

Starting with maths: Patterns and formulas



Starting with maths: Patterns and formulas



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Introduction

Patterns occur everywhere in art, nature, science and especially mathematics. Being able to recognise, describe and use these patterns is an important skill that helps you to tackle a wide variety of different problems. This course explores some of these patterns ranging from ancient number patterns to the latest mathematical research. It also looks at some useful practical applications. You will see how to describe some patterns mathematically as formulas and how these can be used to solve problems both by hand and using a computer spreadsheet. At the end of the course, you can even have a go at an unsolved mathematical problem! Between 2000 and 2002, a \$1 000 000 prize was offered for its solution – that's just to show you that there is still a lot of very exciting mathematics to be discovered and also that everybody – you, me and all great mathematicians – do get stuck with mathematics somewhere!

You won't need a computer to do this though!

This OpenLearn course provides a sample of level 1 study in [Mathematics & Statistics](#)

Learning Outcomes

After studying this course, you should be able to:

- visualise problems using pictures and diagrams
- recognise patterns in a variety of different situations
- use a word formula to help solve a problem
- derive simple word formulas, for example for use in a spreadsheet
- use doing and undoing diagrams to change formulas round.

1 Exploring patterns and processes

Suppose you are tiling a bathroom or kitchen and the last row of square tiles is to be a frieze made up of blank tiles and patterned tiles as shown below.

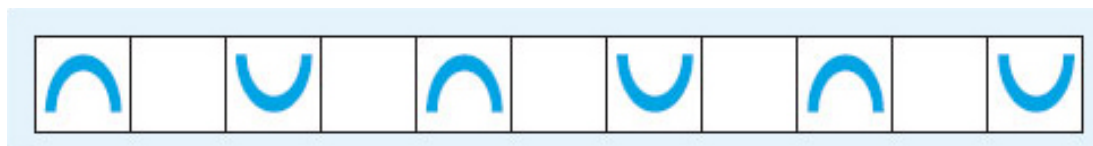


Figure 1

A friend has offered to help. How would you describe the pattern and how to arrange the tiles?

There are lots of ways of tackling this. For example, you might say that you will need some blank tiles and some patterned tiles with the 'bridges' on. Start with a bridge tile, then put a blank tile next to it. Take another bridge tile, but turn it round so that the bridge is upside down, like a smile and put it next to the blank tile in the same line. Then put another blank tile next to the smile tile. This is the pattern, just carry on: bridge, blank, smile, blank, bridge, blank, smile,

Or you may have decided to draw a picture of the tiles, or demonstrate the pattern with the tiles themselves. Whatever you do though, it's probably easier to remember and apply if you have recognised that the pattern is a four-tile repeat with the two different types of tile.

The frieze is an example of a type of geometrical pattern that has many applications in art, crafts and design. The next example is a number pattern which appeared in China and Persia over 700 years ago, but is still used by students in mathematical and statistical problems today and it even appears in chemistry. It is known as Pascal's triangle (named after Blaise Pascal who was a 17th century French mathematician who studied the properties of this triangle) and the first part of it is shown below.

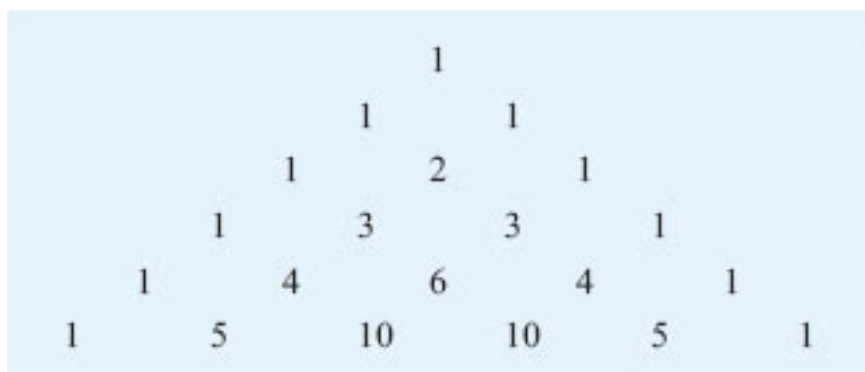


Figure 2

You can continue the triangle indefinitely by following the pattern.

