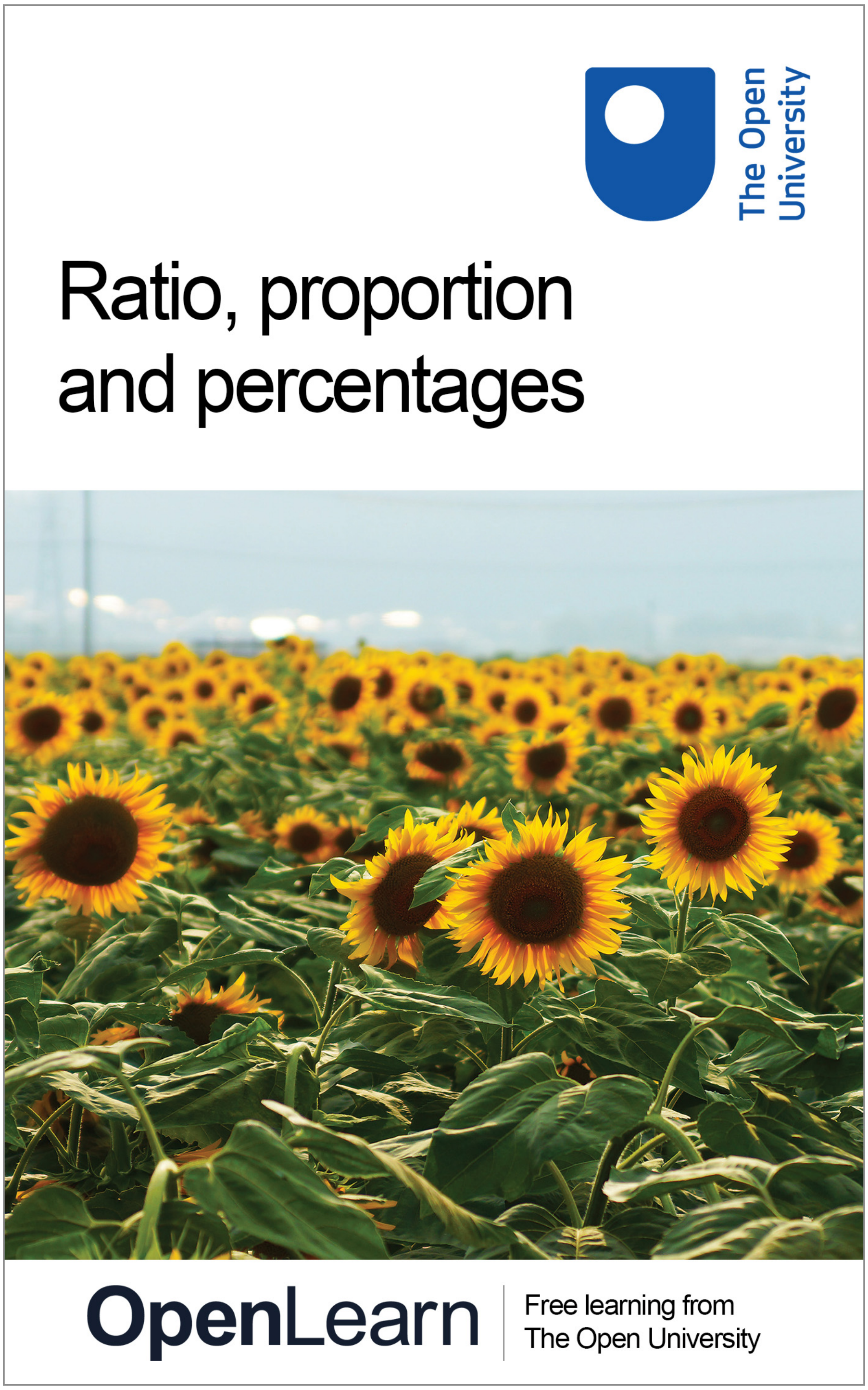
Ratio, proportion and percentages



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About this free course

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Introduction

The topics in this free course, *Ratio, proportion and percentages*, are concerned with dividing something into parts. For example, if there are 200 people living in a small village, and 50 of these are children, this could be expressed as a percentage:

25% of the village population are children;

or as a ratio:

one in every four people is a child *or* there is 1 child for every three adults;

or a proportion:

the proportion of children in the village population is a quarter.

This OpenLearn course provides a sample of level 1 study in Mathematics.

Learning Outcomes

After studying this course, you should be able to:

- work with simple ratios
- convert between fractions, decimals and percentages
- explain the meaning of ratio, proportion and percentage
- find percentages of different quantities
- calculate percentage increases and decreases.

1 Ratio

1.1 Introduction

Ratios crop up often in official statistics. The government wants the teacher–pupil ratio in schools to be increased to one teacher to thirty pupils or less. The birth rate has fallen: the ratio of children to women of child bearing age has gone down. It used to be 2.4 to 1, and now it is 1.9 to 1. Predictions for the ratio of working adults to retired adults is disturbing. Predictions are, that by 2030 the ratio will be two working adults to every retired person, instead of three to one now, and four to one ten years ago.

Often ratios are implicit in the language rather than explicitly referred to: one teacher for 30 pupils; 2.4 children per woman of child bearing age; one retired person per two working adults. The word ‘per’ often indicates that the concept of ratio is being used.

