

**MU120\_4M1   Open Mathematics**

**Numbers, units and arithmetic**

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## Introduction

This free course, Numbers, units and arithmetic, will help you to revise whole numbers, decimals and fractions, both positive and negative. You should know when and how to add, subtract, multiply and divide. Although you may have a calculator, you still need to do simple calculations in your head or on paper. Such calculations could be to estimate the size of an answer, as a check for your calculator work.

This OpenLearn course provides a sample of level 1 study in [Mathematics](http://www.open.ac.uk/courses/find/mathematics?utm_source=openlearn&utm_campaign=ol&utm_medium=ebook).

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## Learning outcomes

After studying this course, you should be able to:

* write whole numbers and decimals in place-value columns and compare their sizes
* multiply and divide whole numbers and decimals by 10, 100, 1000 and so on
* indicate given fractions on a diagram and find equivalent fractions for a given fraction
* mark numbers on a number line
* choose appropriate units for a given purpose.

## 1 The size of numbers

Numbers are used to specify a quantity or amount. For example, people give their ages as a number of years: ‘I am 51 years old’ or ‘I am five and a half’. Votes in elections may be described in thousands. Temperatures are measured in positive and negative degrees Celsius. Numbers smaller than one may be expressed as fractions or decimals. This section considers whole numbers, decimals, fractions and negative numbers and reminds you of the essence of a number – how ‘big’ it is.

## 1.1 Whole numbers

Whole numbers arise from counting: for example the number of sheep in a field or the number of votes in an election.

Start of Box

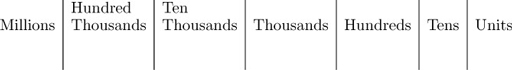
Our everyday number system is the decimal system, where the position of a digit within the number determines whether it represents units, tens, hundreds, thousands etc.

End of Box

For example, the number 1375 means one thousand three hundred and seventy-five. The position of the 3, third from the right, means that it represents 3 hundreds.

A thousand thousand, 1 000 000, is called a million and a thousand million, 1 000 000 000, is called a billion. (A British billion used to be a million million, but now the US convention of a thousand million is normally used.) To compare two numbers, it sometimes helps to write them, or think of them, in columns:

Start of Figure



End of Figure

Start of Box

The position of a digit in the columns is called its **place value**.

End of Box

Start of Example

**Example 1**

A lottery organiser announces that this week’s winnings will be over two million pounds. After the draw, the organisers announce that the winnings were £2 201 995. Was the announcement correct?

The issue is whether the figures represent a number greater than two million, or not. Write the two numbers in a number column table. Start with two million.

[View answer - Example 1](" \l "Session1_Answer1)

End of Example

### 1.2.1 Try some yourself

Start of Activity

**Activity 1**

Start of Question

Can you answer the following questions:

* (a) Write ‘twenty thousand one hundred and forty-four’ as a number.
* (b) Say (or write) the number 31 002 103 in words.

End of Question

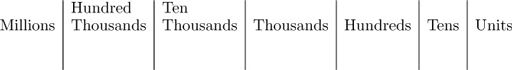
[View answer - Activity 1](" \l "Session1_Answer2)

End of Activity

## 1.2 Using the place value

Writing numbers in place-value columns gives an easy way of multiplying by 10, 100, 1000 and so on. Have a look at the place value table again.

Start of Figure



End of Figure

Notice that moving all the digits one column to the left in the table (and adding a zero to the units column) is the same as multiplying by ten; for instance, ten tens are a hundred, ten hundreds are a thousand. If you move two columns to the left, you multiply by a hundred; for instance, a hundred tens are a thousand. Moving three columns to the left is equivalent to multiplying by a thousand, and so on.

Start of Example

**Example 2**

After local government re-organisation a new council expects to receive cash payments in ten instalments from about 26 000 households. In order to negotiate the cash collection at post offices it needs to know how many payments are made altogether in a year. How many are there likely to be?

[View answer - Example 2](" \l "Session1_Answer3)

End of Example

### 1.3.1 Try some yourself

Start of Activity

**Activity 2**

Start of Question

Write each of the following three numbers in numerals and then place them in ascending order:

* eight hundred and eight thousand
* two million and twenty-four
* nine thousand nine hundred and ninety-eight

End of Question

[View answer - Activity 2](" \l "Session1_Answer4)

End of Activity

Start of Activity

**Activity 3**

Start of Question

Multiply 2490 by 100.

End of Question

[View answer - Activity 3](" \l "Session1_Answer5)

End of Activity

Start of Activity

**Activity 4**

Start of Question

Now try to answer these questions:

* (a) Place the following numbers of votes in an election in order, with the smallest first:

  9 999,   5 001,   49 020

* (b) Multiply 9 494 by 100.

End of Question

[View answer - Activity 4](" \l "Session1_Answer6)

End of Activity

## 1.3 Number lines

It is often useful in mathematics to think of numbers stretched out along the imaginary **number line**. The diagram below shows part of the number line.

Start of Media Content

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End of Media Content

Start of Box

As you move to the right along the number line, the numbers increase.

End of Box

This concept will be useful for picturing other numbers, as well as whole numbers.

## 1.4 Decimals

Quantities can be smaller than one (such as 0.5 kg) or take values between whole numbers (such as a height of 1.65 metres). Numbers smaller than one are expressed as decimals or as fractions. Decimals are often easier to work with (especially when using a calculator). Decimals are explained in this section, and fractions following that (Section 1.7).

Decimals can be indicated on the number line in between whole numbers. 0.5 and 1.65 are indicated on the figure below.

Start of Figure

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_126d9116029a039cc72977cf38bf03489c78cb4f_mu120_ai010i.jpg

End of Figure

Start of Box

Decimals arise when whole numbers are divided by ten, a hundred etc.

End of Box

Start of Example

**Example 3**

What is 3579 cents in dollars? (There are 100 cents in a dollar ($).)

[View answer - Example 3](" \l "Session1_Answer7)

End of Example

### 1.4.1 Try some yourself

Start of Activity

**Activity 5**

Start of Question

When dividing by 1000, move the digits 3 places to the right (past the decimal point). Divide 202.15 by 1000. What is 202.15 metres in kilometres?

End of Question

[View answer - Activity 5](" \l "Session1_Answer8)

End of Activity

## 1.5 Decimals and place value

What do the digits to the right of the decimal point mean? The first column to the right of the decimal point is obtained by dividing the units by 10; this is the ‘tenths’ column. Similarly the next column contains the hundredths, and so on. So here is the extended place value table:

Start of Figure

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_503665df59ec806dd940362be2f96250cccf5ecc_mu120_ai012i.jpg

End of Figure

A tenth is 0.1 or 1 divided by 10, i.e. D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_5fd6d36e464f727dc3ec6423bafd7479f5070e95_mu120_a_i015e.gif.

A hundredth is 0.01 or 1 divided by 100, i.e. D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_8911aaf9a23a0201bdc7e6f2a8e50e37baa3d074_mu120_a_i016e.gif.

A thousandth is 0.001 or 1 divided by 1000, i.e. D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_18401a15620de2d385f5e2a8bc40f0012467d789_mu120_a_i017e.gif.

So 0.101 is one tenth plus one thousandth or D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_e2586c44103edafd0e37a3bae7c6b6ac9662a326_mu120_a_i018e.gif.

Notice that you need to include the zeros after the decimal point in numbers like 0.05 or 0.005, just as you do before the decimal point in numbers like 50 or 500, in order to indicate the place value of the 5. Also, note that often there is a 0 before the decimal point for numbers smaller than one. Whether it is written as 0.1 or .1 the value is still the same (one tenth). 0.1 is more usual when writing, .1 is more usual on calculators. In general, adding the zero before the decimal point is a clearer way of writing the number – the decimal point will then not be missed.

Start of Example

**Example 4**

Two people go to a ‘pick your own’ orchard and gather a large bag of apples each. They weigh the bags and find the first weighs 6.85 kg and the second weighs 6.58 kg. Which is heavier?

[View answer - Example 4](" \l "Session1_Answer9)

End of Example

### Try some yourself

Start of Activity

**Activity 6**

Start of Question

What is 370.76 grams in kilograms? There are 1000 grams in a kilogram.

End of Question

[View answer - Activity 6](" \l "Session1_Answer10)

End of Activity

Start of Activity

**Activity 7**

Start of Question

Place the following numbers of kilometres in order, with the smallest distance first:

  1.013   1.103   1.0103   1.0129

End of Question

[View answer - Activity 7](" \l "Session1_Answer11)

End of Activity

Start of Activity

**Activity 8**

Start of Question

Place the following measurements in ascending order:

0.1704 metres   0.1074 metres   0.0714 metres   0.0741 metres

End of Question

[View answer - Activity 8](" \l "Session1_Answer12)

End of Activity

## 1.6 Fractions

A fraction is written as one number over another (such as D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_7f0ffc3f3a0e8b7e70ec8bb4fa6aab31851f410e_mu120_a_i019e.gif) and means the top number divided by the bottom number. The top number, 3, is called the **numerator** and the bottom number, 10, is called the **denominator**. Whole numbers can be written as fractions with the denominator 1. Thus, 2 can be written as D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_15fd0e1448046def983a26b9326aeeee85510f2c_mu120_a_i020e.gif. The same fraction may be written in different ways, depending upon the context.

Start of Box

The top of a fraction is called the **numerator**.

The bottom of a fraction is called the **denominator**.

End of Box

Click on the tabs below and note that all of the shaded areas represent the same fraction of the circle: one third of it. So

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_8a97fe7c6a88db5c2e7f1934e7a6b3a88e023161_mu120_a_u002e.gif

End of $1

Start of Media Content

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Start of Box

Fractions which are equal in value but are written with different numerators and denominators are called **equivalent fractions**.

End of Box

You can find equivalent fractions for any given fraction by multiplying top and bottom by the same whole number.

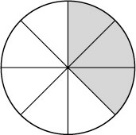
Decimals are themselves a special form of fractions (0.3 is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_7f0ffc3f3a0e8b7e70ec8bb4fa6aab31851f410e_mu120_a_i019e.gif). There are situations where it is simpler to use fractions. For example, if you wanted to divide a circle into six equal pieces, you think of the fraction D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_895ad6ae933cf87355c59db1a82f88242306e510_mu120_a_i026e.gif. There is no exact decimal equivalent of D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_895ad6ae933cf87355c59db1a82f88242306e510_mu120_a_i026e.gif (because dividing 6 into 1.0000 … produces a never-ending string of 6s: 0.1666 …). Recipes often ask for fractions, such as ‘half a pint of milk’ or a cooking time of ‘three-quarters of an hour’.

Start of Example

**Example 5**

What fraction of the circle is shaded in the diagram below?

Start of Figure



End of Figure

[View answer - Example 5](" \l "Session1_Answer13)

End of Example

### 1.6.1 Try some yourself

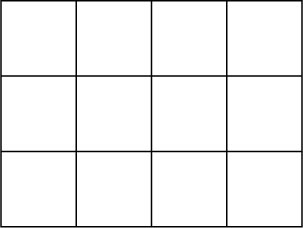
Start of Activity

**Activity 9**

Start of Question

The diagram below shows an oatmeal cake marked into 12 equal portions. I want to give my sister a third of the cake. Where could I cut the cake, and what would be left over?

Start of Figure



End of Figure

End of Question

[View answer - Activity 9](" \l "Session1_Answer14)

End of Activity

Start of Activity

**Activity 10**

Start of Question

Which two of the following fractions are equivalent to each other?

Start of $1

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End of $1

End of Question

[View answer - Activity 10](" \l "Session1_Answer15)

End of Activity

## 1.7 Simplest form of a fraction

The fraction D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_b2a4fe7cf62d3946c00e445baa1e448f5802e0e9_mu120_a_i021e.gif, is the simplest form of all its equivalent fractions, because it cannot be ‘simplified’ further (by dividing top and bottom by the same whole number called a **common factor**); D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ad0664f6bf450aef3a4db888f38cc75855c9d0d4_mu120_a_i022e.gif or D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_6f6566f5cbb7e1de6615020b49dc2dcbac84e3df_mu120_a_i023e.gif, on the other hand, can be simplified to D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_b2a4fe7cf62d3946c00e445baa1e448f5802e0e9_mu120_a_i021e.gif by dividing top and bottom by 2 and 4 respectively.

Start of Box

A fraction is in its **simplest form** if the numerator and the denominator have no **common factors**.

End of Box

6 can be written as 2 × 3. 2 and 3 are called **factors** of 6.

15 can be written as 5 × 3. 5 and 3 are **factors** of 15.

3 is a **common factor** of 6 and 15. Hence D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_15c6118151b2cecb0af00fbdf5a057af70363f57_mu120_a_si036e.gif. This is the **simplest form** of D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_95f0b441d2c53c2f1cfa519848698729896ed4f5_mu120_a_si037e.gif.

Start of Example

**Example 6**

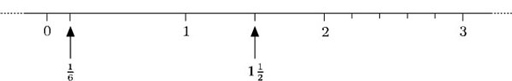
* (a) Find some equivalent fractions for D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_2bd72d1d40aefeb1c2270616bbbafff76ba68cc9_mu120_a_i032e.gif.
* (b) What is the simplest form of D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_60ceb2528138e9de2d2b01cec33314f21fae84c3_mu120_a_i033e.gif?

[View answer - Example 6](" \l "Session1_Answer16)

End of Example

You can indicate fractions along the number line by marking points between the whole numbers. The numbers D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_895ad6ae933cf87355c59db1a82f88242306e510_mu120_a_i038e.gif and D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_530e0bd6067835bd0740739abf2cb3bfb76c80b2_mu120_a_i039e.gif are shown below.

Start of Figure



End of Figure

Note D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_530e0bd6067835bd0740739abf2cb3bfb76c80b2_mu120_a_i039e.gif may also be written as 1.5 or as D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_52999cce607beb392968afb5bdf2aa5ca417844e_mu120_a_i037e.gif.

### Try some yourself

Start of Activity

**Activity 11**

Start of Question

For each of the following shapes, indicate the fraction of the whole shape represented by the shaded parts:

Start of Figure

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_6c2b4a1d8ec7fa9e5efbae61aca1a807e36a0f92_mu120_ai017i.jpg

End of Figure

End of Question

[View answer - Activity 11](" \l "Session1_Answer17)

End of Activity

## 1.8 Negative numbers

Numbers can be positive or negative, i.e. greater than or less than zero. Negative numbers have several uses; for example, to measure temperatures below zero, such as −3°C (‘minus 3 degrees Celsius’). They are also used to represent debts and overdrawn accounts: a bank balance of −£84.33 means ‘overdrawn by £84.33’.

Negative numbers are shown on the number line to the left of 0. The animation below shows −8, −7, −6, etc.

Start of Media Content

Interactive content is not available in this format.

End of Media Content

The numbers always increase as you move to the right along the number line, wherever you start from. 1 is to the right of −2, so 1 is greater than −2. 1° is warmer than −2° and a bank balance of £1 is more than one of −£2 (two pounds overdrawn).