Each row of numbers starts and ends with the number 1. Look at each pair of numbers in the last line above. For each pair, add the two numbers together and write their sum on the line below as shown. This process generates the next row of the triangle.

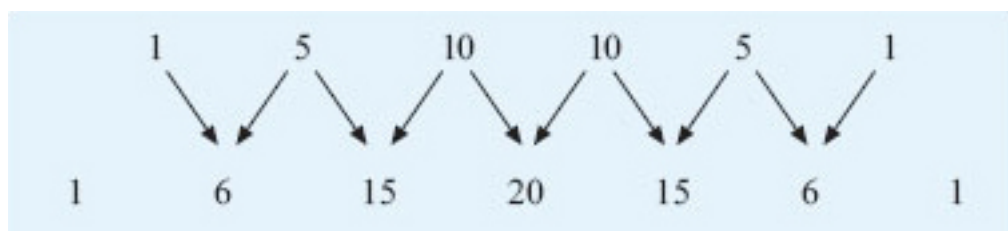


Figure 3

Activity 1: Pascal's triangle

Using the hexagonal paper attached in PDF format below, write down the numbers in the next two rows of Pascal's triangle.

[Hexagon paper](#)

- Can you spot any patterns in the numbers in the triangle?
- What do you notice about the sum of the numbers in each row?
- If this pattern continues, what do you think the total for the 10th row will be?

Keep this paper handy as you will need your triangle later in the course.

The next two rows are 1, 7, 21, 35, 35, 21, 7, 1 and 1, 8, 28, 56, 70, 56, 28, 8, 1.

There are many different patterns. The 1s down the sides of the triangle is probably the easiest one to spot! You can also see the counting numbers 1, 2, 3, 4, 5 in the diagonal rows next to the sides. In the next diagonal row, there is the sequence 1, 3, 6, 10, These numbers are known as the triangular numbers because they create triangular patterns.

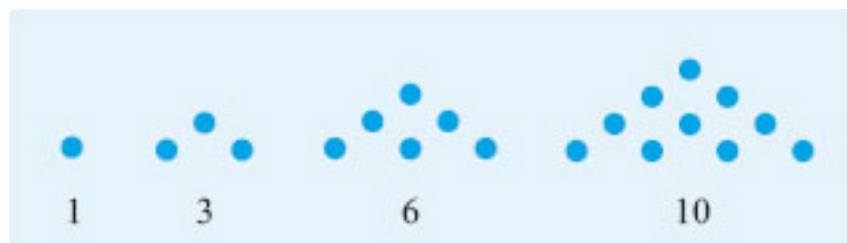


Figure 4

The triangle is symmetrical too. If you draw a vertical line down through 1, 2, 6, ..., one side of the triangle is a mirror image of the other.

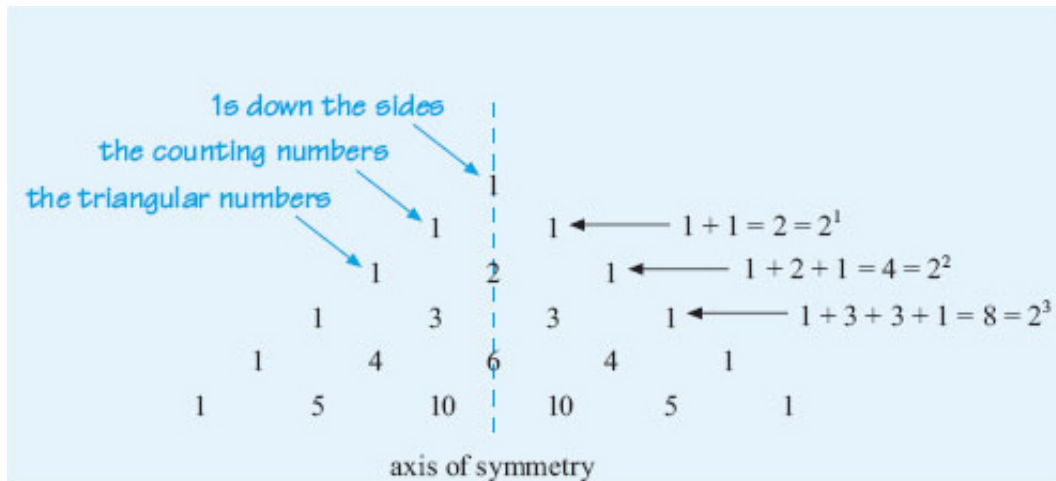


Figure 5

Do not worry if you did not spot all these patterns!

