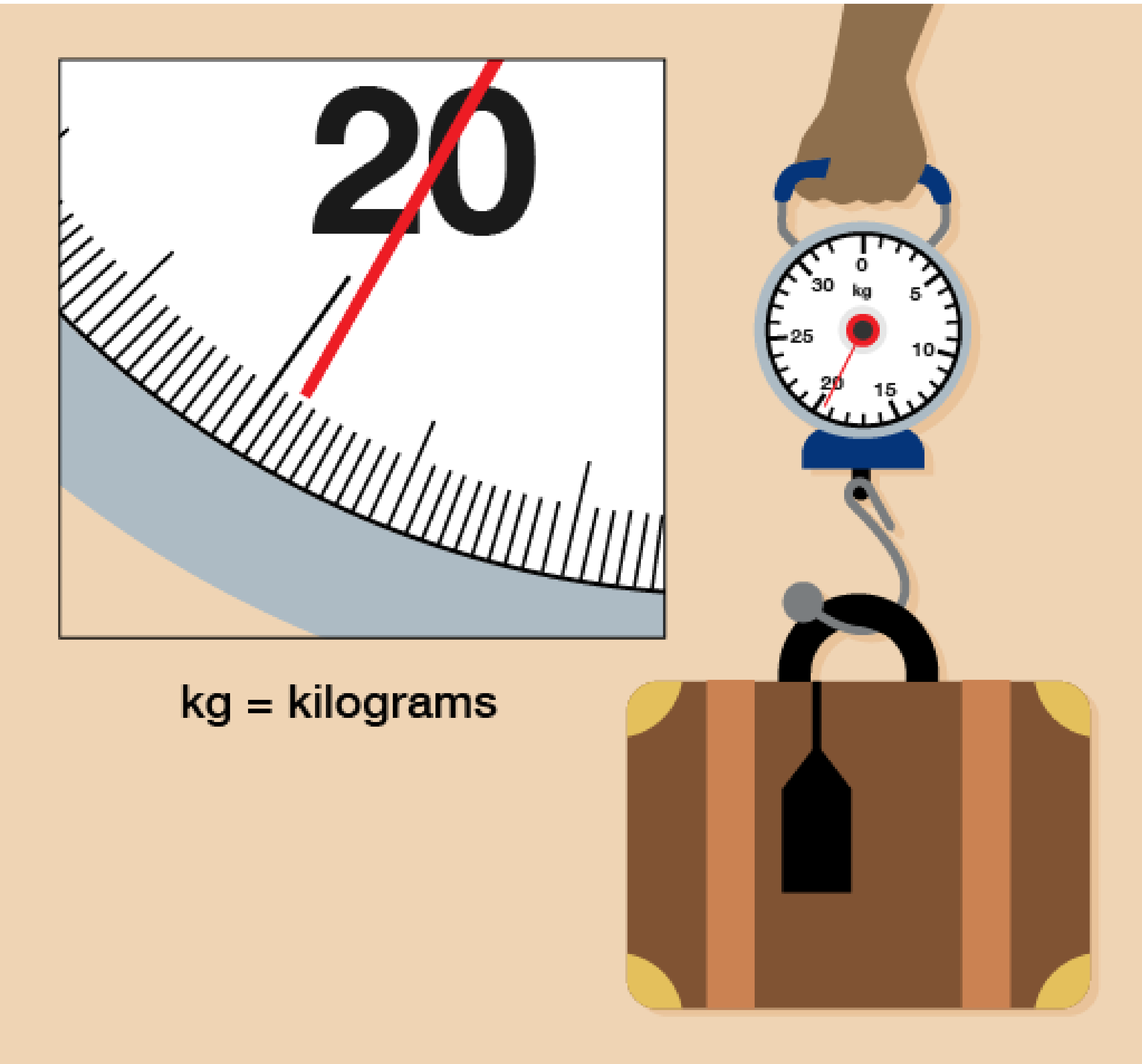


Figure 30 Luggage scales weighing luggage

Answer

2. To answer this question you need to remember that 1 kg = 1000 g.

   The scale is numbered at every 1 kg interval and there are 10 steps between each numbered interval, so each step marks 0.1 kg (1 ÷ 10 = 0.1). You could also think of each marker being 100 g (0.1 kg = 100 g).

   The arrow is almost at 19.8 kg (19 800 g). If Sara has a weight limit of 21 kg then:

   21 kg − 19.8 kg = 1.2 kg (21 000 g − 19 800 g = 1200 g)

   Sara can pack another 1.2 kg (or 1200 g) worth of luggage.

3. Simon needs to weigh out 4 kg of potatoes. Looking at the reading on the scale, how many more grams of potatoes does he need to add to make 4 kg?

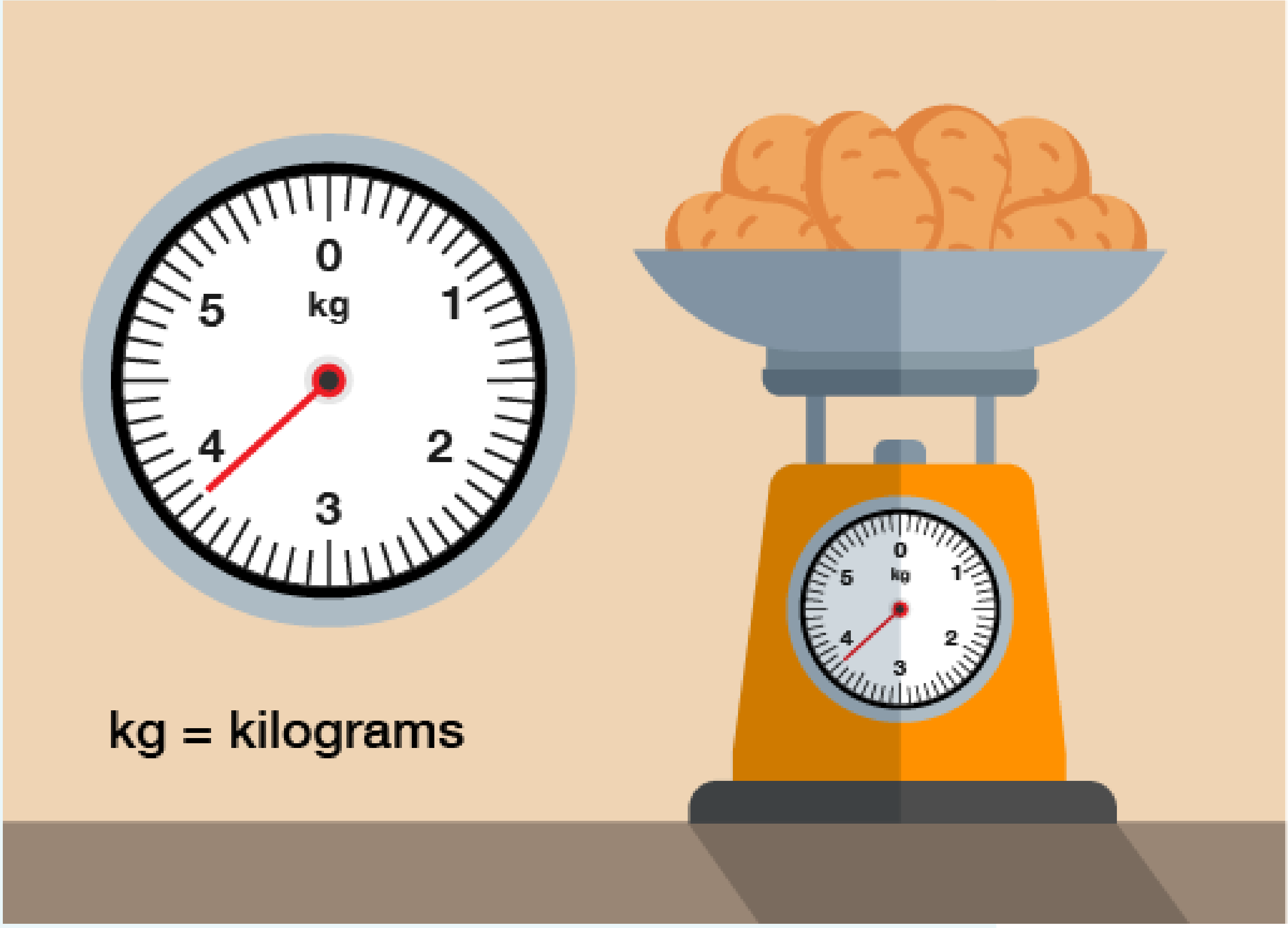


Figure 31 Food scales weighing potatoes

Answer

3. As with Question 2, you need to remember that 1 kg = 1000 g.

   The scale is numbered at every 1 kg interval and there are 10 steps between each numbered marker so each step marks 0.1 kg (1 ÷ 10 = 0.1). You could also think of each step being 100 g (0.1 kg = 100 g).

   The arrow is pointing to 3.8 kg (or 3800 g).

   If Simon needs 4 kg (4000 g) of potatoes then he needs to weigh out another 200 g.

Hopefully you will be feeling confident at reading scales on measuring devices now which leads you nicely onto the next section which looks at conversion scales.

5.3 Using conversion scales

Earlier on in the session you looked at converting between units of measure in different systems by carrying out calculations.

Many measuring instruments (e.g. thermometers, rulers, measuring jugs) have scales which show two or more different units of measure. This means that there may be times where you can compare the scales on the measuring instrument to make a conversion rather than carry out a calculation.

Look at the following example.

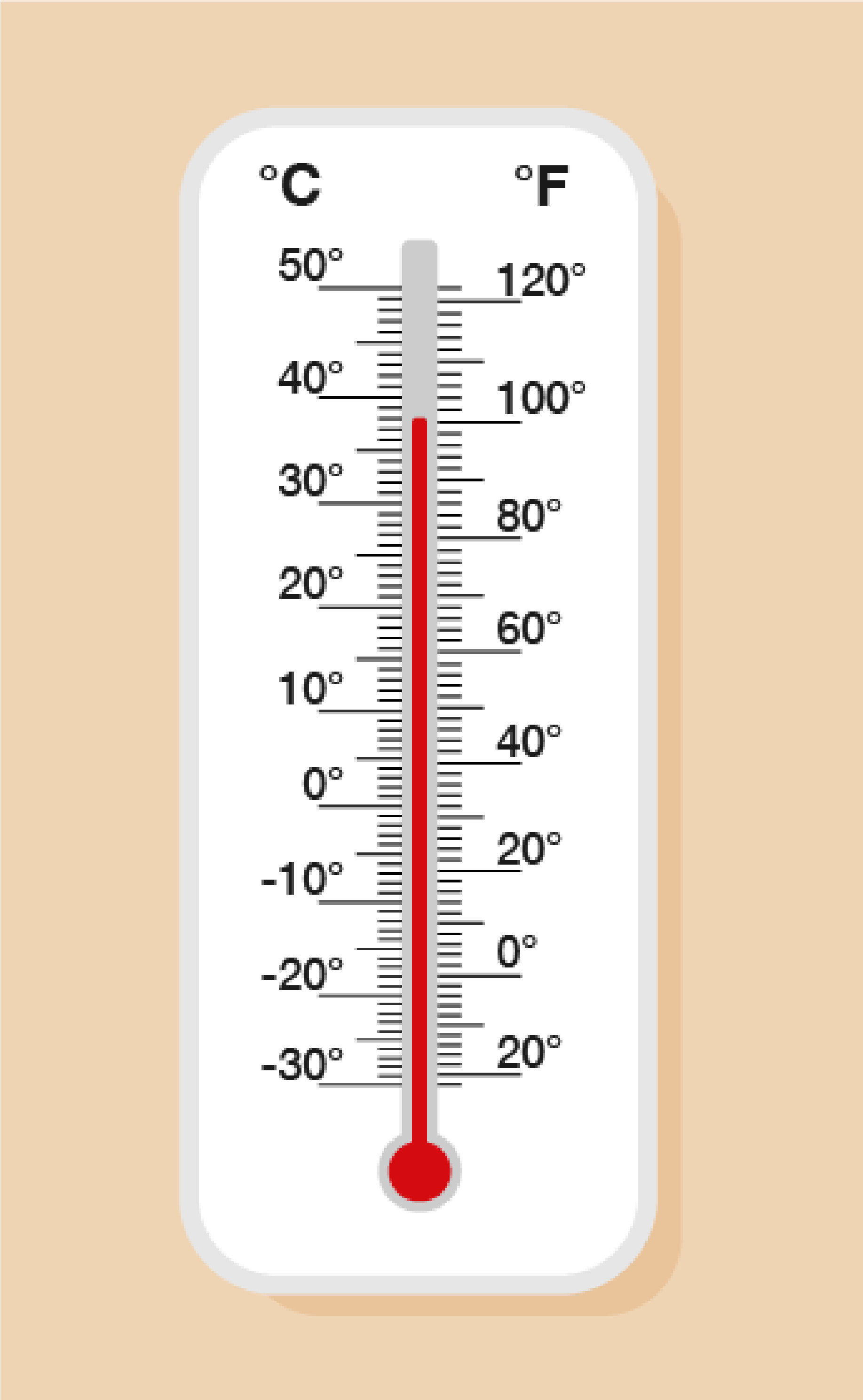


Figure 32 Example – Reading a thermometer

The thermometer above has a scale down the left-hand side which shows degrees Celsius (°C) and a scale on the right-hand side which shows degrees Fahrenheit (°F). This means that you can take a reading on this thermometer in both units of measure, depending on which is needed or which you are more familiar with. It can also help you to look up conversions between units.

You need to be careful with each scale, though – as they are showing different units, they are marked differently and go up in different steps.

On this thermometer, the degrees Celsius scale is going up in steps of 1°C, so the temperature shown is 38°C. If you want to take the reading in degrees Fahrenheit, you can see that it is 100°F (the scale is going up in steps of 2°F). It can be difficult to get a precise comparison between units, but using this thermometer, we can say that 38°C is roughly 100°F.

Now have a go at the following activity.

Activity 12: Using conversion scales

Look at the weighing scales below and answer the questions that follow.

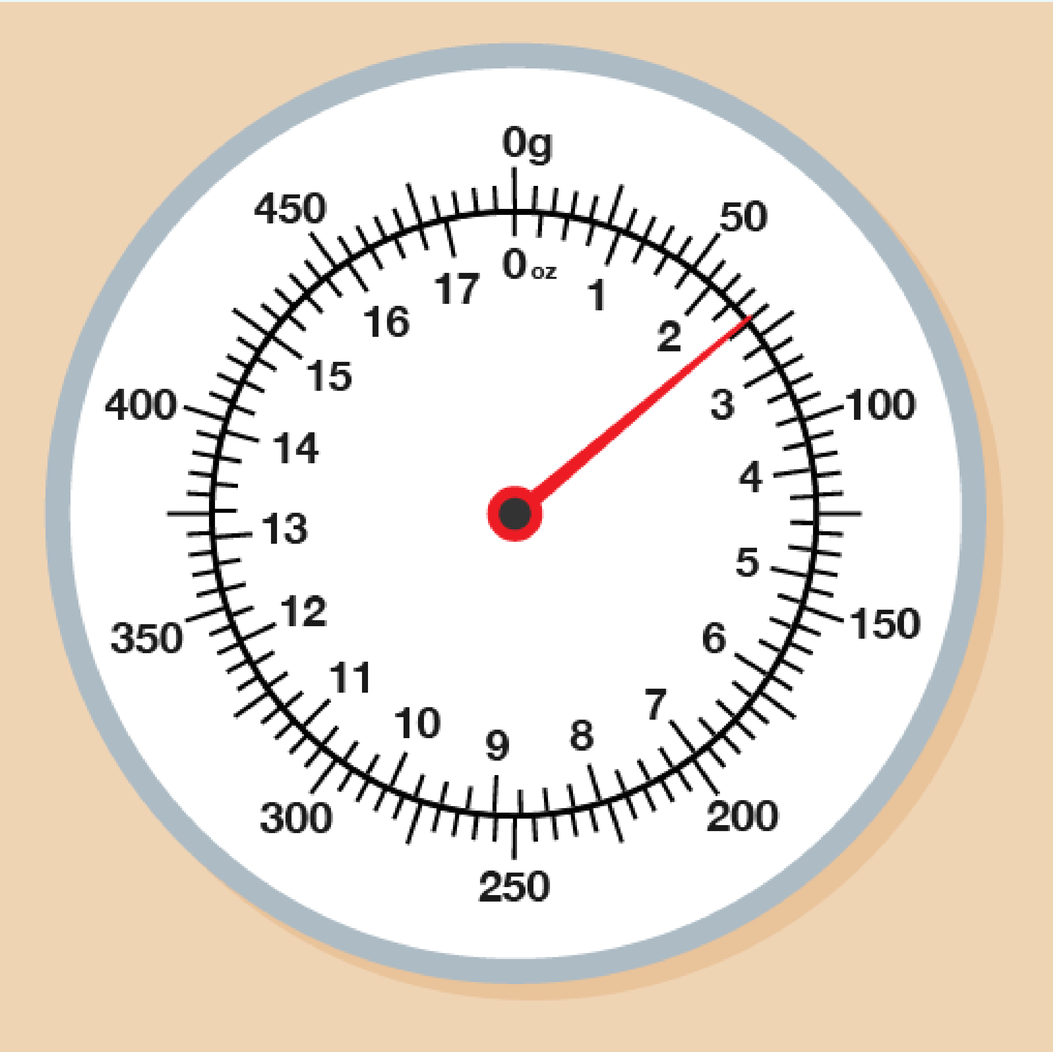


Figure 33 Weighing scales showing two units of measure

1. What is the reading shown by the arrow in grams?
2. How many ounces (oz) is 200 g, to the nearest whole oz?
3. Roughly how many grams is 1 oz, to the nearest 10 g?

4. I see a recipe which states that I need 6 oz of flour. Roughly, how many grams of flour is this?

Answer

Grams (g) are shown on the outside of this scale and ounces (oz) are shown on the inside.

1. The arrow is pointing to 70 g (the scale is going up in steps of 5).
2. You need to look on the outside of the scale to find 200 g and then look on the inside to see how many whole ounces it is nearest to.

   The nearest whole ounce is 7 oz.
3. Find 1 oz on the inside of the scale. Now look on the outside to take this reading in grams. The nearest marker is 30 grams (the grams scale goes up in steps of 5) so 1 oz is approximately 30 g.
4. Look on the inside of the scale for 6 oz. Then take the equivalent gram reading from the outside of the scale. 6 oz is approximately 170 g.

You have now learned all you need to know about units of measures! If you feel unsure on any part of this section, feel free to refer back to the examples or activities again to ensure you feel secure in all areas. All that remains of this section is the end of session quiz. Good luck!

Summary

In this section you have learned to read:

- measuring scales using different intervals
- scales on different measuring instruments
- conversion scales.

6 Session 2 quiz

Now it's time to review your learning in the end-of-session quiz.

Session 2 quiz.

Open the quiz in a new window or tab (by holding ctrl [or cmd on a Mac] when you click the link), then return here when you have done it.

7 Session 2 summary

You have now completed Session 2, 'Units of measure'. If you have identified any areas that you need to work on, please ensure you refer to this section of the course and retry the activities.

You should now be able to:

- understand that there are different units used for measuring and how to choose the appropriate unit
- convert between measurements in the same system (e.g. grams and kilograms) and those in different systems (e.g. litres and gallons)
- use exchange rates to convert currencies
- work with time and timetables
- work out the average speed of a journey using a formula
- convert temperature measurements between Celsius (°C) and Fahrenheit (°F)
- read scales on measuring equipment.

All of the skills listed above will help you with tasks in everyday life, such as measuring for new furniture or redesigning a room or garden. These are essential skills that will help you progress through your employment and education.

You are now ready to move on to Session 3: Shape and space.

Session 3: Shape and space

Introduction

Take a look at the picture below. In order to decorate the room, you would need to know the length of skirting required (perimeter), how much carpet to order and how many tins of paint to buy (area). You would also need to use your rounding skills (as items such as paint and skirting must be bought in full units) and your addition and multiplication knowledge – to work out the cost!

Figure 1 Floor plan of a house

This session of the course will draw upon the skills you learned in the ‘Working with numbers’ and ‘Units of measure’ sessions.

Throughout this session you will learn how to find the perimeter, area and volume of simple and more complex shapes – if you’ve ever decorated a room you will be familiar with these skills already. It’s important to be able to work out area and perimeter accurately to ensure you buy enough of each material (but not too much to avoid wastage).

Once your room is beautifully decorated, you’re going to need to plan the best layout for your furniture (and make sure it fits!). You may well use a scale drawing to achieve this.

By the end of this session you will be able to:

- understand the difference between perimeter, area and volume and be able to calculate these for both simple and more complex shapes
- know that volume is a measure of space inside a 3D object and calculate volumes of shapes in order to solve practical problems
- draw and use a scale drawing or plan.

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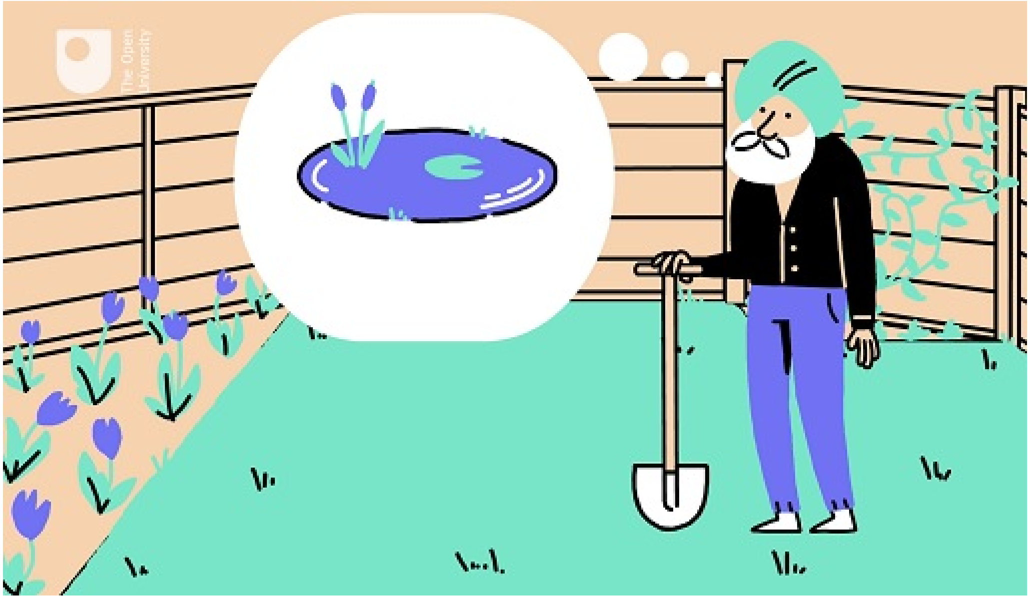

1 Perimeter

As you will see from the picture below, perimeter is simply the distance around the outside of a shape or space. The shape you need to find the perimeter of could be anything from working out how much fencing you need to go around the outside of your garden to how much ribbon you need to go around the outside of a cake. Shapes or spaces with straight edges (rather than curved) are easiest to calculate so let's begin by looking at these.

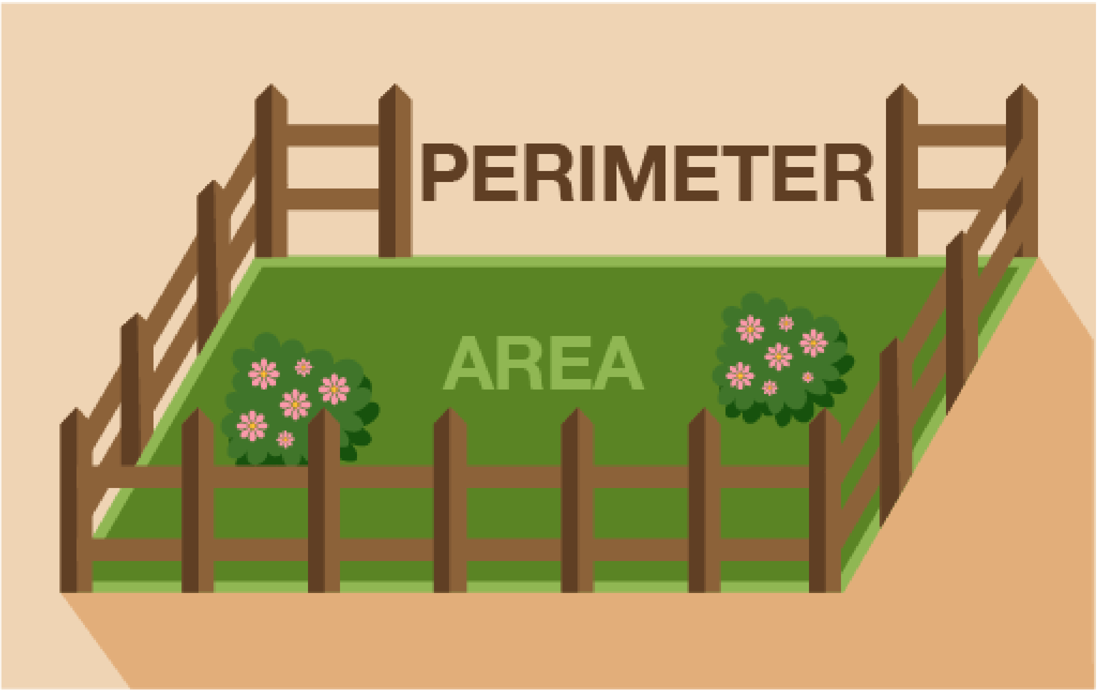

Figure 2 A perimeter fence

1.1 Perimeter of simple shapes

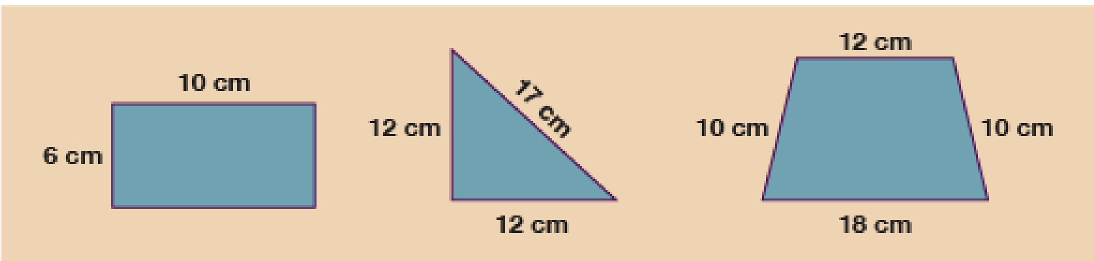

Figure 3 Finding the perimeter of simple shapes

In order to work out the perimeter of the shapes above, all you need to do is simply add up the total length of each of the sides.

- Rectangle: 10 + 10 + 6 + 6 = 32 cm.
- Triangle: 12 + 12 + 17 = 41 cm.
- Trapezium: 10 + 12 + 10 + 18 = 50 cm.