### Try some yourself

Start of Activity

**Activity 12**

Start of Question

Copy the number line below, and mark the following points on your copy:

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_8cbcbf29d88c45b27a8ec3be958a224d34186904_mu120_a_i050e.gif

Start of Figure

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_8c9da0e1d67d57244dd76e77efa6740160cfd5bc_mu120_ai021i.jpg

End of Figure

End of Question

[View answer - Activity 12](" \l "Session1_Answer18)

End of Activity

Start of Activity

**Activity 13**

Start of Question

What are the numbers marked A, B and C on the number line below?

Start of Figure

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_c72047be39e278b96c174939706671c6b1123058_mu120_ai022i.jpg

End of Figure

End of Question

[View answer - Activity 13](" \l "Session1_Answer19)

End of Activity

### 1.9 Negative number notation

It is worth mentioning notation at this point. You may have noticed that the minus sign used to denote a negative number is shorter, closer to the number and raised, compared with the minus sign used to denote subtraction. It is important to distinguish between the two, and it can help to think of −3, say, as ‘negative 3’ rather than ‘minus 3’. In the calculation 8 − 3 (8 subtract 3), the minus sign is an operator, an instruction to subtract; in −3, the sign is part of the number. The two types of minus sign are distinguished on many calculators. If you have a calculator handy, look at the key pad to see if you have two keys. They are often labelled D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_691d92a8eef30e3eea30dcddd84b8771c0fc7996_mu120_a_i019i.jpg and D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_15cc52349d6e52db3947dc1679bc685523a6ccd0_mu120_a_i020i.jpg.

### Try some yourself

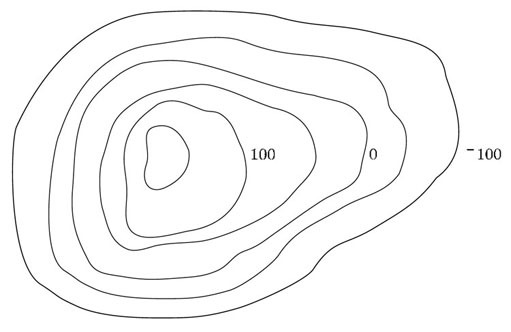
Start of Activity

**Activity 14**

Start of Question

Contour lines on a map show all the points at a given height above sea level. The lines are drawn for each height at 50-metre intervals, and points below sea level are shown by negative heights. The diagram below shows a peak with six such contour lines. The lines for −100, 0 and 100 have been labelled. Copy the diagram and label the other three contour lines.

Start of Figure



End of Figure

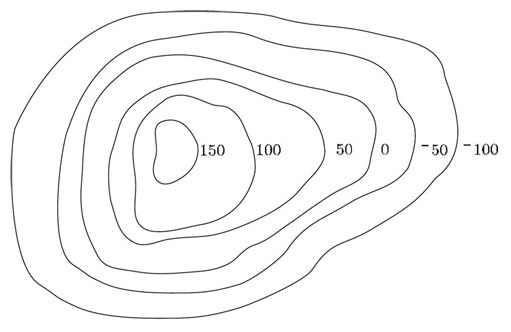
End of Question

[View answer - Activity 14](" \l "Session1_Answer20)

End of Activity

You might like to make some notes on this section, perhaps in your [Learning Journal](http://openlearn.open.ac.uk/mod/oublog/view.php?), before moving on to the next section

Start of Figure



End of Figure

## 2 Units of measurement

Numbers are used to count things (such as 54 sheep in a field) or, together with units of measurement, to quantify something (such as a 2-litre water bottle or a 3-year waiting list for an operation). This section reminds you of some units of measurement and considers appropriate units to use. We also need to make comparisons between quantities given in different units.

## 2.1 Which units to use

It is important to choose appropriate units, both to have a sense of the size you are talking about and also to avoid having to deal with very large or very small numbers.

For example using the UK decimal system, where £1 = 100p (one pound is equivalent to 100 pence), you would probably use pence for prices less than a pound: 50p rather than £0.50 and pounds for larger amounts, e.g. £2.50 instead of 250p.

For distance, mass and liquid measures, the metric system of units is often used. For example, the distance a cyclist cycles in a day is likely to be given in kilometres (sometimes written as the symbol km), a person’s height in metres (m) and a waist measurement in centimetres (cm). The prefixes ‘kilo’ and ‘centi’, together with ‘deci’ and ‘milli’, are used throughout the metric system. Some common prefixes are shown in [Table 1](#tab001_001).

Start of Table

Table 1: Prefixes used in the decimal system

|  |  |  |  |
| --- | --- | --- | --- |
| **Prefix** | **Symbol** | **Meaning** | **Factor** |
| micro- | μ | one millionth | 0.000001 |
| milli- | m | one thousandth | 0.001 |
| centi- | c | one hundredth | 0.01 |
| deci- | d | one tenth | 0.1 |
| kilo- | k | thousand | 1000 |
| mega- | M | million | 1 000 000 |
| giga- | G | thousand million | 1 000 000 000 |

End of Table

So 1 centimetre = D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_d9a6ddcd6b5f51049859d99ff19d96026019afd3_mu120_4m1_1over100.gif metre or 1 cm = D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_d9a6ddcd6b5f51049859d99ff19d96026019afd3_mu120_4m1_1over100.gif m.

This is the same as saying there are 100 centimetres in 1 metre or 100 cm = 1 m.

For mass, kilograms (kg), grams (g) or milligrams (mg) are usually used and a metric tonne is used for very large masses (1000 kg). The word ‘mass’ is scientifically more precise than ‘weight’, although people usually talk about weight rather than mass. For liquid capacity, litres (l), decilitres (dl), centilitres (cl) or millilitres (ml) are used.

### 2.1.1 Units in action

Start of Example

**Example 7**

Imagine a friend is planning a new kitchen in her house. In the kitchen showroom she noticed that the measurements of most of the kitchen units were given in millimetres. One worktop, for instance, is 575 mm deep. What units should she use to measure the large room in the house where the new kitchen will be located?

[View answer - Example 7](" \l "Session2_Answer1)

End of Example

### Try some yourself

Start of Activity

**Activity 15**

Start of Question

Suggest appropriate units for each of the following:

* (a) the age of the kitten when it is weaned;
* (b) the distance between one train station and the next;
* (c) the amount of one ‘active’ ingredient of a prescribed pill;
* (d) the cost of leaving the light on continuously for 24 hours.

End of Question

[View answer - Activity 15](" \l "Session2_Answer2)

End of Activity

Start of Activity

**Activity 16**

Start of Question

Choose appropriate units for each of the following, using metric units where possible:

* (a) your height;
* (b) your weight;
* (c) the length of time to boil an egg;
* (d) the capacity of a large pan.

End of Question

[View answer - Activity 16](" \l "Session2_Answer3)

End of Activity

## 2.2 Converting units

A great advantage of the metric system of units is that conversion between units within the system is particularly easy. For example, ‘£1 is worth 100p’ is converting one pound into pence. To convert pounds to pence, you multiply by 100. So £2 is 200p, and £2.63 is 263p. (Remember that to multiply by 100, you move the digits two places to the left in the place value table.)

To convert from pence to pounds, you need to reverse this process, i.e. to divide by 100 (moving the digits two places to the right) so that 845p becomes £8.45. In the same way, any of the metric units can be converted simply by multiplying or dividing by ten, a hundred or a thousand. For example, the statement ‘1 m = 1000 mm’ tells you that to convert from metres to millimetres you multiply by 1000.

Start of Box

The general rule when converting units is: to convert from a larger unit (e.g. metres) to a smaller one (e.g. millimetres), multiply; to convert from a smaller unit to a larger one, divide.

End of Box

Start of Example

**Example 8**

The measurements of the kitchen units in the example above were given in millimetres – a unit was 575 mm deep. What is this in metres?

[View answer - Example 8](" \l "Session2_Answer4)

End of Example

There is no handy metric system for time. There are 60 seconds in a minute, 60 minutes in an hour, 24 hours in a day and 7 days in a week; a month may be 28, 29, 30 or 31 days long and a year may contain 365 or 366 days.

### 2.2.1 Try some yourself

Start of Activity

**Activity 17**

Start of Question

A person’s height is given as 1.65 m. What is this in centimetres?

End of Question

[View answer - Activity 17](" \l "Session2_Answer5)

End of Activity

Start of Activity

**Activity 18**

Start of Question

Try the following conversions:

* (a) Express 675 mm in centimetres.
* (b) Express 45.2 km in metres.
* (c) Express 3.5 kg in grams.
* (d) Express 167.2 mm in metres.

End of Question

[View answer - Activity 18](" \l "Session2_Answer6)

End of Activity

Start of Activity

**Activity 19**

Start of Question

A kitchen unit is 1200 mm long, 600 mm high and 350 mm deep. Write these measurements in metres.

End of Question

[View answer - Activity 19](" \l "Session2_Answer7)

End of Activity

We will now provide more detail on converting units using interactive activities.

## 2.3 Measuring mass

The basic SI unit for mass is the kilogram, symbol kg

The tonne (t) which is equivalent to 1000 kg and is a metric unit is often used alongside the SI units.

The animation below illustrates how to convert between the most commonly used units of mass, the metric tonne (t); the kilogram (kg); the gram (g); the milligram (mg) and the microgram (μg).

Start of Media Content

Interactive content is not available in this format.

End of Media Content

For example, to convert 480 kg into t, divide by 1000.

So 480 kg = 480 ÷ 1000 t = 0.48 t.

Try some yourself – use the calculator to type in the amount and then choose the units you want to convert. Then choose the operation (multiply or divide) and the factor. Then select check to see if your answer is correct.

Start of Media Content

Interactive content is not available in this format.

End of Media Content

## 2.4 Measuring length

The basic SI unit for length is the metre, abbreviation m.

The animation below illustrates how to convert between the most commonly used units of length, kilometres (km); metres (m); centimetres (cm); millimetres (mm) and micrometres (μm).

Start of Media Content

Interactive content is not available in this format.

End of Media Content

For example, to convert 6.23 km into cm, multiply by 1000 to convert into m, and then mulitply by 100 to convert the measurement into cm.

So 6.23 km = 6.23 × 1000 m = 6230 m = 6230 × 100 cm = 623 000 cm.

Try some yourself – use the calculator to type in the amount and then choose the units you want to convert. Then choose the operation (multiply or divide) and the factor. Then select check to see if your answer is correct.

Start of Media Content

Interactive content is not available in this format.

End of Media Content

## 2.5 Measuring capacity

The basic metric unit for capacity is the litre, usually denoted by the symbol l (though sometimes an uppercase L is used to avoid confusion with the number 1).

In the SI system, units such as cubic metres (m3), cubic centimetres (cm3) and cubic millimetres (mm3) are used. These two systems are linked because:

Start of Quote

1 ml = 1 cm3

End of Quote

The animation below illustrates how to convert between the most commonly used units of capacity:

Start of Media Content

Interactive content is not available in this format.

End of Media Content

Notice that to convert cubic centimetres to cubic millimetres, you need to multiply by 1000, NOT 10. To see this, imagine that you have a box which has internal measurements 1 cm by 1 cm by 1 cm. So, its volume is 1 cubic centimetre.

Now imagine filling it with tiny cubes, each side of which was 1 mm, so the volume of each tiny cube is 1 mm3. You would be able to fit 10 rows each with 10 tiny cubes in the bottom of the box, that is 100 tiny cubes altogether.

But these cubes would only fill the depth of the box to 1 mm. So you would be able to get 10 layers (each with 100 tiny cubes) into the box overall.

So there would be 1000 tiny cubes in the box.

In other words 1 cm3 = 1000 mm3.

Can you explain why 1 m3 = 1 000 000 cm3?

The argument is similar to that above for cm3 and mm3, except that there are 100 cm in a m, so a cubic metre (a 1 m by 1 m by 1 m cube) has 100 by 100 by 100 little cubic cm in it.

Start of Example

**Example 9**

Convert 500 cm3 into litres.

[View answer - Example 9](" \l "Session2_Answer8)

End of Example

Try some yourself – use the calculator to type in the amount and then choose the units you want to convert. Then choose the operation (multiply or divide) and the factor. Then select check to see if your answer is correct.

Start of Media Content

Interactive content is not available in this format.

End of Media Content

## 2.6 Comparing measurements

In order to compare quantities, it is best to express them in the same units.

Start of Example

**Example 10**

Three children have just measured their own heights in metric units. Isaac says ‘My height is 1098’, Jasmine says ‘My height is 112’ and Kim says ‘Mine is 1.1’. What units were they using? Who is the tallest?

[View answer - Example 10](" \l "Session2_Answer9)

End of Example

### Try some yourself

Start of Activity

**Activity 20**

Start of Question

Place the following masses in order, with the lightest first:

  2125 mg   1.9 g   0.02 kg

End of Question

[View answer - Activity 20](" \l "Session2_Answer10)

End of Activity

Start of Activity

**Activity 21**

Start of Question

Place each of the following sets of quantities in order, smallest first.

* (a) X = 0.5 l   Y = 490 cl   Z = 510 ml
* (b) X = 20p   Y = £0.02
* (c) X = 0.039 kg   Y = 4 g   Z = 30 000 mg

End of Question

[View answer - Activity 21](" \l "Session2_Answer11)

End of Activity

## 3 Arithmetic

Although you may have a calculator at your disposal, you need to be able to carry out some calculations (usually with simpler numbers) in order to check your calculator work. Actually handling numbers, on paper or in your head, helps you to understand the nature of the calculations you perform on your calculator and gives you a deeper understanding of the underlying mathematical processes. In some cases, it really is quicker and easier to carry out a calculation by hand rather than key it into your calculator. However, do use your calculator for more cumbersome arithmetic.

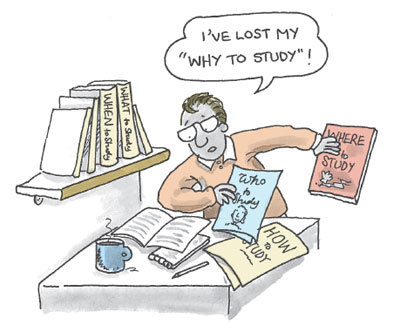
Start of Figure



End of Figure

The arithmetic system works not only for positive whole numbers but also for fractions, decimals and negative numbers. It can also be used when letters represent unknown numbers (algebra). It is therefore important to understand the principles for whole numbers, so you can extend these ideas to include different types of numbers.

Start of Figure



End of Figure

## 3.1 Arithmetic with whole numbers

When you are adding or subtracting whole numbers, an important thing to keep in mind is the **place value** of the figures. It is often a good idea to set out the numbers in columns before doing the arithmetic.

Start of Example

**Example 11**

* (a) There are 4985 people living in a village and 93 people living in a nearby hamlet. How many people live in the village and the hamlet in total?
* (b) Of the total number of people in the village and hamlet, 1352 are under voting age. How many people are eligible to vote?

[View answer - Example 11](" \l "Session3_Answer1)

End of Example

### Try some yourself

Start of Activity

**Activity 22**

Start of Question

Carry out the following calculations, without using a calculator.

* (a) A million pound lottery prize minus a three hundred pound administrative charge.
* (b) The sum of cheques for £25 and £1029, to pay into a bank account.

End of Question

[View answer - Activity 22](" \l "Session3_Answer2)

End of Activity

## 3.2 Multiplying and dividing

To multiply and divide by 10, 100, 1000, etc., write the digits in their place value columns. To multiply, move the digits to the left (replacing the numbers on the right with zeros) and to divide move them to the right (putting in a decimal point, and any zeros necessary for the place value).