Looking at the diagonal row of triangular numbers, if you add any two adjacent numbers together, you get the square numbers. For example, $1 + 3 = 4$ which is the same as 2^2 ; $3 + 6 = 9$ which is the same as 3^2 , and so on.

The pattern in the powers suggests that the first total, 1, might be written as 2^0 . Check on your calculator to see if this is correct. Does any number to the power 0 give you 1?

Now, if you add together the numbers in each horizontal row, you get the following pattern: 1, 2, 4, 8, 16, 32, 64, The total for the next row is double the total for the current row. Assuming this pattern continues, the total for row 8 will be 128, row 9 will be 256 and row 10 will be 512. You might have spotted that these numbers can also be written as $1, 2^1, 2^2, 2^3, 2^4, \dots$

What have these two examples to do with mathematics? Well, recognising patterns in shapes, sets of numbers, processes or problems generally and spotting what is the same and what is different about situations often makes the task easier to solve. You saw how recognising the tiling pattern makes it easier to remember and by using the number patterns in Pascal's triangle, you could work out the sum of each row without adding up the individual numbers. If you can spot a pattern and then describe what happens in general, this can lead to a rule or formula. Provided you can prove that this rule will always work, it can then be used elsewhere. For example, if you can work out the general process for calculating a quarterly electricity bill and then give these instructions to a computer, many electricity bills can be generated, printed and sent out in just a few minutes!

2 Looking for relationships

In this section, we are going to consider the relationship between quantities in two practical situations and see how to describe these relationships by writing down a general rule or a **word formula**.

Suppose you are planning a visit abroad. Your map marks all the distances between places in kilometres rather than miles. How can you work out what these distances are in miles?



Figure 6

The first bit of information you will need is how long one kilometre is, if it is measured in miles.

According to a dictionary, 1 km is equivalent to approximately 0.6214 miles.

The next step is to work out the mathematical process you will need to use to change kilometres into miles. You may be able to see how to convert from kilometres into miles immediately, but, if not, try to visualise a few simple examples first. Drawing a diagram often helps!

Each km is the same as 0.6214 miles as shown below.

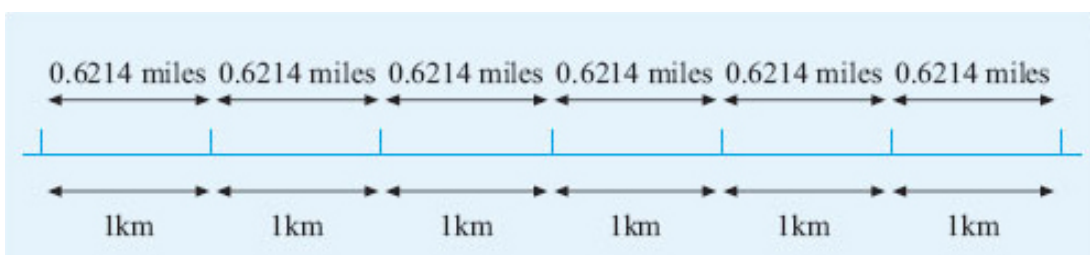


Figure 7

So if the distance is 3 km, you will have 3 lots of 0.6214 miles; if the distance is 10 km, you will have 10 lots of 0.6214 miles; if the distance is 250 km, you will have 250 lots of 0.6214 miles, and so on.

You can write this down mathematically as:

$$250 \text{ km} = 250 \times 0.6214 \text{ miles} = 155 \text{ miles (to the nearest whole number).}$$

Notice that in each of the examples above, the process for calculating the number of miles was the same: multiply the number of kilometres by the conversion rate of 0.6214. This technique will work for any distance and can be written down concisely as the following word formula:

$$\text{distance in kilometres} \times 0.6214 = \text{distance in miles.}$$

This is the same as writing:

$$\text{distance in miles} = \text{distance in kilometres} \times 0.6214.$$

You can use this formula to convert any distance in kilometres into miles. For example, suppose you wanted to convert 500 km. Using the formula and replacing 'distance in kilometres' by 500 gives

$$\text{distance in miles} = 500 \times 0.6214 = 310.7.$$

So 500 km is equivalent to approximately 311 miles.