When you give your answer, make sure you write the units in cm, m, km etc. One other important thing to note is that before you work out the perimeter of any shape, you must make sure all the measurements are given in the same units. If, for example, two lengths are given in cm and one is given in mm, you must convert them all to the same unit before you work out the total. It doesn't usually matter which measurement you choose to convert but it's wise to check the question first because sometimes you might be asked to give your answer in a specific unit.

Activity 1: Finding the perimeter

Work out the perimeters of each of the shapes below.

Remember to give units in your answer and to check that all measurements are in the same units before you begin to add. Carry out your calculations without using a calculator. Remember to check your answers against ours.

1.

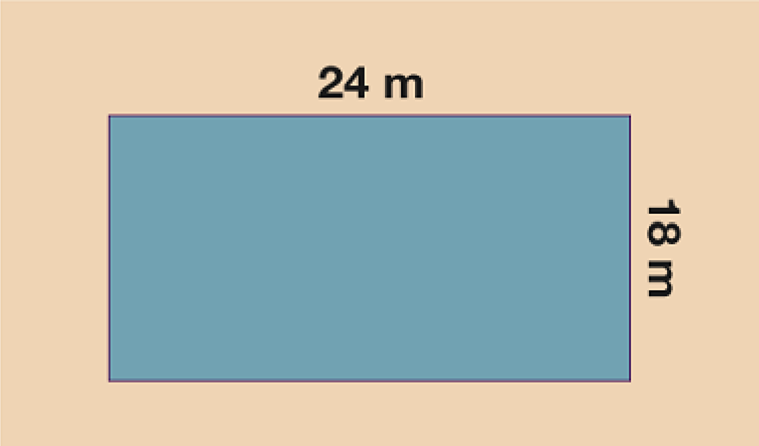

Figure 4 Calculating the perimeter – Question 1

2.

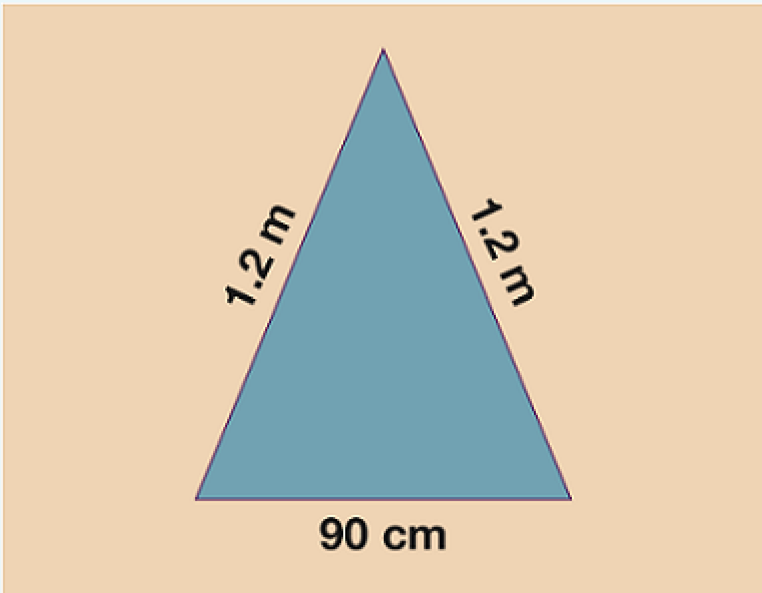

Figure 5 Calculating the perimeter – Question 2

3.

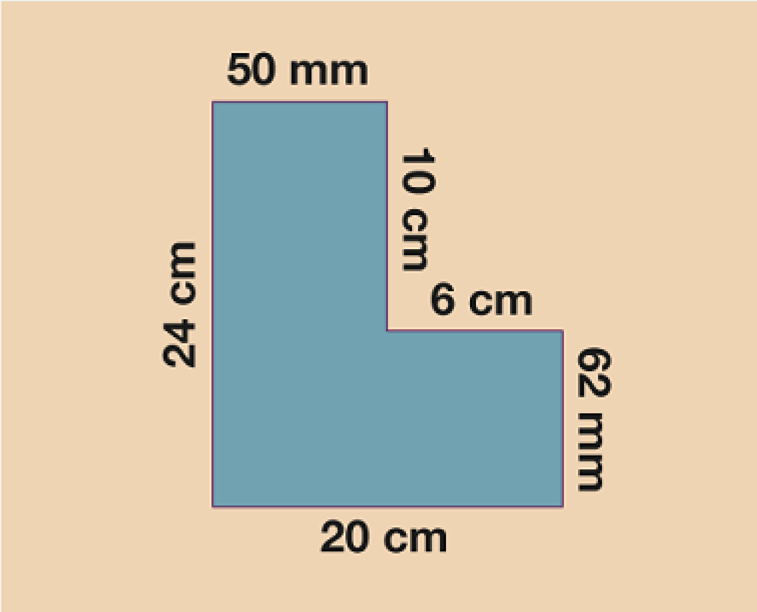

Figure 6 Calculating the perimeter – Question 3

Answer

1. 24 + 18 + 24 + 18 = 84 m.
2. If you worked in cm:

    1.2 m = 120 cm.

    Therefore, the perimeter is 120 + 120 + 90 = 330 cm.

   If you worked in m:

    90 cm = 0.9 m.

    Therefore, the perimeter is 0.9 + 1.2 + 1.2 = 3.3 m.
3. 50 mm = 5 cm, 62 mm = 6.2 cm.

   Therefore, the perimeter is 24 + 5 + 10 + 6 + 6.2 + 20 = 71.2 cm.

Hopefully, you found working out the perimeters of these shapes fairly straightforward.

The next step on from shapes like the ones you have just worked with, is finding the perimeter of shapes where not all the lengths are given to you. In shapes such as a rectangle or regular shapes like squares (where all sides are the same length) this is a simple process. However, in a shape where the sides are not the same as each other, you have a little more work to do.

1.2 Perimeters of shapes with missing lengths

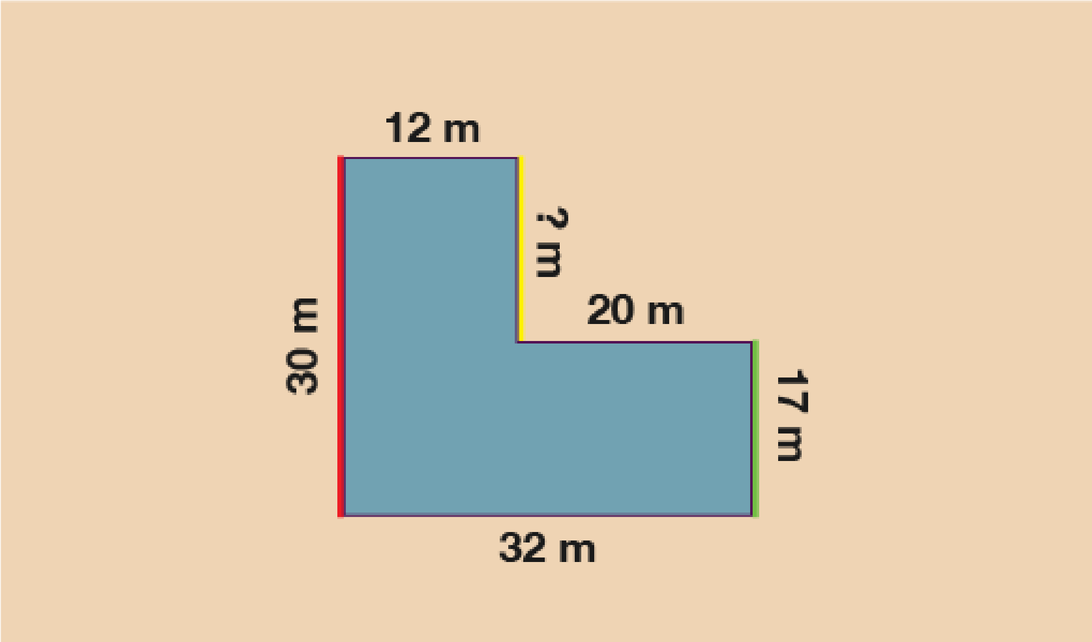

Figure 7 Finding the perimeter when measurements are missing

Look at the shape above. You can see that one of the lengths is missing from the shape. How do you find the perimeter when you don't have all the measurements? You cannot just assume that the missing length (yellow) is half of the red length, so how do you work it out? You'll need to use the information that is given in the rest of the shape.

If you look at all the vertical lengths (red, yellow and green) you can see that we know the length of two of the three. You can also see that green + yellow = red, since the two shorter lengths put together would equal the same as the longest vertical length.

If 17 + ? = 30, then in order to find the missing length you must do 30 − 17 = 13.

The missing length is therefore 13 m. Now that you know this, you can work out the perimeter of the shape in the normal way.

> 30 + 12 + 13 + 20 + 17 + 32 = 124
> So the perimeter of this shape is 124 m.

Let's look at one more example before you try some on your own.

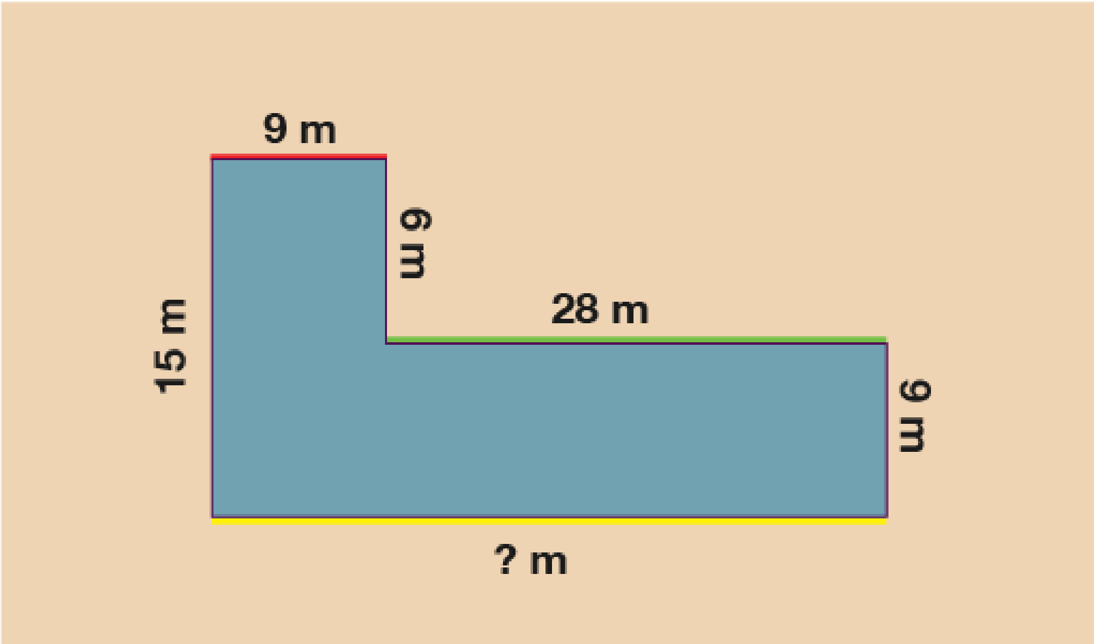

Figure 8 Finding the perimeter when a length is missing

In the example above, you will see that there is again a missing length. This time the missing length (yellow) is a horizontal one and so you need to look at the other two horizontal lengths (red and green) in order to work out the missing side.

You can see that this time that red + green = yellow, since the missing length is the sum of the two shorter lengths. You can now do 9 + 28 = 37, so the missing length is 37 m.

Now that you know all the side lengths, you can work out the perimeter by finding the total of all the lengths.

> 15 + 9 + 6 + 28 + 9 + 37 = 104
>
> So the perimeter of this shape is 104 m.

Activity 2: Perimeters and missing lengths

1. Work out the perimeter of the shape below.

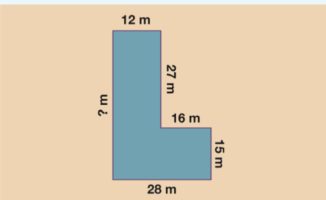

Figure 9 Perimeters and missing lengths – Question 1

2. You are redecorating your living room and need to replace the skirting board. The layout of the room is shown below. Skirting board can only be bought in lengths of 2 m.

   How many lengths should you buy?

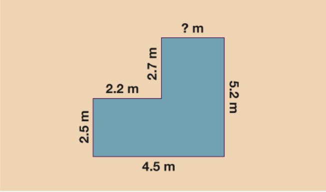

Figure 10 Perimeters and missing lengths – Question 2

Answer

1. Firstly, you need to work out the missing length:

   15 + 27 = 42 m

   So the perimeter is:

   42 + 12 + 27 + 16 + 15 + 28 = 140 m
2. Again, work out the missing length first:

   4.5 − 2.2 = 2.3 m

   Next, work out the perimeter of the room:

   2.5 + 2.2 + 2.7 + 2.3 + 5.2 + 4.5 = 19.4 m

   To see how many 2 m length of skirting you will need do:

   19.4 ÷ 2 = 9.7

   Since we can only buy whole lengths we need to round this up to 10 lengths of skirting.

Good work! You can now work out perimeters of simple and complex shapes, including shapes where there are missing lengths. There is just one more shape you need to consider – circles.

Whether it's ribbon around a cake or fencing around a pond, it's useful to be able to work out how much of a material you will need to go around the edge of a circular shape. The final part of your work on perimeter focusses on finding the distance around the outside of a circle.

1.3 Circumference of a circle

You may have noticed that a new term has slipped in to the title of this section. The term circumference refers to the distance around the outside of a circle – its perimeter. The perimeter and the circumference of a circle mean exactly the same, it's just that when referring to circles you would normally use the term circumference rather than perimeter.

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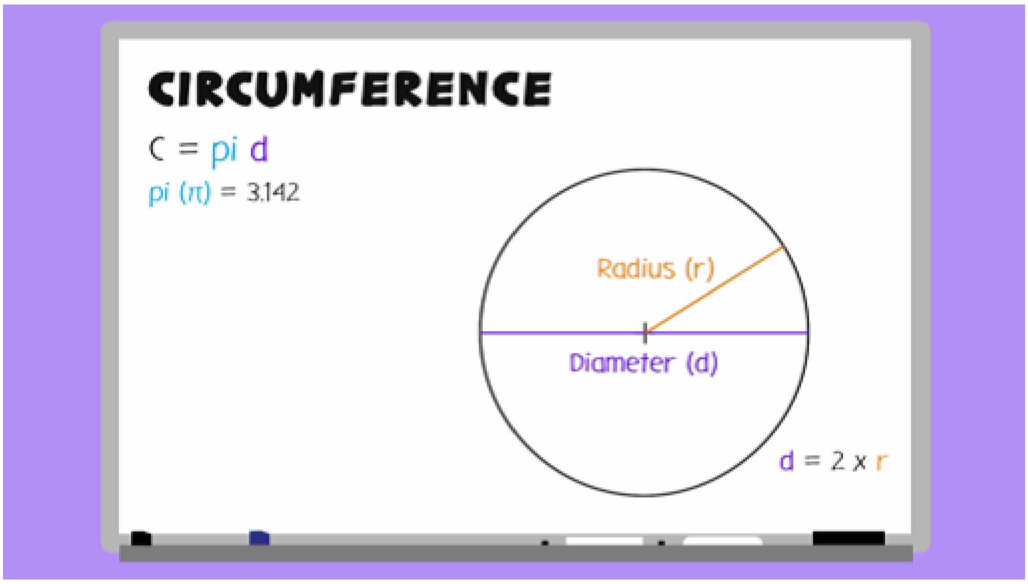


Activity 3: Finding the circumference

1. You have made a cake and want to decorate it with a ribbon.

   The diameter of the cake is 15 cm. You have a length of ribbon that is 0.5 m long. Will you have enough ribbon to go around the outside of the cake?

Figure 11 A round chocolate cake

2. You have recently put a pond in your garden and are thinking about putting a fence around it for safety. The radius of the pond is 7.4 m.

   What length of fencing would you require to fit around the full length of the pond? Round your answer up to the next full metre.

Figure 12 A round garden pond

Answer

1. d = 15 cm

   Using the formula $C = \pi d$

   $C = 3.142 \times 15$

$C = 47.13$ cm

Since you need 47.13 cm and have ribbon that is 0.5 m (50 cm) long, yes, you have enough ribbon to go around the cake.

2. Radius = 7.4 m so the diameter is $7.4 \times 2 = 14.8$ m

   Using the formula $C = \pi d$

   $C = 3.142 \times 14.8$

   $C = 46.5016$ m which is 47 m to the next full metre.

You should now be feeling confident with finding the perimeter of all types of shapes, including circles. By completing Activity 3, you have also re-capped on using formulas and rounding.

The next part of this section looks at finding the area (space inside) a shape or space. As mentioned previously, this is incredibly useful in everyday situations such as working out how much carpet or turf to buy, how many rolls of wallpaper you need or how many tins of paint you need to give the wall two coats.

Summary

In this section you have learned:

- that perimeter is the distance around the outside of a space or shape
- how to find the perimeter of simple and more complex shapes
- how to use the formula for finding the circumference of a circle.

2 Area

The area of a shape is the amount of space inside it. This applies to only two dimensional (flat) shapes. If we are dealing with a three dimensional shape, the space inside this is called the volume. Since perimeter is the measure of a length or distance, the units it is measured in are cm, m, km etc.

As area is a measure of space rather than length or distance, it is measured in square units. This could be square metres, square centimetres, square feet and so on. You will often see these written as cm^2, m^2, ft^2, etc. Over the next few pages you will learn how to find the area of simple shapes, compound shapes and circles.

2.1 Area of simple shapes

The simplest shapes to begin with when looking at area are squares and rectangles. If you look at the rectangle in Figure 13 you can see that it's 6 cm long and 3 cm wide. If you count the squares, there are 18 of them. The area of the shape is 18 cm^2.

It is not always possible (or practical) to count the squares in a shape or space but it's a useful illustration to help you understand what area is.

More practically, to find the area (A) of a square or a rectangle, you would multiply the length (l) by the width (w), so the formula would be:

Area = *length* × *width* or:

$A = lw$ (remember the multiplication sign is not normally written in a formula)

In the example below $A = 6 \times 3 = 18$ cm^2.

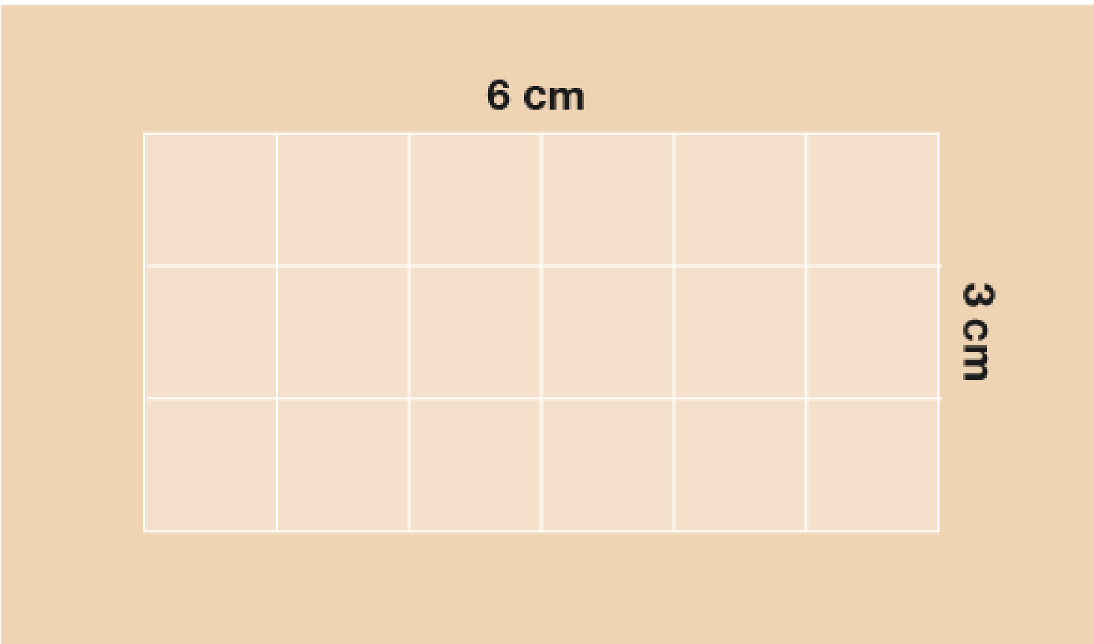


Figure 13 Finding the area of a rectangle

Triangles are another shape that you can find the area of relatively simply. If you think of a triangle, it is really just half of a rectangle. This is easiest to see with a right-angled triangle as shown below. You can see that the actual triangle (in yellow) is simply a rectangle that has been diagonally cut in half.

In order to find the area of the triangle then, you simply multiply the base by the height (as you would for a rectangle) and then halve the answer.

Sometimes this is shown as the formula:

$A = (b \times h) \div 2$

where b is the base of the triangle and h is the vertical height.

This formula may be written as:

$A = \frac{bh}{2}$

For the triangle below then, you would do:

$A = (5 \times 4) \div 2$
$A = 20 \div 2$
$A = 10 \text{ cm}^2$

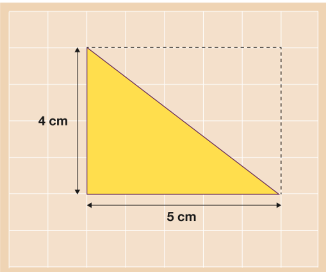


Figure 14 Finding the area of a right-angled triangle

This formula remains the same for any triangle. Take a look at the triangle below. It's a bit less obvious than with the example above but if you were to take the two yellow sections away and put them together, you would end up with a shape exactly the same size as the orange triangle.

The area of this triangle can be found in the same way as the previous one:

$A = (b \times h) \div 2$
$A = (4 \times 7) \div 2$
$A = 28 \div 2$
$A = 14 \text{ cm}^2$

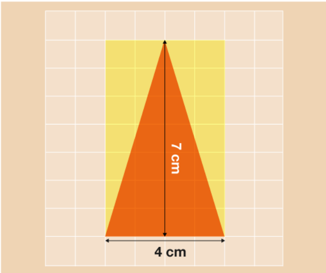


Figure 15 Finding the area of a triangle

Another shape you may need to find the area of is the trapezium. You will need to use a simple formula for this shape (don't panic when you see it, it looks scary but it really is quite easy to use!)

A trapezium looks like any of the shapes below.

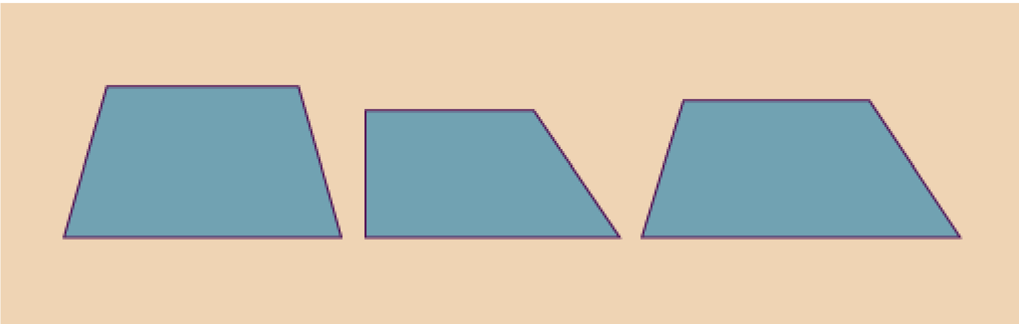


Figure 16 Examples of trapezium shapes

In order to work out the area of a trapezium you just need to know the vertical height and the length of the top and bottom sides. Traditionally, the length of the top is called *'a'*, the length on the bottom is *'b'* and the vertical height is *'h'*. Once this is clear you can then use the formula:

$$A = \frac{(a + b) \times h}{2}$$

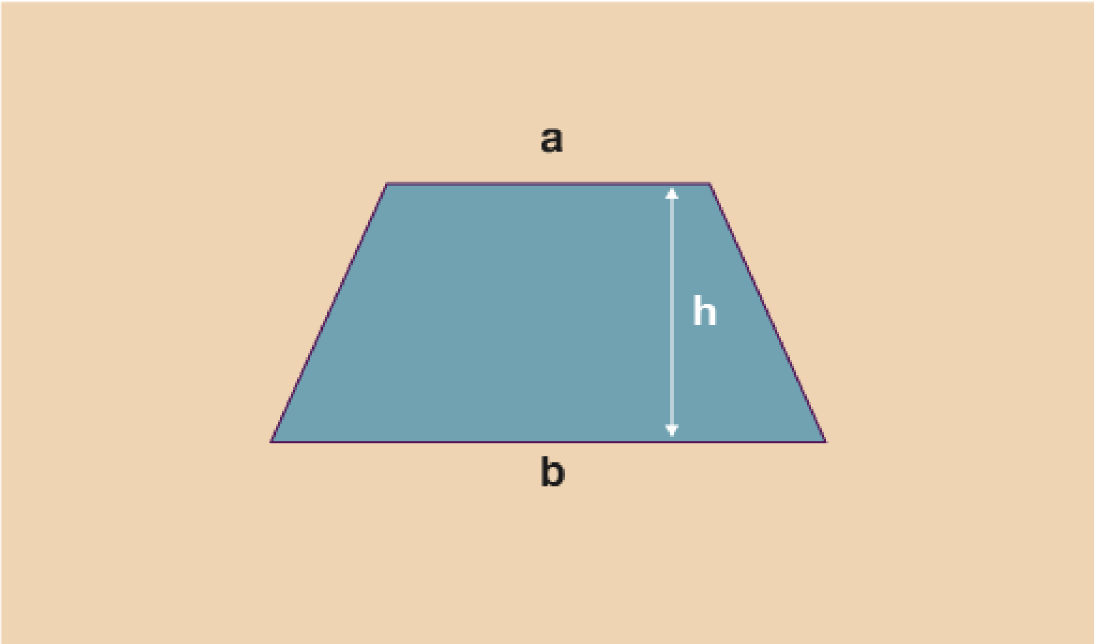


Figure 17 Dimensions of a trapezium

Let's take a look at an example on how to work out the area of the trapezium below. We can see that the top length (*a*) = 12 cm. The bottom length (*b*) = 20 cm, and the height (*h*) = 13 cm.