1.2 Expressing ratios

To make short crust pastry, one recipe book says ‘use one part of fat to two parts of flour’; another recipe says ‘use fat and flour in the ratio of one to two’; and yet another says ‘use half as much fat as flour’. These are different ways of expressing the same ratio. Ratios are often expressed as fractions. So in this case:

$$\frac{\text{amount of fat}}{\text{amount of flour}} = \frac{1}{2}.$$

Since you can multiply top and bottom of a fraction by the same number and get an equivalent fraction, you can use the ratio in a number of ways. If you have 100 grams of fat then

$$\frac{\text{amount of fat}}{\text{amount of flour}} = \frac{1 \times 100}{2 \times 100} = \frac{100}{200}.$$

So you need 200 grams of flour to 100 grams of fat. There are many ways to arrive at this answer. The important point is that a ratio of 100 to 200 is equivalent to 1 to 2.

To make concrete, the instructions are ‘use sand and cement in the ratio three to one’. This means

$$\frac{\text{amount of sand}}{\text{amount of cement}} = \frac{3}{1} = \frac{3 \times 30}{1 \times 30} = \frac{90}{30}$$

If you have 30 kg of cement, then you need 90 kg of sand.

The conversion rates between currencies or different units are often easier to remember as ratios. Many people remember that the ratio of distance in miles to the same distance in kilometres is five to eight.

Example 1

At the time of writing the ratio of prices in pounds sterling to prices in euros is two to three (2 : 3). What is the equivalent price in pounds for a coat costing 150 euros?

Answer

3 euros are equivalent to 2 pounds. This means that

$$\frac{\text{price in pounds}}{\text{price in euros}} = \frac{2}{3}.$$

So price in pounds = 2/3 × price in euros = 2/3 × 150 = 100

The equivalent price of the coat is £100.

1.2.1 Try some yourself

Activity 1

A friend is painting the inside walls of a garage. So far she has used a 2 litre tin of emulsion paint and covered an area of 9 m^2. She needs some more paint. How much more would you advise her to purchase if she intends to paint all the walls and ceiling, which is a total area of 75 m^2.

Answer

An area of 9 m^2 requires 2 litres of paint. The ratio of paint to area is 2 to 9 or $\frac{2}{9}$. So an area of 1 m^2 would require $\frac{2}{9}$ of a litre of paint. She needs to paint an area of (75 − 9) m^2 or 66 m^2. This will require $66 \times \frac{2}{9} = \frac{132}{9} = \frac{44}{3} = 14\frac{2}{3}$ litres of paint (i.e. 14.666 67) litres or approximately 15 litres to the nearest $\frac{1}{2}$ litres. So she needs 15 more litres of paint (rounded up).

Activity 2

If the ratio of distance measured in miles to the same distance measured in kilometres is five to eight, which is the higher speed limit, 70 miles per hour or 110 kilometres per hour?

Answer

70 is 5 × 14, so

$$\frac{\text{distance in miles}}{\text{distance in kilometres}} = \frac{5}{8} = \frac{5 \times 14}{8 \times 14} = \frac{70}{112}$$

70 miles is about the same as 112 kilometres.

Alternatively 1km = $\frac{5}{8}$ miles. So 110 km = 110 × $\frac{5}{8}$ miles = 68.75 miles.

So the two speed limits are very close but with 70 miles per hour being the higher.

1.3 Using ratios

Time conversions are also ratios. The ratio of time measured in minutes to time measured in seconds is one to sixty (1:60), as there are sixty seconds in a minute.

Example 2

Adam's grandfather ran a mile in $4\frac{1}{2}$ minutes. Adam took 260 seconds. Which is greater, 260 seconds or $4\frac{1}{2}$ minutes? Did Adam run faster than his grandfather?

Answer

$4\frac{1}{2}$ minutes is equivalent to $4\frac{1}{2}$ × 60 seconds.

$4\frac{1}{2}$ is equivalent to $\frac{9}{2}$. So this is $\frac{9}{2}$ × 60 = 270 seconds.

So $4\frac{1}{2}$ minutes is just a bit longer than 260 seconds. Therefore Adam has run faster than his grandfather.

When shopping for a bargain, the ratio of price to quantity is often a useful way of comparing prices of different sized packets.

Example 3

A local shop sells ready-made custard at £1.45 for a special offer pack of three 425 g tins. It also sells the same brand of custard in 1 kg cartons costing £1.29 each. Which is the better bargain?

Answer

To compare the prices it would be best to compare the ratio of prices to amounts (measuring amounts in the same units) i.e. prices per kg.

Three 425 g tins will contain 3 × 425 g = 1275 g or 1.275 kg.

£1.45 for 1.275 kg is

$\frac{1.45}{1.275}$ ≈ £1.14 per kg.

The 1 kg carton costs £1.29 per kg.

The three tins are 15 p cheaper per kg. So the three-tin pack is the better bargain.

1.3.1 Try some yourself

Activity 3

A local supermarket sells a popular breakfast cereal in a ‘Large Pack’ and ‘New Extra Large Pack’. They are both being sold at ‘knock down’ prices. The large pack contains 450 g of cereal priced at £1.85. The new extra large pack contains 625 g and is priced at £2.35. Which is the better bargain?

Answer

In order to compare the prices of the two cereal packs it is best to work out the price per gram for each.

The large packet will cost £1.85 for 450 g. This is $\frac{185}{450}$ or 0.411 p per gram.

The New extra large packet will cost £2.35 for 625 g. This is $\frac{235}{625}$ or 0.376 p per gram.