In this example, you used the formula by replacing the phrase 'distance in kilometres' in the right-hand side of the equation by the corresponding value, 500. This is known mathematically as **substituting** the value into the formula.

Activity 2: How many miles?

How far is 350 km in miles?

Substituting 350 for 'distance in km' gives

$$\text{distance in miles} = 350 \times 0.6214 = 217.49.$$

So 350 km is approximately 217 miles.

Activity 3: Exchanging currencies

Suppose you wish to exchange some money in pounds sterling into euros. At the time of writing, one agency had an exchange rate of €1.48 to £1 and did not make any additional charges. (If you prefer, you can look up the current conversion rate in a newspaper or at a bank or travel agency.)

- How many euros would you get for £5?
How many euros would you get for £10?
- Write down a word formula that you can use to convert pounds into euros. Your formula should start 'number of euros = ...'.
- Check that your formula works by using it to convert £5 into euros.

(a) For each pound, you would get €1.48, as shown below.



Figure 8

So, if you exchange £5, you would get five lots of €1.48.

So $£5 = 5 \times €1.48 = €7.40$.

If you exchange £10, you would get 10 lots of €1.48.

So $£10 = 10 \times €1.48 = €14.80$.

(b) To change pounds into euros, you need to multiply the number of pounds by the exchange rate of €1.48. So the word formula is

number of euros = number of pounds \times 1.48.

(c) Substituting 5 for 'number of pounds' gives

number of euros = $5 \times 1.48 = 7.40$.

This agrees with the answer in part (a).

What would happen to the formula if there was also a handling charge?

These two examples illustrate how a word formula can be used to summarise some mathematical process such as converting units of length or currencies. Once a formula has been derived, it can then be used in other situations, both for calculations by hand or by computer, for example for currency transactions in a bank. You will be able to practise writing down some more of your own formulas in [Sections 4](#) and [5](#).

3 Using formulas

3.1 Introduction

In the last section, we considered how a formula could be built up and then how it could be used. This section considers some more complicated formulas, which have already been developed and are used in a variety of different situations – cookery, healthcare, business and archaeology. We hope that these examples illustrate some of the very broad applications of maths and how mathematical relationships can be used in making decisions. As you work through these examples, you may like to consider where else maths might be used in each of these topics!

When you are trying to solve a real-life problem mathematically, you often use formulas that have already been developed, so it is important to have some ideas for how to apply these.

This section will help you to do that – you may like to make your own list of tips as you work through this section and compare it with the box at the end of this section.

3.2 Maths in cookery

The time taken to cook a fresh chicken depends on its mass, as given by the following formula:

$$\text{cooking time in minutes} = 15 + \frac{\text{mass in g}}{500} \times 25$$

Roughly how long will a chicken with a mass of 2.2 kg take to cook?

To use the formula, you need to substitute the mass of the chicken into the right-hand side of the equation and then work out the resulting calculation. However, the formula asks for the mass in grams, so the first step is to convert 2.2 kg into g.

Since there are 1000 g in 1 kg, $2.2 \text{ kg} = 2.2 \times 1000 \text{ g} = 2200 \text{ g}$.

Substituting 2200 for the mass in g gives:

$$\text{cooking time in minutes} = 15 + \frac{2200}{500} \times 25$$

Carrying out the multiplication and division first gives:

$$\text{cooking time in minutes} = 15 + 110 = 125.$$

So the cooking time would be about 125 minutes or 2 hours and 5 minutes.

If you prefer, you can use your calculator, but remember to make an estimate first so that you can check that your answer is reasonable.



Figure 9

Activity 4: A smaller bird

How long would it take to cook a chicken with a mass of 1.6 kg?

Converting 1.6 kg into g: $1.6 \text{ kg} = 1.6 \times 1000 \text{ g} = 1600 \text{ g}$.

So the cooking time in minutes $\text{minutes} = 15 + \frac{1600}{500} \times 25 = 95$.

The chicken would take about 95 minutes or 1 hour 35 mins.

3.3 Maths in healthcare

The body mass index (BMI) is sometimes used to help determine whether an adult is under- or overweight. It is calculated as follows:

$$\text{body mass index} = \frac{\text{mass in kg}}{(\text{height in m})^2}$$

Although care needs to be taken in interpreting the results (for example, the formula isn't appropriate for children, old people or those with a very muscular physique!), a BMI of less than 20 suggests the person is underweight and a BMI of over 25 suggests the person is overweight.