Multiplication and division by whole numbers in general can be carried out by combining this technique with a knowledge of the multiplication tables up to 10.

Start of Example

**Example 12**

A machine in a factory puts tops on bottles at the rate of 1 every second.

* (a) How many bottles does it top in a 10 hour working day?
* (b) How many bottles does it top in a 30 day month, at 10 hours a day?

[View answer - Example 12](" \l "Session3_Answer3)

End of Example

### Try some yourself

Start of Activity

**Activity 23**

Start of Question

Candles are sold in boxes of 40. How many candles are there in 30 boxes?

End of Question

[View answer - Activity 23](" \l "Session3_Answer4)

End of Activity

Start of Activity

**Activity 24**

Start of Question

* **2**(a) A doctor prescribes 3 pills a day and the prescription is to last for 4 weeks. How many pills should the pharmacist supply?
* (b) Apples are sold in a pack of 7 for 95 p or singly 14 p each. Which is the better buy?

End of Question

[View answer - Activity 24](" \l "Session3_Answer5)

End of Activity

Start of Activity

**Activity 25**

Start of Question

Describe, in your own words, the effect of multiplying any number by (a) 1, (b) 0.

End of Question

[View answer - Activity 25](" \l "Session3_Answer6)

End of Activity

Start of Activity

**Activity 26**

Start of Question

Place the following times in order, shortest first:

  four weeks;  29 days;  360 hours.

End of Question

[View answer - Activity 26](" \l "Session3_Answer7)

End of Activity

## 3.3 Division

Division is probably the most awkward of the four arithmetic operations. Since you may have a calculator, you do not need to be able to carry out complicated divisions by hand, but you do need to carry out simple divisions in order to check your calculator calculations. Division is the reverse process of multiplication. The quantity 12 ÷ 3 tells us how many times 3 goes into 12. Since 4 × 3 = 12, 12 ÷ 3 = 4.

Start of Example

**Example 13**

Small candles are sold in boxes of 40. How many boxes do you need in order to have 1000 candles?

[View answer - Example 13](" \l "Session3_Answer8)

End of Example

### Try some yourself

Start of Activity

**Activity 27**

Start of Question

* (a) Divide a £27 000 jackpot prize equally among 9 people in the syndicate.
* (b) Divide a £27 000 jackpot prize equally among 900 people in the syndicate.

End of Question

[View answer - Activity 27](" \l "Session3_Answer9)

End of Activity

## 3.4 Order of calculations

You may have noticed that sometimes the order in which calculations are carried out seems to matter and sometimes it does not. When using a calculator, it is very important to know the order in which it will do calculations. It is not always the order in which you enter them.

Although written English is read from left to right, this is not the case for all written languages (Chinese is read top to bottom, right to left). With mathematics, the order of the written operations does not always indicate the order in which they should be carried out.

Start of Example

**Example 14**

* (a) Multiply 3 by 365. How many days are there in 3 years?
* (b) Divide 366 by 3 to find out how many days there are per term, in a 3-term leap year.
* (c) Does it matter if you interchange the numbers in each of the sums you did above?

[View answer - Example 14](" \l "Session3_Answer10)

End of Example

To see that dividing 3 by 366 is not the same as dividing 366 by 3 note the following:

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_222c570fac2e83d055e7d29c5f53c799db29b2df_mu120_a_i089e.gif gives a value of over a hundred, whereas D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_877d9158cbc07f1c69534865d3bb2722c0160ff0_mu120_a_i090e.gif is much less than 1.

Start of Box

When you multiply two numbers together, it makes no difference which number you write first.

When you divide one number by another, it does matter which number you write first.

End of Box

If you add two numbers, the order does not matter,

  3 + 2 = 2 + 3,

but the same is not true with subtraction,

  3 − 2 ≠ 2 − 3.

(≠ means ‘is not equal to’.)

Start of Box

When adding two numbers together, it makes no difference which number you write first.

When subtracting one number from another, it does matter which number you write first.

End of Box

### Try some yourself

Start of Activity

**Activity 28**

Start of Question

Look at the rules in the boxes above.

Write in your own words the rules for multiplying and dividing, adding and subtracting two numbers, giving an example of each.

End of Question

[View answer - Activity 28](" \l "Session3_Answer11)

End of Activity

Start of Activity

**Activity 29**

Start of Question

Is the following statement true or false?

When adding three numbers together, it doesn’t matter in which order you add them.

End of Question

[View answer - Activity 29](" \l "Session3_Answer12)

End of Activity

## 3.5 Several calculations and using brackets

Sometimes you may want to make several calculations in succession, and the order in which the calculations are performed may or may not be significant. For example, if you want to add 12 + 7 + 13, it makes no difference which of these two processes you adopt:

add the 12 and 7 first, to give 19, and then the 13, to give 32;

or

add the 7 and 13 first, to give 20, and then add this to 12 to give 32 again.

Start of Box

Brackets can be used in a calculation to mean ‘do this first’.

End of Box

Although (7 + 13) + 12 is the same as 7 + (13 + 12), sometimes the order of calculation does make a difference:

  (7 − 12) + 13 is not the same as 7 − (12 + 13).

Start of Example

**Example 15**

Calculate (7 + 3) × 2 and 7 + (3 × 2).

[View answer - Example 15](" \l "Session3_Answer13)

End of Example

### Try some yourself

Start of Activity

**Activity 30**

Start of Question

Carry out the following calculations, without your calculator.

* (a) 3 × (60 + 70).
* (b) (3 × 60) + 70.
* (c) (70 − 60) ÷ 5.

Check your answers using your calculator.

End of Question

[View answer - Activity 30](" \l "Session3_Answer14)

End of Activity

### 3.5.1 With or without brackets

Brackets are not always given in a calculation, and there are rules which tell you in which order to do the calculations in the absence of brackets. The following is the order that scientific calculators normally use.

Start of Box

**Calculations are performed in the following order:**

Brackets;

Powers (e.g. squaring or cubing a number);

Division and Multiplication (performed in the order written, left to right);

Addition and Subtraction (performed in the order written, left to right).

End of Box

If you have a calculator handy, check that it follows these rules.

Start of Example

**Example 16**

Calculate 10 + 3 × 7.

[View answer - Example 16](" \l "Session3_Answer15)

End of Example

Sometimes brackets are implied in division. For example,

Start of $1

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End of $1

### Try some yourself

Start of Activity

**Activity 31**

Start of Question

Insert brackets in the following calculations to emphasise the order in which a scientific calculator would perform them, then do the calculations by hand and on your calculator, with and without the brackets, as a check.

* (a) 3 × 60 + 70.
* (b) 10 − 15 ÷ 5.
* (c) 20 − 2 × 8.
* (d) 3 + 16 − 10.
* (e) 3 × 10 ÷ 5.

End of Question

[View answer - Activity 31](" \l "Session3_Answer16)

End of Activity

Start of Activity

**Activity 32**

Start of Question

In which of the calculations in Question 1 does the order of the calculations make a difference?

End of Question

[View answer - Activity 32](" \l "Session3_Answer17)

End of Activity

Start of Activity

**Activity 33**

Start of Question

**3**

* (a) Calculate D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_d71fb4d01969beace2427d4c58d3944eb30c8960_mu120_a_i091e.gif
* (b) Does the calculation in part (a) represent an exception to the ‘division before addition’ rule?

End of Question

[View answer - Activity 33](" \l "Session3_Answer18)

End of Activity

Start of Activity

**Activity 34**

Start of Question

The target score in a game of darts is obtained by subtracting the score of the three darts thrown in a player’s turn from the current target score on the board. In the game of ‘301’, the target score starts at 301. If a player’s first three darts are a double 19, a single 20 and a treble 17, what is the new target score?

End of Question

[View answer - Activity 34](" \l "Session3_Answer19)

End of Activity

Start of Activity

**Activity 35**

Start of Question

Are the following statements true or false?

* (a) When carrying out an addition and a multiplication, such as 2 + 3 × 4, it doesn’t matter whether you do the addition or the multiplication first.
* (b) Putting brackets into an expression can change the order of the calculations.

End of Question

[View answer - Activity 35](" \l "Session3_Answer20)

End of Activity

## 3.6 Arithmetic with decimals

Arithmetic with decimals is much the same as arithmetic with whole numbers, but continue to take care with place value.

### 3.6.1 Addition and subtraction with decimals

Addition and subtraction can be carried out with decimals just as for whole numbers. Take care to write the numbers in their correct columns and keep the decimal points in line under each other.

Start of Example

**Example 17**

* (a) Add 303.035 kg and 7.77 kg.
* (b) Subtract 7.77 kg from 303.035 kg.

[View answer - Example 17](" \l "Session3_Answer21)

End of Example

### Try some yourself

Start of Activity

**Activity 36**

Start of Question

Without using your calculator, find the following:

* (a) 100.001 + 10.1
* (b) 100.001 − 10.1

End of Question

[View answer - Activity 36](" \l "Session3_Answer22)

End of Activity

Start of Activity

**Activity 37**

Start of Question

Suppose you went shopping with a friend and your combined bill came to £85. You knew that you had spent £47.65. What was your friend’s share of the bill?

End of Question

[View answer - Activity 37](" \l "Session3_Answer23)

End of Activity

## 3.7 Multiplying and dividing with decimals

Multiplication and division of decimal numbers are carried out just as with whole numbers, except that now you can carry out the division even when one number does not divide exactly into the other. You also need to take care about the position of the decimal point. So check your answer is sensible.

Start of Example

**Example 18**

* (a) If somebody’s hair grows at the rate of 0.4 cm a week, how much will it grow in 52 weeks or one year?
* (b) How long would it take to grow their hair 10 cm?
* (c) How long would it take to grow their hair 3 cm?

[View answer - Example 18](" \l "Session3_Answer24)

End of Example

### Try some yourself

Start of Activity

**Activity 38**

Start of Question

Evaluate each of the following:

* (a) 7.9 ×0.8
* (b) 82.3 × 40
* (c) 7.20 ÷ 0.8
* (d) 62.30 ÷ 40

End of Question

[View answer - Activity 38](" \l "Session3_Answer25)

End of Activity

Start of Activity

**Activity 39**

Start of Question

Use your answer to question 1 above to evaluate the following.

* (a) Special cheese is £7.90 per kilo. How much does 0.8 kilos cost?
* (b) Petrol is 82.3p per litre. How much does 40 litres cost?
* (c) The till receipt showed the cheese cost £7.20, and you bought 0.8 kilos. How much was it per kilo?
* (d) The bill was £62.30 for 40 small children’s party hats. How much were they each? Was there an error in the bill or a discount?

End of Question

[View answer - Activity 39](" \l "Session3_Answer26)

End of Activity

## 3.8 Order of decimal calculations

The same rules about the order of calculations apply to decimals as apply to whole numbers.

Start of Box

**Calculations are performed in the following order:**

Brackets;

Powers (e.g. squaring or cubing a number);

Division and Multiplication (performed in the order written, left to right);

Addition and Subtraction (performed in the order written, left to right).

End of Box

Start of Example

**Example 19**

Calculate 3.5 + 0.7 × 8 ÷ (4.6 − 2.6).

[View answer - Example 19](" \l "Session3_Answer27)

End of Example

### Try some yourself

Start of Activity

**Activity 40**

Start of Question

Without using your calculator, find the following.

* (a) 75.6 ÷ 0.6
* (b) 75.6 × 0.6
* (c) 100.001 + 75.6 ÷ 0.6
* (d) (100.001 + 10.1) × 60

End of Question

[View answer - Activity 40](" \l "Session3_Answer28)

End of Activity

Start of Activity

**Activity 41**

Start of Question

Try dividing 10 by 3, first without using your calculator, and giving your answer as a decimal. What is the difficulty? What answer does the calculator give?

End of Question

[View answer - Activity 41](" \l "Session3_Answer29)

End of Activity

Start of Activity

**Activity 42**

Start of Question

Is the following statement true or false?

Multiplying one number by a second number always gives an answer greater than the first number.

End of Question

[View answer - Activity 42](" \l "Session3_Answer30)

End of Activity

## 3.9 Addition and subtraction with fractions

Addition and subtraction with fractions can be quite awkward, particularly when fractions are mixed with whole numbers and when the fractions have different denominators. In such cases you may need to use your calculator. However try to gain confidence in handling simple fractions.

To add two fractions with the same denominator, just add the numerators. For example,

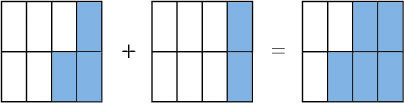
Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_527b41c03b4b8c795e1e87d015f4db800b53bdfa_mu120_a_u007e.gif

End of $1

Just as 3 eggs plus 2 eggs is 5 eggs, so 3 eighths plus 2 eighths is 5 eighths.

Start of Figure



End of Figure

To subtract one fraction from another, where the denominators are the same, just subtract one numerator from the other. For example,

Start of $1

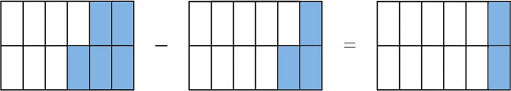
D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_7c76abc851f018643015bd1f05ad001f143842f2_mu120_a_u008e.gif

End of $1

Just as 5 oranges minus 3 oranges is 2 oranges, so 5 twelfths minus 3 twelfths is 2 twelfths,

but D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_cff2c5fa074f5685d8046aea19cf528bfcdeebd2_mu120_a_i097e.gif is the same as D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_45d5eb36a9cc8ed99a534a3d203c779a41d0c803_mu120_a_i098e.gif. So the simplest answer is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_45d5eb36a9cc8ed99a534a3d203c779a41d0c803_mu120_a_i098e.gif.

Start of Figure



End of Figure

However, if the denominators are different, the calculation is a bit more awkward. The first thing to do is to rewrite the fractions as equivalent fractions where the denominators are the same.

**A useful mathematical technique with things you can’t immediately handle is to put them in a form where you can handle them.**

Start of Example

**Example 20**

* Evaluate
* D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_21638a1c023fd1c9958ceea01ae262cb27d05d19_mu120_a_i100e.gif.

[View answer - Example 20](" \l "Session3_Answer31)

End of Example

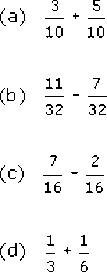
### Try some yourself

Start of Activity

**Activity 43**

Start of Question

Evaluate each of the following.

* 

End of Question

[View answer - Activity 43](" \l "Session3_Answer32)

End of Activity

## 3.10 More difficult fraction sums

Sometimes you have to find equivalent factions for all fractions in the sum in order that the denominators are the same.

Start of Box

To add or subtract fractions, use equivalent fractions to make the denominators the same and then add or subtract the numerators.

End of Box

Start of Example

**Example 21**

Evaluate D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_2822e22634a94fdc96505b86ecfd21b18577b815_mu120_a_i111e.gif.

[View answer - Example 21](" \l "Session3_Answer33)

End of Example

### Try some yourself

Start of Activity

**Activity 44**

Start of Question

Is the following statement true or false?

Adding the same number to the top and bottom of a fraction gives you an equivalent fraction.