Obviously the new extra large pack is the better bargain.

(But the extra large package is 37 cm tall (about 13″). Unluckily it might not fit in the kitchen cupboard.)

Activity 4

Baking potatoes are priced at 75 p for a pack of three (very similar) potatoes or at £2.70 for a 5 kg bag. How heavy does each of the three potatoes in the pack have to be in order for the pack to be a better bargain than the 5 kg bag? Does your answer seem reasonable? (Try to imagine a potato of this size.)

Answer

5 kg costs £2.70. You want to know, at this rate, how much you get for 75 p. First work out how much you get for £1:

if £2.70 buys 5 kg then £1 buys $\frac{5}{2.7}$ kg.

So 75 p should buy 0.75 × $\frac{5}{2.7} \approx$ 1.39 kg.

If three potatoes weigh 1.39 kg each, one weighs about 0.46 kg. So each potato in the pack would need to weigh around half a kilogram for the pack to be a better bargain.

Baking potatoes are big, but it's unlikely that a pack of three baking potatoes would weigh $1\frac{1}{2}$ kg.

1.4 Converting ratios from fractions to decimals

Although ratios are often given as fractions, they can also be expressed as decimals. You need to deal with a mixture of fractions and decimals, and to compare ratios given in either form, so you need to be able to convert between the two forms.

Example 4

The ratio of the circumference of a circle to its diameter is a constant denoted by π (the Greek letter *p*) pronounced pi, it has been approximated by a number of different fractions. One such fraction is $\frac{22}{7}$, another is 355/113. How do these compare with the decimal value from a calculator of 3.141592 654?

Answer

If you have a calculator handy then you could key in 22 ÷ 7 to convert to a decimal. However, if not, you might use long division or an informal method of division. Either way you should get 3.1428 …. So $\frac{22}{7}$ agrees to 3 significant figures.

You will probably find it easier to use a calculator for dividing 355 by 113. The result is 3.14159292, which agrees to 7 significant figures.

Sometimes you are given a ratio as a decimal, but might find it easier to use and/or remember as a fraction.

Example 5

Suppose you had been told that the ratio between a distance measured in miles and the same distance measured in kilometres is about 0.625. Convert this to a fraction.

Answer

$$0.625 = \frac{625}{1000}$$

This fraction is an adequate answer, or you can divide top and bottom by common factors to reach a simpler equivalent fraction.

Divide top and bottom by 5 to get $\frac{125}{200}$.

Divide top and bottom by 5 again to get $\frac{25}{40}$.

Divide top and bottom by 5 again to get $\frac{5}{8}$.

So the fraction is $\frac{5}{8}$, which is the fraction used earlier.

An alternative method is do this in one line cancelling $\frac{\cancel{125}\ \cancel{25}^{5}}{\cancel{200}\ \cancel{40}_{8}}$

1.4.1 Try some yourself

Activity 5

Convert each of the following fraction ratios to decimal ratios.

(a) $\frac{1}{5}$

(b) $\frac{1}{3}$

(c) $1\frac{1}{2}$

(d) $2\frac{1}{4}$

Answer

(a) $\frac{1}{5}$= 0.2

(b) $\frac{1}{3}$ = 0.3333 ... this repeats indefinitely. As a decimal, you need to terminate after some finite number of places, e.g. 0.3333 to four decimal places.

(c) $1\frac{1}{2} = \frac{3}{2}$ = 1.5

(d) $2\frac{1}{4} = \frac{9}{4}$ = 2.25

Alternatively, 2 is 2.00, $\frac{1}{4}$ is 0.25, and adding them together gives 2.25.

Activity 6

Convert each of the following decimal ratios to fraction ratios.

(a) 0.25
(b) 1.135
(c) 0.064

Answer

(a) 0.25 = $\frac{25}{100} = \frac{1}{4}$

(b)1.135 = $\frac{1135}{1000} = \frac{227}{200}$

(c)0.064 = $\frac{64}{1000} = \frac{8}{125}$

Activity 7

A recipe for a casserole involves soaking dried beans. The beans require $1\frac{3}{4}$ litres of water per kilogram of beans. If you have $2\frac{1}{2}$ kilograms of beans, how much water is required?

Answer

You need to multiply $2\frac{1}{2}$ by $1\frac{3}{4}$. First convert to top-heavy fractions or decimals:

$$2\frac{1}{2} \times 1\frac{3}{4} = \frac{5}{2} \times \frac{7}{4} = \frac{35}{8} = 4\frac{3}{8}$$

Follow-up in Section 3.1.2
Alternatively 2.5 × 1.75 = 4.375.
So $4\frac{3}{8}$ or 4.375 litres of water is required.

1.5 Speeds

Speed is the ratio of distance travelled to time taken. A runner's speed may be quoted in metres per second, miles per hour or kilometres per hour. The units are given as:

unit of distance *per* unit of time.

When you have a distance covered (such as a mile) and a time taken (such as four minutes) the average speed is defined as

$\frac{\text{distance travelled}}{\text{time taken}}$.

The formula for average speed applies even over a journey made up of several stages.

Example 6

In 1999, Hicham El Guerrouj held the record for running a mile. He covered the distance in just over 3 minutes 43 seconds. In the same year he also held the world record for the 1500 m race. He completed this distance in 3 minutes 26 seconds.