Using these values and the formula:

$$A = \frac{(a + b)\times h}{2}$$

$$A = \frac{(12 + 20)\times 13}{2}$$

$$A = \frac{416}{2}$$

$A = 208\ \text{cm}^2$

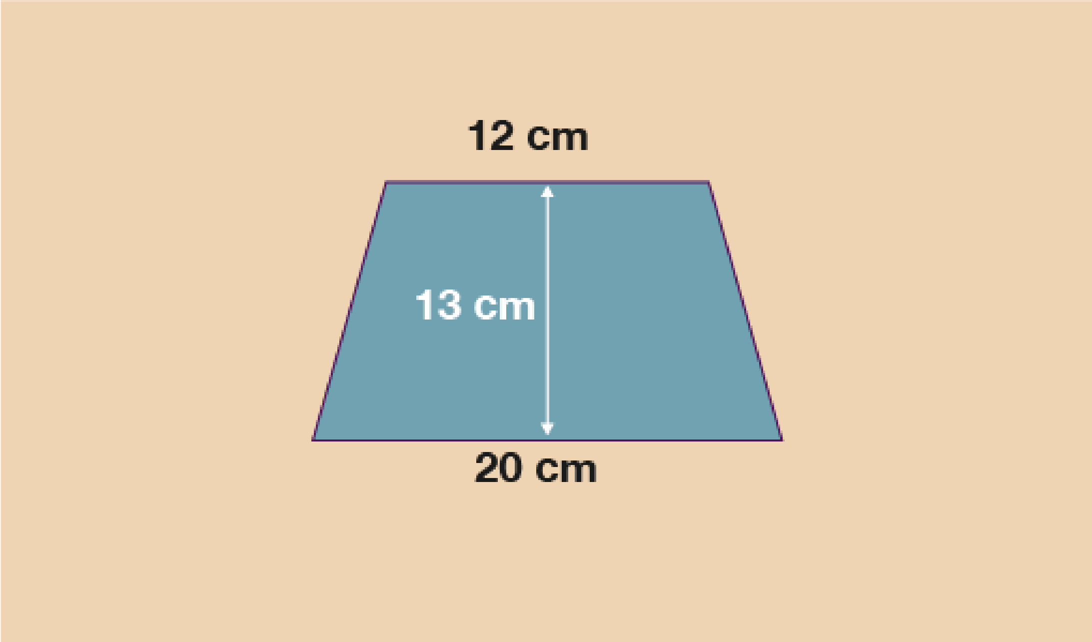


Figure 18 Finding the area of a trapezium

Now that you have seen how to work out the area of several basic shapes, it's time to put your skills to the test. Have a go at the activity below. Don't forget that, just as with perimeter, before you can begin any calculations you must make sure all measurements are in the same units.

Activity 4: Finding the area

Work out the area of each of the shapes below.

1.

Figure 19 Finding the area – Question 1

Answer

1. You need to convert the measurements into the same units before you can work out the area.

   If working in cm:

   1.6 m = 160 cm, so

   A = 160 × 95 = 15 200 cm^2

   If working in m:

   95 cm = 0.95 m, so

   A = 1.6 × 0.95 = 1.52 m^2

2.

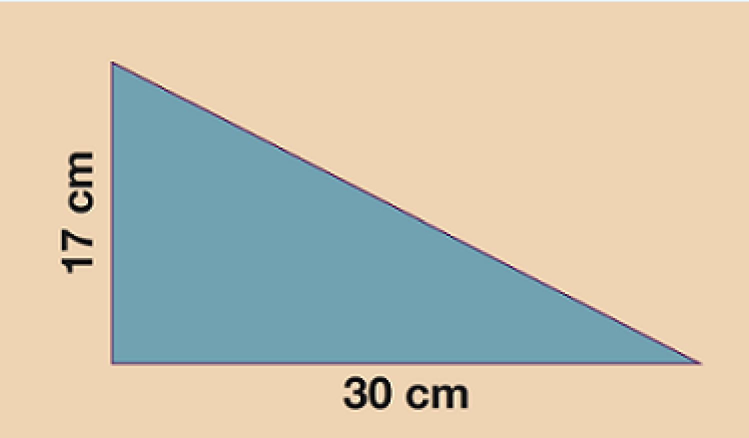


Figure 20 Finding the area – Question 2

Answer

2. The two measurements we need for the triangle are the base (30 cm) and the vertical height (17 cm). Don't be fooled by the diagonal length of 46 cm, you do not need it for the area!

   $A = (b \times h) \div 2$

   $A = (30 \times 17) \div 2$

   $A = 510 \div 2$

   A = 255 cm^2

3.

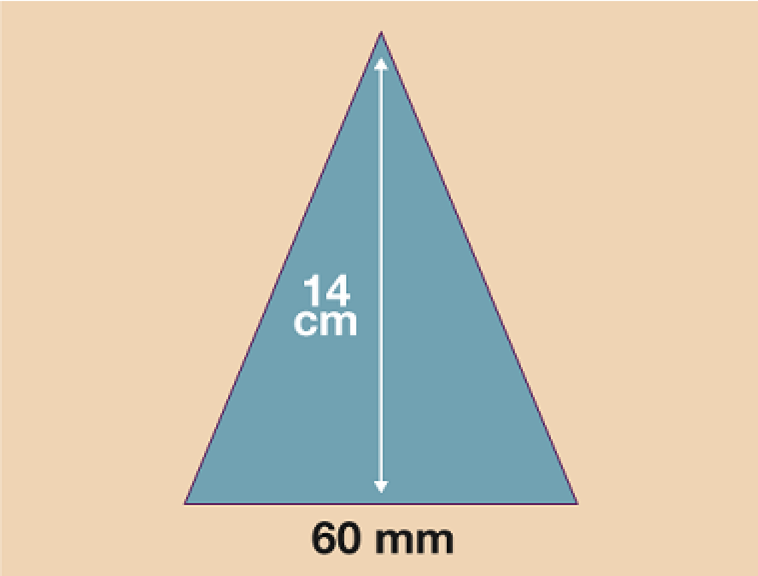


Figure 21 Finding the area – Question 3

Answer

3. You need to convert the measurements into the same units before you can work out the area.

   If working in mm: 14 cm = 140 mm

   $A = (b \times h) \div 2$
   $A = (60 \times 140) \div 2$
   $A = 8400 \div 2$
   $A = 4200\ \text{mm}^2$

   If working in cm: 60 mm = 6 cm

   $A = (b \times h) \div 2$
   $A = (6 \times 14) \div 2$
   $A = 84 \div 2$
   $A = 42\ \text{cm}^2$

4.

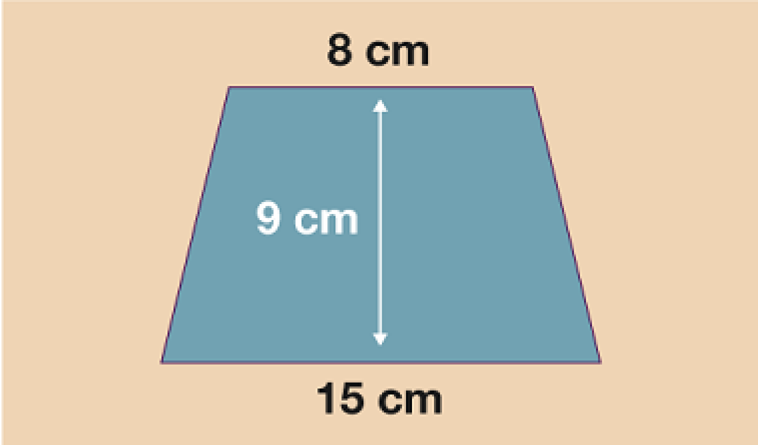


Figure 22 Finding the area – Question 4

Answer

4. Top length (a) = 8 cm

   Bottom length (b) = 15 cm
   Height (h) = 9 cm
   Using the formula for the area of a trapezium:

$$A = \frac{(8+15)\times 9}{2}$$

$$A = \frac{(23)\times 9}{2}$$

$$A = \frac{207}{2}$$

$$A = 103.5\ \text{cm}^2$$

Now that you've mastered finding the area of basic shapes, it's time to look at compound shapes.

A compound shape is just a shape that is made from more than one basic shape. Rarely will you find a floor space, garden area or wall that is perfectly rectangular. More often than not it will be a combination of shapes. The good news is that, to find the area of compound shapes, you simply split them up into their basic shapes, find the area of each of these, and then add them up at the end!

2.2 Area of compound shapes

Take a look at the shape below; this is an example of a compound shape. Whilst you cannot find the area of this shape as it is by using a formula as you have done previously, you can split it into two basic shapes (rectangles) and then use your existing knowledge to work out the area of each of these shapes.

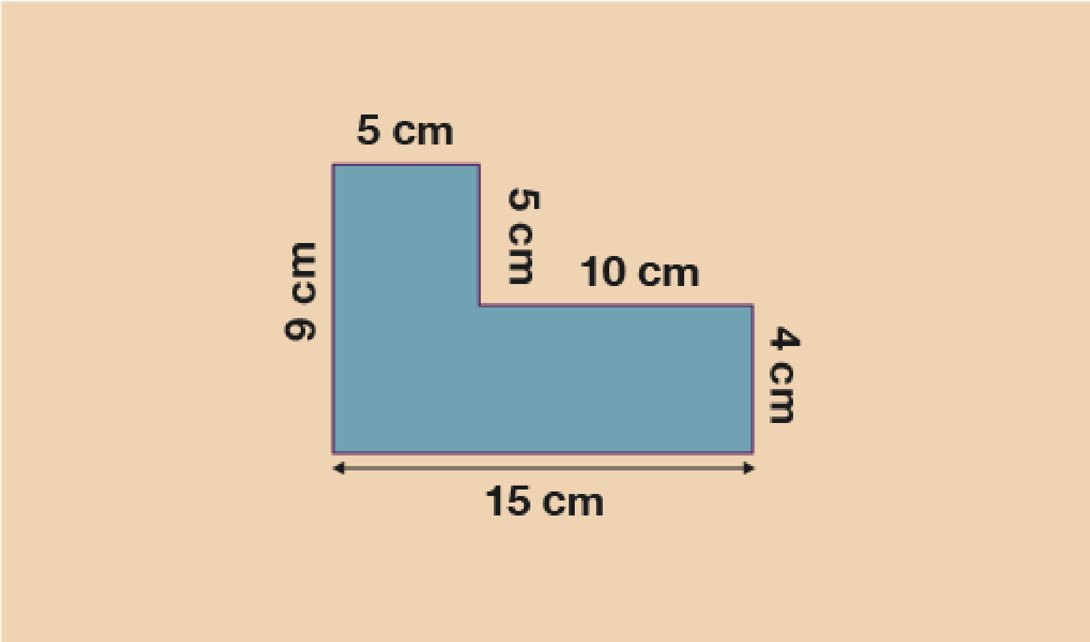


Figure 23 Finding the area of a compound shape

You should be able to see that you can split this shape into two rectangles. It does not matter which way you split it – you will get the same answer at the end.

You could split it like this:

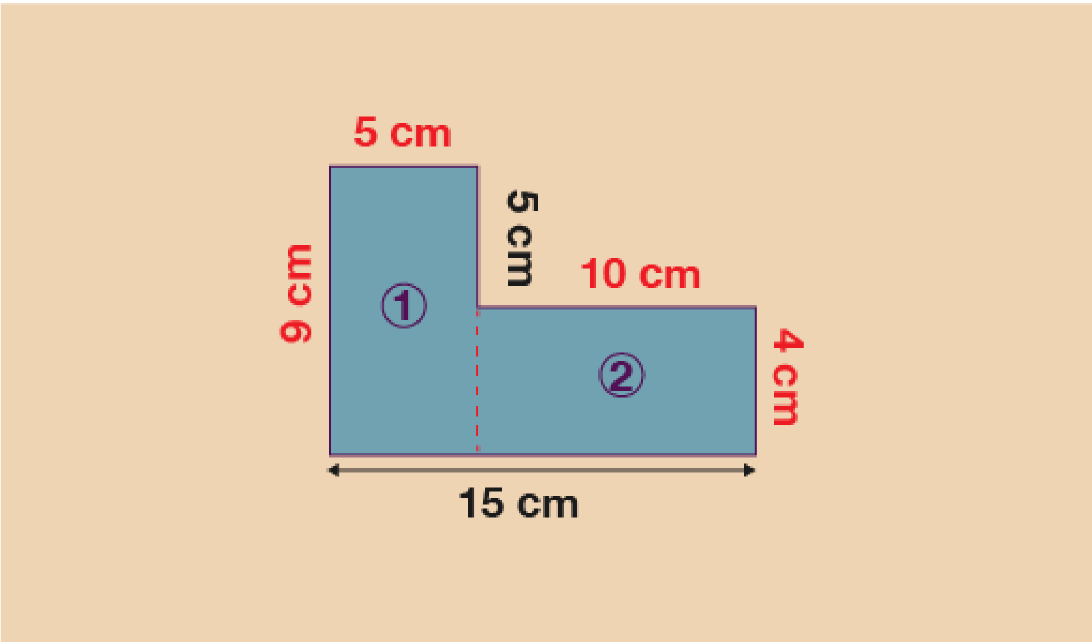


Figure 24 Splitting a compound shape horizontally to find the area

You now have two rectangles. To work out the area of rectangle ①, you do $A = 9 \times 5 = 45$ cm^2.

To work out the area of rectangle ②, you do $A = 10 \times 4 = 40$ cm^2.

Now that you have the area of both rectangles, simply add them together to find the area of the whole shape:

45 + 40 = 85 cm^2

You need to be careful that you are using the correct measurements for the length and width of each rectangle (the measurements in red). In this example, the lengths of 15 cm and 5 cm (in black) are not required.

Alternatively, you could split the shape like this:

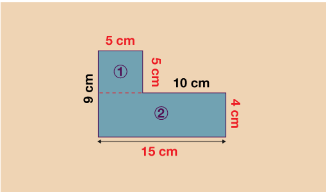


Figure 25 Splitting a compound shape vertically to find the area

Again, you now have two rectangles. To work out the area of rectangle ①, you do A = 5 × 5 = 25 cm^2.

To work out the area of rectangle ②, you do A = 15 × 4 = 60 cm^2.

Now that you have the area of both rectangles, simply add them together to find the area of the whole shape:

25 + 60 = 85 cm^2

Again, you need to be careful that you are using the correct measurements for the length and width of each rectangle (the measurements in red). In this example, the lengths of 9 cm and 10 cm (in black) are not required.

You will notice that regardless of which way you choose to split the shape, you arrive at the same answer of 85 cm^2.

The best way for you to practise this skill is to try a few examples for yourself. Have a go at the activity below and then check your answers.

Activity 5: Finding the area of compound shapes

Work out the area of the shapes below.

1.

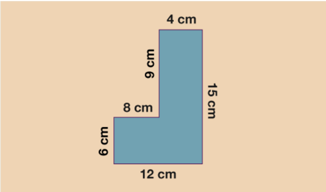


Figure 26 Finding the area of compound shapes – Question 1

Answer

1. The area of the whole shape is 108 cm^2

   Depending on how you split the shape you may have done:

   $6 \times 8 = 48\ cm^2$
   $15 \times 4 = 60\ cm^2$
   $48 + 60 = 108\ cm^2$

   or:

   $12 \times 6 = 72\ cm^2$
   $9 \times 4 = 36\ cm^2$
   $72 + 36 = 108\ cm^2$

2.

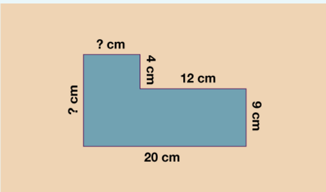


Figure 27 Finding the area of compound shapes – Question 2

Hint: You'll need to find some missing lengths on this shape before you can work out the area.

Answer

2. The missing vertical length is 13 cm (9 cm + 4 cm) and horizontal length is 8 cm (20 cm − 12 cm). The area of the whole shape is 212 cm^2.

   Depending on how you split the shape you may have done:

   $13 \times 8 = 104\ cm^2$
   $12 \times 9 = 108\ cm^2$
   $104 + 108 = 212\ cm^2$

   or:

   $20 \times 9 = 180\ cm^2$
   $4 \times 8 = 32\ cm^2$

$180 + 32 = 212\ \text{cm}^2$

Now that you can calculate the area of basic and compound shapes, there is just one other shape you will practise finding the area of: circles. Similarly to finding the perimeter of a circle, you'll need to use a formula involving the Greek letter π.

2.3 Area of a circle

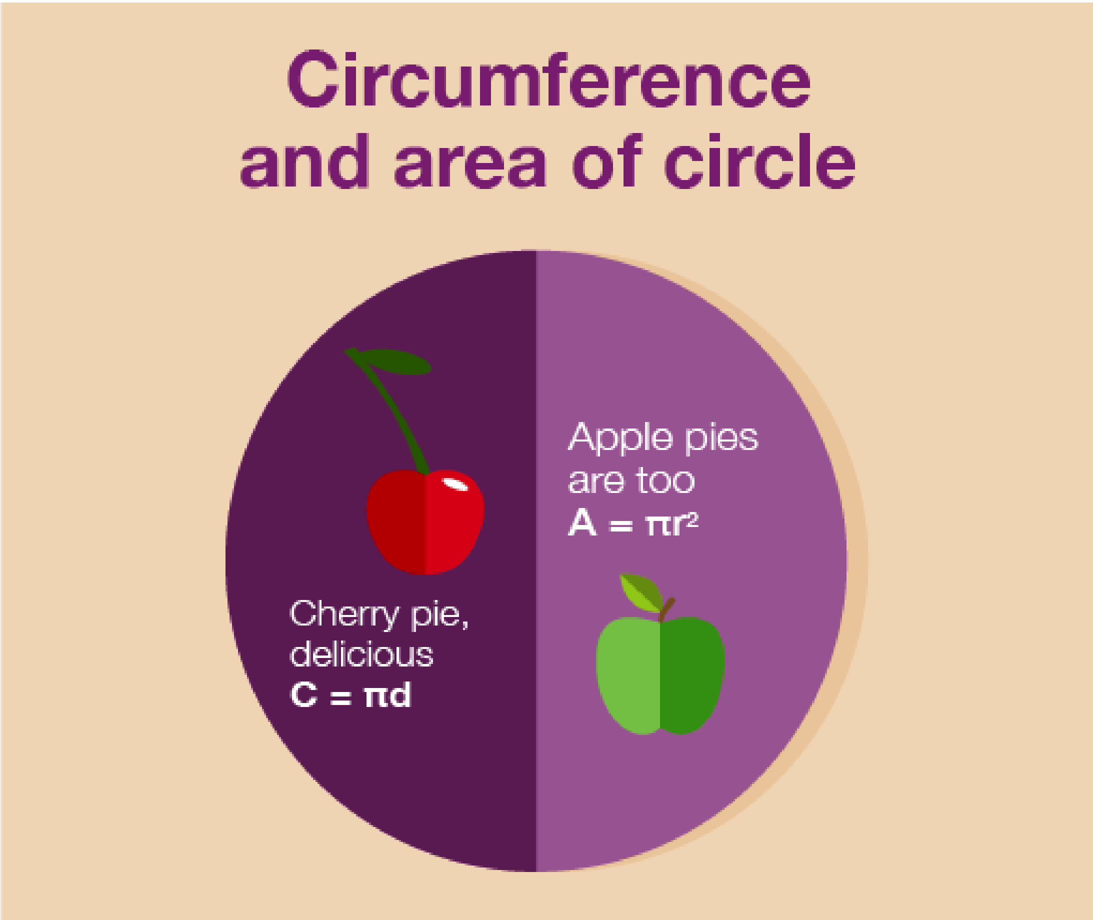


Figure 28 The circumference and area of a circle

You have already practised using the formula to find the circumference of a circle. You will now look at using the formula to find the area of one.

To find the area of a circle you need to use the formula:

Area of a circle = pi × radius^2

This can also be written as:

$A = \pi r^2$

where:

A = area

π = pi

r = radius
r^2!Warning! Cambria Math not supported means r squared.

Hint: Remember that when you square a number you simply multiply it by itself, *radius*^{2} is therefore simply *radius* × *radius*.

Let's look at an example.

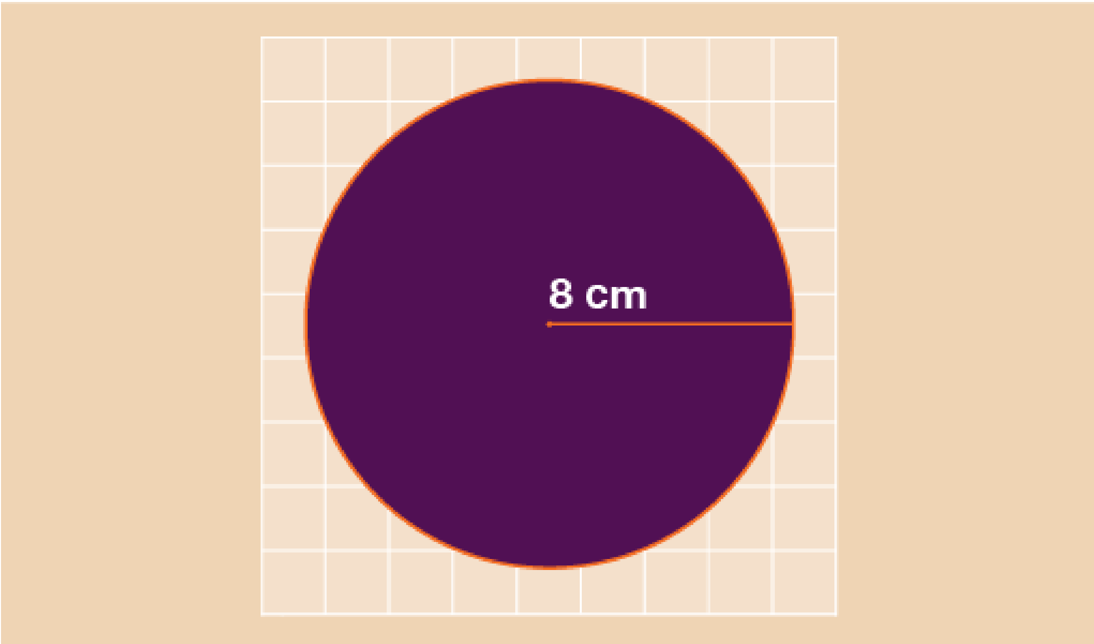


Figure 29 The radius of a circle

In the circle above you can see that the radius is 8 cm. For these tasks we will use the figure of 3.142 for π.

To find the area of the circle we need to do:

$A = \pi r^2$
$A = 3.142 \times 8 \times 8$
$A = 201.088 \text{ cm}^2$

Before you try some on your own, let's take a look at one further example.

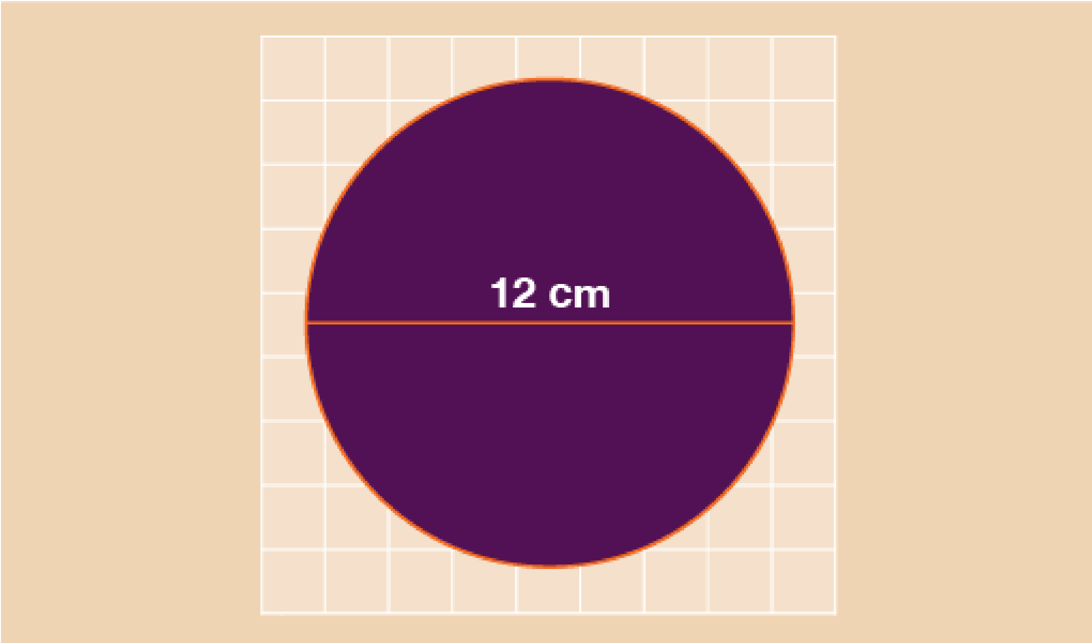


Figure 30 The diameter of a circle

This circle has a diameter of 12 cm. In order to find the area, you first need to find the radius. Remember that the radius is simply half of the diameter and so in this example radius = 12 ÷ 2 = 6 cm.

We can now use:

$A = \pi r^2$
$A = 3.142 \times 6 \times 6$
$A = 113.112\ \text{cm}^2$

Try a couple of examples for yourself before moving on to the next part of this section.