End of Question

[View answer - Activity 44](" \l "Session3_Answer34)

End of Activity

Start of Activity

**Activity 45**

Start of Question

Explain in your own words how to add two fractions, as if to someone who does not know how to do it.

End of Question

[View answer - Activity 45](" \l "Session3_Answer35)

End of Activity

## 3.11 Multiplication of fractions

Multiplication by a whole number is really just repeated addition, 4 × 3 is 4 times 3:

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_7256ce07633d33319f206bdcdb1160c837756029_mu120_a_u010e.gif

End of $1

In the same way,

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_011257e9cc1e7eeab1d14777b57ccbf75d450a2d_mu120_a_u011e.gif

End of $1

**Just add the numerators since the denominators are the same.**

You may have noticed that the answer is the same as if you had multiplied the numerator of D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_6e098b9f3c2885d58794d49c7c1f3b134082d91d_mu120_a_i115e.gif by 4:

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_623f34268ba35fda78668892366f44acb1d925ce_mu120_a_u012e.gif

End of $1

Now look at an example where two fractions are multiplied.

Start of Example

**Example 22**

Isaac and Jasmine are helping themselves to some pizza. Isaac takes D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_57876b0497f91b8ee4621d249266171dead70b55_mu120_a_i116e.gif of the pizza and gives D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ac422cfa44c77f5c79e9faf8a1ed703e827c1f4e_mu120_a_i069e.gif of this portion to Jasmine. How much pizza does Jasmine get?

[View answer - Example 22](" \l "Session3_Answer36)

End of Example

Notice that in the example above, when multiplying two fractions together you could just multiply the numerators and multiply the denominators, i.e.

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_c66cdc7253a8d945fd5bc1dd254683ebbd9dc733_mu120_a_u014e.gif

End of $1

This is true in general.

To give the answer in its simplest form, divide top and bottom by the common factor 3 to give

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_d8d5142837f7a42e11ec40ab664cb6062fe6af46_mu120_a_u015e.gif

End of $1

You can confirm from the diagram above that Jasmine’s share is a quarter of the whole pizza. There is a quicker way of getting to the answer in its simplest form, known as **cancelling**. Look again at the fraction

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_e336a5f1d91393ce27aee906336914f0850941f9_mu120_a_u016e.gif

End of $1

Divide top and bottom by 3, and show this by crossing off, or cancelling, the threes,

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_e6d15fbec96b06b1ee79b033742108f173d4f7f4_mu120_a_u017e.gif

End of $1

Start of Box

To multiply fractions, multiply the denominators together and the numerators together, cancelling any **factors** which the numerators and denominators have in common.

End of Box

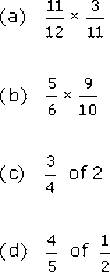
### Try some yourself

Start of Activity

**Activity 46**

Start of Question

Evaluate each of the following.

* 

End of Question

[View answer - Activity 46](" \l "Session3_Answer37)

End of Activity

Start of Activity

**Activity 47**

Start of Question

A school contains 720 pupils. D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_5b11d78a2be85724b863db5e51d0ced41093580a_mu120_a_i169e.gif are girls. How many girls are there in the school?

End of Question

[View answer - Activity 47](" \l "Session3_Answer38)

End of Activity

Start of Activity

**Activity 48**

Start of Question

If you divide a cake into 12 equal pieces and then eat 3 pieces, what fraction of the cake is left?

End of Question

[View answer - Activity 48](" \l "Session3_Answer39)

End of Activity

Start of Activity

**Activity 49**

Start of Question

A recipe specifies a cooking time of D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_b4f59de9d34fd4f1454dd6910a4973337674a90e_mu120_a_i059e.gif hour and suggests checking and basting D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_28e45ab7c57b572a6ea4161bc0c31d671a7c1cca_mu120_a_i060e.gif of the way through the cooking time. After how long should you check it?

End of Question

[View answer - Activity 49](" \l "Session3_Answer40)

End of Activity

## 3.12 Division by fractions

Before considering division of fractions, it is helpful to think about division of whole numbers.

6 ÷ 2 asks for the number of twos in 6: 6 ÷ 2 = 3, since three twos are six (3 × 2 = 6).

In a similar way, 6 ÷ D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_dff3d8f7bc0951b88b254c3b7381f2a0d330786c_mu120_a_i081e.gif is asking for the number of halves in 6. Suppose a friend is making salad decorations for plates of sandwiches. He has 6 tomatoes and wants to know how many half tomatoes this will give. This will be 6 ÷ D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_dff3d8f7bc0951b88b254c3b7381f2a0d330786c_mu120_a_i081e.gif So think of the 6 as 6 whole tomatoes.

Start of Media Content

Interactive content is not available in this format.

End of Media Content

Each tomato contains two half-tomatoes, so 6 tomatoes contain 6 × 2 half-tomatoes. Hence

Start of $1

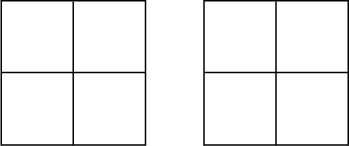
D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_3d61a72671c7c46b517a2bc224e7f55c887507ba_mu120_a_u018e.gif

End of $1

Thus dividing by D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_dff3d8f7bc0951b88b254c3b7381f2a0d330786c_mu120_a_i081e.gif is the same as multiplying by 2.

Similarly 2 ÷ D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_920eaded157992e7ff463a17c81537445e0e7a31_mu120_a_i129e.gif is asking for the number of quarters in 2. Think of two cakes.

Start of Figure



End of Figure

Each cake contains 4 quarters: the two cakes contain 8 quarters.

Expressed in figures this is

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_be453f2bc9ce7272eaed0593683a529966607166_mu120_a_u019e.gif

End of $1

Hence dividing by D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_920eaded157992e7ff463a17c81537445e0e7a31_mu120_a_i129e.gif is the same as multiplying by 4.

This illustrates the rule for dividing by fractions of the form D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_e100ee46baa7264890e76a770b76c834f1fcee4f_mu120_a_i131e.gif.

Start of Box

To divide by D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_e100ee46baa7264890e76a770b76c834f1fcee4f_mu120_a_i131e.gif, multiply by n (n can be any number except zero).

End of Box

### Try some yourself

Start of Activity

**Activity 50**

Start of Question

Evaluate 3 ÷ D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_b2a4fe7cf62d3946c00e445baa1e448f5802e0e9_mu120_a_i021e.gif

End of Question

[View answer - Activity 50](" \l "Session3_Answer41)

End of Activity

## 3.13 More division with fractions

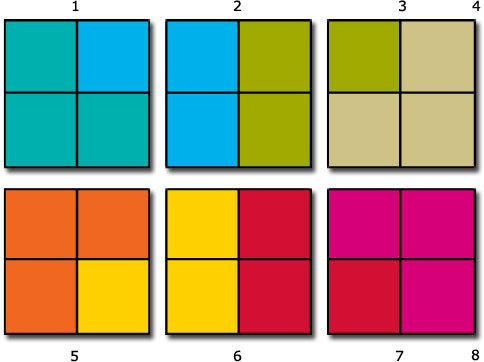
How about other fractions? What is 6 ÷ D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ac422cfa44c77f5c79e9faf8a1ed703e827c1f4e_mu120_a_i133e.gif? This means how many D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ac422cfa44c77f5c79e9faf8a1ed703e827c1f4e_mu120_a_i133e.gif are there in 6? Consider six squares, divided into quarters, and count how many groups of three quarters there are.

Start of Activity

**Activity 51**

Start of Question

Start of Figure



End of Figure

End of Question

[View answer - Activity 51](" \l "Session3_Answer42)

End of Activity

Three quarters is 3 × D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_920eaded157992e7ff463a17c81537445e0e7a31_mu120_a_i134e.gif? So dividing by D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ac422cfa44c77f5c79e9faf8a1ed703e827c1f4e_mu120_a_i133e.gif is the same as dividing by 3 and by D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_920eaded157992e7ff463a17c81537445e0e7a31_mu120_a_i134e.gif. Dividing by D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_920eaded157992e7ff463a17c81537445e0e7a31_mu120_a_i134e.gif is multiplying by 4. So dividing by D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ac422cfa44c77f5c79e9faf8a1ed703e827c1f4e_mu120_a_i133e.gif is the same as dividing by 3 and multiplying by 4, which is multiplying by D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_512b13f393fe41b00de108b964091e66c251b77d_mu120_a_i139e.gif. So 6 ÷ D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ac422cfa44c77f5c79e9faf8a1ed703e827c1f4e_mu120_a_i133e.gif = 6 × D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_512b13f393fe41b00de108b964091e66c251b77d_mu120_a_i139e.gif = D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_176793bd81841856ec0dd7d06a05b57ab98cf753_mu120_a_i142e.gif = 8.

The general rule for dividing by a fraction can be stated as follows.

Start of Box

To divide by a fraction, turn the fraction upside down and multiply.

End of Box

Does this also work for dividing by D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_e100ee46baa7264890e76a770b76c834f1fcee4f_mu120_a_i131e.gif? Yes, because multiplying by n is the same as multiplying by D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_1677ba1e3a23ab08bc0a37f624865433787e81ee_mu120_a_i144e.gif. It also works for division of whole numbers: dividing by 4 (= D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_0fe5ec4b43cb74d2d16e55c035a3cfd3f928eed3_mu120_a_i145e.gif) is the same as multiplying by D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_920eaded157992e7ff463a17c81537445e0e7a31_mu120_a_i134e.gif.

Start of Example

**Example 23**

Evaluate:

* D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_cc32e24c81ae419d9efe310bc73733444258f0d4_mu120_a_i147e.gif

[View answer - Example 23](" \l "Session3_Answer43)

End of Example

Turning a fraction upside down is called finding its **reciprocal**.

The reciprocal of D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_7e73999aa93a467b4aac77984479eb4e0f275653_mu120_a_i155e.gif is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_f1cc1319a833ed1ef5bfbbabf071e47e15a32982_mu120_a_i156e.gif. The reciprocal of D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_45d5eb36a9cc8ed99a534a3d203c779a41d0c803_mu120_a_i157e.gif is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_e08a988592d7ea3de508e74dac37cb690d3cec76_mu120_a_i158e.gif, i.e. 6.

So an alternative formulation of the rule for dividing by a number can be stated as follows.

Start of Box

Division by a number is the same as multiplication by its reciprocal.

End of Box

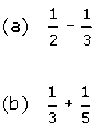
### Try some yourself

Start of Activity

**Activity 52**

Start of Question

Evaluate each of the following.

* 

End of Question

[View answer - Activity 52](" \l "Session3_Answer44)

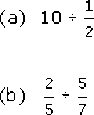
End of Activity

Start of Activity

**Activity 53**

Start of Question

Evaluate each of the following, giving each answer in its simplest form.

* 

End of Question

[View answer - Activity 53](" \l "Session3_Answer45)

End of Activity

Start of Activity

**Activity 54**

Start of Question

Explain how to multiply fractions, to somebody who does not know how to do it.

End of Question

[View answer - Activity 54](" \l "Session3_Answer46)

End of Activity

## 3.14 Mixed numbers

In order to do arithmetic with mixed numbers like D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_91443372b9e74dc758bf4f8bb8d3de54338e23d3_mu120_4m1_ie001i.gif, it is often best to write them as a simple fraction, that is, one number over another.

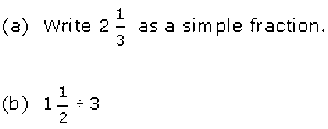
Start of Box

Multiplication or division involving mixed numbers is done by converting them to **fractions** first.

End of Box

Start of Example

**Example 24**

* 

To enter D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_3809e79a630fdb7c35df001d4f4757e20a87617d_mu120_a_i105e.gif on a calculator, you can input D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_c62f7969f1a15e42d158f5706bd795ece1d839c1_mu120_a_i106e.gif.

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_3809e79a630fdb7c35df001d4f4757e20a87617d_mu120_a_i105e.gifis called a ‘mixed number’.

[View answer - Example 24](" \l "Session3_Answer47)

End of Example

### Try some yourself

Start of Activity

**Activity 55**

Start of Question

Evaluate the following, giving your answer in its simplest form.

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_a4cf78f442fa6dacfb7a18a3919af73a71b28d5e_mu120_a_i172e.gif

End of Question

[View answer - Activity 55](" \l "Session3_Answer48)

End of Activity

## 3.15 Arithmetic with negative numbers

In order to understand arithmetic with negative numbers, it is helpful to see how arithmetic can be represented on the number line. The strategy is to start with simple examples of whole positive numbers and then generalise to negative numbers. The same principles must apply!

### Addition on the number line

For example, to find 8 + 3 start at 8 and move 3 units to the right.

Start of Figure



End of Figure

giving 8 + 3 = 11. Adding a positive number means moving to the right along the number line.

Another way of considering the arithmetic of positive and negative numbers is to consider them as the total value of the contents of a piggy bank, belonging to a child (Thomas). The numbers on the above number line can represent the value of Thomas’s piggy bank. Calculations represent transactions involving the piggy bank. When Thomas gets pocket money or gifts of money, he adds them into his piggy bank. If Thomas had £8 in his piggy bank and adds £3 (from his pocket money) the value of the piggy bank in pounds is 8 + 3 = 11.

When he spends money (on toys usually) he takes money out (subtracts). If there is not enough money in the piggy bank, he needs to borrow money. When he borrows money from his family, they note the debt on a piece of paper headed ‘IOU’ (I owe you). Thomas puts the ‘IOUs’ into his piggy bank. IOUs represent a negative amount of money (or debts).

Now suppose, at another time, Thomas’s piggy bank contains an IOU for £3 when Thomas receives a gift of £5 to add to his piggy bank. £3 of this pays off the IOU and he has £2 left in the piggy bank. The transaction is represented by the calculation of the new value (in pounds) of the piggy bank

−3 + 5 = 2.

How is this represented on the number line? Adding a positive number is moving to the right.

Start of Figure

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_10c64014185a2621e82ad92f62f4fc68b26e203a_mu120_a_i031i.jpg

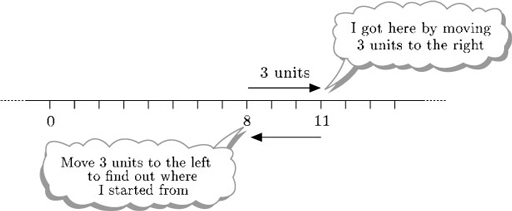
End of Figure

Moving 5 units to the right from −3 gets us to 2. So −3 + 5 = 2.

### Subtraction on the number line

Now what about subtraction? You can think of subtraction as undoing addition: adding 3 to 8 gets you 11, and so subtracting 3 from the answer, 11, gets you back to 8. Therefore, in terms of the number line, subtracting 3 from 11 means starting at 11 and moving 3 units to the left.

Start of Figure



End of Figure

So 11 − 3 = 8. Subtracting a positive number means moving to the left along the number line. Thinking in terms of the contents of the piggy bank: if Thomas had £11 in his piggy bank and subtracts £3 (to buy a toy), he is left with £8.

Suppose on one occasion Thomas has £2 in his piggy bank. He wants to buy a toy for £5. 2 − 5 means starting at 2 and moving 5 units to the left.