Work out:

(a) his (average) speed in miles per hour for the 1 mile race;

(b) his (average) speed in kilometres per hour for the 1500 metre race;

(c) compare (a) and (b). In which race was his (average) speed faster assuming that 1.61 km = 1 mile?

Answer

(a) To find the speed in miles per hour, you need the ratio of the distance in miles to time in hours. In 223 seconds (3 minutes 43 seconds) he ran 1 mile. Therefore in 1 second he would run 1/223 miles. In 3600 seconds (i.e. 1 hour), he would run 3600/223 miles i.e. 16.14 miles (to 2 d.p.). So the athlete's speed is 16.14 miles per hour (to 2 d.p.).

(b) Hicham ran 1500 metres in 3 minutes 26 seconds (i.e. 206 seconds), so he ran $\frac{1500}{206}$ metres in 1 second. In 3600 seconds (1 hour) he runs 3600 × $\frac{1500}{206}$ metres. That is, 26 213.59 metres per hour. To convert to kilometres: divide by 1000 which gives 26.21 km per hour (to 2 d.p.).

(c) To compare speeds in different units, you need to convert one to the other, say miles per hour to kilometres per hour. Since 1 mile is 1.61 km, 16.14 miles is 16.14 × 1.61 = 25.99 km (to 2 d.p.). Hence 16.14 miles per hour is the same as 25.99 km per hour. This is a lower speed than 26.21 km per hour. So Hicham ran the 1500 metre race faster, which is not surprising as it is a shorter distance.

1.5.1 Try some yourself

Activity 8

Which is greater, 1.2 minutes or 70 seconds?

Answer

There are at least three ways of answering this:

(a) 70 seconds is $\frac{70}{60} = \frac{7}{6}$ = 1.1666 ... minutes, which is less than 1.2 minutes.

(b) 1.2 minutes is $1\frac{2}{10}$ = $1\frac{1}{5}$ minutes. 70 seconds is 1 minute 10 seconds, i.e. $1\frac{10}{60}$ = $1\frac{1}{6}$ minutes. Since $\frac{1}{6}$ is less than $\frac{1}{5}$, 70 seconds is less than 1.2 minutes.

(c) 1.2 minutes is 1.2 × 60 = 72 seconds, which is greater than 70 seconds. So 1.2 minutes is greater than 70 seconds.

Activity 9

Answer the following questions:

(a) A cheetah is the fastest land animal over short distances. It can run 400 metres in 15 seconds. What would be its speed in metres per second and kilometres per hour?

(b) The slowest moving land animal is the three-toed sloth from tropical America. It takes 16 hours to travel a mile. What is this speed in kilometres per hour and in metres per second? (5 miles is approximately 8 km.)

Answer

(a) Speed is the ratio of distance travelled to time taken, which in the cheetah's case is 400 metres to 15 seconds. So its speed is $\frac{400}{15} \simeq$ 27 metres per second (to 2 s.f.).

To determine this speed in kilometres per hour, both units need changing.

400 metres are $\frac{400}{1000}$ kilometres

15 seconds are $\frac{15}{3600}$ hours.

So $\frac{\text{distance}}{\text{time}}$ in the new units is

$$\begin{aligned}\frac{400}{1000} \div \frac{15}{3600} &= \frac{400}{1000} \times \frac{3600}{15} \\ &= \frac{\overset{8}{\cancel{400}}}{\cancel{1000}} \times \frac{\cancel{3600}^{12}}{\cancel{15}\cancel{5}} \\ &= 8 \times 12 \\ &= 96 \text{ kilometres per hour}\end{aligned}$$

So the cheetah's speed is 96 kilometres per hour (over a distance of 400 metres).

(b) In 16 hours the sloth moves 1 mile.

It will take 5 × 16 = 80 hours to move 5 miles which is 8 km.

So it will take 80 hours to move 8 km.

In 1 hour it will move 8/80 km or 0.1 km.

So the sloth will move at a speed of 0.1 km/h (assuming it does not fall asleep!).

To convert this speed to metres per second,

0.1 km is 100 metres

1 hour is 3600 seconds

So speed is $\frac{100}{3600} \simeq$ 0.028 metres per second (to 2 s.f.)

Activity 10

Which is longer, 11 minutes or 0.17 hours?

Answer

As a fraction of an hour, 11 minutes is 11/60 hours.

To convert this to a decimal, divide 11 by 60 to get 0.18333….

This is greater than 0.17, so 11 minutes is longer.

(Alternatively, 0.17 hours is 60 × 0.17 = 10.2 minutes, so 11 minutes is longer.)

Activity 11

A van driver averages 50 km per hour travelling on ordinary roads and 70 km per hour on motorways. Estimate:

(i) how far the van will travel in $2\frac{1}{2}$ hours;

(ii) how long it will take to travel 160 km;

when travelling on (a) ordinary roads and (b) motorways.

Answer

(a)

(i) In 1 hour the van travels 50 km so in $2\frac{1}{2}$ hours the van travels

50 × $2\frac{1}{2}$ = 125 km.

(ii) The van travels 50 km in 1 hour so it travels 1 km in $\frac{1}{50}$ hour. It therefore travels 160 km in 160 × $\frac{1}{50}$ hours, i.e. $\frac{16}{5}$ hours or $3\frac{1}{5}$ hours (3 hours 12 minutes).

(b)

(i) 70 ×$2\frac{1}{2}$ = 175 km.

(ii) $\frac{160}{70} \simeq$ 2.29 hours (2 hours 17 minutes).