Activity 6: Finding the area of a circle

1. Find the area of the circle shown below. Give your answer to one decimal place.

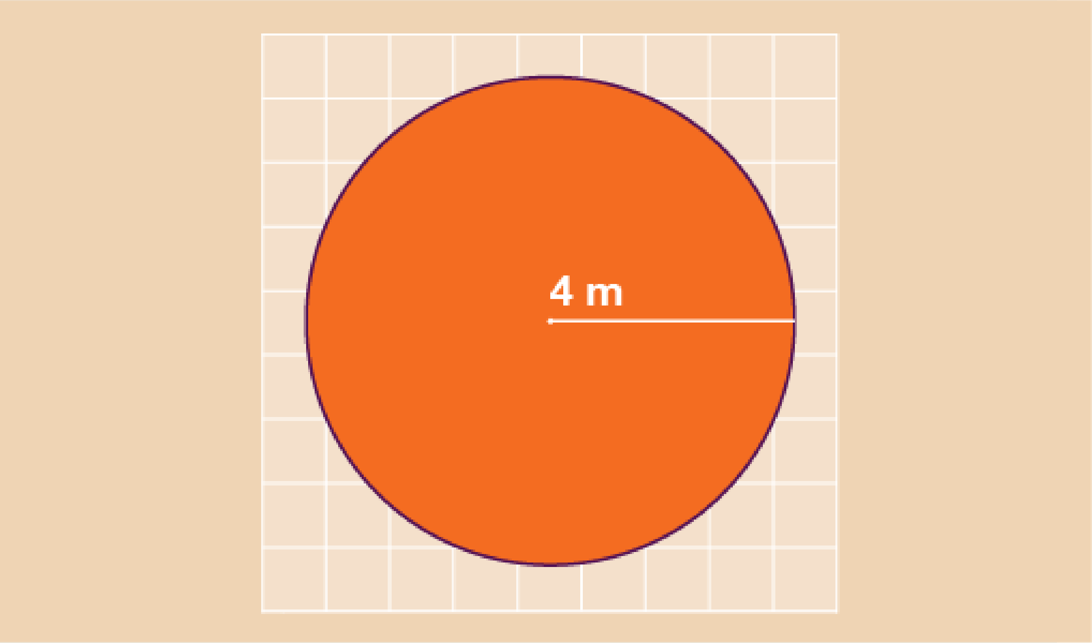


Figure 31 Finding the area of a circle – Question 1

2. Find the area of the circle shown below. Give your answer to 1 decimal place.

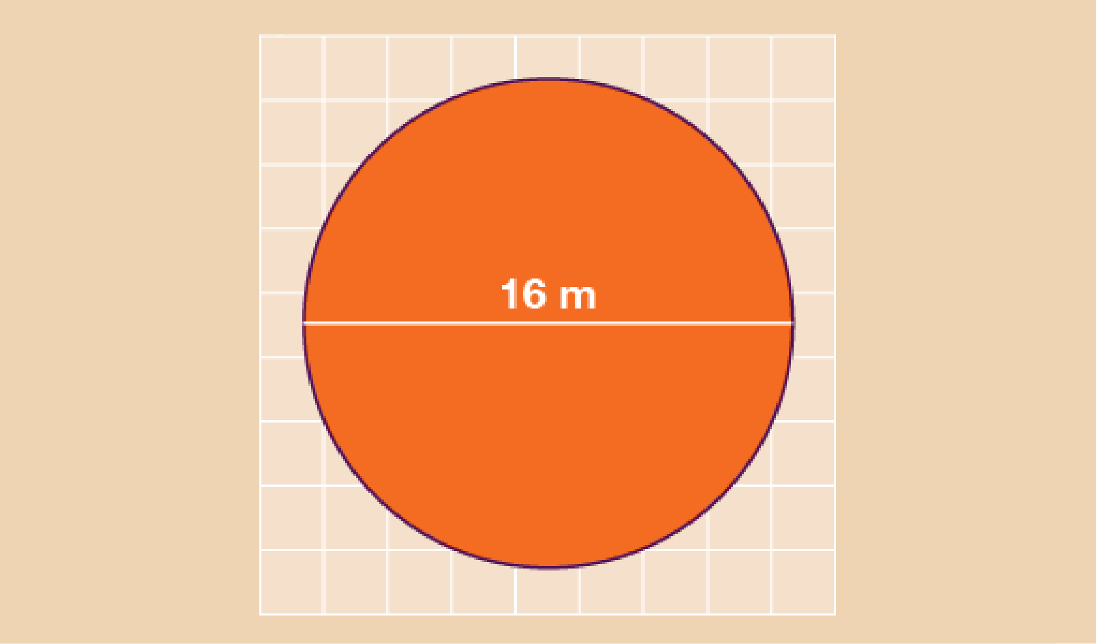


Figure 32 Finding the area of a circle – Question 2

3. You are designing a mural for a local school and need to decide how much paint you need. The main part of the mural is a circle with a diameter of 10 m as shown below. Each tin of paint will cover an area of 5 m^2. You will need to use two coats of paint. How many tins of paint should you buy?

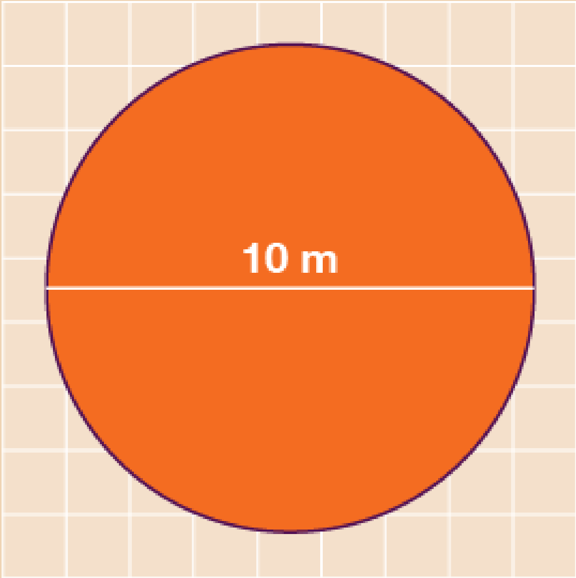


Figure 33 Finding the area of a circle – Question 3

Answer

1. To find the area of the circle you need to do:

   $A = \pi r^2$

   $A = 3.142 \times 4 \times 4$
   $A = 50.272\ m^2$
   $A = 50.3\ m^2$ to 1 d. p.

2. You need to find the radius of the circle first. Since the diameter of the circle is 16 cm, the radius is 16 cm ÷ 2 = 8 cm. Now you can find the area:

   $A = \pi r^2$
   $A = 3.142 \times 8 \times 8$
   $A = 201.088\ cm^2$
   $A = 201.1\ cm^2$ to one d.p.

3. You need to find the area of the circle first. Since the diameter of the circle is 10 m, the radius is 10 m ÷ 2 = 5 m.

   Now you can find the area of the circle:

   $A = \pi r^2$

   $A = 3.142 \times 5 \times 5$
   $A = 78.55\ m^2$

   So the area of the circle you need to paint is 78.55 m^2

   Since you need to give 2 coats of paint, you need to double this number:

   $78.55 \times 2 = 157.1\ m^2$

   You now need to work out how many tins of paint you need. As one tin of paint covers 5 m^2 you need to do:

   $157.1 \div 5 = 31.42$ tins

   Since you must buy whole tins of paint, you will need to buy 32 tins.

You have now learned all you need to know about finding the area of shapes! The last part of this section is on finding the volume of solid shapes – or three-dimensional (3D) shapes.

Summary

In this section you have learned:

- that area is the space inside a two-dimensional (2D) shape or space
- how to find the area of rectangles, triangles, trapeziums and compound shapes
- how to use the formula to find the area of a circle.

3 Volume

The volume of a shape is how much space it takes up. You might need to calculate the volume of a space or shape if, for example, you wanted to know how much soil to buy to fill a planting box or how much concrete you need to complete your patio.

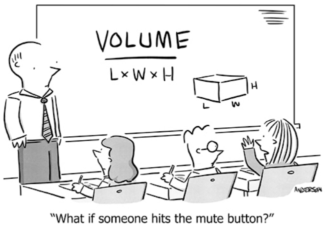
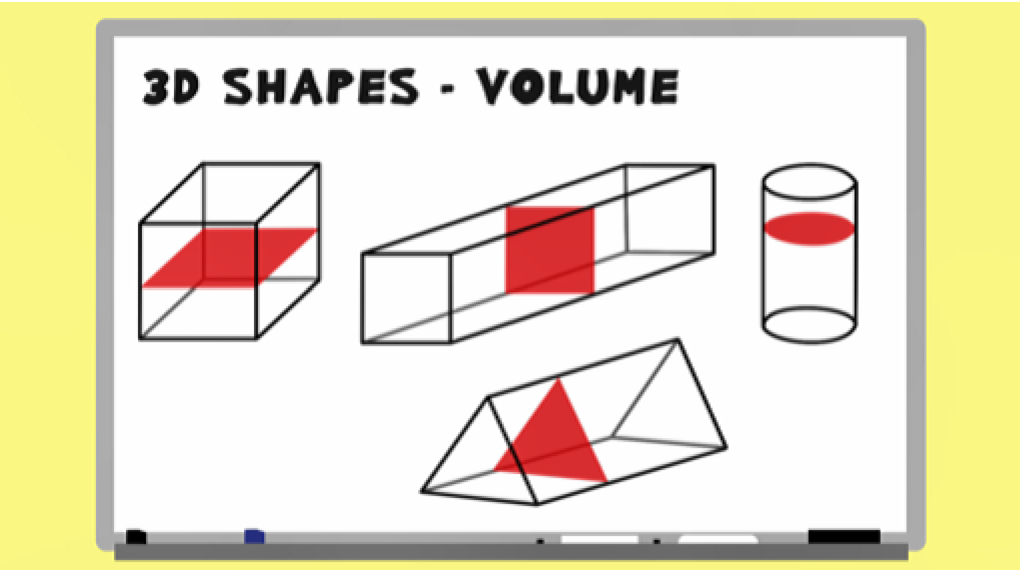


Figure 34 A volume cartoon

You'll need your area skills in order to calculate the volume of a shape. In fact, as you already know how to calculate the area of most shapes, you are one simple step away from being able to find the volume of most shapes too!

Video content is not available in this format.



Example: Calculating volume

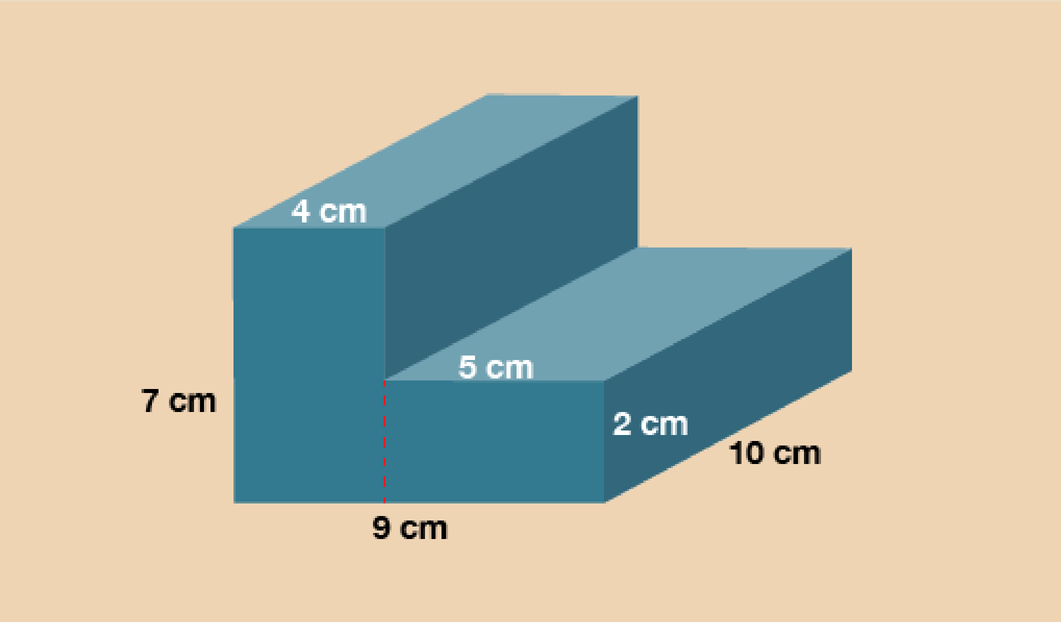


Figure 35 Calculating volume

The cross-section on this shape is the L shape on the front. In order to work out the area you'll need to split it up into two rectangles as you practised in the previous part of this section.

Rectangle 1 = 7 × 4 = 28 cm^2

Rectangle 2 = 5 × 2 = 10 cm^2

Area of cross-section = 28 + 10 = 38 cm^2

Now you have the area of the cross-section, multiply this by the length to calculate the volume.

$V = 38 \times 10 = \underline{380\ cm^3}$

Example: Calculating cylinder volume

The last example to look at is a cylinder. The cross-section of this shape is a circle. You'll need to use the formula to find the area of a circle in the same way you did in the previous part of this section.

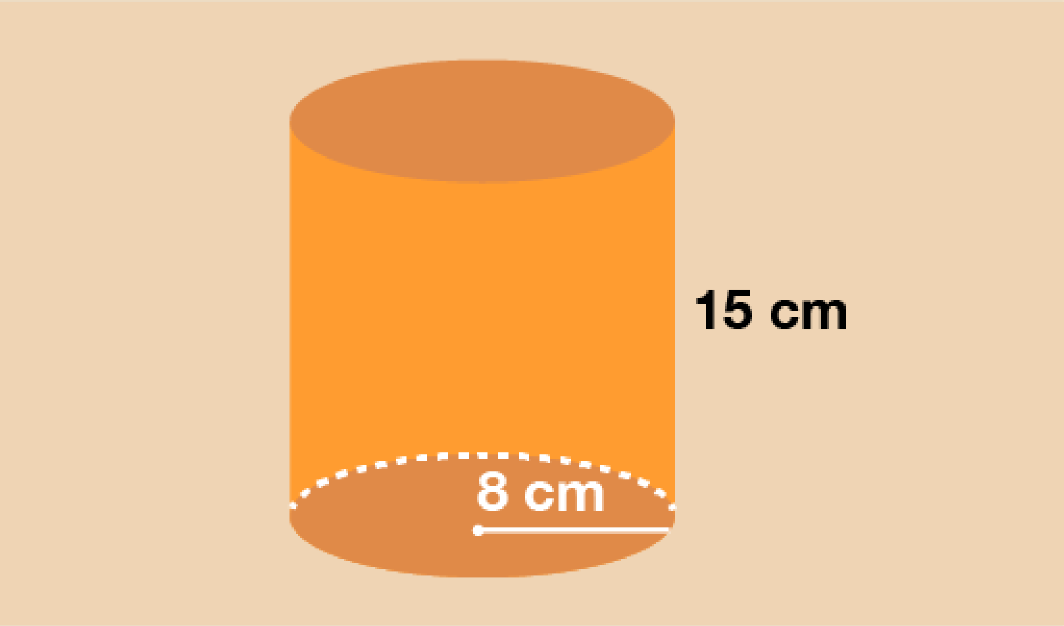


Figure 36 Calculating cylinder volume

You can see that the circular cross-section has a radius of 8 cm. To find the area of this circle, use the formula:

$A = \pi r^2$

$A = 3.142 \times 8 \times 8$

$A = 201.088\ \text{cm}^2$

Now you have the area, multiply this by the length of the cylinder to calculate the volume.

$V = 201.088 \times 15 = \underline{3016.32\ \text{cm}^3}$

Now try the following questions. Please carry out the calculations without a calculator. You can double check with a calculator if needed and remember to check your answers against ours.

Activity 7: Volume

Find the volume of the following shapes without using a calculator. The shapes are not drawn to scale.

1.

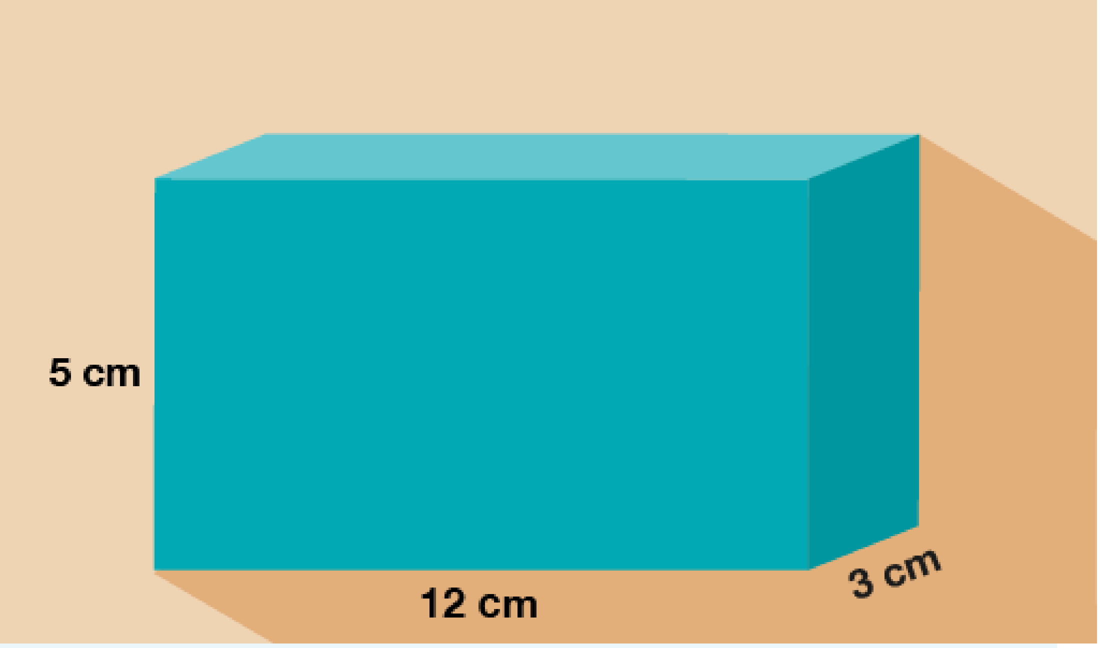


Figure 37 Rectangular prism (cuboid)

Answer

1. The area of the rectangular cross-section is:

   $5 \times 12 = 60\ \text{cm}^2$

   To get the volume, you now need to multiply the area of the cross-section by the length of the shape:

   $60 \times 3 = \underline{180\ \text{cm}^3}$

2.

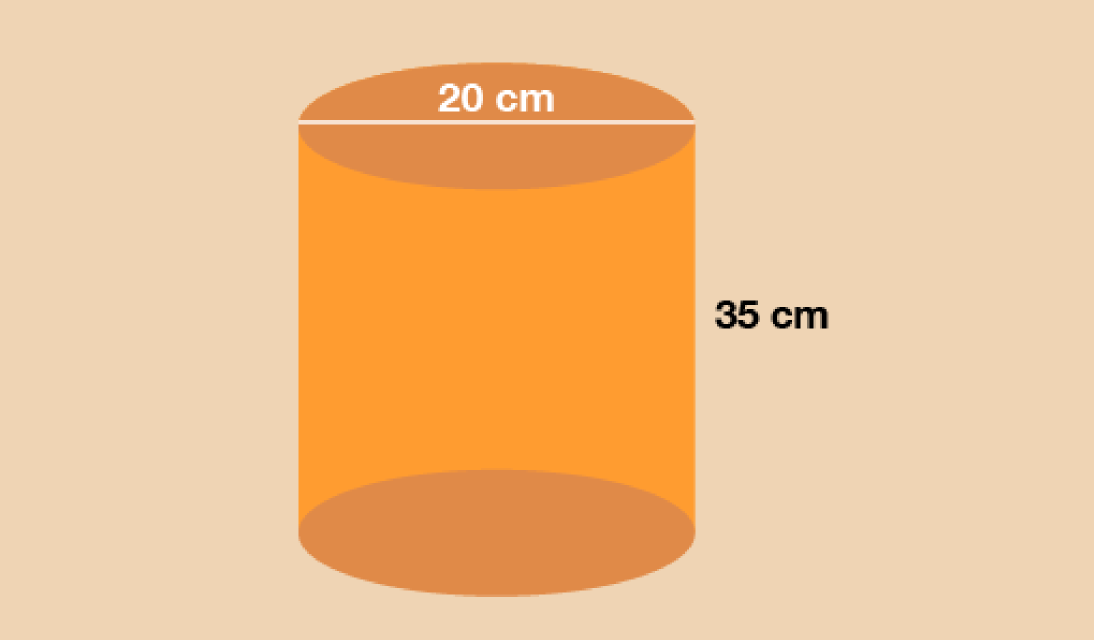


Figure 38 Circular prism (cylinder)

Answer

2. You can see that the circular cross-section has a diameter of 20 cm. To find the area of this circle, you need to find the radius first:

   radius = 20 ÷ 2 = 10 cm

   To find the area of the circular cross-section, use the formula:

   $A = \pi r^2$

   $A = 3.142 \times 10 \times 10$

   $A = 314.2\ \text{cm}^2$

   Now you have the area, multiply this by the length of the cylinder to calculate the volume:

   $V = 314.2 \times 35 = \underline{10\ 997\ \text{cm}^3}$

3.

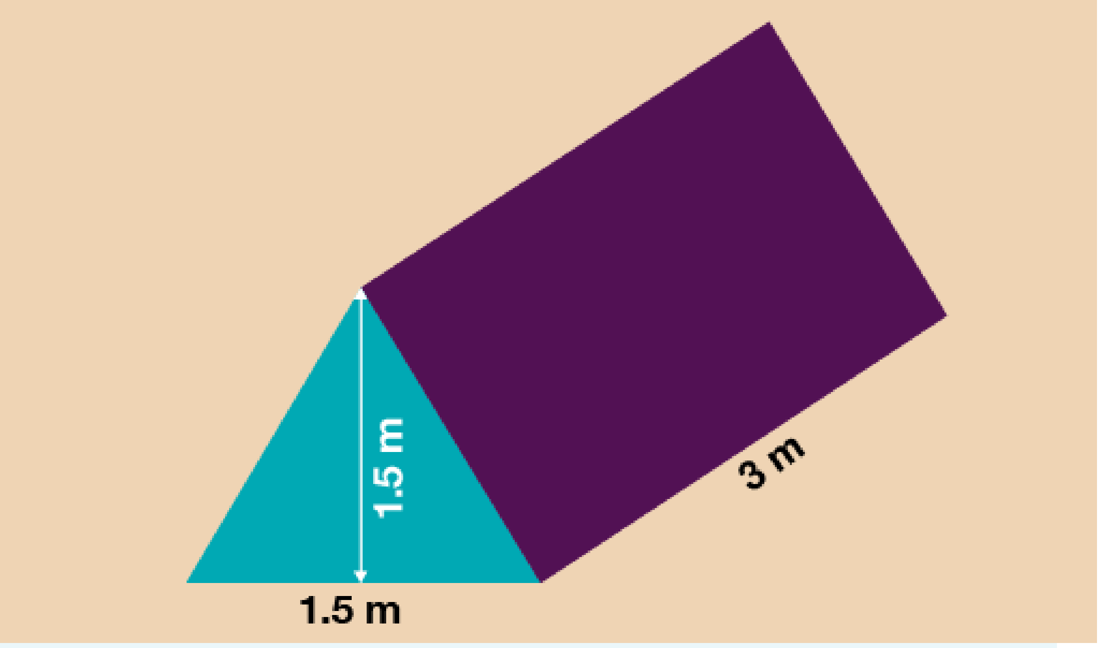


Figure 39 Triangular prism

Answer

3. The cross-section on this shape is the triangle at the front of the shape. To find the area of a triangle you use the formula:

   $A = (b \times h) \div 2$

   In this case then:

   $A = (1.5 \times 1.5) \div 2$

   $A = 2.25 \div 2$

   $A = 1.125\ m^2$

   Now you know the area of the cross-section, you multiply it by the length of the shape to find the volume (V):

   $V = 1.125 \times 3 = \underline{3.375\ m^3}$

4.

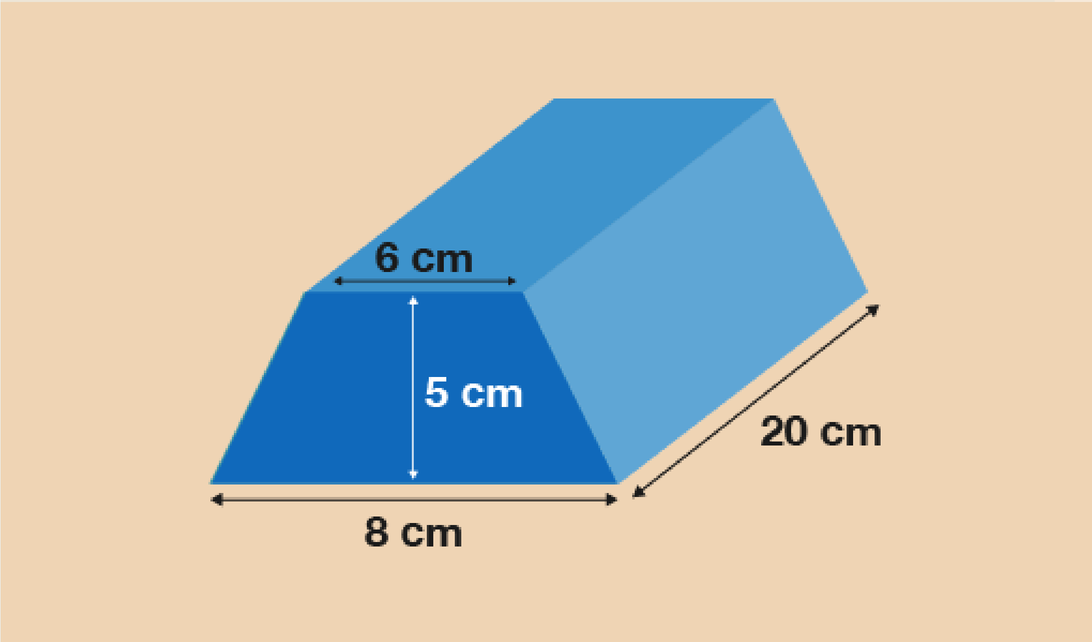


Figure 40 Trapezoidal prism

Hint: Go back to the section on area and remind yourself of the formula to find the area of a trapezium.

Answer

4. The cross-section is a trapezium so you will need the formula:

$$A = \frac{(a + b) \times h}{2}$$

$$A = \frac{(6 + 8) \times 5}{2}$$

$$A = \frac{14 \times 5}{2}$$

$$A = \frac{14}{2}$$

$A = 35 \text{ cm}^2$

Now you have the area of the cross-section, multiply it by the length of the prism to calculate the volume:

$V = 35 \times 20 = \underline{700 \text{ cm}^3}$

Summary

In this section you have learned:

- that volume is the space inside a 3D shape or space
- how to find the volume of prisms such as cuboids, cylinders and triangular prisms.

4 How many will fit?

You may need to work out how many tiles to buy to tile an area of wall or how many tins you can pack in a box. We will use the example below to illustrate.

Example 1: Tiling a wall

You are going to buy tiles measuring 0.75 m by 0.75 m to tile this area of wall:

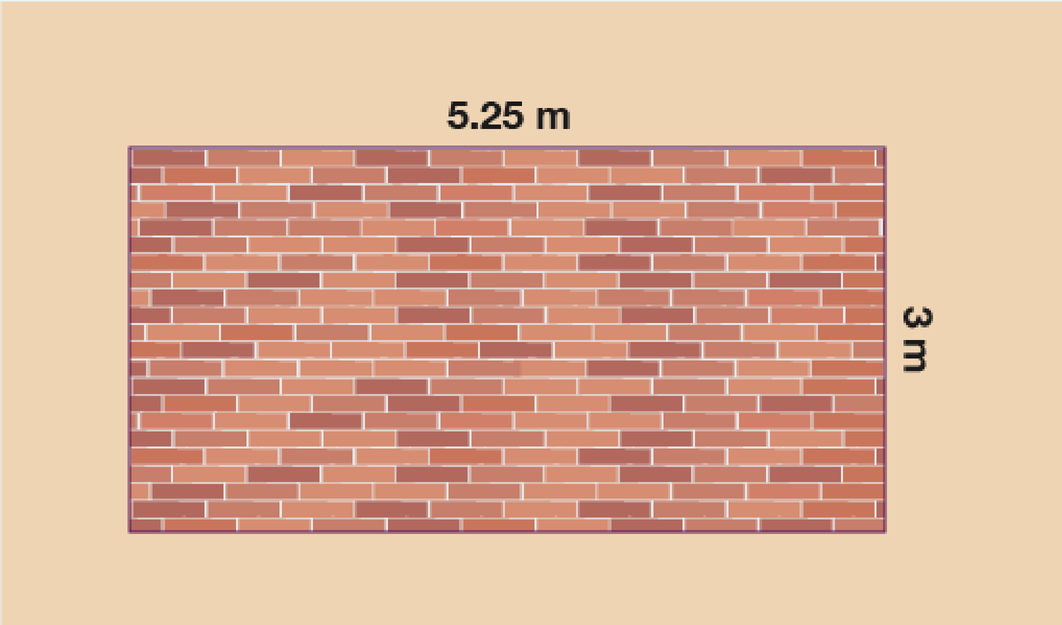


Figure 41 Wall measurements for tiling

How many tiles do you need to buy?