In this formula, the units have been included in the expression on the right-hand side of the equals sign. It is important to change any measurements into these units before you substitute the values into the formula. (No units have been included for the BMI on the left-hand side, in line with current health-care practice. However, strictly the unit for the BMI is

$\frac{\text{kg}}{\text{m}^2}$ which is the same as $\frac{\text{kg}}{\text{m}^2}$ or 'kilograms per square metre'.)

Activity 5: Body mass index

If an adult man is 176 cm tall and has a mass of 85 kg, calculate his BMI and decide whether he is overweight.

The formula needs the mass in kilograms and the height in metres. So convert the height into metres first.

Since there are 100 cm in a metre, $176 \text{ cm} = 176 \div 100 \text{ m} = 1.76 \text{ m}$.

Substituting the height and the mass into the formula for the BMI gives

$$\text{BMI} = \frac{85}{1.76^2} = \frac{85}{3.0976} \approx 27.$$

Since his body mass index is over 25, the man is probably overweight.

Can you work out what weight he should be?

Note that, if you study science courses, scientists include the units throughout the calculation writing the solution like this:

$$\text{BMI} = \frac{85 \text{ kg}}{(1.76 \text{ m})^2} = \frac{85 \text{ kg}}{3.0976 \text{ m}^2} \approx 27 \text{ kg/m}^2.$$

3.4 Maths in business

One of the advantages of identifying the general features of a calculation and then describing it mathematically is that the formula can then be used in either a computer or a calculator program to work out many different calculations quickly and efficiently. Many utility suppliers (gas, water, electricity, telephone) have tariffs based on a fixed daily (or monthly or quarterly) charge and a further charge based on how much you have used during the billing period.

For example in 2005, a mobile phone network charged £15 per month for 30 minutes (or less) of phone calls. Extra calls above the 30 minutes were charged at 10p per minute.

Assuming that more than 30 minutes were used per month, the formula for the total monthly cost was given by

$$\text{total monthly cost in } \pounds = 15 + (\text{total number of minutes used} - 30) \times 0.1.$$

Activity 6: Understanding the formula

Suppose that one month 75 minutes of phone calls were made. Explain how you would calculate the cost for the extra minutes (above the 30-minute allowance) and hence how to calculate the total cost for that month. Can you explain how the formula has been derived?

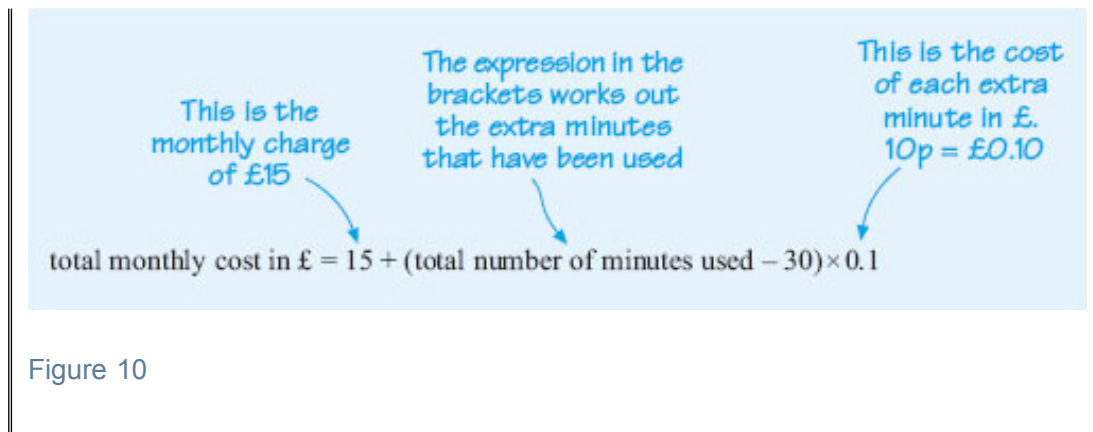
As 30 minutes are included in the £15 charge, the number of minutes that are charged separately is

$$75 - 30 = 45.$$

Each extra minute costs £0.1, so 45 extra minutes will cost $45 \times \pounds 0.1 = \pounds 4.50$.