2 Proportion

2.1 Introduction

Proportion is another way of expressing notions of part and whole. You might say that the proportion of village inhabitants who are children is a quarter, or that the proportion of fruit juice in the punch is two thirds, or that the proportion of sand in the concrete is three quarters.

All these examples involve the fractions $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{4}$. Problems involving proportions are best handled by manipulating fractions, generally, by division or multiplication. The task is to decide which fraction to manipulate, in what way, and at what stage!

2.2 Direct proportion

In a recipe the quantity of each ingredient needed depends upon the number of portions. As the number of portions increases, the quantity required increases. The quantity per portion is the same. This is called direct proportion. The quantity is said to be **directly proportional** to the number of portions. If 2 potatoes are required for one portion, 4 will be required for two portions etc. A useful method for direct proportion problems is to find the quantity for one and multiply by the number you want.

Example 7

John lives with three cats. His daughter asks him to look after her cat for a week while she goes away. John normally buys two tins of cat food a day for the three cats. How many tins should he buy for the four cats for a week?

Answer

There are several ways to do this. Here is one.

3 cats eat 2 tins a day

1 cat eats $\frac{2}{3}$ tin a day

4 cats eat $4 \times \frac{2}{3} = \frac{8}{3}$ tins a day

4 cats eat $7 \times \frac{8}{3} = \frac{56}{3} = 18\frac{2}{3}$ tins a week

So John should buy 19 tins for the week.

Note it helped to simplify the problems by considering 1 cat, rather than going straight to 4 cats.

Example 8

Debbie is checking her phone bill. Her mobile phone calls have all been charged at the same rate, 30 pence per minute. (Call charges are rounded to the nearest penny and charged to the nearest second.)

(a) She wants to check the cost of a call to her friend. The call lasted 7 minutes and 34 seconds. How much should she have been charged?

(b) How long, at this rate, can she speak to her friend if the call charge is to cost no more than £2.50?

Answer

(a) 1 minute cost 30 p. (i.e. 60 seconds for 30 p).

1 second for $\frac{30}{60}$ p = 0.5 p.

7 minutes 34 seconds is 454 seconds, costing 454 × 0.5 p = 227 p.
The call should have been charged at £2.27.

(b) 0.5 p will allow her to talk for 1 second.

1 p will allow her to talk for 2 seconds.
£2.50 (250 p) will allow her to talk for 250 × 2 seconds, = 500 seconds, i.e. 8 minutes 20 seconds.

2.2.1 Try some yourself

Activity 12

A recipe for four people calls for $\frac{3}{4}$ teaspoon of mustard powder. How much should you use for ten people?

Answer

For one person you need $\frac{3}{4} \div 4 = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$ teaspoons.

For ten people you need $\frac{3}{16} \times 10 = \frac{30}{16} = \frac{15}{8} = 1\frac{7}{8}$ teaspoons.

So almost 2 teaspoons of mustard are needed.

Activity 13

The length of time it takes to cook a Christmas pudding in a pressure cooker depends on the weight of the pudding. Kim has forgotten the time per pound but remembers that her $1\frac{1}{2}$ kg pudding takes 5 hours.

How long will it take to cook a 2 kg pudding?

Answer

If it takes 5 hours to cook $1\frac{1}{2}$ kg then the time to cook 1 kg is 5 ÷ $1\frac{1}{2}$ hours.

$5 \div \frac{3}{2} = 5 \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$ hours.

So a 2 kg pudding takes $2 \times 3\frac{1}{3} = 6\frac{2}{3}$ hours (6 hours 40 mins), assuming that cooking time is proportional to weight.

2.3 Inverse proportion

In Section 2.2 you saw that direct proportion described relationships between two quantities, where as one increased, so did the other. Sometimes as one quantity increases the other decreases instead of increasing. This is called indirect proportion. Team tasks are often an example of this. The time taken to do a job is indirectly proportional to the number of people in the team.

A difficulty with the real-life context of such problems is that, in many cases, it is hard to believe that people working in a team will work at the same rate regardless of the size of the team, unless the team work independently, i.e. 'in parallel'. The main idea behind this type of problem is that increasing the number of people working decreases the time taken to complete the task. (An obvious exception to this is decision-making in a committee: if two people can reach a decision in an hour, four people are liable to take twice as long!)

Such problems can be compared with certain problems involving speed: doubling the number of people working is the same as doubling the speed at which the team work. In either case the time is halved. It is useful to find out how long it would take one person to do the whole job, then divide by the number of people sharing the work. This is a good approach to most indirect proportion problems.

Example 9

A team of five people can deliver leaflets to every house on a housing estate in three hours. How long will it take a team of just two people?

Answer

Take the same approach as in Examples 7 and 8, and first work out how long it would take *one* person to deliver leaflets to the estate.

It will take one person five times as long as a team of five people. (If you find this hard to accept, imagine that the estate consists of five streets, and that each person delivers leaflets to one of these streets in the three hours.) So each street takes 3 hours to leaflet. It would take one person 5 × 3 hours to leaflet all five.

So it takes one person 15 hours to deliver leaflets to the whole estate. Two people will take half this time, so two people take $7\frac{1}{2}$ hours.

(As a check: you would expect two people to take longer than five.)

2.3.1 Try some yourself

Activity 14

A piece of computer software is to be developed by a team of programmers. It is estimated that a team of four people would take a year. Which of the following times is the length of time taken by three programmers?