Start of Figure

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_1b4281f7bbe03ad630c636551b2036cc133d38cd_mu120_a_i033i.jpg

End of Figure

This takes you back to −3, so 2 − 5 = −3.

So Thomas needs to borrow £3 and gets an IOU (from his mother) which he puts in his piggy bank. His piggy bank is worth −£3.

## 3.16 Addition of negative numbers

Now think about adding a negative number, by looking at the number line again. Suppose you start at 0. Since 0 + −3 is the same as −3, you would expect that adding −3 to 0 on the number line should take you to the point marked −3 on the number line. So, 0 + −3 = −3.

Suppose Thomas’s piggy bank were empty and he added an IOU for £3 to it. The value of the piggy bank would be −3.

Start of Figure

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_75760dac5bf7b48c66efe56f0b86a0b1161e497f_mu120_a_i034i.jpg

End of Figure

To reach −3 from 0, move 3 units to the left. In general to add a negative number you move to the left along the number line.

Start of Box

Adding a negative number is the same as subtracting the corresponding positive number.

End of Box

In terms of Thomas’s piggy bank, a negative number is an IOU. Adding an IOU is the same as taking money out (subtracting a positive number).

Start of Example

**Example 25**

Evaluate each of the following:

* (a) 11 + −3
* (b) 2 + −5

[View answer - Example 25](" \l "Session3_Answer49)

End of Example

### Try some yourself

Start of Activity

**Activity 56**

Start of Question

Evaluate each of the following and give an example from everyday life to illustrate the sum (e.g. Thomas’s piggy bank).

* (a) −4 − 6
* (b) 3 + −5
* (c) −4 + −7
* (d) −5 + 8

End of Question

[View answer - Activity 56](" \l "Session3_Answer50)

End of Activity

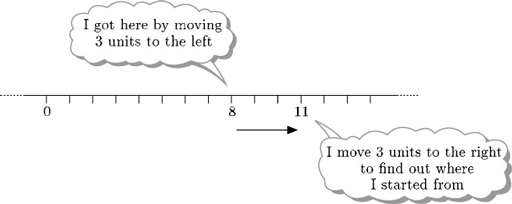
## 3.17 Subtraction of negative numbers

Next consider subtraction of a negative number. In terms of Thomas’s piggy bank, subtracting a negative number is the same as taking away one of his IOUs. If his mother says ‘you have been a good boy today so I’ll take away that IOU for £3’ this is equivalent to him being given £3.

So, − (−3) = 3. Does this correspond with the number line interpretation of subtracting a negative number?

Consider the evaluation of 8 − −3. Continue to think of subtraction as undoing addition: adding −3 to 11 gives 8, and so subtracting−3 from the answer, 8, gets back to 11. So, in terms of the number line, subtracting −3 from 8 means starting at 8 and moving 3 units to the right.

Start of Figure



End of Figure

But moving 3 units to the right is the same as adding 3. So

   8 − −3 = 8 + 3 = 11.

If on one occasion Thomas’s piggy bank is worth £8 in total (cash and IOUs) and his mother takes away a £3 IOU, the value of the piggy bank is 8 − −3 = 8 + 3 = 11.

Start of Box

Subtracting a negative number is the same as adding the corresponding positive number.

End of Box

### Try some yourself

Start of Activity

**Activity 57**

Start of Question

Evaluate each of the following and give an example from everyday life to illustrate the sum (e.g. Thomas’s piggy bank).

* (a) 4 − −2
* (b) −3 − −5

End of Question

[View answer - Activity 57](" \l "Session3_Answer51)

End of Activity

## 3.18 Practical examples of negative numbers

Negative numbers occur in financial matters, in temperature or height measurements and many other practical situations.

Start of Example

**Example 26**

* (a) If the value of a painting increases by £20 a year and it is worth £200 today, how much will it be worth in a year’s time? How much was it worth a year ago?
* (b) Describe in words how to calculate the value of an object like a picture one year in the future or one year ago, given a constant annual increase.
* (c) If the value of a washing machine decreases by £20 a year and it is worth £200 today, how much will it be worth in a year’s time? How much was it worth a year ago?
* (d) If you regard a decrease as a negative increase, does your answer to (b) apply to the washing machine in (c)?

[View answer - Example 26](" \l "Session3_Answer52)

End of Example

**So adding a negative increase is the same as subtracting the decrease.**

**Subtracting a negative increase is the same as adding the decrease.**

### Try some yourself

Start of Activity

**Activity 58**

Start of Question

Answer the following questions:

* (a) If you had £3 in your bank account and drew out £10, how much would you have left?
* (b) The temperature was −5 °C on Monday and dropped overnight by 6 °C. What was the temperature on Tuesday morning?

End of Question

[View answer - Activity 58](" \l "Session3_Answer53)

End of Activity

Start of Activity

**Activity 59**

Start of Question

Evaluate each of the following:

* (a) −3 + −12
* (b) −4 − −11

Think of a financial context where each might be an appropriate calculation (bear in mind that negative numbers can represent debts).

End of Question

[View answer - Activity 59](" \l "Session3_Answer54)

End of Activity

Start of Activity

**Activity 60**

Start of Question

Kim was walking in Israel. She started at 37 metres below sea level and ended up at 42 metres above sea level. How far had she climbed up?

End of Question

[View answer - Activity 60](" \l "Session3_Answer55)

End of Activity

## 3.19 Multiplication with negative numbers

Now that you have rules for addition and subtraction of negative numbers, think about multiplication and division.

Start of Example

**Example 27**

Describe each of the following in terms of the number line and the value of Thomas’s piggy bank:

* (a) the multiplication of 3 times 5 (that is, 3 × 5);
* (b) the multiplication of 3 times −5 (that is, 3 × −5).

[View answer - Example 27](" \l "Session3_Answer56)

End of Example

So multiplying a negative number by a positive number gives a negative answer. But what does it mean to multiply by a negative number – how can you add something a negative number of times?

Think of multiplication by a positive number as repeated addition, and of multiplication by a negative number as repeated subtraction. If Thomas’ mother says she’ll take away three of his £5 IOUs, it is equivalent to giving him three £5 notes. So −3 × −5 = 3 × 5 = 15. He is £15 better off.

Start of Example

**Example 28**

Evaluate each of the following in terms of the number line and the value of Thomas’s piggy bank.

* (a) 5 × −3
* (b) −5 × −3

[View answer - Example 28](" \l "Session3_Answer57)

End of Example

## 3.20 Multiplication rules

The rules for multiplying positive and negative numbers can be illustrated by the table below.

Start of Box

Multiplying a **positive** number by a **positive** number gives a **positive** answer.

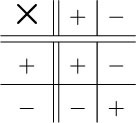
Multiplying a **negative** number by a **positive** number gives a **negative** answer.

Multiplying a **positive** number by a **negative** number gives a **negative** answer.

Multiplying a **negative** number by a **negative** number gives a **positive** answer.

End of Box

Start of Figure



End of Figure

The painting and the washing machine from [Example 26](#exa026) can be used again, to illustrate these rules.

Start of Example

**Example 29**

A painting increases in value at £20 a year and a washing machine decreases in value at £20 a year. Each is worth £200 today.

* (a) How much will each be worth in four years’ time?
* (b) How much was each worth four years ago?

[View answer - Example 29](" \l "Session3_Answer58)

End of Example

### Try some yourself

Start of Activity

**Activity 61**

Start of Question

Evaluate each of the following and where possible give an example from everyday life to illustrate the calculation.

* (a) 4 × −3
* (b) −2 × −7
* (c) −3 × 5

End of Question

[View answer - Activity 61](" \l "Session3_Answer59)

End of Activity

## 3.21 Division rules

Lastly consider division. Dividing 8 by 2 means ‘How many times does 2 go into 8?’ or ‘What must you multiply 2 by to get 8?’. The answer is 4.

So to find 8 ÷ −2, you need to ask ‘What do I have to multiply −2 by to get 8?’. The answer is −4, since −2 × −4 = 8.

So 8 ÷ −2 = −4.

Similarly, to find −8 ÷ −2 you need to ask ‘what do I have to multiply −2 by to get −8?’ and the answer is 4, since −2 × 4 = −8.

So −8 ÷ −2 = 4.

Using this sort of argument you can work out the rules for division of and by negative numbers. They are remarkably similar to those for multiplication (as you might expect, since they are inverse processes).

Start of Box

Multiplying or dividing a **positive** number by a **positive** number gives a **positive** answer.

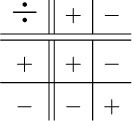
Multiplying or dividing a **negative** number by a **positive** number gives a **negative** answer.

Multiplying or dividing a **positive** number by a **negative** number gives a **negative** answer.

Multiplying or dividing a **negative** number by a **negative** number gives a **positive** answer.

End of Box

Start of Figure



End of Figure

The last one is the most difficult to remember. If you are in doubt, you can always use your calculator to check.

These rules may be summarised as follows.

Start of Box

Multiplying or dividing two numbers of the same sign gives a positive answer.

Multiplying or dividing two numbers of different signs gives a negative answer.

End of Box

### Try some yourself

Start of Activity

**Activity 62**

Start of Question

Give examples of all the rules for the division of and by positive and negative numbers.

End of Question

[View answer - Activity 62](" \l "Session3_Answer60)

End of Activity

Start of Activity

**Activity 63**

Start of Question

Evaluate each of the following and where possible give an example from everyday life to illustrate the calculation.

* (a) 24 ÷ −4
* (b) −40 ÷ −4
* (c) −45 ÷ 15

End of Question

[View answer - Activity 63](" \l "Session3_Answer61)

End of Activity

Start of Activity

**Activity 64**

Start of Question

Evaluate each of the following.

* (a) 7 × −6
* (b) −56 ÷ −7

Think of a financial context where each might be an appropriate calculation (bear in mind that negative numbers can represent debts).

End of Question

[View answer - Activity 64](" \l "Session3_Answer62)

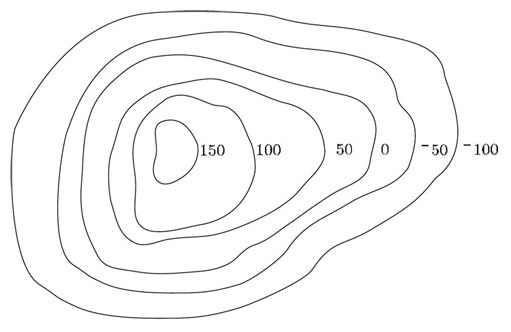
End of Activity

**If you have a calculator check that it gives the same answers as yours.**

## 4 OpenMark quiz

You might like to make some notes on the course for your own use later. Here is an example of a student’s notes.

Start of Figure

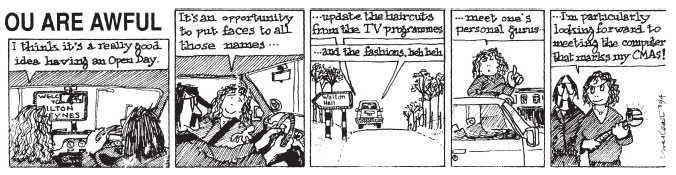


End of Figure

Now check your own learning by attempting the [interactive quiz](https://students.open.ac.uk/openmark/mu120-ol.module1/).

The computer will mark your answer and give you helpful feedback after each attempt.

Start of Figure



End of Figure

## Conclusion

This free course provided an introduction to studying Mathematics. It took you through a series of exercises designed to develop your approach to study and learning at a distance and helped to improve your confidence as an independent learner.

**Get careers guidance**

The [National Careers Service](https://nationalcareers.service.gov.uk/find-a-course/the-skills-toolkit?utm_source=openlearn&utm_medium=referral&utm_campaign=skillstoolkit_return) can help you decide your next steps with your new skills.

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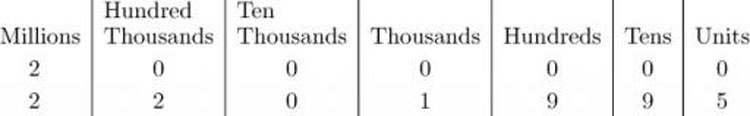
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## Solutions

## Example 1

#### Answer

Start of Figure



End of Figure

Now look at the numbers from the left. Both numbers have a 2 in the millions column, so move to the next place. The first number has a 0 in the next column (i.e. hundred thousands) whereas the second number has a 2. So the second number is larger, the announcement was correct and more than two million pounds was paid out.

In practice you probably won’t want to write the headings in the columns each time. But do keep their meanings in mind.

[Back to - Example 1](" \l "Session1_Example1)

## Activity 1

#### Answer

**1** Your answers should be

* (a) The number is 20 144 (or it could be written 20,144).
* (b) Thirty-one million two thousand one hundred and three.

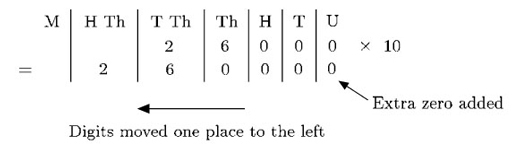
[Back to - Activity 1](" \l "Session1_Activity1)

## Example 2

#### Answer

The total number of payments is the number of households times the number of payments per household, i.e. 26 000 × 10. This can be worked out by moving all the digits of 26 000 one place to the left, and adding a zero at the end:

Start of Figure



End of Figure

So 26 000 × 10 = 260 000.

A total of 260 000 cash payments should be expected.

[Back to - Example 2](" \l "Session1_Example2)

## Activity 2

#### Answer

**1** Written in numerals, the numbers are:

  808 000,   2 000 024,   9 998

So, in ascending order, we have:

  9 998,    808 000,   2 000 024.

[Back to - Activity 2](" \l "Session1_Activity2)

## Activity 3

#### Answer

2490 × 100 = 249 000

Notice that the digits have moved 2 places to the left and two zeroes have been added at the right.

[Back to - Activity 3](" \l "Session1_Activity3)

## Activity 4

#### Answer

**3** Your answers should be:

* (a) 5 001,   9 999,   49 020.
* (b) 949 400.

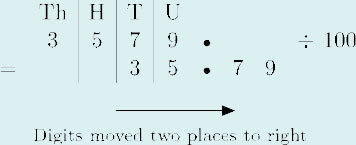
[Back to - Activity 4](" \l "Session1_Activity4)

## Example 3

#### Answer

You need to divide 3579 by 100. To multiply a number by 100, you shift the digits two places to the left (and add zeros to fill the spaces). To reverse the process, i.e. to divide by 100, move the digits two places to the right. To show this, you need to mark the end of the units column, and this is done by means of a **decimal point**.

Start of Figure



End of Figure

So the answer is 35.79, or in words, thirty five **point** seven nine (not seventy nine).

So 3579 cents = $35.79.

[Back to - Example 3](" \l "Session1_Example3)

## Activity 5

#### Answer

0.202 15. So 202.15 metres are 0.202 15 kilometres.

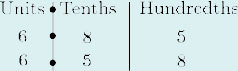
[Back to - Activity 5](" \l "Session1_Activity5)

## Example 4

#### Answer

The point at issue is which number is larger, 6.85 or 6.58. To determine this, write the digits in a place value table and then compare the values in each column:

Start of Figure



End of Figure

Again, work from the left. The digits in the units column are the same, so look at the next column. The first number has 8 tenths and the second number has 5 in this column (5 tenths), so the first number is bigger which means that the first bag is heavier.