The easiest way to tackle a question such as this is to work out how many tiles will fit across the wall and how many will fit down the wall. You can then work out how many you need altogether.

The length across the wall is 5.25 m so we can divide by the length of the tile to work out how many will fit:

> 5.25 ÷ 0.75 = 7 tiles across

You may have preferred to convert the measurements into centimetres to avoid dividing by a decimal:

> 1 m = 100 cm, so
> 5.25 × 100 = 525 cm
> 0.75 × 100 = 75 cm
> 525 ÷ 75 = 7 tiles (you can see that the number of tiles is the same.)

Down the wall the measurement is 3 m, so down the wall you will fit:

> 3 ÷ 0.75 = 4 tiles (you may have converted to cm again and done
> 300 ÷ 75 = 4 tiles)

If 7 tiles will fit across the wall and 4 will fit down then altogether you will need:

7 × 4 = 28 tiles.

Example 2: Packing calculations

A shop is packing tins of varnish into boxes for delivery. The tins are cylindrical with a diameter of 7.5 cm and a height of 10 cm. The packing box is 60 cm long, 45 cm wide and 30 cm high.

How many tins can be packed into each box?

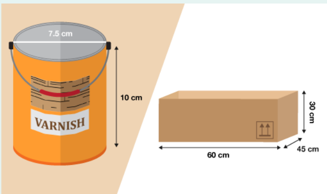


Figure 42 Dimensions of varnish tins and packing box

The easiest way to tackle this type of question is to work out how many tins you can fit in a single layer first and then to work out how many layers you can have. To work out how many tins will fit in a single layer use the same method you used above for working out how many tiles would fit.

The box is 60 cm long and tins are 7.5 cm wide:

60 ÷ 7.5 = 8 so 8 tins will fit across the box.

Hint: make sure you are working with the diameter and not the radius of the cylinder.

The box is 45 cm wide:

45 ÷ 7.5 = 6
so 6 tins would fit.

This would give 6 rows of 8 tins:

8 × 6 = 48
so 48 tins would fit in 1 layer.

Next you need to work out how many layers you could fit. The tins are 10 cm high and the box is 30 cm high:

30 ÷ 10 = 3
so the tins can be stacked 3 high:

3 × 48 = 144

The box would hold 144 tins.

Now try the following questions. Please carry out the calculations without a calculator. You can double check with a calculator if needed and remember to check your answers against ours.

Activity 8: How many will fit?

1. You want to re-carpet your floor using carpet tiles. Your floor measures 4.5 m by 6 m. The tiles you like measure 0.5 m by 0.5 m and cost £1.89 per tile.
   a. How many tiles will you need to buy?
   b. How much will they cost altogether?

Answer

1.
   a. You need to work out how many tiles will fit across the length of the floor and across the width of the floor.

      Length

      The floor is 6 m long. Each tile is 0.5 m by 0.5 m so to work out how many will fit do:

      6 ÷ 0.5 = 12 tiles

      You could also convert to centimetres:

      6 m = 6 × 100 = 600 cm

      0.5 m = 0.5 × 100 = 50 cm

      600 ÷ 50 = 12 tiles

      Width

      The floor is 4.5 m wide. Each tile is 0.5 m by 0.5 m so to work out how many will fit do:

      4.5 ÷ 0.5 = 9 tiles

      You could also convert to centimetres:

      4.5 m = 4.5 × 100 = 450 cm

      0.5 m = 0.5 × 100 = 50 cm

      450 ÷ 50 = 9 tiles

      So altogether you need:

      12 × 9 = 108 tiles

   b. Each tile costs £1.89 and you need 108 tiles so the total cost of the tiles will be:

      1.89 × 108 = £204.12

2. How many 25 cm square tiles would you need to tile the floor area below?

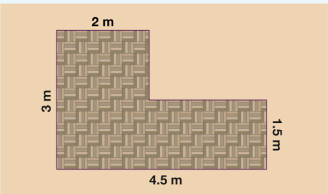


Figure 43 Area of floor for tiling

Answer

2. You need to split the floor up into 2 rectangles. You could do this in a couple of different ways.

   First method for splitting the floor as follows:

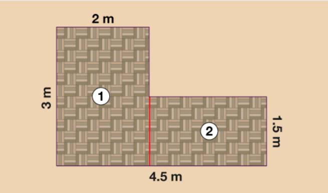


Figure 44 First method for splitting the floor

Rectangle 1

Rectangle ① is 3 m by 2 m. Convert the dimensions into centimetres:

3 m × 100 = 300 cm
2 m × 100 = 200 cm

The tiles are 25 cm square (25 cm by 25 cm) so you will need:

300 ÷ 25 = 12
200 ÷ 25 = 8
so for this part of the floor you will need:

12 × 8 = 96 tiles.

Rectangle 2

Rectangle ② is 1.5 m by 2.5 m (4.5 m − 2 m = 2.5 m). Convert the dimensions into centimetres:

1.5 m × 100 = 150 cm
2.5 m × 100 = 250 cm

The tiles are 25 cm square (25 cm by 25 cm) so you will need:

150 ÷ 25 = 6
250 ÷ 25 = 10
so for this part of the floor you will need:

6 × 10 = 60 tiles.

So altogether you will need:

60 + 96 = 156 tiles to tile the floor.

You may have split the floor area up differently.

Second method for splitting the floor as follows:

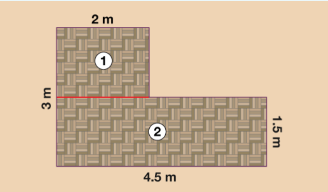


Figure 45 Second method for splitting the floor

Rectangle 1

Here rectangle ① is 1.5 m by 2 m (3 − 1.5 m = 2 m). Convert the dimensions into centimetres:

1.5 m × 100 = 150 cm

2 m × 100 = 200 cm

The tiles are 25 cm square (25 cm by 25 cm) so you will need:

150 ÷ 25 = 6

200 ÷ 25 = 8

so for this part of the floor you will need:

6 × 8 = 48 tiles

Rectangle 2

Here rectangle ② is 1.5 m by 4.5 m. Convert the dimensions into centimetres:

1.5 m × 100 = 150 cm

4.5 m × 100 = 450 cm

The tiles are 25 cm square (25 cm by 25 cm) so you will need:

150 ÷ 25 = 6

450 ÷ 25 = 18

so for this part of the floor you will need:

6 × 18 = 108 tiles

So altogether you will need:

48 + 108 = 156 tiles to tile the floor.

3. You are packing boxes of chocolates. Each chocolate box measures 23 cm long by 15 cm wide by 4 cm high and you are packing them into a storage box measuring 48 cm long by 30 cm wide by 34 cm high.

   What is the **maximum** number of chocolate boxes that can be packed in one storage box?

   Hint: The chocolate boxes need to be packed flat (horizontally) but you may want to try turning the chocolate box around.

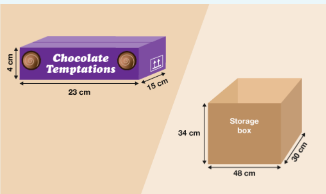


Figure 46 Chocolate boxes to fit into a storage box

Answer

3. The chocolate boxes need to be packed flat (horizontally). However, the boxes could be packed in 2 different ways:

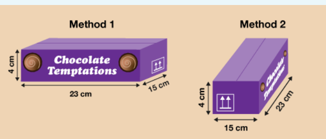


Figure 47 Method 1 and 2 for packing the chocolate boxes

Remember the dimensions of the storage box are as follows:

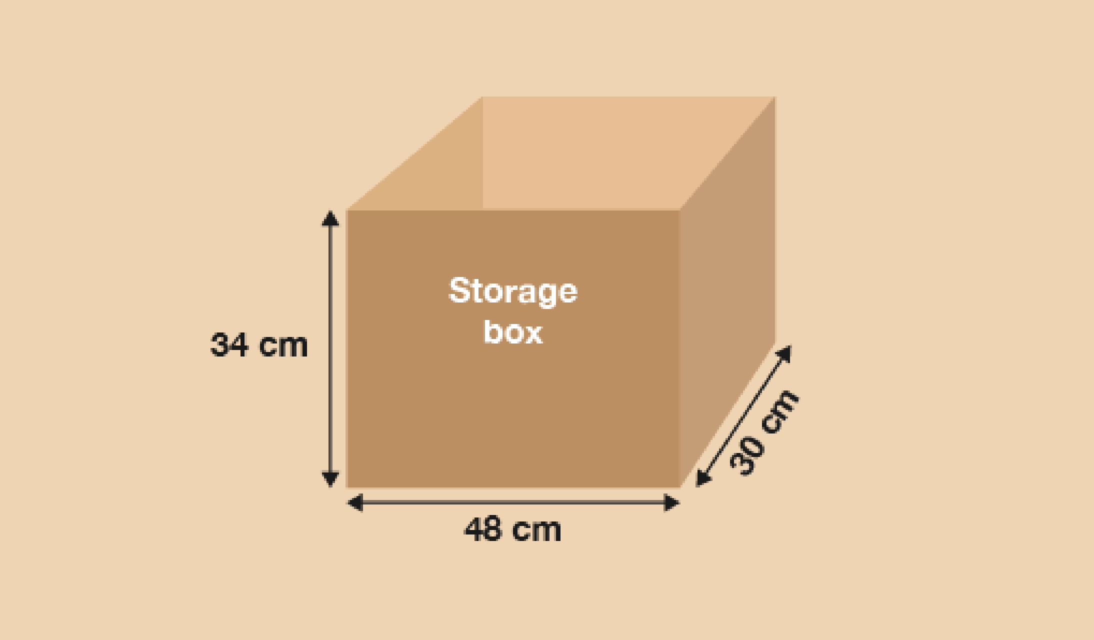


Figure 48 Storage box dimensions

Method 1

If you packed the chocolate boxes this way the storage box is 48 cm long and the chocolate boxes are 23 cm long so the calculation would be:

48 ÷ 23 = 2.1 (rounded to one d.p.)

so only 2 chocolate boxes will fit.

The storage box is 30 cm wide and the chocolate boxes are 15 cm wide so:

30 ÷ 15 = 2

so 2 chocolate boxes would fit.

This would give 2 rows of 2 chocolate boxes:

2 × 2 = 4

so 4 chocolate boxes would fit in 1 layer.

Next you need to work out how many layers you could fit. The storage box is 34 cm high and the chocolate boxes are 4 cm high:

34 ÷ 4 = 8.5

so 8 chocolate boxes will fit.

The chocolate boxes can be stacked 8 high so:

8 × 4 = 32

If using Method 1 the storage box would hold 32 chocolate boxes.

Method 2

If you packed the chocolate boxes this way the calculation would be as follows:

48 ÷ 15 = 3.2 so 3 boxes will fit

30 ÷ 23 = 1.3 (rounded to one d.p.) so only 1 will fit

In one layer, you could fit 3 × 1 = 3 chocolate boxes.

Next you need to work out how many layers you could fit. The storage box is 34 cm high and the chocolate boxes are 4 cm high:

34 ÷ 4 = 8.5

so only 8 chocolate boxes will fit.

The chocolate boxes can be stacked 8 high:

$8 \times 3 = 24$

If using Method 2 the storage box would hold 24 chocolate boxes.

Therefore, the maximum number of chocolate boxes that could fit into the storage box is 32 and you would have to pack them the first way (Method 1) to achieve this.

4. A factory producing jars of jam packs the jars into boxes that measure 42 cm long, 42 cm wide and 45 cm high. The jars of jam have a radius of 3.5 cm and a height of 11 cm.

   How many jars of jam can be packed into a box? The jars must be packed upright.

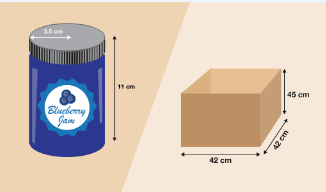


Figure 49 Jam jars (left) to fit into a packaging box (right)

Answer

4. First you need to calculate the diameter (width) of the jars. The radius of the jars is 3.5 cm:

   $3.5 \times 2 = 7$

   so the diameter of the jars is 7 cm.

   The box is 42 cm wide and jars are 7 cm wide:

   $42 \div 7 = 6$

   so 6 jars will fit across the box.

   The length of the box is also 42 cm so you know that there will be 6 rows of 6 jars on the bottom layer.

   $6 \times 6 = 36$

   so 36 jars will fit in 1 layer.

   Next you need to work out how many layers you could fit. The jars are 11 cm high, and the box is 45 cm high:

   $45 \div 11 = 4.09$ to two d.p.

   so the tins can be stacked 4 high.

   $4 \times 36 = 144$

   So 144 jars of jam would fit in each box.

The final part of his section will look at scale drawings and plans. It will require your previous knowledge of ratio as scale drawings are really just another application of ratio. The good news then, is that you already know how to do it!

Summary

In this section you have:

- learned to calculate how many smaller shapes or items will fit into larger areas or spaces.

5 Scale drawings and plans

If you completed *Everyday maths 1*, you will be familiar with the idea of scale. Scales are found on drawings, plans and maps and they are often written with the units indicated. Let's look at an example.

A football pitch is drawn to the scale of 1 cm to 5 m.

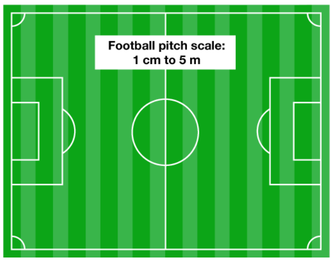


Figure 50 A football pitch drawn at 1 cm to 5 m

This means that every 1 cm measured on the plan is 5 m in real life.

If the plan is drawn with the length being 18 cm and the width being 9 cm, what are the dimensions of the football pitch in real life?

Write down the scale first:

1 cm to 5 m

You know the drawing dimensions so you need to work with these one at a time. Let's start with the length of 18 cm:

If the scale is 1 cm to 5 m then
18 cm = ? m.
If you have been given the drawing measurement and need to know the real life measurement, you multiply:
18 × 5 = 90 m
so the length is 90 m.

Note: If you have been given the actual measurement and need to find the drawing measurement you would need to divide.

Now you can work out the width measurement:

If the scale is 1 cm to 5 m then
9 cm = ? m.
Again, you need to multiply:
9 × 5 = 45 m
so the width is 45 m.

A lot of scales are written differently, without the units indicated. The scale of 1 cm to 5 m could also be written as 1:500.

This is the scale expressed as a ratio and it is independent of any units. A scale of 1:500 means that the actual real-life measurements are 500 times greater than those on the plan or map. This means that it does not matter whether you take the measurements on the plan in millimetres (mm), centimetres (cm) or metres (m) – the measurements will be 500 times as much in real life.

To write a scale as a ratio, you often have to convert. Let's look at the football pitch example again:

---

1 cm to 5 m

At the moment, the units of the scale are different. The plan side is given in centimetres (cm) and the real-life side is given in metres (m).

To express this as a ratio, you need to convert both sides to the same units. It is usually easiest to convert the real-life side of the scale into the same unit as the drawing side, so in this case it is easiest to convert 5 m into cm:

5 × 100 = 500 cm

So you can now write the scale as a ratio:

1:500

It is standard to try to write the ratio in the simplest form possible, ideally with a single unit (a '1') on the drawing side of the ratio. This will make any calculations you do using the scale easier.

Now have a go a converting scales to ratios.

Activity 9: Writing a scale as a ratio

Rewriting these scales as a ratio in their simplest form:

1. 1 cm to 2 m
2. 2 cm to 5 m
3. 10 mm to 20 m
4. 1 cm to 1 km
5. 5 cm to 2 km

Answer

1. It is easiest to change the 2 m into cm:

   2 × 100 = 200 cm so the scale expressed as a ratio would be **1:200**.
2. It is easiest to change the 5 m into cm:

   5 × 100 = 500 cm so the scale expressed as a ratio could be written as:

   2:500

   However, we usually try to get the drawing side of the ratio down to a single unit (1) to make calculations easier. Therefore, you need to simplify the ratio. To do this here, divide both sides by 2:

   2 ÷ 2 = 1

500 ÷ 2 = 250 so the scale can be written as:

1:250

3. It is easiest to change the 20 m into mm. It might be easiest to do this in stages:

   Convert to cm first –

   1 m = 100 cm so 20 × 100 = 2000 cm

   Now convert to mm –

   1 cm = 10 mm so 2000 × 10 = 20 000 mm

   This makes the scale:

   10:20 000

   This can be simplified by dividing both sides by 10 to get:

   1:2000

4. Change the 1 km into cm. Again, this will be easiest to do in stages:

   Convert to m first –

   1 km = 1000 m so 1 × 1000 = 1000 m

   Now convert to cm –

   1 m = 100 cm so 1000 × 100 = 100 000 cm

   This means the scale should be written as:

   1:100 000

5. Change the 2 km into cm. In stages this can be done as follows:

   Convert to m first –

   1 km = 1000 m so 2 × 1000 = 2000 m

   Now convert to cm –

   1 m = 100 cm so 2000 × 100 = 200 000 cm

   This makes the scale:

   5:200 000

   This can be simplified by dividing both sides by 5 to get:

   1:40 000

Now you will look at using ratio scales to work out measurements.

5.1 Scale drawing method and problems

First watch the scale drawings example video.

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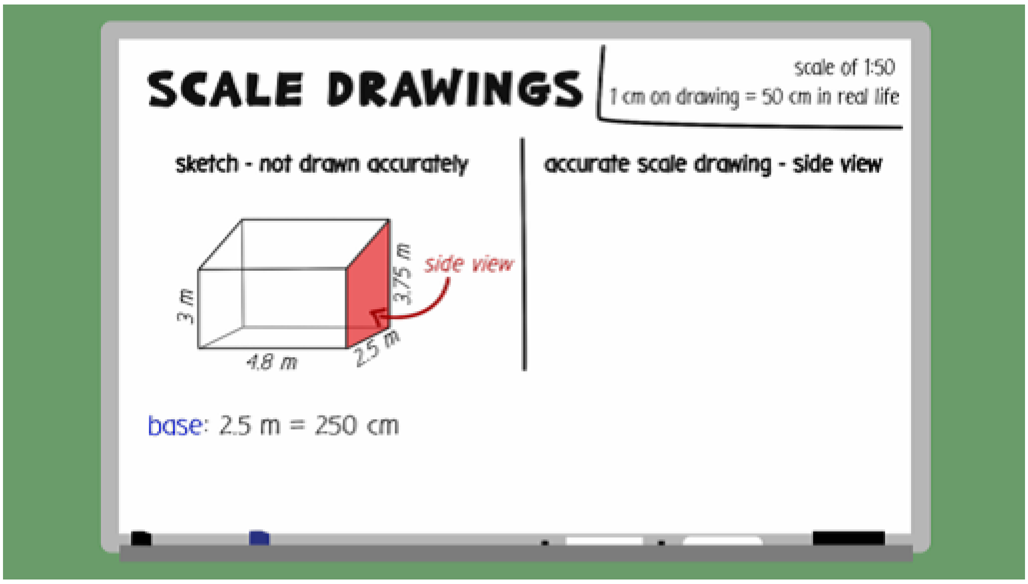

Summary of method

Remember if you are given the scale drawing measurement and asked to work out the real-life size, you need to *multiply*.

If you are given the real-life size and asked to work out the drawing measurement, you need to *divide*.

Example 1: Plan of a room

Rhodri has drawn a scale diagram of his living room. The scale he has used is 1:20.

On his diagram his living room is 30 cm long and 20 cm wide. What are the actual dimensions of his living room?

You need to work out the length and width separately.

Length

The scale he has used is 1:20, which means that everything is 20 times bigger in real life than on his diagram. You know the drawing measurement (30 cm) so you need to multiply by 20 to find the actual length of the room:

30 cm × 20 = 600 cm

It is a good idea to express the actual measurement in metres:

600 cm = 6 m

Width

You need to use the same scale of 1:20. If the width of the living room on the drawing is 20 cm then it will be 20 times as great in real life:

20 cm × 20 = 400 cm (400 cm = 4 m)

So Rhodri's actual living room measures 6 m by 4 m.

Now have a go at solving these problems involving scale. Do the calculations without a calculator. You may double-check on a calculator if you need to and make sure you check your answers against ours.

Activity 10: Scale problems

1. A scale drawing has been drawn below of a shed that a garden planner wants to build. The scale used for the drawing is 1:25.

   The area that the shed will be built on is a rectangle which measures 5.1 m by 4 m. Will the shed fit into the space allocated?

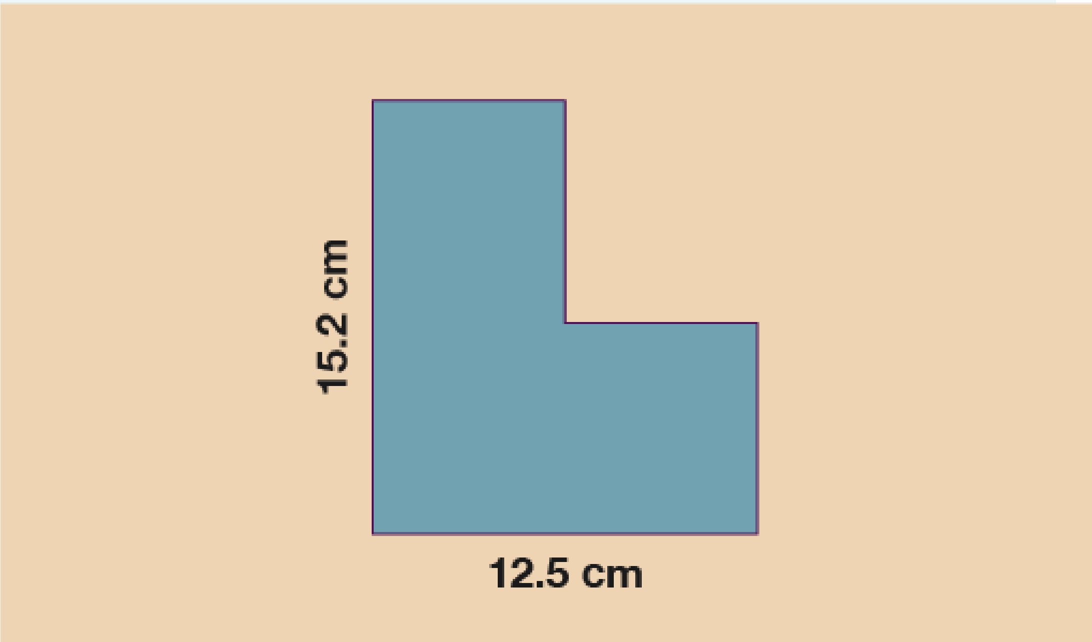

Figure 51 Calculating scale 1

Answer

1. You know the scale is 1:25, so 1 cm on the diagram represents 25 cm in real life. You have been given the measurements on the diagram and want to work out the actual measurements so you need to multiply.

   Horizontal length:

   12.5 × 25 = 312.5 cm = 3.125 m

   Vertical length:

   15.2 × 25 = 380 cm = 3.8 m

   Since both lengths for the shed are shorter than the lengths given for the area of land, you know the shed will fit.

2. A landscaper wants to put a wild area in your garden. She makes a scale plan of the wild area:

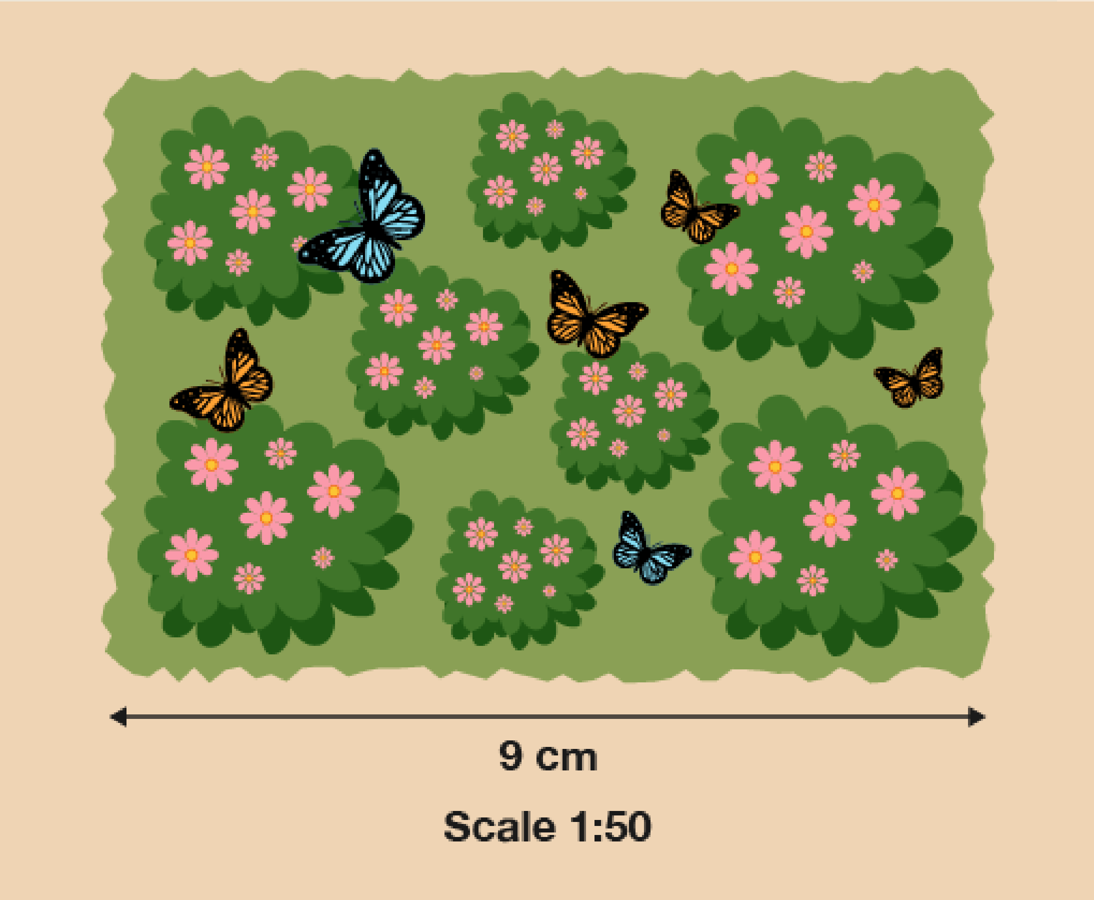

Figure 52 Calculating scale 2

What is the length of the longest side of the actual wild area in metres?

Answer

2. The length on the drawing is 9 cm, and the scale is 1:50. This means that 1 cm on the drawing is equal to 50 cm in real life. So to find out what 9 cm is in real life, you need to multiply it by 50:

   $9 \times 50 = 450$ cm

   The question asks for the length in metres, so you need to convert centimetres into metres:

   $450 \div 100 = 4.5$ m

   The actual length of the wild area will be 4.5 m.