A 1 year 3 months **B** 9 months **C** 1 year 4 months

Answer

C: 4 programmers take 1 year.

So 1 programmer would take 4 years.

3 programmers would take $\frac{4}{3}$ years = $1\frac{1}{3}$ or 1 year 4 months.

Activity 15

A 10 kg bag of potatoes lasts for a week when used in catering for 7 people.

(a) How long will it last for 2 people, assuming everybody eats the same amount?

(b) If, instead of buying a 10 kg bag (which might not keep well), you want to buy fresh potatoes every week, how much per week should you buy for 2 people?

Answer

(a) First work out how long it would last one person, remembering that it will last longer for one person:

it lasts 1 week for 7 people, so it lasts 7 weeks for 1 person.

It will last less than this for 2 people – you need to divide by 2: i.e. it lasts $\frac{7}{2} = 3\frac{1}{2}$ weeks for 2 people.

(b) You know that in $3\frac{1}{2}$ weeks 2 people get through 10 kg of potatoes, so in 1 week 2 people get through 10 ÷ $3\frac{1}{2}$= 2.86 kg. In practice you would probably buy 3 kg most weeks, and 2 kg when there seemed to be a lot of potatoes left over from the previous week.

Activity 16

Two workers in the Open University warehouse take 20 minutes to stick labels on 500 packages for an MU120 mailing. There are still 4000 more packages. How many workers are required, if the job is to be done in about a further hour?

Answer

One worker should take twice as long as two. So 1 worker takes 40 mins for 500 labels. 4000 is 8 times 500 so 1 worker takes 40 × 8 min = 320 mins. 1 hour is 60 mins. So you need $\frac{320}{60}$ workers $= 5\frac{1}{3}$.

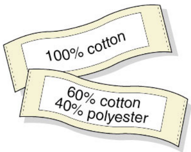
Practically, you could have 5 workers for $\frac{320}{5}$ = 64 mins (just over an hour) or 6 workers for $\frac{320}{6}$ = 53 mins (just under an hour).

3 Percentages

3.1 What are percentages?

Percentages are used, particularly in newspaper articles, to indicate fractions (as in ‘64% of the population voted’) or to indicate changes (as in ‘an increase of 4%’).

Percentages often indicate proportions. For example, labels in clothes indicate the various proportions of different yarns in the fabric. ‘Per cent’ means ‘per hundred’ and is denoted by the symbol %. 100% is the same as the whole, or one hundred per hundred.



100% cotton indicates that the fabric is made entirely from cotton. (100 parts out of 100 parts).

60% cotton means that $\frac{60}{100}$ (or 0.6) of the fabric is cotton.

40% polyester means that $\frac{40}{100}$ (or 0.4) is polyester.

The percentages on the label should total 100%, just as the corresponding fractions add up to 1, because the total (100%) refers to the whole garment. 60% + 40% = 100%, 0.6 + 0.4 = 1.

Percentages can also be manipulated as either fractions or decimals.

Example 10

A building society offers 90% mortgages to first-time buyers. How much would the Smiths get on a house valued at £150 000?

Answer

They want to find 90% of £150 000. 90% = $\frac{90}{100}$ = 0.9.

$$0.9 \times 150\,000 = 135\,000$$

So the Smiths would get £135 000.

3.1.1 Try some yourself

Activity 17

Express each of the following percentages as fractions:

(a) 40%

(b) 8%

(c) 70%

(d) $5\frac{1}{2}\%$

Answer

(a) 40%= $\frac{40}{100} = \frac{2}{5}$

(b) 8%= $\frac{8}{100} = \frac{2}{25}$

(c) 70%= $\frac{70}{100} = \frac{7}{10}$

(d) $5\frac{1}{2}\% = \frac{11}{2} \times \frac{1}{100} = \frac{11}{200}$

Activity 18

Express each of the following percentages as decimals:

(a) 50%

(b) 85%

(c) 7%

(d) $17\frac{1}{2}\%$

Answer

(a) 50% = $\frac{50}{100}$ = 0.5

(b) 85% = $\frac{85}{100}$=0.85

(c) 7% = $\frac{7}{100}$= 0.07

(d) $17\frac{1}{2}\% = 17.5\% = \frac{17.5}{100} = 0.175$

Activity 19

A survey was carried out on 840 couples to investigate family income. 75% of the women were in paid work; 90% of the men were in paid work; 18% of the couples had a joint income of less than £160 per week.

(a) How many women were in paid work?

(b) How many men were in paid work?

(c) How many couples had a joint income of less than £160 per week?

Answer

(a) 75% of 840 = 0.75 × 840 = 630,

so 630 women were in paid work.

(b) 90% of 840 = 0.9 × 840 = 756,

so 756 men were in paid work.

(c) 18% of 840 = 0.18 × 840 = 151.2.

But 151.2 is not a whole number. So 151 couples had a joint income of less than £160 per week. (The survey result must have been rounded, as a percentage, to the nearest whole number.)

3.2 Converting to a percentage

Fractions and decimals can also be converted to percentages, by multiplying by 100%. So, for example, 0.17, 0.3 and $\frac{3}{4}$ can be expressed as percentages as follows:

0.17 × 100% = 17%;

0.3 × 100% = 30%;

$\frac{3}{4} \times 100\% = 75\%$

Decimals or fractions bigger than 1 correspond to percentages greater than 100%. For example,

$1\frac{3}{4}$ = 1.75, which as a percentage is 1.75 × 100% = 175%.