[Back to - Example 4](" \l "Session1_Example4)

## Activity 6

#### Answer

370.76 ÷ 1000 = 0.370 76.

So 370.76 g = 0.370 76 kg.

[Back to - Activity 6](" \l "Session1_Activity6)

## Activity 7

#### Answer

Writing the numbers out in columns with the decimal points lined up (as in a place value table), gives:

1.013

1.103

1.0103

1.0129

Working from the left shows that 1.0103 is the smallest, follwed by 1.0129. So in ascending order they are: 1.0103,  1.0129,  1.013,  1.103.

[Back to - Activity 7](" \l "Session1_Activity7)

## Activity 8

#### Answer

Writing the numbers out in columns with the decimal points lined up (as in a place value table), gives:

0.1704

0.1074

0.0714

0.0741

Working from the left, shows that 0.0714 is the smallest, followed by 0.0741. So in ascending order (in metres) they are:

  0.0714,  0.0741,  0.1074,  0.1704.

[Back to - Activity 8](" \l "Session1_Activity8)

## Example 5

#### Answer

The fraction is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_a623233ea6cbb5c6f5101a2e7aa347d9f497c9a3_mu120_a_i029e.gif. The circle is divided into 8 equal sections, so each section is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_7472c5a0318bc71ae15a7e2dee0bba7fcb22e8b3_mu120_a_i030e.gif. Three of them are shaded so the shaded part is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_b6bf1bc8a3a1f8caba3e06b4d62ed4d149e86967_mu120_a_i031e.gif of the circle.

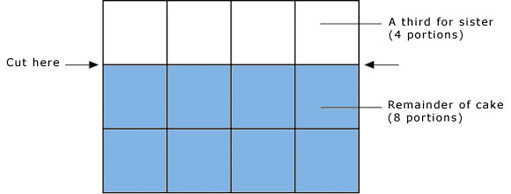
However there are other equivalent fractions which represent the same shaded area, obtained by multipying the numerator and the denominator by the same whole number, e.g. D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_0bb72311aeff44ae49a218f414eee87cfb1e34d8_mu120_a_i157616e.gif.

[Back to - Example 5](" \l "Session1_Example5)

## Activity 9

#### Answer

Start of Figure



End of Figure

(There are several other ways of cutting the cake into one-third (4 portions) and two-thirds (8 portions).)

[Back to - Activity 9](" \l "Session1_Activity9)

## Activity 10

#### Answer

Only D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_557e64acbe987a439d7ed1c4e6691dee98114eb2_mu120_a_i004e.gif and D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_89babd205dca5c48db3fccf5a13b72258691c31a_mu120_a_i005e.gif are equivalent fractions, since D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_d82f312cbd09dea9fdbafc0ef68b13f04dae8d44_mu120_a_i006e.gif.

(Notice that neither D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_535b6b4781db6348538950a98057a2a7e69d61c6_mu120_a_i007e.gif, which is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_bd7a70e56b53bcb2176f93c14b33afa6a655f9d8_mu120_a_i008e.gif, nor D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_9a7b0f6a83759bf2f4544bb73536a5d5689bbf24_mu120_a_i009e.gif, which is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_0e8f6992588327d52d2b84a77a9002407bf54c13_mu120_a_i010e.gif, are equivalent to D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_557e64acbe987a439d7ed1c4e6691dee98114eb2_mu120_a_i004e.gif.)

[Back to - Activity 10](" \l "Session1_Activity10)

## Example 6

#### Answer

Multiply or divide top and bottom by the same whole number to give an equivalent fraction.

* (a) Some examples are D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_5d82a4f5a7f24e806951969dcad0a7fc07d18325_mu120_a_i034e.gif
* (b) Top and bottom are divisible by 5. So

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_46a8d1e4450122dbbfa8cf507994bab71569856e_mu120_a_u003e.gif

End of $1

3 and 4 have no common factors so D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_20237704918ba379451f68c6302b58f80e49bc1c_mu120_a_i035e.gif is the simplest form.

[Back to - Example 6](" \l "Session1_Example6)

## Activity 11

#### Answer

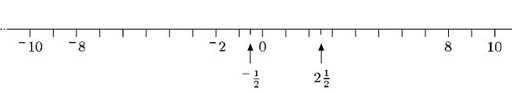
* (a) 1 part is shaded out of 4 (equal parts). So the fraction is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_2bd72d1d40aefeb1c2270616bbbafff76ba68cc9_mu120_a_si001e.gif
* (b) 7 parts are shaded out of 12 (equal parts). So the fraction is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_997ff14462a5b50cb1fed468bb0291feeb111c9e_mu120_a_si002e.gif
* (c) 3 parts are shaded out of 7 (equal parts). So the fraction is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_c6d6512b141721c92a001775d10c03639ce21bf2_mu120_a_si003e.gif

[Back to - Activity 11](" \l "Session1_Activity11)

## Activity 12

#### Answer

Start of Figure

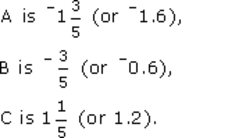


End of Figure

[Back to - Activity 12](" \l "Session1_Activity12)

## Activity 13

#### Answer

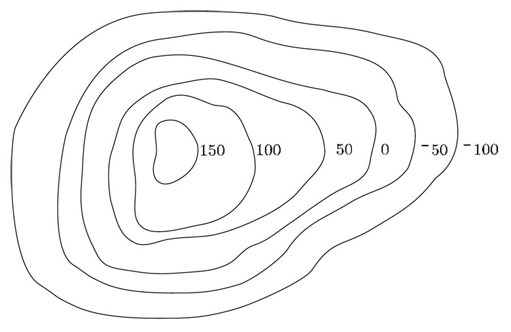


[Back to - Activity 13](" \l "Session1_Activity13)

## Activity 14

#### Answer

Start of Figure



End of Figure

[Back to - Activity 14](" \l "Session1_Activity14)

## Example 7

#### Answer

She could measure in millimetres, but that would give large numbers. It is difficult to visualise the room in terms of millimetres – 1 mm is approximately the thickness of a piece of wire. Can you imagine how many of those would fit along one wall! But thinking of it in terms of metres may be easier. A single bed is about one metre wide so you could visualise how many would fit side by side along each wall. This will help check measurements. So she might measure each side of the room in metres (e.g. 2.82 m by 4.25 m). To avoid using decimals, she could have used centimetres instead (giving 282 cm by 425 cm). Either of these would be sensible. It would be unhelpful (to say the least!) to quote the dimensions in kilometres (e.g. 0.00282 km by 0.00425 km).

[Back to - Example 7](" \l "Session2_Example1)

## Activity 15

#### Answer

* (a) The age of a young kitten would be measured in weeks or days.
* (b) The distance between train stations could be measured in kilometres (or miles).
* (c) The active ingredients of a pill are usually measured in milligrams.
* (d) The cost of leaving a light on for 24 hours will vary according to the type of bulb and the price of electricity, but it is likely to be measured in pence.

[Back to - Activity 15](" \l "Session2_Activity1)

## Activity 16

#### Answer

* (a) Your height could be measured in metres, e.g. 1.55 m, or centimetres, e.g. 155 cm. It may be that you only know your height in feet and inches, e.g. 5 ft 1 in.
* (b) Your weight might be given in kilograms, e.g. 58.06 kg, or perhaps in stones and pounds, e.g. 9 st 2 lb.
* (c) The length of time to boil an egg is usually given in minutes, e.g. 3 min.
* (d) Pans are described in terms of the amount of liquid they hold, measured in litres (or pints), e.g. a 2‑litre pan.

[Back to - Activity 16](" \l "Session2_Activity2)

## Example 8

#### Answer

To convert millimetres to metres, divide by 1000. So to convert 575 mm to metres, move the digits three places to the right:

  575.0 ÷ 1000 = 0.575.

So the answer is 0.575 m.

[Back to - Example 8](" \l "Session2_Example2)

## Activity 17

#### Answer

1.65 m is 165 cm (multiply by 100).

[Back to - Activity 17](" \l "Session2_Activity3)

## Activity 18

#### Answer

* (a) To convert millimetres to centimetres, divide by 10. So 675 mm is 67.5 cm.
* (b) 45.2 km is 45 200 m (multiply by 1000).
* (c) 3.5 kg is 3500 g (multiply by 1000).
* (d) 167.2 mm is 0.1672 m (divide by 1000).

[Back to - Activity 18](" \l "Session2_Activity4)

## Activity 19

#### Answer

To convert millimetres to metres, divide by a thousand, i.e. move the numbers three places to the right in the place value table. In terms of metres, the kitchen unit is then 1.2 m long, 0.6 m high and 0.35 m deep.

[Back to - Activity 19](" \l "Session2_Activity5)

## Example 9

#### Answer

To convert 500 cm3 into l, divide by 10 to convert into cl, and then divide by 100 to convert the measurement into l.

So, 500 cm3 = 500 ÷ 10 cl = 50 cl = 50 ÷ 100 l = 0.5 l.

[Back to - Example 9](" \l "Session2_Example3)

## Example 10

#### Answer

First decide what units the children were using and then convert all the measurements to the same units. It is reasonable to guess that the heights are between 1 and 2 metres. (A height between 10 cm and 20 cm would mean they were not yet born, whereas a height between 10 m and 20 m would make them as tall as a house!) So Isaac has given his height in millimetres and his height in metres is 1098 ÷ 1000 = 1.098 m. Jasmine has given her height in centimetres, so her height in metres is 112 ÷ 100 = 1.12 m. Kim’s height is in metres already, so Kim is 1.1 m tall. So, the heights in metres are:

Start of Table

|  |  |
| --- | --- |
| Isaac | 1.098 |
| Jasmine | 1.120 |
| Kim | 1.100 |

End of Table

(Inserting a 0 to the right of Jasmine’s 1.12, and two 0s to the right of Kim’s 1.1, has no effect on the value but is useful for comparisons as here.)

Comparing from the left, all have 1s in the units column. Isaac has a 0 in the tenths column, but Jasmine and Kim have 1s in the tenths column. So Jasmine and Kim are both taller than Isaac. In the hundredths column, Jasmine has 2 and Kim has 0. So Jasmine is the tallest.

[Back to - Example 10](" \l "Session2_Example4)

## Activity 20

#### Answer

To compare these masses, convert them all to the same units (it doesn’t matter which unit you choose, so long as all are converted to the same). Choosing grams gives 2.125 g, 1.9 g and 20 g. Placing these in order gives:

  1.9 g,    2.125 g,    20 g.

In the original units, this is

  1.9 g,    2125 mg,    0.02 kg.

[Back to - Activity 20](" \l "Session2_Activity6)

## Activity 21

#### Answer

* (a) First convert to centilitres, say:

  X is 50 cl;  Y is 490 cl;  Z is 51 cl.

So the order is X, Z, Y.

* (b) First convert to pence, say:

  X is 20p;  Y is 2p.

So the order is Y, X.

* (c) First convert to grams, say:

  X is 39 g;  Y is 4 g;  Z is 30 g.

So the order is Y, Z, X.

[Back to - Activity 21](" \l "Session2_Activity7)

## Example 11

#### Answer

* (a) There are 5078 people altogether.
* 
* An alternative method is to note that 93 is 7 less than 100. So add 100 to 4985 (5085) and subtract 7 (5078).
* (b) 3726 people are eligible to vote.
* 

[Back to - Example 11](" \l "Session3_Example1)

## Activity 22

#### Answer

* (a) £999 700 remains.
* 
* (b) The total is £1054.
* 

[Back to - Activity 22](" \l "Session3_Activity1)

## Example 12

#### Answer

* (a) There are 60 seconds in a minute and so it puts on 60 tops a minute. There are 60 minutes in an hour so it puts on 60 × 60 tops in an hour, which gives 3600.
* There are many ways to do this e.g. think of 60 × 60 as 6 × 10 × 6 × 10, which is 6 × 6 × 10 × 10, i.e. 36 × 100, giving 3600 as before.
* However you tackle it, the important thing is to remember the place values of the numbers.
* So it tops 3600 bottles an hour. In 10 hours it tops 3600 × 10 = 36 000 bottles.
* It tops 36 000 bottles in a 10 hour day.
* (b) To find out how many bottles it tops in 30 days, multiply 36 000 by 30.
* This is 36 000 × 3 × 10 = 1 080 000.
* So the factory machine tops 1 080 000 bottles a month (just over a million).
* The factory puts on 1 080 000 bottle tops in 30 days.

[Back to - Example 12](" \l "Session3_Example2)

## Activity 23

#### Answer

One box has 40 candles.

10 boxes have 40 × 10 candles = 400 candles.

30 boxes have 40 × 3 × 10 candles or 40 × 10 × 3 or 400 × 3 or 1200 candles.

[Back to - Activity 23](" \l "Session3_Activity2)

## Activity 24

#### Answer

* (a) There are 7 days in a week, so 4 weeks is 7 × 4 = 28 days.
* The number of pills required is 28 × 3 = 84.
* (b) The apples in packs cost D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_7774587f69eda0c6bd68cc98c3318130cf94fb09_mu120_a_i061e.gif p each.
* Dividing 95 by 7 gives 13 D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_7de35b2f710e7cedbbed2f7119549d3467c59b2a_mu120_a_i062e.gif,
* which is less than the 14p per apple sold singly.
* Alternatively, you could compare the cost of 7 apples: 7 apples at 14p each would cost 98p, which is more than 95p. So the apples in the pack would appear to be the better buy. (However, if you do not want 7 apples, and are likely to leave some of them to rot, you would be better off just buying the number of apples you want!)

[Back to - Activity 24](" \l "Session3_Activity3)

## Activity 25

#### Answer

* (a) Multiplying by 1 gives an answer the same as the original number. (There are many possible ways of wording this. You may have said it ‘leaves the number unchanged’ or ‘has no effect’.)
* (b) Multiplying by 0 always gives 0.

[Back to - Activity 25](" \l "Session3_Activity4)

## Activity 26

#### Answer

Convert all the times to days, say:

  four weeks is 4 × 7 = 28 days;

  29 days is already in days.

To convert 360 hours to days, divide by 24.

Writing it as D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_394cec13c966c9aa3e99ec3926d9687b9e11f816_mu120_a_si031e.gif you may spot that you can divide top and bottom by 12 to give D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_89fc3ac01006678f9c7264af765360923521ef22_mu120_a_si032e.gif = 15.

(Or you can divide 360 by 12 and then by 2 to give the same result as dividing by 24.

Alternatively you may say 10 days is 240 hours, 5 days is 120 hours, so 15 days is 360 hours.)

So, in order:

  360 hours, 4 weeks, 29 days.

[Back to - Activity 26](" \l "Session3_Activity5)

## Example 13

#### Answer

To solve the problem, you need to know how many times 40 goes into 1000.

There are several ways to do this. One is to divide 1000 by 40:

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_4df114d0bab3629e22ad5271dc372c487355b377_mu120_a_i088e.gif

So its value is unchanged if top and bottom are multiplied or divided by the same number.

Here, notice that top and bottom are both divisible by 10.

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_176d0f781a7f54bd7de1532ec226d418a2779963_mu120_a_u004e.gif

End of $1

So you need 25 boxes of candles.