3. Here is a scale drawing showing one disabled parking space in a supermarket car park. The supermarket plans to add two more disabled parking spaces either side of the existing one.

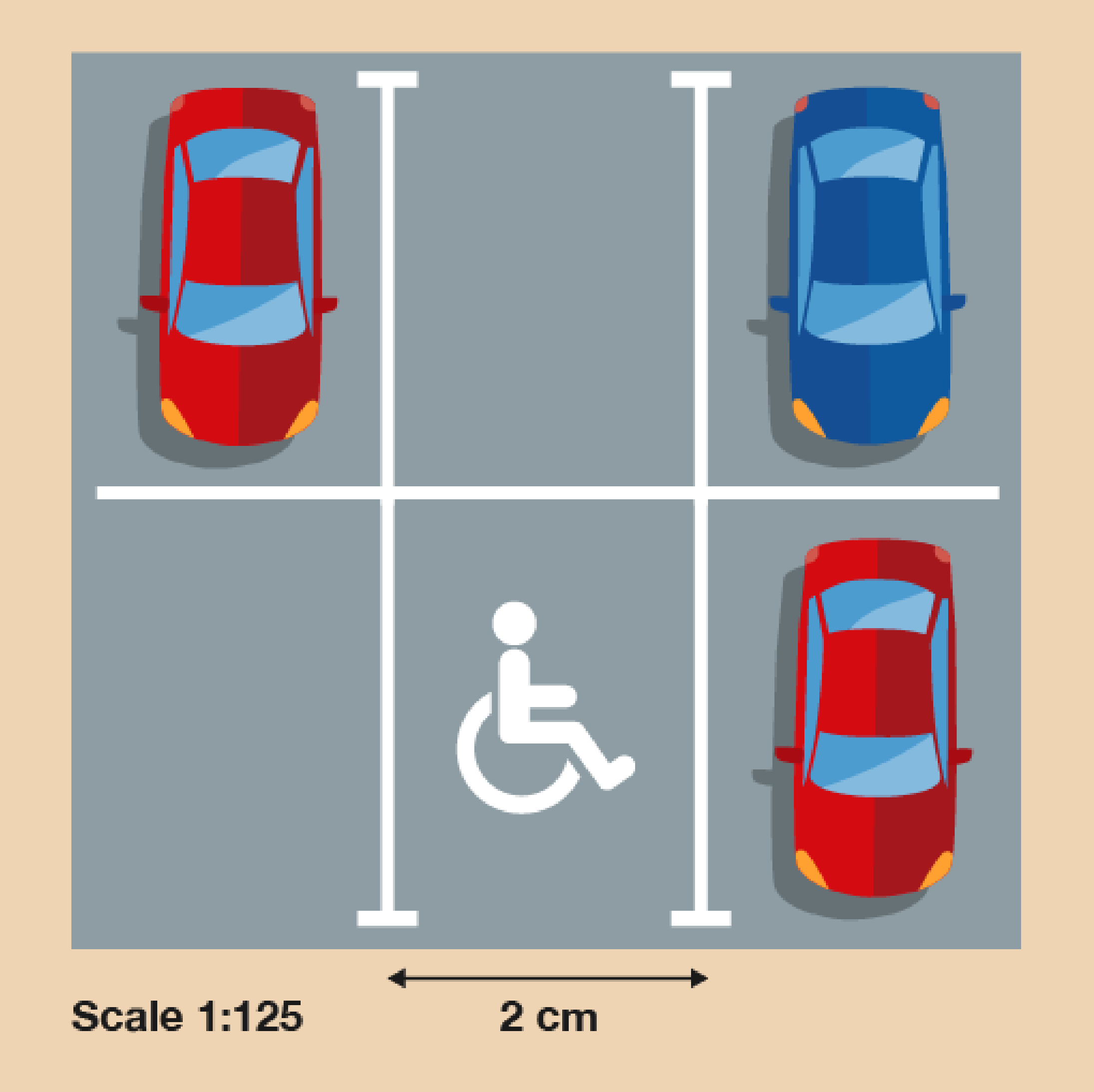

Figure 53 Calculating scale 3

What will be the total actual width of the three disabled parking spaces in metres?

Answer

3. You need to find out the width of three disabled parking spaces. The width of one parking space on the scale drawing is 2 cm, so first you need to multiply this by 3:

   $2 \times 3 = 6$ cm

   The scale is 1:125. This means that 1 cm on the drawing is equal to 125 cm in real life. So to find out what 6 cm is in real life, you need to multiply it by 125:

   $6 \times 125 = 750$ cm

   The question asks for the length in metres, so you need to convert centimetres into metres:

   $750 \div 100 = 7.5$ m

   The actual width of all three parking bays will be 7.5 m.

4. For this question you will need a pen or pencil, paper and a ruler.

   Jane has a raised vegetable patch. She plans to build a slope leading up to the vegetable patch. Jane will cover the slope with grass turf.

   She draws this sketch of the cross-section of the slope. The measurements indicated are the actual measurements.

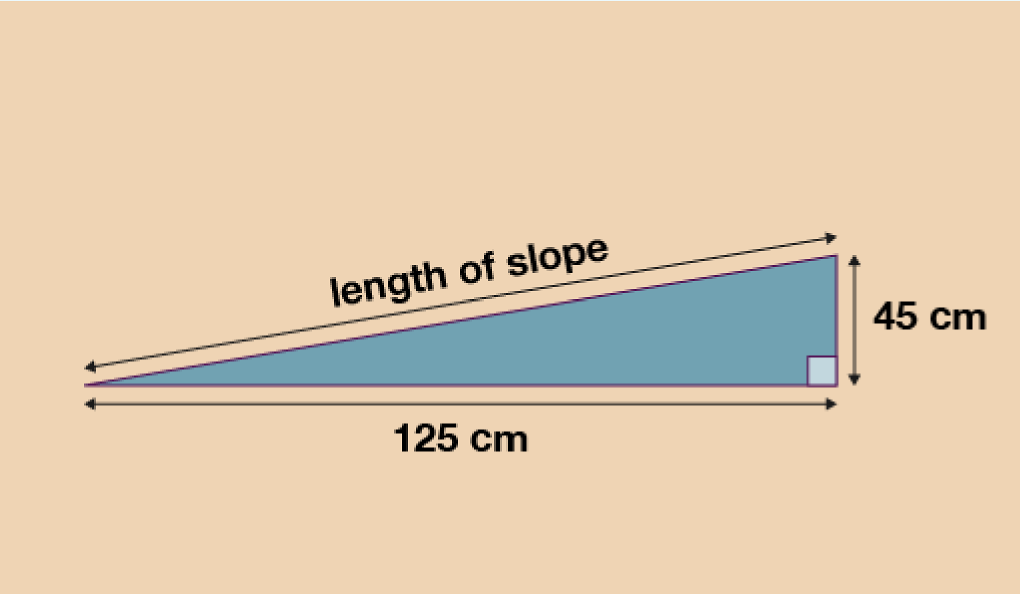

Figure 54 Finding the length of a slope

Jane will use a scale diagram to work out the length of the slope. She wants to use a scale of 1:10.

d. What will the measurements of the base and height of the slope be on her diagram?
e. Draw her scale diagram using the measurements you calculated in part (a).
   (i) What will the length of the slope be on the diagram?
   (ii) What will it be in real life?

Answer

4.

d. The scale is 1:10 and you want to work out the measurements for the scale diagram, so you need to divide the real-life measurements by 10:

   Base of patch = 125 ÷ 10 = 12.5 cm

   Height of patch = 45 ÷ 10 = 4.5 cm

e. (i) Draw the scale diagram with the base measuring 12.5 cm and the height measuring 4.5 cm.

   Now draw in your slope linking the end points of the base and height together. If you measure the length of the slope with a ruler it should measure around 13.2 cm on the diagram.

   (ii) To work out the length of the actual slope, you need to use the scale of 1:10 again, but this time you need to multiply by 10 to work out the actual slope length:

13.2 cm × 10 = 132 cm in real life.

Your answer may vary slightly from ours but should be within a reasonable range.

Summary

In this section you have:

- applied your ratio skills to the concept of scale plans and drawings
- interpreted scale plans.

Well done! You have now completed this section of your course. You are now ready to test the knowledge and skills you've learned in the end of session quiz. Good luck!

6 Session 3 quiz

Now it's time to review your learning in the end-of-session quiz.

Session 3 practice quiz.

Open the quiz in a new window or tab (by holding ctrl [or cmd on a Mac] when you click the link), then return here when you have done it.

7 Session 3 summary

Well done! You have now completed Session 3 ‘Shape and space’. If you have identified any areas that you need to work on, please ensure you refer back to this section of the course.

You should now be able to:

- understand the difference between perimeter, area and volume and be able to calculate these for both simple and more complex shapes
- know that volume is a measure of space inside a 3D object and calculate volumes of shapes in order to solve practical problems
- draw and use a scale drawing or plan.

All of the skills above will help you with tasks in everyday life. Whether you are at home or at work, number skills are essential skills to have.

You are now ready to move on to Session 4: Handling data.

Session 4: Handling data

Session 4: Handling data

Introduction

Data is a big part of our lives and can be represented in many different ways. This session will take you through a number of these representations and show you how to interpret data to find specific information. Before delving into the world of charts, graphs and averages though, it is important to make the distinction between the two different types of data:

> **Qualitative data** – this is data that is not numerical e.g. eye – colours, favourite sports, models of cars.
>
> **Quantitative data** – this is numerical data related to things that can be counted or measured e.g. temperatures, house prices, GCSE grades. Quantitative data can be *discrete* or *continuous*.

By the end of this session you will be able to:

- identify different types of data
- create and use tally charts, frequency tables and data collection sheets to record information
- draw and interpret bar charts, pie charts and line graphs
- understand there are different types of averages and be able to calculate each type
- understand that probability is about how likely an event is to happen and the different ways that it can be expressed.

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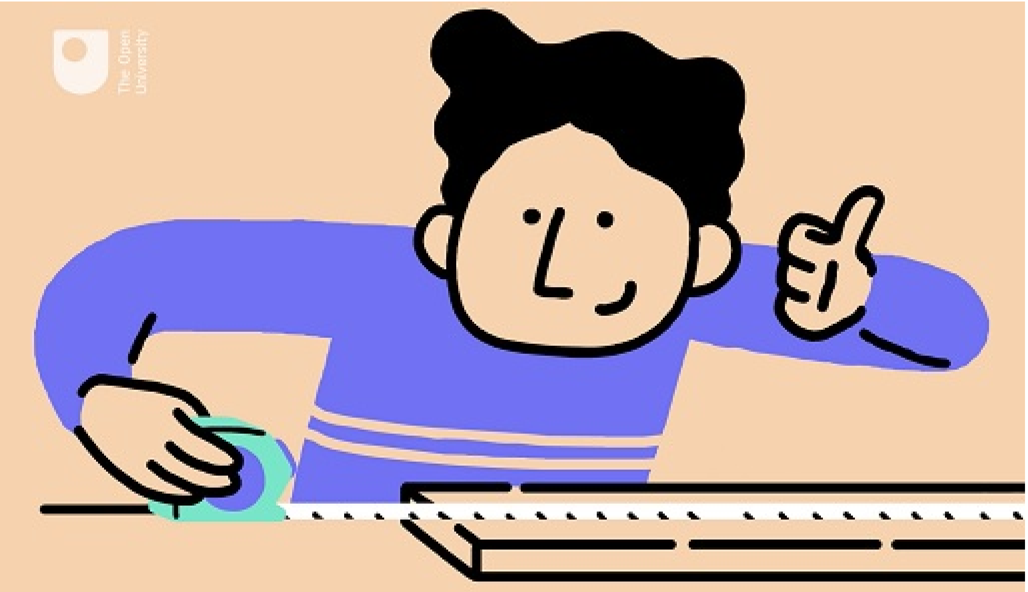


1 Discrete and continuous data

Discrete data is information that can only take certain values. These values don't have to be whole numbers (a child might have a shoe size of 3.5 or a company may make a profit of £3456.25 for example) but they are fixed values – a child cannot have a shoe size of 3.72!

The number of each type of treatment a salon needs to schedule for the week, the number of children attending a nursery each day or the profit a business makes each month are all examples of discrete data. This type of data is often represented using tally charts, bar charts or pie charts.

Continuous data is data that can take any value. Height, weight, temperature and length are all examples of continuous data. Some continuous data will change over time; the weight of a baby in its first year or the temperature in a room throughout the day. This data is best shown on a line graph as this type of graph can show how the data changes over a given period of time. Other continuous data, such as the heights of a group of children on one particular day, is often grouped into categories to make it easier to interpret.

You will have looked at different ways of presenting data in *Everyday maths 1*. Have a go at the next activity to see what you can remember.

Activity 1: Presenting discrete and continuous data

Match the best choice of graph for the data below.

1. Chart to show the heights of children in a class.
2. Chart to show favourite drink chosen by customers in a shopping centre.
3. Chart to show the temperature on each day of the week.
4. Chart to show percentage of each sale of ticket type at a concert.

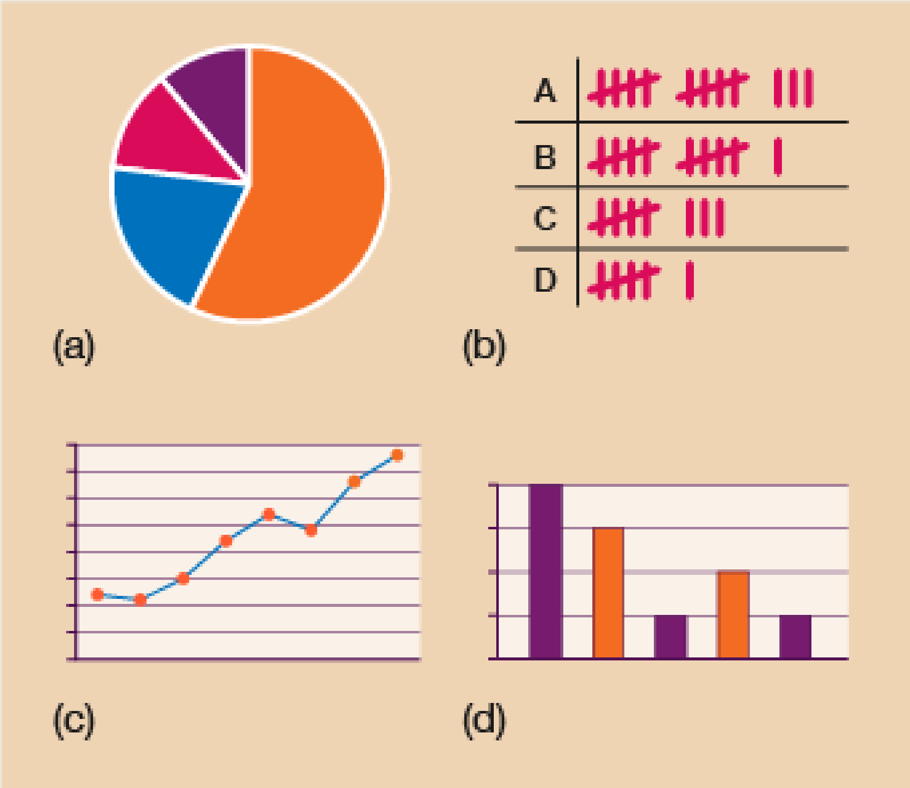


Figure 1 Different types of charts and graphs

Answer

1. Chart to show the heights of children in a class.

   The best choice here is **(d)** the bar chart as it can show each child's height clearly.
2. Chart to show favourite drink chosen by customers in a shopping centre.

   The best choice here is **(b)** the tally chart since you can add to this data as each customer makes their choice. A bar or pie chart would also be suitable.
3. Chart to show the temperature on each day of the week.

   The only choice here is **(c)** the line graph as it shows how the temperature changes over time.
4. Chart to show percentage of each sale of ticket type at a concert.

   The best choice here is probably **(a)** the pie chart since it shows clearly the breakdown of each type of ticket sale. A bar chart would also represent the data suitably.

Now that you are familiar with the two different types of data let's look in more detail at the different types of chart and graph; how to draw them accurately and how to interpret them.

Summary

In this section you have:

- learned about the two different types of data, discrete and continuous, and when and why they are used.

2 Tally charts, frequency tables and data collection sheets

Have you ever been stopped in the street or whilst out shopping and asked about your choice of mobile phone company or to sample some food or drink and give your preference on which is your favourite? If you have, chances are, the person who was conducting the survey was using a tally chart to collect the data.

Figure 2 Favourite fruit tally chart

Tally charts are convenient for this type of survey because you can note down the data as you go. Once all the data has been collected it can be counted up easily because, as shown in the picture above, every fifth piece of data for a choice is marked as a diagonal line. This allows you to count up quickly in fives to get the total. A frequency or total column can then be filled out to make the data easier to work with.

Take a look at the example below:

Table 1

Method of Travel	Tally	Frequency
Walk	𝍸 IIII	9

Bike	III	3
Car	𝍸 I	6
Bus	𝍸 𝍸 II	12
	TOTAL	30

You can see each category and its total, or frequency, clearly. Whilst this is a very simple example it demonstrates the purpose of a tally chart well. A tally chart may often be turned into a bar chart for a more visual representation of the data but they are useful for the actual data collection.

You can also use a tally chart for collecting grouped data. If for example, you want to survey the ages of clients or customers, you would not ask for each person's individual age, you would ask them to record which age group they came within. If you want to set yourself up a tally chart for this data it might look similar to the below:

Table 2

Age	Tally	Frequency
0–9		
10–19		
20–29		
30–39		
40–49		
50–59		
60–69		
70+		

Note that the age groups do not overlap; a common mistake would be to make the groups 0–10, 10–20 and so on. This is incorrect because if you were aged 10, you would not know which group you should place yourself in.

A more complex example of a tally chart can be seen below. In this example you can see that the information that has been collected is split into more than one category. These are sometimes called data summary sheets or data collection tables. It does not matter where each category is placed on the chart as long as all aspects are included.

Table 3

	fewer than 6 trips		6 trips or more	
	under 26 years	26 years and over	under 26 years	26 years and over

male	I (1)	(0)	III (3)	~~IIII~~ I (6)
female	III (3)	(0)	I (1)	IIII (4)

If you want to design a data collection or data summary sheet, you first need to know which categories of information you are looking for. Let's take a look at an example of how you might do this.

Imagine you work in a hotel and want to gather some data on your guests. You want to know the following information:

- Rating given by the guest: excellent, good, or poor.
- Length of stay: under 5 days, or 5 days or more.
- Location: from the UK, or from abroad.

A data summary sheet for this information could look like this:

Table 4

	Stayed for under 5 days			**Stayed for 5 days or more**		
	Excellent	Good	Poor	Excellent	Good	Poor
From the UK						
From abroad						

Have a go at the activity below and try creating a data collection sheet for yourself.

Activity 2: Data collection

You work at a community centre and want to gather some data on the people who use your services. You would like to know the following information:

- whether they are male or female
- if they use the centre during the day or during the evening
- which age category they are in: 0–25, 26–50, 51+.

Design a suitable data collection sheet to gather the information.

Answer

Your chart should look something like the example below. You may have chosen to put the categories in different rows and columns, which is perfectly fine. As long as all the options are covered your data collection sheet will be correct.

Table 5

	Daytime Visitor	**Evening Visitor**

	0–25	26–50	51+	0–25	26–50	51+
Male						
Female						

Now that you know the different ways in which you can collect your data, it's time to look at how you can put this information into various charts and graphs.

Summary

In this section you have:

- explored the differences between tally charts, frequency tables and data collection sheets and understood their usefulness in data collection.

3 Bar charts

There are two main types of bar charts (also known as bar graphs) – single and dual. The most common type is a single bar chart. These bar charts use vertical or horizontal bars to display data as seen below.

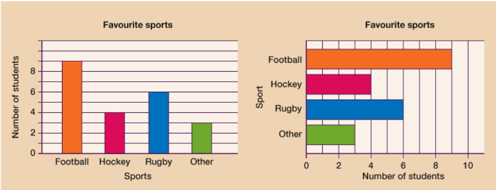


Figure 3 Vertical and horizontal single bar charts

A comparative bar chart (also known as a dual bar chart) shows a comparison between two or more sets of data. Whilst the chart below shows the favourite sports of a group of students, it further breaks down the data and gives a comparison between boys and girls; there are two bars for each sport. Depending on the type of data you are collecting, you could have more than two bars under each category.

Looking at the example comparative bar chart below, we can see that 6 girls said their favourite sport was football compared to only 3 boys.

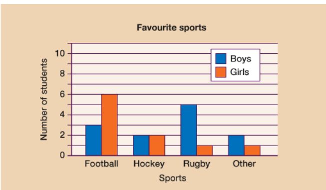


Figure 4 Dual bar chart

A third way of presenting data is a component bar chart. Instead of using separate bars, all the data is represented in a single bar. Each component, or part, of the bar is coloured differently. A key is again needed to show what each colour represents. Look at the example below:

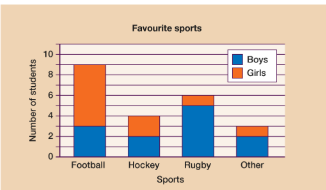


Figure 5 Single (component) bar chart

A component bar chart is useful for displaying data where we want to see the overall total together with a breakdown. In the chart above, we can easily see that in total 9 students said that football was their favourite sport. However, we can also see the data broken down to show us that 3 boys and 6 girls said that football was their favourite sport.

3.1 Features of a bar chart

Regardless of which type of bar chart you are drawing, there are certain features that all charts should have. Look at the diagram below to learn what they are.

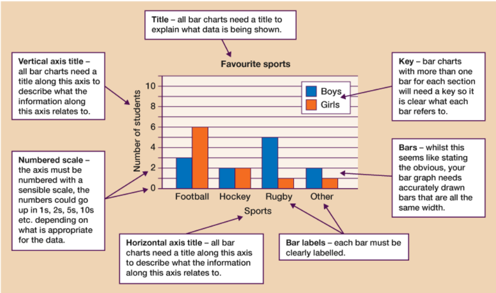


Figure 6 Features of a bar chart

If you are drawing a graph or chart by hand, you will need to use graph paper. However, creating a graph on a computer is much quicker and more convenient than doing it by hand. The computer will select the scale for the graph and it will plot it accurately. All you need to do is make sure you choose the best type of graph for your data and remember to label it all clearly and appropriately.

Activity 3: Drawing a bar chart

1. Draw a bar chart to represent the information shown in the table below. You may hand-draw your graph or create it using a computer package.

Table 6 Leisure centre users

	January	February	March	April
Male	175	154	120	165
Female	205	178	135	148

Remember to choose a suitable scale for your chart and ensure that it has a title and axis labels. It will also need a key.

Answer

1. Your bar chart should look something like the example below. You may have chosen to use a slightly different numbering scale but your chart should have the following features:
    - a title
    - labels on both the horizontal and vertical axes
    - bars labelled with the month
    - accurately drawn bars of the same width
    - a numbered scale on the vertical axis
    - a key to show which bars are for males and which are for females.

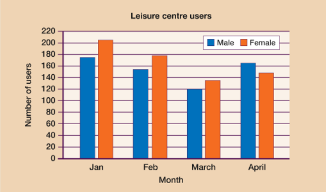


Figure 7 A bar chart showing numbers of leisure centre users

2. Draw a bar chart to represent the data shown in the table below. The numbers are much bigger than in the previous example so if you are drawing your chart by hand, you will need to consider your scale carefully.

Table 7 Food sales at a three-day summer music festival

Food types	Number of portions sold
Sandwiches and baguettes	37 000
Burgers	25 000
Fish and chips	16 000
Ice cream cones	12 000
Other	8 500

Answer

2. Your bar chart should look something like the example below.

   Note the use of gridlines to help with interpreting the data in the graph. Also note how the scale and label on the vertical axes (see the second example on the right-hand side of the graph) are slightly different. Both formats are shown here as different examples and either would be acceptable.

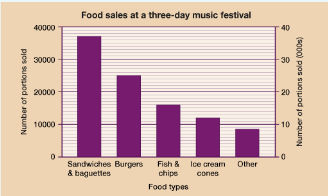


Figure 8 A bar chart showing food sales at three-day music festival

Your bar chart should be similar to the above and include:

- a title
- labels on both the horizontal and vertical axes
- bars labelled with the food types
- accurately drawn bars of the same width
- a numbered scale on the vertical axis, which goes up in equal intervals.

Now that you understand the features of bar charts and are able to draw them, let's look at how to interpret the information they show.

3.2 Interpreting bar charts

If you see a bar chart that is showing you information, it's useful to know how to interpret the information given. The most important thing you need to read to understand a bar chart correctly is the scale on the vertical axis.

As you know from drawing bar charts yourself, the numbers on the vertical axis can go up in steps of any size. You therefore need to work out what each of the smaller divisions is worth before you can use the information. The best way for you to learn is to practise looking at, and reading from, different bar charts.

Have a go at the activity below and see how you get on.

Activity 4: Interpreting a bar chart

The chart below shows the average daily viewing figures for Sky Cinema channels during 11–17 March 2019. Answer the questions that follow. Choose the correct answer in each case.

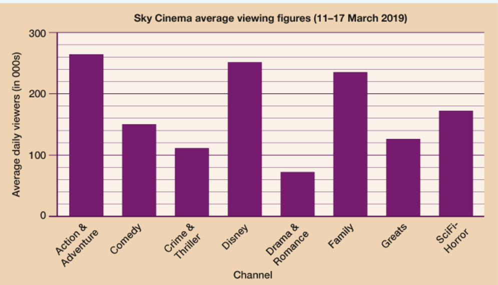


Figure 9 Sky Cinema average viewing figures (11–17 March 2019)

1. Which was the most popular channel?
    a. Action & Adventure
    b. Disney
    c. Family

Answer

a. Action & Adventure

2. Approximately how many viewers tuned in to Sky Cinema Disney?
    b. 250 viewers
    c. 25 000 viewers
    d. 250 000 viewers

Answer

c. 250 000 viewers – note how the vertical axis label states that the figures are in 000s (thousands) which means that the scale goes up in intervals of 20 000. This means the Disney bar goes up to 250 000.

3. Approximately how many more viewers watched Sky Cinema SciFi-Horror than Sky Cinema Drama & Romance?
   c. 100 000
   d. 80 000
   e. 100

Answer

a. 100 000 – Sky Cinema SciFi-Horror had appoximately 175 000 viewers whilst Sky Cinema Drama & Romance had approximately 75 000 viewers, making a difference of 100 000 viewers.

Activity 5: Interpreting a comparative bar chart

The chart below shows the average house prices by local authority in Wales for 2017 and 2018. Answer the questions that follow.