So the total charge for that month is $\pounds 15 + \pounds 4.50 = \pounds 19.50$.

The formula says that the fixed charge is £15, and then each minute in excess of the 30 minutes allowed costs 10p.



Activity 7: Phone bills

On this phone tariff, what would the monthly bill have been, if the total time for the calls were the following?

- (a) 48 minutes
- (b) 25 minutes

(a) Substituting 48 for the total number of minutes gives:

$$\begin{aligned} \text{total monthly cost in £} &= 15 + (48 - 30) \times 0.1 \\ &= 15 + 18 \times 0.1 \\ &= 15 + 1.8 = 16.8. \end{aligned}$$

Hence the bill for the month is £16.80.

Note how this calculation is set out with a concluding sentence that answers the question precisely.

(b) The formula only applies if more than 30 minutes of calls are made, so it cannot be used in this case. This is an important step in using formulas – you do need to check that they apply in your particular situation before using them! Here, the charge is £15 for up to 30 minutes of calls, so the charge for this month is £15 overall.

3.5 Maths in archaeology

In several different parts of the world, footprints from prehistoric human civilisations have been found preserved in either sand or volcanic ash. From these tracks it is possible to measure the foot length and the length of the stride. These measurements can be used to estimate both the height of the person who made the footprint and also whether the person was walking or running by using the following three formulas:

$$\begin{aligned} \text{height} &= 7 \times \text{length of foot} \\ \text{relative stride length} &= \frac{\text{stride length}}{\text{hip height}} \\ \text{hip height} &= 4 \times \text{length of foot} \end{aligned}$$

Note that no units have been included in these formulas, so it is important to make sure that the same units, for example centimetres, are used throughout the calculation. If the value of the relative stride length is less than 2, the person was probably walking and if the value is greater than 2.9, the person was probably running.

Activity 8: Footprints in the sand

From one set of footprints the length of the foot is measured as 21.8 cm and the stride length as 104.6 cm. What does the data suggest about the height and the motion of the person who made these footprints?

The height of the person is estimated to be 7×21.8 cm or approximately 153 cm.

To work out the relative stride length, we need to calculate the hip height and then substitute it into the relative stride length formula.

The hip height is 4×21.8 cm or 87.2 cm.

So the relative stride length = $\frac{104.6}{87.2} = 1.2$ (1 d.p.).

As the relative stride length is less than 2, the person was probably walking.

Notice that the relative stride length has no units. The stride length and the hip height are both measured in cm, and when we divide one by the other, these units cancel each other out, just like cancelling numbers. If you go on to study physics or other sciences, you will find lots of examples like this.

How do you think these formulas have been derived? How reliable do you think these estimates are?

Notice that to find the relative stride length here, we tackled the problem by using the two formulas step by step, but you could also have combined these two formulas into one:

$$\text{relative stride length} = \frac{\text{stride length}}{4 \times \text{foot length}}$$

and used this new formula directly.

Activity 9: What's wrong?

A student used the formula: $\text{relative stride length} = \frac{\text{stride length}}{4 \times \text{foot length}}$ to calculate the relative stride length for the person in [Activity 8](#). Use this formula to work out the relative stride length and check that your answer agrees with the result in the last activity. The student entered the key sequence for the calculation as $104.6 \div 4 \times 21.8$. This gave an answer of approximately 570, so the student concluded that the person was probably running very fast indeed. Can you explain where the student made his mistake?

The calculator will perform this calculation from left to right, using the BIDMAS (**B**rackets, **I**ndices, **D**ivision, **M**ultiplication, **A**ddition and **S**ubtraction) rules, which treat multiplication and division as equally important. So it will first divide 104.6 by 4 to get 26.15 and then multiply by 21.8 to get approximately 570. However, this is not the correct calculation from the formula. The stride length (104.6) should be divided by $(4 \times \text{foot length})$, so the calculation should be $104.6 \div (4 \times 21.8)$ or alternatively $104.6 \div 4 \div 21.8$. Enter both these expressions into your calculator directly, and check that these both do give the correct answer.

Can you explain why these two calculations are equivalent?

Note that the student should have been expecting an answer between about 0 and 5, so an answer of 570 is suspicious! Also if the student had worked out an estimate for the answer first, the mistake might have been spotted. (If you round all the values to 1 significant figure, an estimate for the answer is $\frac{100}{4 \times 20}$ which is just over 1.)