Alternatively, you might say 1 box gives 40, so 10 boxes give 400, 20 boxes give 800, 5 boxes give 200, so 25 boxes give 1000.

[Back to - Example 13](" \l "Session3_Example3)

## Activity 27

#### Answer

* (a) 27 000 ÷ 9 = 3000.
* They get £3000 each.
* (b) D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_f22d1b08de4ced333127cc24bd02d794f5f4418b_mu120_a_si030e.gif.
* They get £30 each.

[Back to - Activity 27](" \l "Session3_Activity6)

## Example 14

#### Answer

* (a) Multiplying by 365 is quite hard work, whereas multiplying by 3 is comparatively easy. Since 3 lots of 365 are the same as 365 lots of 3, it makes no difference whether you multiply 3 by 365 or multiply 365 by 3.

  3 × 365 = 365 × 3 = 1095.

Since 3 × 365 = 1095, there are 1095 days in 3 years, provided one is not a leap year. If it were there would be 1096.

* (b) 366 ÷ 3 = 122. So there will be 122 days.
* (c) 3 × 365 = 365 × 3
* So you can interchange the numbers in (a).
* But 366 ÷ 3 does not give the same answer as 3 ÷ 366.
* So you cannot interchange the numbers in (b).

[Back to - Example 14](" \l "Session3_Example4)

## Activity 28

#### Answer

When adding two numbers together, or multiplying two numbers together, it makes no difference which number you write first. You get the same answer either way round.

  e.g. 3 + 4 = 4 + 3, and 3 × 4 = 4 × 3.

When subtracting one number from another or dividing one number by another, it does matter which number you write first. You get different answers the other way round.

  e.g. 8 − 4 = 4 is not the same as 4 − 8 = −4; and 8 ÷ 4 = 2 is not the same as 4 ÷ 8 = 1/2.

[Back to - Activity 28](" \l "Session3_Activity7)

## Activity 29

#### Answer

True. For example 2 + 3 + 4 is the same as 4 + 2 + 3.

[Back to - Activity 29](" \l "Session3_Activity8)

## Example 15

#### Answer

To find (7 + 3) × 2, first do the calculation in brackets to get 7 + 3 = 10.

Then multiply by 2 to get 10 × 2 = 20.

For the second calculation, to find 7 + (3 × 2), first calculate 3 × 2 to give 6.

Then add to 7 to give 13. So (7 + 3) × 2 ≠ 7 + (3 × 2).

[Back to - Example 15](" \l "Session3_Example5)

## Activity 30

#### Answer

* (a) 3 × (60 + 70) = 3 × 130 = 390.
* (b) (3 × 60) + 70 = 180 + 70 = 250.
* (c) (70 − 60) ÷ 5 = 10 ÷ 5 = 2.

[Back to - Activity 30](" \l "Session3_Activity9)

## Example 16

#### Answer

Multiplication is done before addition (unless brackets tell you otherwise) so

  10 + 3 × 7 = 10 + 21 = 31.

[Back to - Example 16](" \l "Session3_Example6)

## Activity 31

#### Answer

* (a) 3 × 60 + 70 = (3 × 60) + 70 = 180 + 70 = 250.
* (b) 10 − 15 ÷ 5 = 10 − (15 ÷ 5) = 10 − 3 = 7.
* (c) 20 − 2 × 8 = 20 − (2 × 8) = 20 − 16 = 4.
* (d) 3 + 16 − 10 = (3 + 16) − 10 = 19 − 10 = 9.
* (e) 3 × 10 ÷ 5 = (3 × 10) ÷ 5 = 30 ÷ 5 = 6.

[Back to - Activity 31](" \l "Session3_Activity10)

## Activity 32

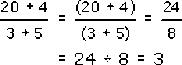
#### Answer

The order makes a difference in calculations (a), (b) and (c) of Question 1.

[Back to - Activity 32](" \l "Session3_Activity11)

## Activity 33

#### Answer

* (a) 
* (b) No, because there are implicit brackets around each of the top and bottom line of the fraction. When you write one number over another, you should treat the expressions above and below the line as if they were each in brackets. So the fact that the additions in part (a) were carried out before the division does not represent an exception to the ‘division before addition’ rule; it simply follows the ‘brackets before division’ rule.

[Back to - Activity 33](" \l "Session3_Activity12)

## Activity 34

#### Answer

The player scores 2 × 19, 20 and 3 × 17, which gives 38 + 20 + 51 = 109.

301 − 109 gives a new target score of 192.

[Back to - Activity 34](" \l "Session3_Activity13)

## Activity 35

#### Answer

* (a) False. Carrying out the addition first gives 2 + 3 = 5, then multiplying by 4 gives 20. Carrying out the multiplication first gives 3 × 4 = 12, then adding to 2 gives 14 (which is the correct procedure here).
* (b) True. For example writing brackets around 2 + 3 in the expression 2 + 3 × 4 gives (2 + 3) × 4 = 20, whereas without the brackets you should carry out the multiplication first to give 14.

[Back to - Activity 35](" \l "Session3_Activity14)

## Example 17

#### Answer

Remember to supply a ‘0’ at the end of 7.77 to keep the number of decimal places the same.

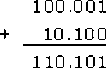
* (a) The answer is 310.805 kg.
* 
* (b) The answer is 295.265 kg.
* 

(With an awkward subtraction like this, a useful check is to add 7.77 to 295.265 to make sure that you get back to 303.035.)

[Back to - Example 17](" \l "Session3_Example7)

## Activity 36

#### Answer

* (a)
* 
* (b)
* 

[Back to - Activity 36](" \l "Session3_Activity15)

## Activity 37

#### Answer

Your friend’s share of the bill is £85 − £47.65 = £37.35. There are many ways to do this subtraction. Setting out as a formal sum is one approach, but counting on (as an assistant may do to give change) is another (from £47.65 a further 35 p is needed to make £48, then £2 to make £50 and £35 to make £85 gives £37.35).

[Back to - Activity 37](" \l "Session3_Activity16)

## Example 18

#### Answer

* (a) In 52 weeks it grows 52 × 0.4 cm.

Start of Quote

To multiply by 0.4, multiply by 4 and shift the digits one place to the right,

  52 × 4 = 208. So 52 × 0.4 = 20.8.

So the hair will grow 20.8 cm in a year.

(0.4 is just under a half and half of 52 is 26, so the answer is sensible.)

End of Quote

**Notice that multiplying by 0.4 gives an answer smaller than 52.**

* (b) You need to know how many times 0.4 cm goes into 10 cm, so divide 10 by 0.4,

Start of Quote

  10÷ 0.4 = D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_fd5124c56cc3e901f9c4314346ab4043bf840946_mu120_a_i094e.gif = D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_3da9f20b239722a6d68728a262687e5ebedb6485_mu120_a_i095e.gif (multiply top and bottom by 10).

Now carry out the division: 100 ÷ 4 = 25.

So it would take 25 weeks to grow 10 cm.

(Check: 10 cm is about half of the previous answer of 20.8 cm which took a year, so will take about half a year. 25 weeks is near enough half a year, so the answer is sensible.)

End of Quote

* (c) There are several ways of doing this. You might do it formally as

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_4b44676a11677ad2f5197c3cf77e7fa95d266f18_mu120_a_u006e.gif

End of $1

Start of Quote

Carrying out the division gives 7.5.

Alternatively you might do it more informally: it grows 4 cm in 10 weeks,

and 3 cm is three-quarters of 4 cm,

so the answer is three-quarters of 10 weeks,

i.e. D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ac422cfa44c77f5c79e9faf8a1ed703e827c1f4e_mu120_a_i096e.gif × 10 = 7.5 weeks.

Therefore it would take 7.5 weeks to grow 3 cm.

(Check: 3 cm is a bit less than a third of 10 cm which takes 25 weeks, so will take a bit less than a third of 25 weeks. 7.5 weeks is near enough a third of 25 weeks, so the answer is sensible.)

End of Quote

[Back to - Example 18](" \l "Session3_Example8)

## Activity 38

#### Answer

* (a) 7.9 × 0.8 = 6.32
* (multiply by 8 then shift the digits one place to the right).
* (b) 82.3 × 40 = 3292
* (multiply by 4 then shift the digits one place to the left).
* (c)D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_8a10eebf1baf48153f909f7933c87d1eada9e6bb_mu120_a_i063e.gif
* (d) D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_9046fbd9564f5723f4670401193ed3a8c230bc1f_mu120_a_i066e.gif

[Back to - Activity 38](" \l "Session3_Activity17)

## Activity 39

#### Answer

* (a) 0.8 kilos of cheese costs 0.8 × £7.90 = £6.32.
* (b) 40 litres of petrol costs 40 × 82.3p = 3292p = £32.92.
* (c) The cheese costs £7.20 ÷ 0.8 = £9 per kilo.
* (d) Each hat costs £62.30 ÷ 40 = £1.5575 each, but they must be a whole number of pence each so only two decimal places are meaningful. Hence there was either an error or some discount.

[Back to - Activity 39](" \l "Session3_Activity18)

## Example 19

#### Answer

Brackets first: 4.6 − 2.6 = 2.0, so calculate 3.5 + 0.7 × 8 ÷ 2.0.

Multiply next: 0.7 × 8 = 5.6, so calculate 3.5 + 5.6 ÷ 2.0.

Divide next: 5.6 ÷ 2 = 2.8, so calculate 3.5 + 2.8.

Lastly add: 3.5 + 2.8 = 6.3.

So the answer is 6.3.

[Back to - Example 19](" \l "Session3_Example9)

## Activity 40

#### Answer

* (a) D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_8ac8b35fe5316608d81df88e600f5221597cb3b1_mu120_a_si038e.gif
* (b) 75.6 × 0.6 = 75.6 × D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_3011b82ff22daa75ebb08fc2862562072841f3d6_mu120_a_si039e.gif = 7.56 × 6 = 45.36.
* (c) Carrying out the division first, use the result of part (a) to give:
* 100.001 + 75.6 ÷ 0.6 = 100.001 + 126
* = 226.001.
* (d) Carry out the addition in brackets first:
* (100.001 + 10.1) × 60 = 110.101 × 60 = 1101.01 × 6 = 6606.06.

[Back to - Activity 40](" \l "Session3_Activity19)

## Activity 41

#### Answer

10 ÷ 3 gives

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_10e9bb6fbc3d1bd60fabb34e90d3d97411ec4c82_mu120_a_si040e.gif and so on.

Each time you divide by 3 you get a remainder 1, which gives 10 in the next column, so the decimal carries on forever. This is called a recurring decimal, because the 3 recurs repeatedly.

A scientific calculator gives 3.333333333 (it rounds to 10 digits).

[Back to - Activity 41](" \l "Session3_Activity20)

## Activity 42

#### Answer

False. For example, multiplying 7.9 × 0.8 in Question 1(a) gave 6.32, which is less than 7.9. In general multiplying a positive number by a number less than one gives a smaller number.

[Back to - Activity 42](" \l "Session3_Activity21)

## Example 20

#### Answer

* You cannot add quarters and eighths directly. You need to change them into the same thing.
* It is possible to change quarters to eighths.
* D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_1f4d003f5cf6402886656d2b089dadd4cacaed98_mu120_a_i102e.gifis equivalent to D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_920eaded157992e7ff463a17c81537445e0e7a31_mu120_a_i103e.gif: D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_69b576c50a7cb383618f4dc1525514818ae60d8a_mu120_a_i104e.gif.
* So make the denominators the same, i.e. both 8.

Start of $1

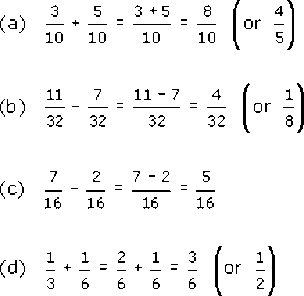
D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_91e09a777208ef3d3613f5865c07dfef2a5affbe_mu120_a_u009e.gif

End of $1

[Back to - Example 20](" \l "Session3_Example10)

## Activity 43

#### Answer

* 

[Back to - Activity 43](" \l "Session3_Activity22)

## Example 21

#### Answer

You cannot change halves to thirds or vice versa, but you can change them both to sixths.

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_a92687f00f00e624e4aa853a7ccaf4d5d94336d7_mu120_a_i112e.gif   and   D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_d1d6a4ac4872eb67091c84fde8d44b4e94115664_mu120_a_i113e.gif. So D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_0027f564234f1b0661def7b6b27fdaafe76693aa_mu120_a_i114e.gif.

[Back to - Example 21](" \l "Session3_Example11)

## Activity 44

#### Answer

False. For example adding 2 to the top and bottom of D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_dff3d8f7bc0951b88b254c3b7381f2a0d330786c_mu120_a_i068e.gif gives D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ac422cfa44c77f5c79e9faf8a1ed703e827c1f4e_mu120_a_i069e.gif. But D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ac422cfa44c77f5c79e9faf8a1ed703e827c1f4e_mu120_a_i069e.gif is not equivalent to D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_dff3d8f7bc0951b88b254c3b7381f2a0d330786c_mu120_a_i068e.gif.

[Back to - Activity 44](" \l "Session3_Activity23)

## Activity 45

#### Answer

There are obviously many different ways of explaining this. For example one student said:

Start of Quote

‘First look at the bottom number of each fraction. If these are the same, then this is also the bottom number of the answer. To get the top number in the answer, add the two top numbers in the given fractions.

‘If the bottom numbers are different, multiply the **top** and **bottom** of the first fraction by the **bottom number** of the second. Then multiply the **top** and **bottom** of the second fraction by the **bottom** number of the **first**. Now the two fractions have the **same bottom number** so they can be added as above.’

End of Quote

Your wording may have been very different but it should have given instructions for the case when the denominators are the same and when they are not. Note that the method described here will always work, when bottom numbers are different, although in some cases a quicker method is possible.

[Back to - Activity 45](" \l "Session3_Activity24)

## Example 22

#### Answer

She gets D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ac422cfa44c77f5c79e9faf8a1ed703e827c1f4e_mu120_a_i069e.gif of D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_57876b0497f91b8ee4621d249266171dead70b55_mu120_a_i116e.gif, which just means D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_41d5b591484f780593875034d43705a4e206d426_mu120_a_i120e.gif.

Isaac’s portion is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_9a2a03b9ad92da225d784096acf29a64ed008385_mu120_a_i121e.gif of the pizza. Divide the D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_57876b0497f91b8ee4621d249266171dead70b55_mu120_a_i116e.gif (i.e. D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_801a6c0d25b10585ad1eba1bb6e9a3c6dc590bde_mu120_a_i123e.gif) into 4 portions and Jasmine gets 3 of them. Each piece is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_0134a3c4a567f3135a05a176137e75c9e8254f68_mu120_a_i124e.gif of the whole pizza, and three of these pieces constitute D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_fe53af259aac21fa847963e70bfda444b74886b4_mu120_a_i125e.gif of the whole pizza. So the answer is that Jasmine gets three twelfths of the pizza. The calculation to which this corresponds is

Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_158cd98d182fe29c9a819e3ffb8d7d755b5799eb_mu120_a_u013e.gif

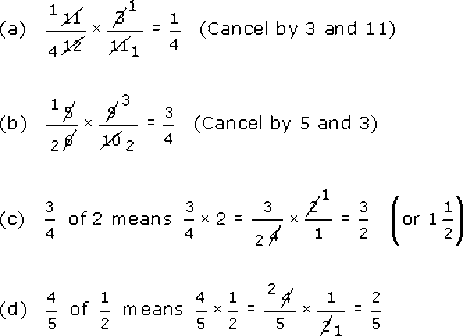
End of $1

Notice that D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_1a1c97fd87e7f2f45c454b1820a3e58830b7bd34_mu120_a_i126e.gif, so an equivalent answer is that Jasmine gets a quarter of the pizza.