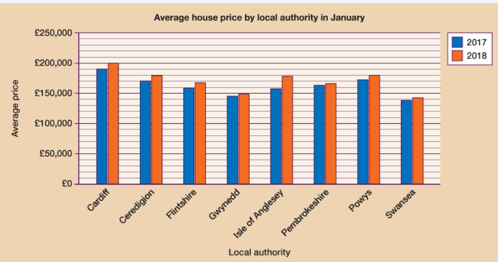


Figure 10 Average house price by local authority in January

1. Which local authority had the biggest average house price increase between 2017 and 2018?

Answer

Isle of Anglesey

2. Which of the following is the best estimate of the difference between Cardiff and Swansea's 2018 average house prices?
   b. £50 000
   c. £60 000
   d. £120 000

Answer

b. £60 000 – note how the scale goes up in intervals of £10 000. In 2018 Cardiff's average house price was approximately £200 000 compared to Swansea's average house price of approximately £140 000, making a difference of £60 000.

Activity 6: Interpreting a component bar chart

The following chart shows the number of enrolments for different college courses. Answer the questions that follow.

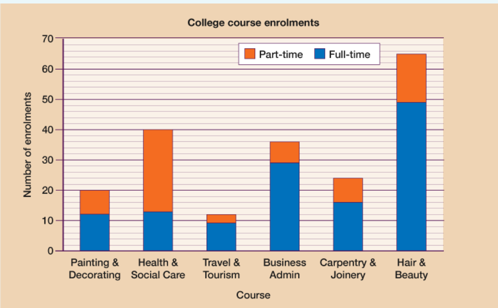


Figure 11 College course enrolments

1. Which course had the fewest number of part-time enrolments?

Answer

Travel & Tourism

2. Which course had more part-time than full-time enrolments?

Answer

Health & Social Care

3. How many full-time enrolments did Hair & Beauty have?

Answer

49.

Note how the scale goes up in 2s and the full-time component of the Hair & Beauty bar goes up to halfway between 48 and 50.

4. How many part-time enrolments did Business Admin have?

Answer

7

5. Approximately how many course enrolments were there in total?

Answer

197 – You need to count the total enrolments for all of the separate courses and then add them together:

- Painting & Decorating = 20
- Health & Social Care = 40
- Travel & Tourism = 12
- Business Admin = 36
- Carpentry & Joinery = 24
- Hair & Beauty = 65

20 + 40 + 12 + 36 + 24 + 65 = 197

You should now be feeling confident with both drawing and interpreting bar charts so it's time to move on to look at another type of chart – pie charts.

Summary

In this section you have learned:

- about the different features of a bar chart
- to use single and comparative bar charts to present data
- to interpret the information shown on different types of bar chart.

4 Pie charts

A pie chart is the best way of showing the proportion (or fraction) of the data that is in each category. It is very easy to visually see the largest and smallest sections of the chart and how they compare to the other sections.

This section will help you to understand when a pie chart should be used over other presentation methods. In the first part of this section you'll learn how to draw and then move on to interpreting pie charts.

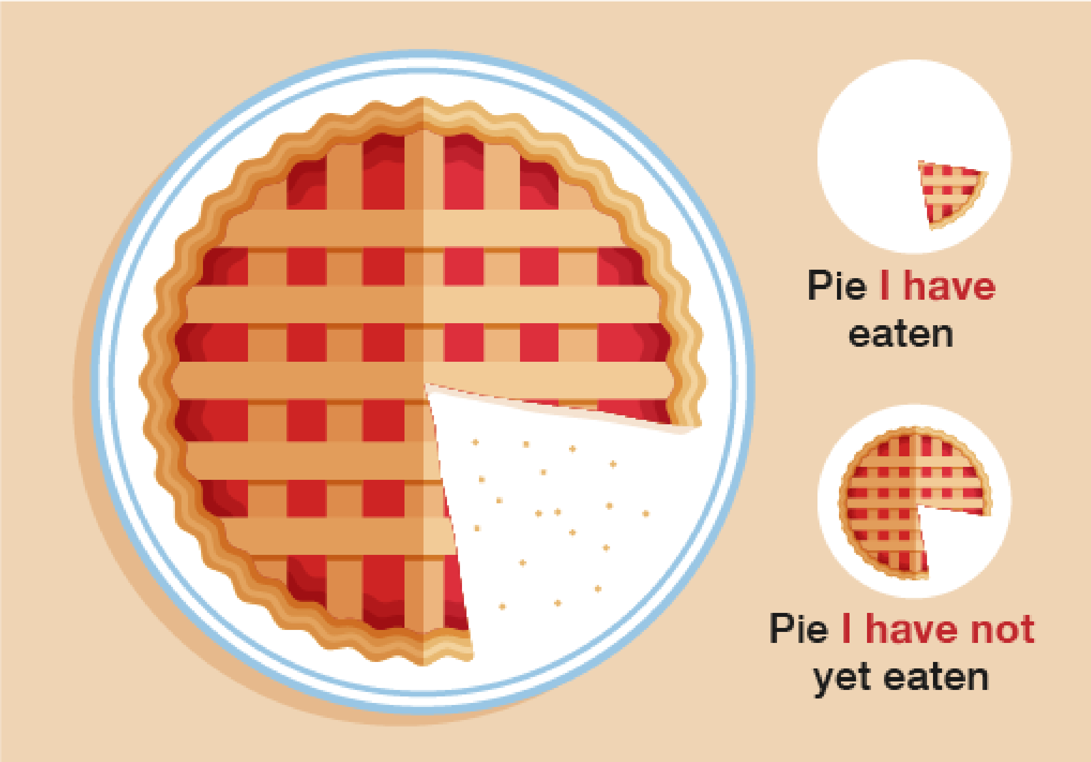


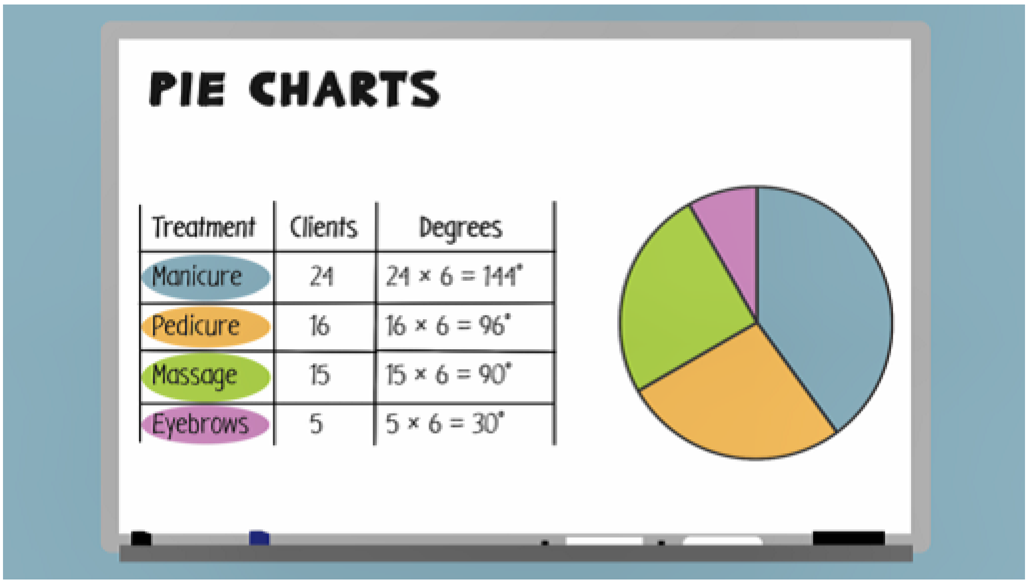
Figure 12 A partially eaten pie

In the first part of this section you'll learn how to draw and then move on to interpreting pie charts.

4.1 Drawing pie charts

The best way to understand the steps involved in drawing a pie chart is to watch the worked example in the video below.

Video content is not available in this format.



Now have a go at drawing a pie chart for yourself.

Activity 7: Drawing a pie chart

1. A leisure centre wants to compare which activities customers choose to do when they visit the centre. The information is shown in the table below. Draw an accurate pie chart to show this information. You may hand-draw your pie chart or create it using a computer package.

Table 8(a)

Activity	Number of customers
Swimming	26
Gym	17
Exercise class	20
Sauna	9

Answer

1. Firstly, work out the total number of customers:

   $26 + 17 + 20 + 9 = 72$

   Now work out the number of degrees that represents customer:

   $360^\circ \div 72 = 5^\circ$ per customer

Table 8(b)

Activity	Number of customers	Number of degrees
Swimming	26	$26 \times 5 = 130^\circ$
Gym	17	$17 \times 5 = 85^\circ$
Exercise class	20	$20 \times 5 = 100^\circ$
Sauna	9	$9 \times 5 = 45^\circ$

Now use this information to draw your pie chart. It should look something like this:

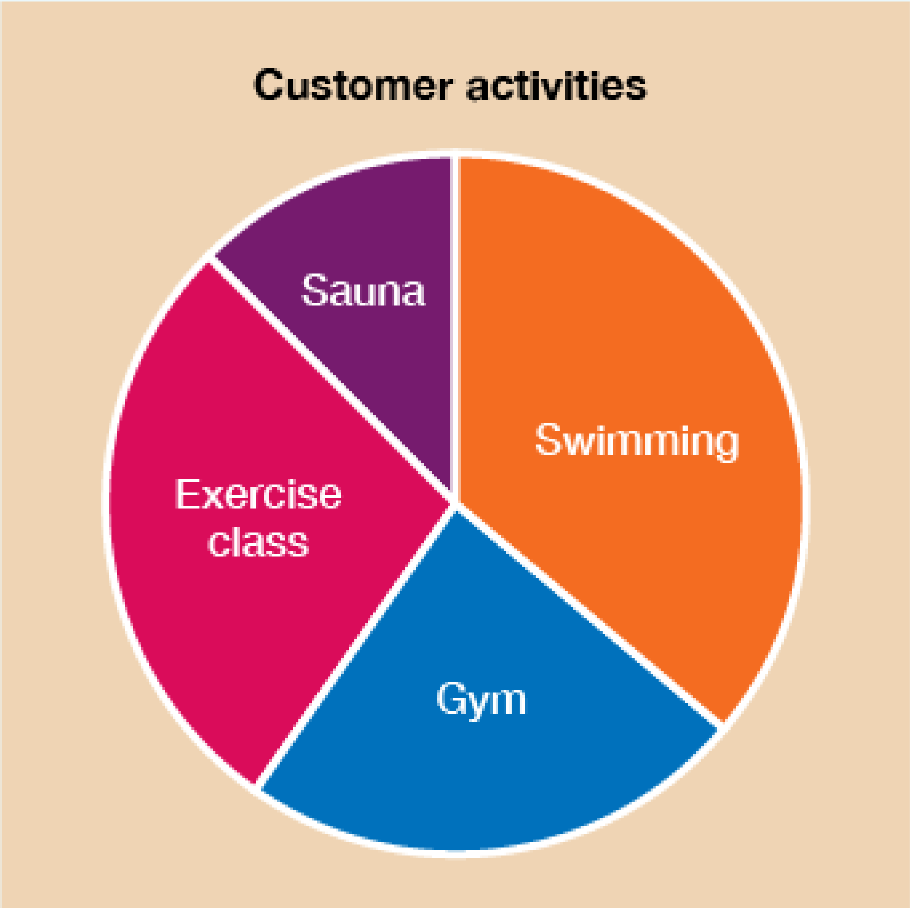


Figure 13 Pie chart for customer leisure centre activities

2. The table below shows the sandwich sales over one year for sandwich company, Belinda's Butties. Draw a pie chart to illustrate the data. You may hand-draw your pie chart or create it using a computer package.

Table 9(a)

Sandwich Type	Sales (000s)
Cheese and onion	20
Egg and cress	17
Prawn	11
Coronation chicken	12
Tuna mayonnaise	9
Ham	8
Beef and tomato	13

Answer

2. Firstly, work out the total number of sandwich sales:

   20 + 17 + 11 + 12 + 9 + 8 + 13 = 90 (000s)

   Now work out the number of degrees that represents each sale:

   360 ÷ 90 = 4˚ per sale

Table 9(b)

Sandwich Type	Sales (000s)	Number of Degrees
Cheese and onion	20	20 × 4 = 80°
Egg and cress	17	17 × 4 = 68°
Prawn	11	11 × 4 = 44°
Coronation chicken	12	12 × 4 = 48°
Tuna mayonnaise	9	9 × 4 = 36°
Ham	8	8 × 4 = 32°
Beef and tomato	13	13 × 4 = 52°

Now use this information to draw your pie chart. It should look something like the one below.

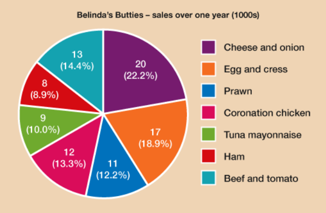


Figure 14 Belindas Butties – sales over one year (000s)

Now that you can accurately draw a pie chart, it's time to look at how to interpret them. You won't always be given the actual data, you may just be given the total number represented by the chart or a section of the chart and the angles on the pie chart itself. It's useful to know how to use your maths skills to work out the actual figures.

Here's a reminder of the degrees of a circle which will be useful when you come to read from pie charts.

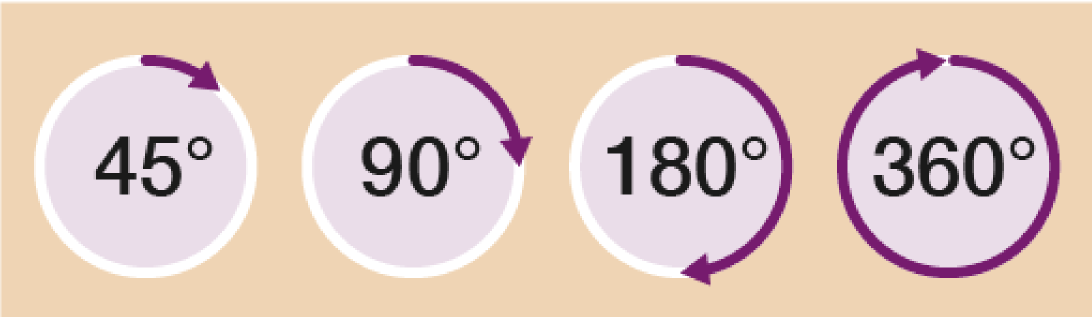


Figure 15 Degrees of a circle

4.2 Interpreting pie charts

Imagine you've been presented with the pie chart below. The chart shows the ages of students competing at an athletic event.

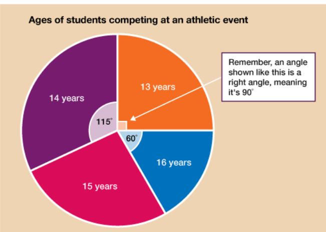


Figure 16 Ages of students competing at an athletic event

There are two possible pieces of information you could be given. You could be given the total number of students that were at the event or, you could be given the number of students in one of the age categories.

Example: Reading a pie chart 1

Let's say you were told that 72 people attended the competition. Since you know that 360˚ has been shared equally between all 72 people, you do 360 ÷ 72 = 5˚ per person.

Once you know this, if you wanted to know how many students that took part were 16 years old, you would look at the degrees on the chart for 16-year-olds which in this example is 60˚.

You then do 60 ÷ 5 = 12 students.

If you wanted to work out the number of 15-year-olds, you would first need to work out the missing angle; you know that all the angles will add up to 360˚ so just do:

360 – 115 – 90 – 60 = 95˚

And now do the same as before 95 ÷ 5 = 19 students who were 15 years old.

Example: Reading a pie chart 2

Using the same pie chart, let's say that all you were told was that 23 students took part who were 14 years old.

You can see that the angle for 14-year-olds is 115˚ and you've been told that this represents 23 students.

To find out how many degrees each student gets, you do 115 ÷ 23 = 5˚ per student.

Once you know this you can find out how many students are represented by each other section in the same way as we did in example 1. For example, the 13-year-olds have an angle of 90˚.

To find out how many there are you do 90 ÷ 5 = 18 students who were 13 years old.

Pie charts are very similar to ratio. In ratio questions you are always looking to find out how much 1 part is worth; in pie chart questions you are looking to find how many degrees represents 1 person (or whatever object the pie chart is representing).

As well as being closely linked with ratio, pie charts also involve the use of your fractions skills. If, for example, you were asked what fraction of the students were 16 years old, you can show this as $\frac{60}{360}$ since the 16-year-olds are 60 degrees out of the total 360 degrees.

Using your fractions skills however, the fraction $\frac{60}{360}$ can be simplified to $\frac{1}{6}$.

It's time for you to practise your skills at interpreting pie charts. Have a go at the activity below and then check your answers with the feedback given.

Activity 8: Interpreting pie charts

1. The pie chart below shows how long a gardener spent doing various activities over a month.

Figure 17 Time spent doing gardening activities

   a. What fraction of the time was spent planting? Give your answer in its simplest form.
   b. 5 hours were spent digging. How long was spent on cutting the grass?

Answer

1.

   a. Planting = $\frac{80}{360}$ = $\frac{2}{9}$ in its simplest form.

   b. Digging is 100° and you know that that was 5 hours.

      100° ÷ 5 = 20° for each hour.

      Since cutting the grass has an angle of 40°, you do 40 ÷ 20 = 2 hours cutting the grass.

2. 120 adults participating in an online course were asked if they felt there were enough activities for them to complete throughout the course.

   The pie chart below shows the results.

Figure 18 Pie chart of opinions about an online course

   b. What fraction of the adults thought there were too many activities? Give your answer in its simplest form.
   c. How many adults thought there were enough activities?

Answer

2.

   b. Too much = $\frac{105}{360}$ = $\frac{7}{24}$ in its simplest form.

c. You know that 120 adults took part in the survey. To find out how many degrees represents each adult, you do 360 ÷ 120 = 3 degrees per person.

Next you need to know the angle for those who said there were enough activities. For this, you do:

$360 - 105 - 60 - 45 = 150^{\circ}$

Now you know this, you can do:

150 ÷ 3 = 50 adults thought there were enough activities.

Well done! You can now draw and interpret bar charts and pie charts; both of which are good ways to represent discrete data. In the next part of this session, you will learn how to draw and interpret line graphs.

Summary

In this section you have learned:

- what types of information can be represented effectively on a pie chart
- how to use and interpret a pie chart
- how to draw an accurate pie chart when given a set of data.

5 Line graphs

Line graphs are a very useful way to spot patterns or trends over time. You will have looked at how to plot and interpret single line graphs from single data sources in *Everyday maths 1*. Now you will be looking at line graphs that show the results of two data sources.

The example below shows the monthly foreign exchange rate of £1 against the US dollar and the euro. You can clearly see how the pound was dropping in value against the euro and US dollar up to October 2016. Now that you understand how line graphs can be used and why they are useful, next you'll learn how to draw and interpret them.

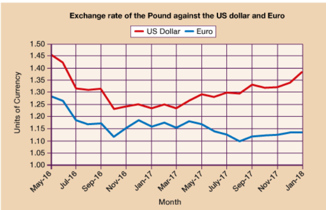


Figure 19 Monthly foreign exchange rate of £1 against the US dollar and the euro from May 2016 to January 2018.

Now that you understand how useful line graphs can be and how they can be used, next you'll learn how to draw and interpret them.

5.1 Drawing line graphs

Drawing a line graph is very similar to drawing a bar chart, and they have many of the same features.

Line graphs need:

- a title
- a label for the vertical axis (e.g. units of currency)
- a numbered scale on the vertical axis
- a label on the horizontal axis (e.g. month) so that it is clear to the reader what they are looking at.

The main difference when drawing a line graph rather than a bar chart, is that rather than a bar, you put a dot or a small cross to represent each piece of information. You then join each dot together with a line. There is significant debate over whether the dots should be joined with a curve or with straight line. Whilst the issue is (believe it or not!) hotly contested, the general consensus seems to be that dots should be joined with straight lines.

Activity 9: Drawing a line graph

1. Have a go at drawing a line graph to represent the data below.

   The table below shows the temperatures in Tenby during the first two weeks of July 2018.

Table 10

Date	Temperature High ˚C	Temperature Low ˚C
01/07/2018	25	15
02/07/2018	28	17
03/07/2018	29	12
04/07/2018	24	15
05/07/2018	26	13
06/07/2018	22	12
07/07/2018	23	12
08/07/2018	27	12
09/07/2018	26	14
10/07/2018	24	14
11/07/2018	22	14
12/07/2018	22	7
13/07/2018	24	12
14/07/2018	25	20

Answer

1. Your line graph should look similar to the one shown below with a title, key and axis labels.

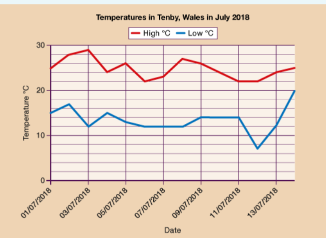


Figure 20 A line graph of temperatures in Tenby, Wales in July 2018

2. A clothing store has outlets in Llandudno and Aberystwyth. Use the data in the table below to draw a line graph comparing monthly sales between January and June.

Table 11

Month		Jan	Feb	Mar	Apr	May	Jun
Sales (£000s)	Llandudno	29	15	19	20	23	24
	Aberystwyth	25	10	16	16	21	26

Answer

2. Your line graph should look similar to the one shown below with a title, key and axis labels.

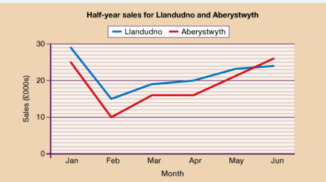


Figure 21 A line graph showing half-year sales for Llandudno and Aberystwyth

5.2 Interpreting line graphs

Interpreting line graphs is also very similar to the way you interpret bar charts; it's just about using the scales shown on the graph to find out information.

Given that you've already learned and practised interpreting bar charts, you can jump straight into an activity on interpreting line graphs.

Activity 10: A population line graph

1. The graph shows information about the population of a village in thousands over a period of time.

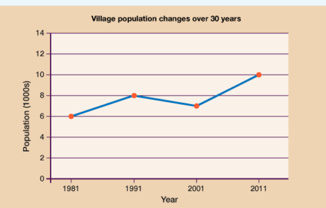


Figure 22 Line graph showing village population over time

   a. What was the population of the village in 1991?
   b. What was the increase in population from 1981 to 2011?
   c. By how much did the population drop between 1991 and 2001?

Answer

1.

   a. In 1991 the population was 8000.
   b. In 1981 the population was 6000, in 2011 the population was 10 000. This is an increase of 10 000 − 6000 = 4000.
   c. In 1991 the population was 8000 and by 2001 it was 7000. 8000 − 7000 = 1000. So the population dropped by 1000.

2. Jen's Gym kept a record of member numbers during 2018.

   The graph below shows information about the results.

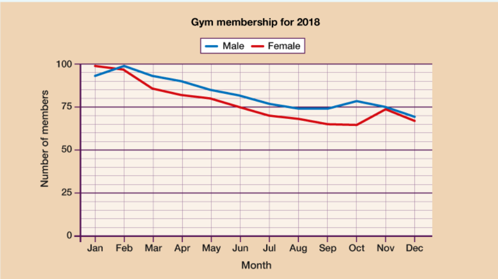


Figure 23 A line graph showing gym membership numbers for 2018

   b. Which was the only month when there were more female members than male members?
   c. Estimate the difference in membership numbers for October.
   d. In what month was the difference between male and female members smallest?

Answer

2.

   b. January
   c. 13 (+/− 1) allowed as this question is estimation
   d. November

How did you get on? Hopefully you were able to answer all the questions without too much difficulty. As long as you have worked out the scale correctly and read the question carefully, there's nothing too tricky involved.

You have now covered each drawing and interpreted each different type of chart and graph so it's time to move on to look at other uses for data: averages and range.

Summary

In this section you have learned:

- which types of data can be suitably represented by a line graph and which are best suited to other types of charts.

- how to interpret the information shown on a line graph
- how to draw an accurate line graph for a given set of data.

6 Mean, median, mode and range

The three types of averages that you will be focussing on in this part of the section are mean, median and mode. You will also be looking at range. For Level 2 Essential Skills Wales, you need to be able to calculate each of these without using a calculator.

6.1 Range

Figure 24 A mountain range of different sized peaks

Much like this stunning mountain range is made up of a variety of different sized mountains, a set of numerical data will include a range of values from smallest to biggest. The range is simply the difference between the biggest value and the smallest value. It shows how spread out a set of data is and can be useful to know because data sets with a big difference between the highest and lowest values can imply a certain amount of risk.

Let's say there are two basketball players and you are trying to choose which player to put on for the last quarter. If one player has a large range of points scored per game (sometimes they score a lot of points but other times they score very few – meaning their scoring is **variable**) and the other player has a smaller range (meaning they are more **consistent** with their point scoring) it might be safest to choose the more consistent player.

Take a look at the example below.

A farmer takes down information about the weight, in kg, of apples that one worker collected each day on his apple farm.

Table 12

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
56 kg	70 kg	45 kg	82 kg	67 kg	44 kg	72 kg

In order to find the range of this data, you simply find the biggest value (82 kg) and the smallest value (44 kg) and find the difference:

82 – 44 = 38 kg

The range is therefore 38 kg.

Now let's compare this worker to another worker whose information is shown in the table below.

Table 13

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
56 kg	60 kg	58 kg	62 kg	65 kg	49 kg	58 kg

This worker has a highest value of 65 kg and a lowest value of 49 kg. The range for this worker is therefore 65 – 49 = 16 kg. The second worker has a lower range than the first worker and is therefore a more consistent apple picker than the first worker, who is a more variable picker.

Now try one for yourself.

Activity 11: Finding the range

1. The table below shows the sales made by a café on each day of the week:

Table 14

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
£156.72	£230.54	£203.87	£179.43	£188.41	£254.70	£221.75

What is the range of sales for the café over the week?

Answer

1. Simply find the highest value: £254.70, and lowest value: £156.72, then find the difference:

   £254.70 − £156.72 = £97.98

2. A bowling team want to compare the scores for their players. The table below shows their results.

Table 15

Name	Andy	Bilal	Caz	Dom	Ede
Highest score	176	175	162	170	150
Lowest score	148	145	142	165	116

Which player is the most consistent? Give a reason for your answer.

Answer

2. You need to look at the range for each player:

   Andy: 176 − 148 = 28
   Bilal: 175 − 145 = 30
   Caz: 162 − 142 = 20
   Dom: 170 − 165 = 5
   Ede: 150 − 116 = 34

   The player with the smallest range is Dom and so Dom is the most consistent player.

3. Outside temperatures at a garden centre were taken daily over four weeks in January and displayed in the following table.

Table 16 Temperatures for January in ˚C

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Week 1	2	4	5	5	1	−1	−3
Week 2	−4	0	0	3	6	5	6
Week 3	3	2	−1	0	3	2	0
Week 4	0	4	7	8	3	−1	−2

   a. Which day of the week showed the most variable temperature range?
   b. Which day of the week showed the most consistent temperature range?
   c. Which days had the same range?