Care needs to be taken when talking in percentages. A percentage is a percentage of a given quantity: 50% of the voters, 25% of the budget, 10% of the population. In newspapers, it is not always clear what quoted percentages are percentages of. Politicians too can give misleading statements. 'We are giving an increase in funding of 10% and 5% of this has no strings attached'. In this statement it is not clear whether the 5% is 5% of the original funding or 5% of the 10% increase in funding (which would be $\frac{5}{100}$ × 10% = 0.5% of the original funding).

To avoid this confusion some people use the term percentage points, when they are comparing the percentages of different quantities. In elections the percentage of the vote at one election for a given party is often compared with the percentage of the vote at the previous election. The difference is referred to as the 'swing' and expressed in percentage points.

Example 11

In a local election 2540 votes are cast for the Purple party out of a total of 5000 votes. At the previous election 42.1% of votes had been for the Purple party. What is the swing to the Purple party in percentage points?

Answer

2540 out of 5000 as a percentage is $\frac{2540}{5000}$ × 100 = 50.8%

Swing = 50.8 − 42.1 = 8.7 percentage points.

3.2.1 Try some yourself

Activity 20

Convert each of the following to percentages. Round off the percentages to whole numbers.

(a)

- (i) 0.8
- (ii) 0.21
- (iii) 0.70
- (iv) 2.4

(b)

- (i) $\frac{1}{2}$
- (ii) $\frac{1}{8}$
- (iii) $\frac{1}{9}$
- (iv) $1\frac{1}{7}$

Answer

(a)

- (i) 0.8 = 80%
- (ii) 0.21 = 21%
- (iii) 0.70 = 70%
- (iv) 2.4 = 240%

(b)

- (i) $\frac{1}{2}$ = 0.5= 50%
- (ii) $\frac{1}{8}$ = 0.125=12.5% $\simeq$ 13%
- (iii) $\frac{1}{9}$ $\simeq$ 0.1111 $\simeq$ 11%
- (iv) $1\frac{1}{7}$ $\simeq$ 1.143 $\simeq$ 114%

Activity 21

In the election in Example 11 the Average party gets 2100 votes. In the previous election they had 10%. Find the swing.

Answer

2100 out of 5000 is

$\frac{2100}{5000} \times 100\% = 42\%$.

The swing is 42 − 10 = 32 percentage points.

3.3 Percentage increase and decrease

3.3.1 Increasing by a percentage

Our everyday experience of percentages includes percentage increases (like VAT at $17\frac{1}{2}\%$, or a service charge of 15%) and percentage decreases (such as a discount of 15%). For example, £8 plus $17\frac{1}{2}\%$ VAT means you actually have to pay

£8 + ($17\frac{1}{2}\%$ of £8).

$17\frac{1}{2}\%$ of 8 is $\frac{17.5}{100} \times 8 = \frac{140}{100} = 1.4$

so the VAT is £1.40, and the sum you pay is £8 + £1.40 = £9.40.

There is another way of doing this calculation:

100% of £8 plus $17\frac{1}{2}\%$ of £8 = (100% + $17\frac{1}{2}\%$) of £8,

that is 117.5% of £8.

117.5% of £8 = £(1.175 × 8)

= £9.40 (as before).

Example 12

A restaurant bill comes to £76 before VAT and the service charge are added. VAT is added at $17\frac{1}{2}$ and the restaurant also adds a service charge of 15%. Does it make any difference to what you have to pay if the VAT is added first then the service charge or vice versa? Does it make a difference to the amount of VAT paid?

Answer

Adding VAT first gives 100% + $17\frac{1}{2}\%$ = $117\frac{1}{2}\%$

$117\frac{1}{2}\%$ of £76 = 1.175 × £76 = £89.30

Adding the service charge on to £89.30 gives 100% + 15% = 115%

115% of £89.30 = 1.15 × £89.30 = £102.70 (correct to the nearest penny).

So the total bill is £102.70.

Adding these extras the other way round:

115% of £76 = 1.15 × £76 = £87.40;

$117\frac{1}{2}\%$ of £87.40 = 1.175 × £87.40

= £102.70 (correct to the nearest penny).

In the first case 1.15 × 1.175 × £76, and in the second 1.175 × 1.15 × £76. The order doesn’t matter in multiplication: 1.175 × 1.15 = 1.15 × 1.175.

The total bill is the same. The order does not matter from the customer's point of view. However, it does from the VAT collector's point of view (and indeed from that of the restaurant management, who receive a bigger service charge if it is calculated last). If VAT is added first, VAT is $17\frac{1}{2}\%$ of £76. If VAT is added last, VAT is $17\frac{1}{2}\%$ of £87.40. Hence legally VAT must be added last!

3.3.2 Try some yourself

Activity 22

Answer the following questions

(a) How much will this tennis racquet cost if VAT at $17\frac{1}{2}\%$ has to be added?

(b) The racquet is put in the sale at 10% discount. What is its sale price, not including VAT?

(c) If customers pay VAT and get the discount, how much do they pay? Does it make any difference to the customer whether the VAT is added first then the discount subtracted, or vice versa? Give a reason for your answer.

Answer

(a) $117\frac{1}{2}\%$ of £35.75 = 1.175 × £35.75 = £42.006 25. In practice this would be rounded to the nearest penny, i.e. £42.01 (or £42.00 perhaps).