[Back to - Example 22](" \l "Session3_Example12)

## Activity 46

#### Answer

* 

[Back to - Activity 46](" \l "Session3_Activity25)

## Activity 47

#### Answer

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_1aa94922ab5a4c6469f3d585a647e18a1df12799_mu120_a_si054e.gif

Alternatively D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_e244e303e39c15177fc0c00f15df2be504bad442_mu120_a_si056e.gif of 720 is 90. So D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_5b11d78a2be85724b863db5e51d0ced41093580a_mu120_a_si057e.gif of 720 = 3 × 90 = 270.

[Back to - Activity 47](" \l "Session3_Activity26)

## Activity 48

#### Answer

You have eaten 3 × D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_0134a3c4a567f3135a05a176137e75c9e8254f68_mu120_a_i072e.gif=D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_fe53af259aac21fa847963e70bfda444b74886b4_mu120_a_i073e.gif (or D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_920eaded157992e7ff463a17c81537445e0e7a31_mu120_a_i074e.gif) of the cake. So there is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_7bbdb6dfc6115eda6235fc52f34a59e712dca62f_mu120_a_i075e.gif (or D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_b4f59de9d34fd4f1454dd6910a4973337674a90e_mu120_a_i059e.gif) of the cake left.

[Back to - Activity 48](" \l "Session3_Activity27)

## Activity 49

#### Answer

Half an hour. There are several ways to do this calculation.

For example, one third of three quarters is one quarter (in symbols, D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_66fb41823c4d4abb086aa57acbe09aea1f501e29_mu120_a_i077e.gif). Hence, two thirds (of three quarters) is two quarters, or a half: D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_7087724f815ade5ee8bb647e2ac2a108d651ef53_mu120_a_i078e.gif.

Alternatively D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_596a0c8911542016b8158408e797fbc0b3c47a93_mu120_a_i079e.gif gives D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_d178b2c64899b55bb4486bde3a19833058269495_mu120_a_i080e.gif, i.e. D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_dff3d8f7bc0951b88b254c3b7381f2a0d330786c_mu120_a_i081e.gif.

Another method is to cancel to give: D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_518df469c523775aed26371fb600f6a49a3e8798_mu120_a_i082e.gif.

You might also have converted hours to minutes:

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_ac422cfa44c77f5c79e9faf8a1ed703e827c1f4e_mu120_a_i083e.gifnow is 45 minutes, and D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_2430e9c700f6499b19029bf6629414e8ffca3dd8_mu120_a_i084e.gif of 45 minutes is D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_c58fb03d055dbff3deed20866cfcd0ef193eb17a_mu120_a_i085e.gifminutes.

So check it after half an hour.

[Back to - Activity 49](" \l "Session3_Activity28)

## Activity 50

#### Answer

3 ÷ D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_b2a4fe7cf62d3946c00e445baa1e448f5802e0e9_mu120_a_i021e.gif = 3 × 3 = 9

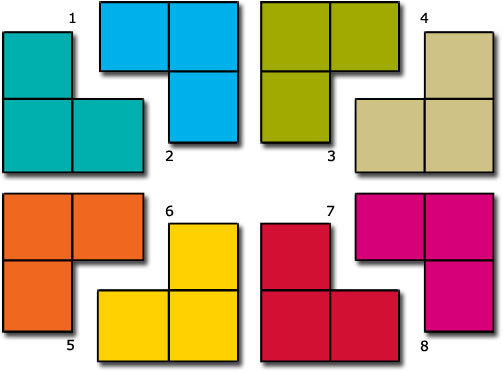
There are 9 thirds in 3.

[Back to - Activity 50](" \l "Session3_Activity29)

## Activity 51

#### Answer

Start of Figure



End of Figure

[Back to - Activity 51](" \l "Session3_Activity30)

## Example 23

#### Answer

* Using the rule above,
* Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_3a93c079a06b98df979a6f420973abdbb2f70b97_mu120_a_u020e.gif

End of $1

* This is
* D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_703c7b4fd4db27040d2f0bd5d718b2df8e630b0f_mu120_a_i149e.gif(divide top and bottom by 3).
* Alternatively divide top and bottom by 3 by cancelling by 3 to give
* Start of $1

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_c34914a1de6029a375eb7277b4bfc5c416a2bb64_mu120_a_u021e.gif

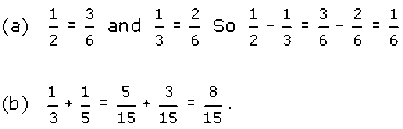
End of $1

* So D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_73b458bb43c8c9c678b682049e5dd4aab49c752c_mu120_a_i150e.gif.

[Back to - Example 23](" \l "Session3_Example13)

## Activity 52

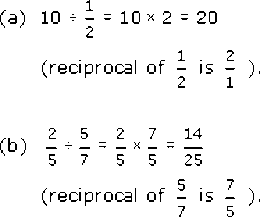
#### Answer

* 

[Back to - Activity 52](" \l "Session3_Activity31)

## Activity 53

#### Answer

* 

[Back to - Activity 53](" \l "Session3_Activity32)

## Activity 54

#### Answer

There are several ways of explaining this. For example:

Start of Quote

‘To multiply two fractions, multiply the top two numbers together (the numerators) and multiply the bottom two numbers together (the denominators). To give the answer in its simplest form, check whether top and bottom numbers have a factor in common and, if so, divide through by it.’

End of Quote

Or

Start of Quote

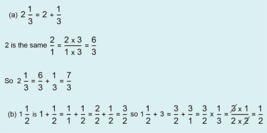
‘To multiply two fractions by cancelling, find a number that goes into (divides) a number on the top and a number on the bottom i.e. cancel. Repeat this until there are no numbers left which will cancel with each other. Then multiply together the top two numbers (the numerators) and the bottom two numbers (the denominators). This fraction should be in its simplest form.’

End of Quote

[Back to - Activity 54](" \l "Session3_Activity33)

## Example 24

#### Answer



In multiplication or division involving mixed numbers like D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_69b54e43bdc1fd15de4b341da0fd174362c500d6_mu120_a_i154e.gif, convert the mixed number to a (top heavy) fraction.

[Back to - Example 24](" \l "Session3_Example14)

## Activity 55

#### Answer

D:\AaaF\OUT\httpswwwopeneduopenlearn_cmid13470_2023-02-24_11-51-10_679dc0091c7140428b5b1be4c9193dac\word\assets\_9b52c1fa2d1b9b6eb4644baf53381021788e431c_mu120_a_si064e.gif

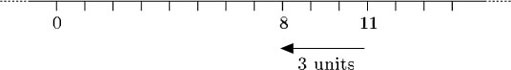
[Back to - Activity 55](" \l "Session3_Activity34)

## Example 25

#### Answer

* (a) In piggy bank terms, this represents contents of £11 plus a £3 IOU. So the overall value is 11 + −3 = 11 − 3 = 8.

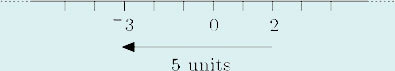
Start of Figure



End of Figure

* (b) Thomas’s piggy bank has £2 in it plus a £5 IOU. So it is worth 2 + −5 = 2 − 5 = −3.

Start of Figure



End of Figure

[Back to - Example 25](" \l "Session3_Example15)

## Activity 56

#### Answer

* (a) −4 − 6 = −10.
* If you have an overdraft of £4 and take out £6 you have an overdraft of £10.
* (b) 3 + −5 = 3 − 5 = −2.
* Thomas’s bank has £3 plus a £5 IOU, which is equivalent to a £2 IOU.
* (c) −4 + −7 = −4 − 7 = −11.
* A debit of £4 plus a debit of £7 is equivalent to a debit of £11.
* (d) −5 + 8 = 3.
* An overdraft of £5 plus a deposit of £8 results in a balance of £3.

[Back to - Activity 56](" \l "Session3_Activity35)

## Activity 57

#### Answer

* (a) 4 − −2 = 4 + 2 = 6. .
* A gift of £4 plus taking away a £2 debt is equivalent to a gift of £6.
* (b) −3 − −5 = −3 + 5 = 2.
* Acquiring a debt of £3 and taking away a debt of £5 results in being £2 better off.

[Back to - Activity 57](" \l "Session3_Activity36)

## Example 26

#### Answer

* (a) The value of the painting in a year’s time is £200 + £20 = £220. The value of the painting a year ago was £200 − £20 = £180.
* (b) To work out the value a year in the future, add the annual increase to the current value. To work out the value a year in the past, subtract the annual increase from its current value.
* (c) The value of the washing machine in a year’s time is £200 − £20 = £180.

The value of the washing machine a year ago was £200 + £20 = £220.

* (d) Yes. Thinking about the annual decrease as a negative increase, apply the rules in part (b) to carry out the calculation. The value of the washing machine in a year’s time is current value + annual increase, i.e. £200 + −£20 = £180.

The value of the washing machine a year ago is current value − annual increase, i.e. £200 − −£20 = 200 + 20 = £220.

[Back to - Example 26](" \l "Session3_Example16)

## Activity 58

#### Answer

* (a) 3 − 10 = −7,
* so you would have −£7 left, i.e. you would have a £7 overdraft.
* (b) −5 − 6 = −11,
* so the temperature was −11 °C.

[Back to - Activity 58](" \l "Session3_Activity37)

## Activity 59

#### Answer

* (a) −3 + −12 = −15 (a debt of £3 plus a debt of £12 gives a debt of £15).
* (b) −4 − −11 = −4 + 11 = 7 (incurring a debt of £4 and being let off a debt of £11 results in being £7 better off).

[Back to - Activity 59](" \l "Session3_Activity38)

## Activity 60

#### Answer

**3** Kim started at −37 m and ended at 42 m. You want 42 − −37 = 42 + 37 = 79. So Kim climbed 79 m.

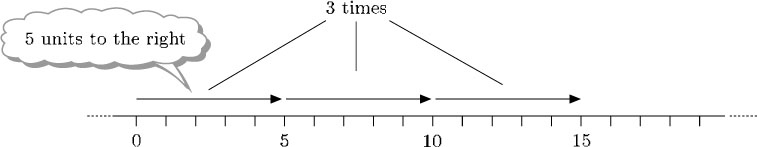
[Back to - Activity 60](" \l "Session3_Activity39)

## Example 27

#### Answer

* (a) Three times 5 means adding three lots of 5, or do the move ‘5 units to the right’ three times.

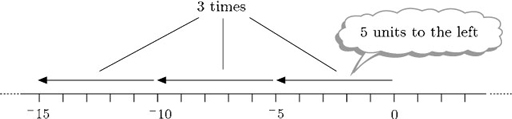
Start of Figure



End of Figure

* Thomas’s piggy bank has three £5 notes in it. The value of the piggy bank in pounds is 3 × 5 = 15.
* (b) 3 times −5 means adding three lots of −5, or do 3 times ‘move 5 units to the left’.

Start of Figure



End of Figure

* Thomas’s piggy bank has 3 IOUs each for £5 in it. The value of the piggy bank in pounds is 3 × −5 = −15.

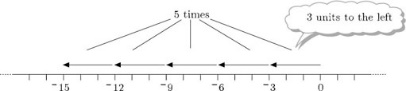
[Back to - Example 27](" \l "Session3_Example17)

## Example 28

#### Answer

* (a) Think of 5 times −3 as meaning ‘5 lots of −3’. So 5 × −3 means: do 5 times ‘move 3 units to the left’.

Start of Figure

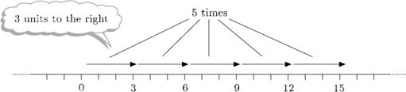


End of Figure

* So 5 × −3 = −15.
* In terms of the piggy bank Thomas has five £3 IOUs in it. The value in pounds is 5 × −3 = −15, which is equivalent to one £15 IOU.
* (b) Thinking in terms of repeated subtraction, do 5 times ‘subtract −3’. Subtracting −3 means adding 3, or moving 3 units to the right. So do 5 times ‘move 3 units to the right’.

**Subtracting −3 is the same as adding 3.**

Start of Figure



End of Figure

* So −5 × −3 = 5 × 3 = 15.
* Thomas’s mother takes away five £3 IOUs. This is equivalent to giving him £15.
* The calculation in pounds is −5 × −3 = 5 × 3 = 15.

[Back to - Example 28](" \l "Session3_Example18)

## Example 29

#### Answer

* (a) To find the value in four years’ time, in each case add the annual increase (which is negative in the case of the washing machine) four times. So the value of the painting will be
* £200 + 4 × £20 = £200 + £80 = £280
* and the value of the washing machine will be
* £200 + 4 × −£20 = £200 − £80 = £120.
* (b) To find the value four years ago, subtract the annual increase from today’s value four times. So the value of the painting four years ago was
* £200 − 4 × £20 = £120
* and the value of the washing machine four years ago was
* £200 − 4 × −£20.
* But this is a matter of repeatedly subtracting a negative number, which is the same as repeatedly adding the corresponding positive number. So
* £200 − 4 × −£20 = £200 + 4 × £20 = £280.

[Back to - Example 29](" \l "Session3_Example19)

## Activity 61

#### Answer

* (a) 4 × −3 = −12.
* Four debts of £3 is a debt of £12.
* (b) −2 × −7 = 14.
* Thomas’s mother taking away two £7 IOUs is equivalent to giving him £14.
* (c) −3 × 5 = −15.
* Thomas gives away 3 five pound notes, which is equivalent to giving away £15.

[Back to - Activity 61](" \l "Session3_Activity40)

## Activity 62

#### Answer

Your list should look something like this.

   Dividing a positive number by a positive number gives a positive answer.

   4 ÷ 2 = 2.

   Dividing a negative number by a positive number gives a negative answer.

−4 ÷ 2 = −2.

   Dividing a positive number by a negative number gives a negative answer.

   4 ÷ −2 = −2.

   Dividing a negative number by a negative number gives a positive answer.

−4 ÷ −2 = 2.

[Back to - Activity 62](" \l "Session3_Activity41)

## Activity 63

#### Answer

* (a) 24 ÷ −4 = −6
* Thomas has lots of £4 IOUs. A gift of £24 will subtract 6 of them.
* (b) −40 ÷ −4 = 10
* An IOU of £40 divided into smaller IOUs of £4 each (which could be paid off more easily from Thomas’s pocket money) gives 10 smaller IOUs.
* (c) −45 ÷ 15 = −3
* A debt of £45 shared between 15 people means each owes £3.

[Back to - Activity 63](" \l "Session3_Activity42)

## Activity 64

#### Answer

* (a) 7 × −6 = −42
* (7 debts of £6 result in a debt of £42).
* (b) −56 ÷ −7 = 8
* (a £56 debt divided into £7 debts gives eight £7 debts).

[Back to - Activity 64](" \l "Session3_Activity43)