Answer

3.
   c. Sundays had a lowest temperature of −3˚C and a highest temperature of 6˚C so the difference was 9˚C, giving Sundays the highest range and making temperatures more variable.
   d. Tuesdays had a lowest temperature of 0˚C and a highest temperature of 4˚C so the difference was 4˚C, giving Tuesdays the lowest range and making temperatures more consistent.
   e. Wednesdays and Thursdays had the same range.

      Wednesdays had a lowest temperature of −1˚C and a highest temperature of 7˚C so the difference was 8˚C.

      Thursdays had a lowest temperature of 0˚C and a highest temperature of 8˚C so the difference was 8˚C.

As you have seen, finding the range of a set of data is very simple but it can give some useful insights into the data. The most commonly used average is the ‘mean average’ (or sometimes just the mean) and you’ll look at this next.

6.2 Mean average

You will now learn about:

- finding the mean when given a set of data
- finding the mean from a frequency table

Figure 25 Below average and mean average

The mean is a good method to use when you want to compare a large set of data, for example:

- the average amount spent by customers in a shop
- the average cost of a house in a certain area
- the average time taken for your chosen breakdown service to get to your car.

The mean average can help us make comparisons between sets of data which can then help you when making a decision.

6.3 Finding the mean from a set of data

To find the mean of a simple set of data, all you need to do is find the total, or sum, of all the items together and then divide this total by how many items of data there are.

Table 13 (repeated)

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
56 kg	60 kg	58 kg	62 kg	65 kg	49 kg	58 kg

Look again at the weight, in kg, of apples that one worker collected each day on an apple farm (shown above). If you want to calculate the mean average weight of apples collected, you first find the sum or total of the weight of apples collected in the week:

56 + 60 + 58 + 62 + 65 + 49 + 58 = 408 kg

Next, divide this total but the number of data items, in this case, 7:

408 ÷ 7 = 58.3 kg (rounded to one d.p.)

It is important to note that the mean may well be a decimal number even if the numbers you added together were whole numbers.
Another important thing to note here is that the two sums (the addition and then the division) are done as two separate sums. If you were to write:

56 + 60 + 58 + 62 + 65 + 49 + 58 ÷ 7

this would be incorrect (remember BIDMAS from Session 1?). Unless you are going to use brackets to show which sum needs to be done first (56 + 60 + 58 + 62 + 65 + 49 + 58) ÷ 7, it is accurate to write two separate calculations.

Have a go at calculating the mean for yourself by completing the activity below.

Activity 12: Finding the mean

1. The table below shows the sale price of ten, 2 bedroom semi-detached houses in a town in Liverpool.

Table 16

1	2	3	4	5	6	7	8	9	10
£70 000	£65 950	£66 500	£71 200	£68 000	£62 995	£70 500	£68 750	£59 950	£67 900

What is the mean house price in this area?

....

Answer

1. First find the total of the house prices:

    £70 000 + £65 950 + £66 500 + £71 200 + £68 000 + £62 995 + £70 500 + £68 750 + £59 950 + £67 900 = £671 745

    Now divide this total by the number of houses (10):

    £671 745 ÷ 10 = **£67 174.50**

2. The table below shows the units of gas used by a household for the first 6 months of a year.

Table 17

January	February	March	April	May	June
1650	1875	1548	1206	654	234

Calculate the mean amount of gas units used per month.

Answer

2. Find the total number of units used:

   1650 + 1875 + 1548 + 1206 + 654 + 234 = 7167 units

   Now divide this total by the number of months (6):

   7167 ÷ 6 = **1194.5** units

This method of finding the mean is fine if you have a relatively small set of data. What about if the set of data you have is much larger? If this was the case, the data would probably not be presented as a list of numbers, it’s much more likely to be presented in a frequency table.

In the next part of this section, you will learn how find the mean when data is presented in this way.

6.4 Finding the mean from a frequency table

Large groups of data will often be shown as a frequency table, rather than as a long list. This is a much more user friendly way to look at a large set of data. Look at the example below where there is data on how many times, over a year, customers used a gardening service.

Table 18

Number of visits in a year	Number of customers
1	6
2	10
3	11
4	16
5	4
6	3

We could write this as a list if we wanted to:

1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3 and so on

but it's much clearer to look at in the table format. But how would you find the mean of this data? Well, there were 6 customers who had 1 visit from the gardening company, that's a total of 1 × 6 = 6 visits. Then there were 10 customers who had 2 visits, that's a total of 2 × 10 = 20 visits. Do this for each row of the table, as shown below.

Table 19

Number of visits in a year	Number of customers	Total visits
1	6	1 × 6 = 6
2	10	2 × 10 = 20
3	11	3 × 11 = 33
4	16	4 × 16 = 64
5	4	5 × 4 = 20
6	3	6 × 3 = 18
	Total = 50 customers	Total = 161 visits

Finally, work out the totals for each column (highlighted in a lighter colour on the table above).

Now you have all the information you need to find the mean; the total number of visits is 161 and the total number of customers is 50 so you do 161 ÷ 50 = 3.22 visits per year as the mean average.

Warning! Many people trip up on these because they will find the total visits (161) but rather than divide by the total number of customers (50) they divide by the number of rows in the table (in this example, 6).

If you do 161 ÷ 6 = 26.83, logic tells you, that since the maximum number of visits any customer had was 6, the mean average cannot be 26.83. Always sense check your answer to make sure it is somewhere between the lowest and highest values of the table. For this example, anything below 1 or above 6 must be incorrect!

Have a go at a couple of these yourself so that you feel confident with this skill.

Activity 13: Finding the mean from frequency tables

1. The table below shows some data about the number of times children were absent from school over a term.

   Work out the mean average number of absences.

Table 20(a)

Number of absences	Number of children
1	26
2	13
3	0
4	35
5	6

Answer

1. First, work out the total number of absences by multiplying the absence column by the number of children. Next, work out the totals for each column.

Table 20(b)

Number of absences	Number of children	Total number of absences
1	26	1 × 26 = 26
2	13	2 × 13 = 26
3	0	3 × 0 = 0
4	35	4 × 35 = 140
5	6	5 × 6 = 30
	Total = 80	Total = 222

   To find the mean do: 222 ÷ 80 = 2.775 mean average.

2. You run a youth club for 16–21-year-olds and are interested in the average age of young people who attend.

   You collect the information shown in the table below. Work out the mean age of those who attend. Give your answer rounded to one decimal place.

Table 21(a)

Age	Number of young people
16	3
17	8
18	5

19	12
20	6
21	1

Answer

2. Again, first work out the totals for each row and column.

Table 21(b)

Age	Number of young people	Total number
16	3	16 × 3 = 48
17	8	17 × 8 = 136
18	5	18 × 5 = 90
19	12	19 × 12 = 228
20	6	20 × 6 = 120
21	1	21 × 1 = 21
	Total = 35	Total = 643

To find the mean you then do:

- 643 ÷ 35 = 18.4 years (rounded to one d.p.)

If you are given all the data in a set and asked to find the mean, it's a relatively simple process.

For the Confirmatory Test element of Essential Skills Wales (ESW), you are expected to complete all of the necessary calculations without using a calculator, so remember to use alternative methods for checking e.g. inverse checks.

6.5 Calculating the median

The next type of average you look at briefly is called the median. Put very simply, the median is the middle number in a set of data and as it is in the middle, it is not affected by abnormally high or low data values. The important thing you need to remember is to put the numbers in size order, smallest to largest, before you begin. Firstly let's look at two simple examples.

Example: Finding the median 1

Find the median of this data set:

Method

5, 10, 8, 12, 4, 7, 10

Firstly, order the numbers from smallest to largest:

4, 5, 7, 8, 10, 10, 12

Now, find the number that is in the middle:

4, 5, 7, **8**, 10, 10, 12

8 is the number in the middle, so the median is 8.

Example: Finding the median 2

Find the median of this data set:

Method

24, 30, 28, 40, 35, 20, 49, 38

Again, you firstly need to order the numbers:

20, 24, 28, 30, 35, 38, 40, 49

And then find the one in the middle:

20, 24, 28, **30**, **35**, 38, 40, 49

In this example there are actually two numbers that are in the middle. You therefore find the middle of these two numbers by adding them together and then halving the answer:

(30 + 35) ÷ 2
75 ÷ 2 = 37.5

The median for this set of data is 37.5. As there are two numbers that are in the middle, your answer does not appear in the original set of data.

The example below is a little more complicated.

Example: Finding the median 3

Tracy did a survey of the number of cups of coffee her colleagues had drunk during one day. The frequency table shows her results.

Table 22

Number of cups of coffee	Frequency
2	1
3	5

4	3
5	4
6	6

First you need to calculate the number of colleagues by adding together the numbers in the frequency column:

$$1 + 5 + 3 + 4 + 6 = 19$$

You then need to work out the median or middle value in the number of cups of coffee. To do this you can list the number of cups of coffee in a line:

2 3 3 3 3 3 4 4 4 **5** 5 5 5 6 6 6 6 6 6

You know there are 19 colleagues, which is an odd number, so you will be able to find the exact midpoint:

$$9 + 9 = 18$$

so count in from either side until you reach the 10th number which can be seen as 5 in the list above.

Another way to do this is to calculate the number of colleagues, which you know is 19. You then find the midpoint by adding the numbers in the frequency table:

$$1 + 5 + 3 = 9$$

and the exact midpoint is the 10th colleague which the table shows as 4 in the frequency column.

If you look under the Number of cups of coffee column, you can see that the answer is 5 cups of coffee, so the median is 5.

Activity 14: Calculating the median

Now calculate the median of the following:

1. The ages of a group of students on a course are:
   16, 44, 32, 67, 25, 18, 22
2. The heights of a group of children in a gymnastics class are:
   1.24 m, 1.27 m, 1.20 m, 1.15 m, 1.26 m, 1.17 m
3. The frequency table below shows the number of televisions that a group of students on a media course have at home.

Table 23

Number of televisions	Number of students
0	**1**
1	**4**

2	**8**
3	**9**
4	**3**

Calculate the median number of televisions.

Answer

1. First you need to list the data in order of size so:

   16, 18, 22, 25, 32, 44, 67

   Now find the middle value. In this case there are 7 data values so you will be able to find the exact middle.

   16, 18, 22, **25**, 32, 44, 67

   The middle value is 25 so this is the median age of the group of students.
2. First you need to list the heights in order of size so:

   1.15 m, 1.17 m, 1.20 m, 1.24 m, 1.26 m, 1.27 m

   Now find the middle value. In this case there are 6 data values so first find the middle two values.

   1.15 m, 1.17 m, **1.20 m**, **1.24 m**, 1.26 m, 1.27 m

   Now add together the two middle values:

   1.20 m + 1.24 m = 2.44 m

   Then halve the answer:

   2.44 m ÷ 2 = 1.22 m

   So the median height of the students is 1.22 m.
3. First you need to calculate the number of students.

   To do this you add up each number in the frequency table:

   1 + 4 + 8 + 9 + 3 = 25 students

   You then need to work out the median or middle value in the number of televisions. To do this you can list the number of televisions in a line:

   0 1 1 1 1 2 2 2 2 2 2 2 2 **2** 3 3 3 3 3 3 3 3 3 4 4

   As you know there are 25 students, you need to find the midpoint which will be 13. Count in until you find the 13th number which is 2 in your list of televisions, so the median is 2.

   Another way to do this is to calculate the number of students, which you know is 25. You then find the midpoint as the 13th student using the frequency table. If you count up the 'Number of students' column in the frequency table, the 13th value is 2 televisions, so the median is 2.

If you want to see some more examples, or try some for yourself, use the link below:

https://www.mathsisfun.com/median.html

6.6 Calculating the mode

The final type of average to look at is the mode, which is the most common value in a data set. There can sometimes be more than one mode, which occurs when two or more values are equally common. Sometimes there will be no mode as each data value occurs only once. When you calculate the mode you will always find that it is one of the values in your original data set. The mode is also called the modal value.

Example: Finding the mode 1

What is the mode of the following numbers?

3, 7, 5, 6, 4, 5, 6, 5, 7, 5

First group the same numbers together by listing them in size order:

3, 4, 5, 5, 5, 5, 6, 6, 7, 7

It is then easy to identify the modal value as 5 as this number occurs the most.

Note: To remember how to calculate this average use *Mode* = *Most*.

Example: Finding the mode 2

What is the mode of the following amounts of money?

£8.99 £16.45 £17.50 £36.20 £6.75 £9.35 £12.99 £8.95

First list them in size order from lowest amount to highest:

£6.75 £8.95 £8.99 £9.35 £12.99 £16.45 £17.50 £36.20

You can now see that there is no mode (or modal value) as each amount of money occurs only once.

Example: Finding the mode 3

Below is a set of data showing the number of people attending a yoga class each week over a year. What is the mode?

17 17 13 16 18 15 12 16 16 17 18 13

First list them in size order from lowest amount to highest:

12 13 13 15 <u>16 16 16 17 17 17</u> 18 18

You can now see that two numbers occur the same number of times. 16 and 17 both occur three times, so in this set of data there are two modes or modal values – 16 and 17.

Example: Finding the mode 4

To find the mode from a frequency table you need to find the value with the highest frequency. The results of a survey on a block of flats are shown in the frequency table below. The highest frequency, which can be seen in the 'Number of flats' column, is 18. This means that the mode or modal number of occupants is 3.

Table 24

Number of occupants	Number of flats
0	2
1	9
2	13
3	**18**
4	6

Activity 15: Calculating the mode

Now calculate the mode of the following:

1. 3, 6, 5, 7, 3, 5, 6, 6, 3, 4, 9, 6
2. 13, 19, 11, 28, 17, 29, 16, 24, 15, 18
3. 81 cm, 53 cm, 74 cm, 62 cm, 53 cm, 70 cm, 81 cm, 74 cm, 42 cm, 90 cm
4. The table below shows the number of tries scored by a school rugby team during one month. What is the modal number of tries scored?

Table 25(a)

Number of tries	Frequency
0	5
1	8
2	6
3	3

Answer

1. First list them in size order from lowest amount to highest:

   3, 3, 3, 4, 5, 5, 6, 6, 6, 6, 7, 9

   It is then easy to identify the modal value as 6 as this number occurs the most.

2. First list them in size order from lowest amount to highest:

   11, 13, 15, 16, 17, 18, 19, 24, 28, 29

   You can now see that each value occurs only once so there is no mode/modal value.
3. First list them in size order from lowest amount to highest:

   42 cm, 53 cm, 53 cm, 62 cm, 70 cm, 74 cm, 74 cm, 81 cm, 81 cm, 90 cm

   You can now see that three numbers occur the same number of times. 53 cm, 74 cm and 81 cm occur twice, so in this set of data there are three modes or modal values.
4. To find the mode you need to look for the highest frequency in the table. In this case it is 8 which shows that the modal number of tries scored is 1.

Table 25(b)

Number of tries	Frequency
0	5
1	**8**
2	6
3	3

6.7 Choosing the best average

For some sets of data it may be better to use one type of average over another as it will be more representative of the data type.

Here are some of the advantages and disadvantages of each type of average.

Mean

Advantages

- Uses all the data values.

Disadvantages

- May not always be one of the values in the set of data or a value that does not make sense for the data, e.g. 1.6 people.
- The mean may not be useful for a set of data which has a value a lot higher or lower than the others, e.g. If you were to include the salary of the Managing Director with the wages of shop floor staff when calculating an average salary, it is likely that the mean would be distorted by the higher salary of the Managing Director, which means that it would not represent the data very well.

Median

Advantages

- It is the middle value so is not affected by very high or very low values. It is a useful type of average for data sets with such values.

Disadvantages

- Sometimes it will not be one of the values in the data set.

Mode

Advantages

- It will always be one of the values in the data set (if there is a mode).
- It is very good for certain types of data, e.g. finding the most common shoe size.

Disadvantages

- There may not be a mode.
- There may be several modes.
- It may be at one end of the data distribution.

Now have a go at finding the mean, median and mode.

Activity 16: Finding different averages

A bridal shop records the sizes of wedding dresses that it sells in one month. The table below shows the results.

1. Find the following:
   a. the mean
   b. the median
   c. the mode.
2. Which of the averages gives the most useful information for this data set?

Table 26(a)

Dress size	No. of dresses sold
8	2
10	8
12	11

14	12
16	5
18	2

Answer

1. (a) **The mean**
   - First, work out the total number of dresses sold by multiplying the dress size column by the number of dresses sold.
   - Add a frequency column to display your calculations.
   - Next, work out the totals for the number of dresses sold column and the frequency column.

Table 26(b)

Dress size	No. of dresses sold	Frequency
8	2	8 × 2 = 16
10	8	10 × 8 = 80
12	11	12 × 11 = 132
14	12	14 × 12 = 168
16	5	16 × 5 = 80
18	2	18 × 2 = 36
Totals	**40**	**512**

   - Finally calculate the mean by dividing the frequency by the number of dresses sold:

     512 ÷ 40 = 12.8

     Mean = 12.8

1. (b) **The median**
   - First you need to calculate the total number of dresses sold. To do this add up each number in the frequency table:

     2 + 8 + 10 + 13 + 5 + 2 = 40 dresses sold.
   - Now find the middle value by listing the quantity of each size of dress from smallest to biggest:

8 8 10 10 10 10 10 10 10 10 12 12 12 12 12 12 12 12 12 **12 12** 14 14 14 14 14 14 14 14 14 14 14 14 16 16 16 16 16 18 18

- The midpoint of the 40 dresses sold is between values 20 and 21 so you need to find both of these values. In this example they are 12 and 12.

  You calculate the median by adding 12 + 12 and dividing by 2:

  12 + 12 = 24

  24 ÷ 2 = 12

  As you can see, in this example the median is size 12.
- A quicker way to do this is to calculate the number of dresses, which you know is 40.
- You then use the frequency table to find the midpoint which is between the 20th and the 21st dress size. If you count up the number of dresses sold in the frequency table:

  2 + 8 + 11 = 21

  you can see that the 20th and 21st values fall within size 12, so the median is size 12.

  Median = 12

1. (c) **The mode**
   - To find the mode you need to look for the highest number of dresses sold. In this case it is 12 which shows that the mode dress size is 14.

     Mode = 14

Table 26(c)

Dress size	No. of dresses sold
8	2
10	8
12	11
14	**12**
16	5
18	2

2. In this case the mean is size 12.8 which does not exist as a dress size so this is not useful.

   The median result is size 12 which is a dress size, but it is still not the most commonly sold dress size.

   The mode is dress size 14 which is the most commonly sold dress size and so this gives the most useful information.

Well done! You have now learned all you need to know about mean, median, mode and range. The final part of this section, before the end-of-course quiz, looks at probability.

Summary

In this section you have learned:

- that there are different types of averages that can be used when working with a set of data – mean, median and mode
- range is the difference between the largest data value and the smallest data value and is useful for comparing how consistently someone or something performs
- mean is what is commonly referred to when talking about the average of a data set
- how to find the mean from both a single data set and also a set of grouped data
- what the median of a data set is and how to find it for a given set of data
- what the mode of a data set is and how to find it for a given set of data
- how to choose the 'best' type of average for a given set of data.

7 Probability

You will use probability regularly in your day-to-day life:

- Should you take an umbrella out with you today?
- What are the chances of the bus being on time?
- How likely are you to meet your deadline?

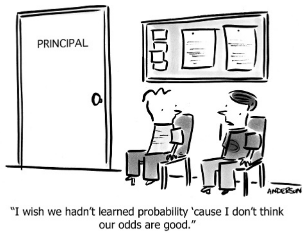


Figure 26 Probability – the odds are you are already using it

Probability is all about how likely, or unlikely, something is to happen. When you flip a coin for example, the chances of it landing on heads is $\frac{1}{2}$ or 50% or 0.5 (do you remember from your work in Session 1 how fractions, decimals and percentages can be converted into one another?).

The probability, or likelihood, of an event happening is perhaps most easily expressed as a fraction to begin with. Then, if you want to express it as a percentage or a decimal you can just convert it.

Let's look at an example.

Example: Chocolate probability

A box of chocolates contains 15 milk chocolates, 5 dark chocolates and 10 white chocolates. If the box is full and you choose a chocolate at random, what is the likelihood of choosing a dark chocolate?

Method

There are 5 dark chocolates in the box. There are 15 + 5 + 10 = 30 chocolates in the box altogether.

The probability of choosing a dark chocolate is therefore:

$$\frac{5}{30} = \frac{1}{6}$$

You could also be asked the probability of choosing either a dark or a white chocolate. For this you just need the total of dark and white chocolates:

5 + 10 = 15

The total number of chocolates in the box remains the same so the likelihood of choosing a dark or a white chocolate is:

$$\frac{15}{30} = \frac{1}{2}$$

You could even be asked what the likelihood is of an event not happening. For example, the likelihood that you will not choose a white chocolate. In this case, the total number of chocolates that are not white is 15 + 5 = 20.

Again, the total number of chocolates in the box remains the same and so the probability of not choosing a white chocolate is:

$$\frac{20}{30} = \frac{2}{3}$$

Now have a go yourself by completing the short activity below.

Activity 17: Calculating probability

1. You buy a packet of multi-coloured balloons for a children's party. You find that there are 26 red balloons, 34 green balloons, 32 yellow balloons and 28 blue balloons.

   You take a balloon out of the packet without looking. What is the likelihood of choosing a green balloon?

   Give your answer as a fraction in its simplest form.
2. At the village fete, 350 raffle tickets are sold. There are 20 winning tickets. What is the probability that you will not win the raffle?

   Give your answer as a percentage rounded to two decimal places.

Answer

1. There are 34 green balloons. The total number of balloons is 26 + 34 + 32 + 28 = 120.

   The likelihood of choosing a green balloon is therefore:

   $\frac{34}{120} = \frac{17}{60}$ in its simplest form.
2. If there are 20 tickets that are winning ones, there are 350 − 20 = 330 tickets that will not win a prize.

   As a fraction this is: $\frac{330}{350}$

   To convert to a percentage, you do 330 ÷ 350 × 100 = 94.29% rounded to two d.p.

You sometimes need to calculate the probability of more than one event occurring. In this case, the events might be:

- **independent** – which means that the outcome of one event does not affect the other
- **dependent** – which means that the outcome of one event does affect the other.

In either case, you can use **tree diagrams** or **tables** to help you to picture and solve these problems.

Example: Gender probability

If a couple has two children, the gender of the first child will not affect the gender of the second child. All possibilities can be shown in the form of a table:

B = boy
G = girl

Table 27 First and second child gender probability table

		First child	
		B	G
Second child	B	BB	GB
	G	BG	GG

Alternatively, it can be shown as a tree diagram:

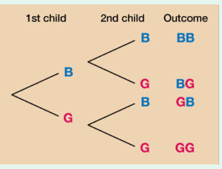


Figure 27 A tree diagram showing first and second child gender outcomes

Both the table and the tree diagram show that there are four possibilities:

1. **BB** – boy then another boy
2. **BG** – boy then a girl
3. **GB** – girl then a boy
4. **GG** – girl then another girl

Out of the possibilities there is 1 in 4 or $\frac{1}{4}$ (1 quarter/25%) chance of having 2 boys or 2 girls.

There is a 2 in 4 or $\frac{2}{4}$ ($\frac{1}{2}$/1 half/50%) chance of having one child of each gender.

Activity 18: Using diagrams and tables to calculate probability

1. Complete the missing details in the following tree diagram:

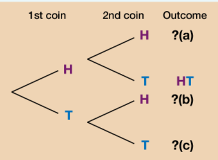


Figure 28 A tree diagram showing first and second coin toss outcomes

2. Draw a table showing all of the possibilities when 2 coins are tossed.
3. What is the probability of getting 2 tails?

Answer

1.
    a. HH
    b. TH
    c. TT
2. Your table should look like the one below.

Table 28 Coin toss probability table

		First coin	
		H	**T**
Second coin	**H**	**HH**	**TH**
	T	**HT**	**TT**

3. Out of the possibilities there is 1 in 4 or $\frac{1}{4}$ (1 quarter/25%) chance of getting 2 tails.

Remember that when you are checking your answers, you may have gone about the question in a different way. In a real exam it is always important to show your working as even if you don't arrive at the correct answer you can still gain marks.

You've now completed Session 4 of your course, congratulations!

Summary

In this section you have learned:

- that the probability of an event is how likely or unlikely that event is to happen and that this can be expressed as a fraction, decimal or percentage.

- how to use a table or tree diagram to show the different outcomes of two or more events.

8 End-of-course quiz

Now it's time to complete end-of-course quiz (Session 4 compulsory badge quiz). It's similar to previous quizzes, but in this one there will be 15 questions.

End-of-course quiz

Open the quiz in a new window or tab then come back here when you're done.

Remember, this quiz counts towards your badge. If you're not successful the first time, you can attempt the quiz again in 24 hours.

9 Session 4 summary

Well done! You have now completed ‘Handling data’, the fourth and final session of the course. If you have identified any areas that you need to work on, please ensure you refer back to this section of the course.

You should now be able to:

- identify different types of data
- create and use tally charts, frequency tables and data collection sheets to record information
- draw and interpret bar charts, pie charts and line graphs
- understand there are different types of averages and be able to calculate each type
- understand that probability is about how likely an event is to happen and the different ways that it can be expressed.

All of the skills listed above will help you when booking a holiday, budgeting, reading the news or analysing data at work.

You are now ready to test the knowledge and skills you’ve learned throughout each section in the end-of-course quiz (Session 4 compulsory badge quiz). Good luck!

10 Bringing it all together

Congratulations on completing *Everyday maths 2*. We hope you have enjoyed the experience and now feel inspired to develop your maths skills further.

Throughout this course you have developed your skills within the following areas:

- understanding and using whole numbers and decimal numbers, and understanding negative numbers in the context of money and temperature
- solving problems requiring the use of the four operations and rounding answers to a given degree of accuracy
- understanding and using equivalences between common fractions, decimals and percentages
- working out simple and more complex fractions and percentages of amounts
- calculating percentage change
- adding, subtracting, multiplying and dividing decimals up to three decimal places
- solving ratio problems where the information is presented in a variety of ways
- understanding the order of operations and using this to work with formulas
- solving problems requiring calculations with common measures, including money, time, length, weight, capacity and temperature
- converting units of measure in the same system and those in different systems
- extracting and interpreting information from tables, diagrams, charts and graphs
- collecting and recording discrete data, and organising and representing information in different ways
- finding the mean, median, mode and range for sets of data
- using data to assess the likelihood of an outcome and expressing this in different forms
- working with area, perimeter and volume, scale drawings and plans.

11 Next steps

If you would like to achieve a more formal qualification, please visit one of the centres listed below with your OpenLearn badge. They'll help you to find the best way to achieve the Level 2 Essential Skills Wales qualification in Application of Number, which will enhance your CV.

- **Coleg Cambria** • https://www.cambria.ac.uk/ • 0300 30 30 007
- **Addysg Oedolion Cymru | Adult Learning Wales** • https://www.adultlearning.wales/ • 03300 580845
- **Coleg Gwent** • https://www.coleggwent.ac.uk/ • 01495 333777
- **NPTC Group of Colleges** • https://www.nptcgroup.ac.uk/

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