(b) A 10% discount would reduce it to 90% of its previous selling price, i.e. 0.9 × £35.75 = £32.175 ≈ £32.18.

(c) If VAT is added first you get 0.9 × £42.01 = £37.809 ≈ £37.81. If the discount is taken first you get 1.175 × £32.18 = £37.8115 ≈ £37.81. Thus it makes no difference to the final price whether you take the discount or the VAT first. The reason for this is, essentially, that the order in which you do successive multiplication doesn't matter:

$0.9 \times 1.175 \times 35.75 = 1.175 \times 0.9 \times 35.75$

Activity 23

A commuter pays £1260 for a season ticket. The train company announces an increase of 7.4% on all its fares. How much will the season ticket cost after the increase?

Answer

If increase on fares is 7.4% then you would need to find

(100% + 7.4%) i.e. 107.4% of £1260.

$1.074 \times 1260 = 1353.24$.

So the season ticket will cost £1353.24 after the increase.

3.4 Decreasing by a percentage

Discount can be calculated in the same way as an increase by a percentage. For example, £8 with 15% discount means you actually pay

£8 less (15% of £8)

15% of 8 = $\frac{15}{100} \times 8 = \frac{120}{100} = 1.2$. So the discount is £1.20

£8 − £1.20 = £6.80

Alternatively the actual amount can be calculated in another way, as follows:

(100% − 15%) of £8 is 85% of £8.

85% of £8 = £(0.85 × 8)

= £6.80 (as before).

Example 13

(a) A carpet store is offering 20% off all its oriental rugs. What would the sale price be for a Chinese rug originally priced at £245?

(b) The business fails to do well and decides to close down. It makes a further reduction on all its stock of 12%. What would the same Chinese rug be sold for now?

Answer

(a) In the first sale the Chinese rug would cost (100% − 20%) of £245, 100% − 20% = 80%, 0.80 × £245 = £196.

(b) In the final closing sale the rug would cost (100% − 12%) of £196 or 0.88 × £196 = £172.48.

3.4.1 Try some yourself

Activity 24

A new train operator boasts 'Train times reduced by 12%'. Decrease 90 minutes by 12%. Give your answer as minutes and seconds.

Answer

A 12% decrease would reduce it to 88% of the original time, i.e. 0.88 × 90 = 79.2 minutes.

0.2 of a minute = 0.2 × 60 = 12 secs.

So 79.2 minutes = 79 minutes and 12 secs.

Activity 25

The population of a small town is 4650. It is predicted that the population will decrease by approximately 4% each year. What is the population likely to be after (a) one year, (b) two years?

Answer

(a) Population after 1 year is likely to be

(100% − 4%) of 4650,

i.e. 96% of 4650,

which is 0.96 × 4650 = 4464 people.

(b) Population after 2 years is likely to be

96% of 4464,

i.e. 0.96 × 4464 = 4285.44 people,

but you can't have 0.44 of a person. So approximately 4285 people.

3.5 More examples of percentages

In lots of everyday situations percentages are used to make predictions and comparisons.

Example 14

The number of casualties handled by the outpatients department of a hospital increases by approximately 8% per year. The number of casualties this year was 1920. Make a prediction for the number of casualties handled (a) next year, (b) in two years' time.

Answer

(a) Casualties would be 100% + 8% of 1920, that is, 108% of 1920.

This gives 1.08 × 1920 = 2073.6 = 2074 (correct to nearest whole number) as a prediction for the number of casualties next year.

(b) Repeat this calculation again on next year's figures to find the predicted figure for the number of casualties in two years' time.

This is 108% of 2074. 1.08 × 2074 = 2239.92 $\approx$ 2240 casualties.

3.5.1 Try some yourself

Activity 26

Answer the following questions

(a) What is 40% as a fraction?

(b) What is $\frac{3}{4}$ as a percentage?

(c) What is 1.26 as a percentage?

Answer

(a) 40% = $\frac{40}{100}$ = $\frac{2}{5}$.

(b) $\frac{3}{4}$ = $\frac{3}{4}$ × 100% = 3 × 25% = 75%.

(c) 1.26 = 1.26 × 100% = 126%.

Activity 27

In an election 1248 votes out of 3000 are cast for the Gold party. What is this as a percentage? If in the previous election 35.1% voted Gold, what is the swing to Gold in percentage points?

Answer

$\frac{1248}{3000}$ × 100% = 41.6%.

The swing is 41.6 − 35.1 = 6.5 percentage points.

Activity 28

A quote from a builder for some home improvements is £2200 plus VAT.

(a) If VAT is $17\frac{1}{2}$%, what is the amount including VAT?

(b) If the builder gives 10% reductions for prompt payment, what would the amount be then (including VAT)?

Answer

(a) £2200 × $\frac{117.5}{100}$= £2585.

(b) £2585 × $\frac{90}{100}$ = £2326.50.

Activity 29

An airline decides to increase its fares from London to Europe by 17% and its fares to North America by 15%. Before the increase a special offer European fare from London to Geneva was £165 and the North American fare from London to New York was £376. What would the new fares be for these trips after the increases?

Answer

The London to Geneva fare is increased by 17%. So it will now cost 117% of £165, i.e. 1.17 × £165 = £193.05.

The London to New York fare is increased by 15%. So it will now cost 115% of £376, i.e. 1.15 × £376 = £432.40.

4 OpenMark quiz

Now try the quiz, and see if there are any areas you need to work on.

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