Now, have a look back over the formulas you have used so far and also any others that you might be familiar with from your home, work or hobbies. What are the common steps in using a formula? If you were asked to jot down a list of tips for using a formula for a friend, what would you say? Here are a few suggestions – feel free to add your own to the Comments section below!

Tips for using a formula

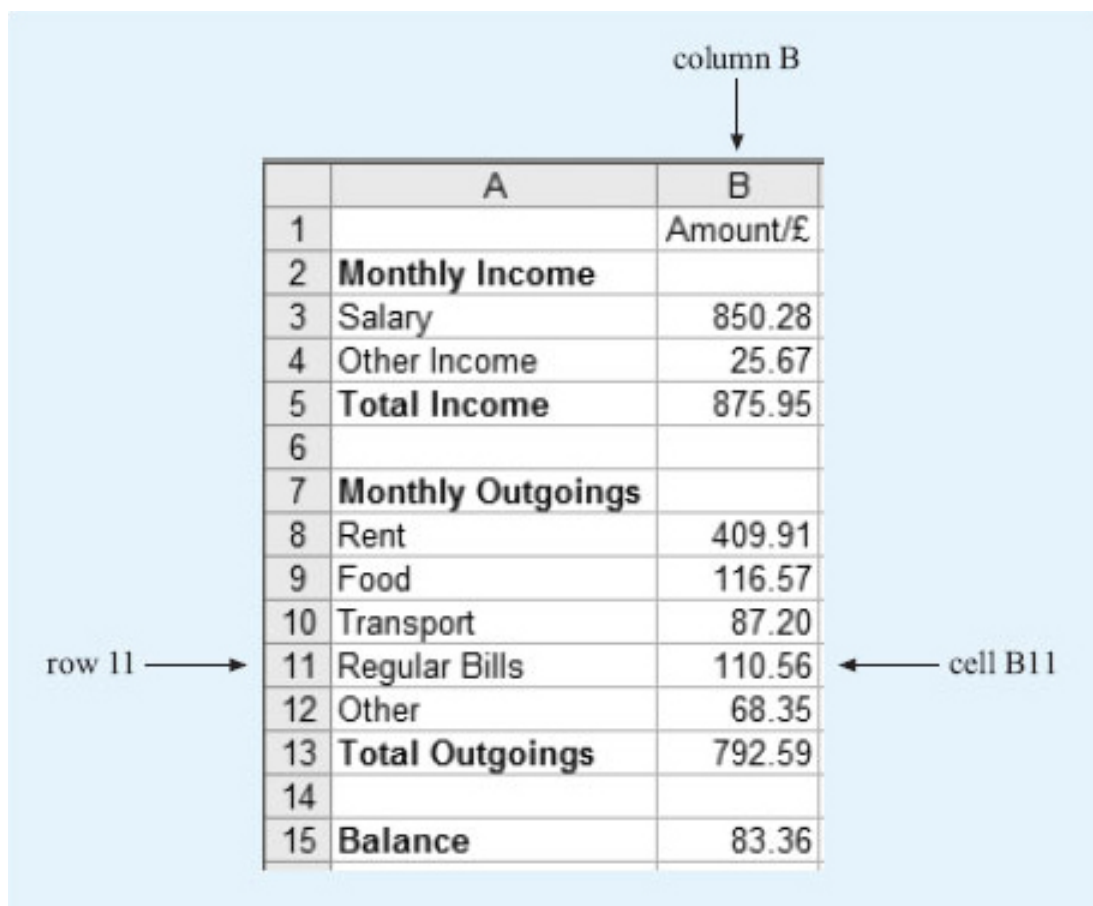
- First, check the formula can be applied to your particular problem!
- Check what values you need to substitute and also what units these should be measured in. Convert the measurements if necessary.
- Substitute the values into the formula carefully.
- Make a rough estimate for the answer.
- Use BIDMAS to work out the resulting calculation, step by step.
- Explain your steps carefully using words like ‘substituting’ and ‘converting’.
- Check that your answer seems reasonable, both practically and from the estimate.
- Round your answer appropriately.
- Remember your concluding sentence and the units!

4 Your formulas – using a spreadsheet

Although there may be many occasions when you are given a formula to use, sometimes you may need to devise your own formulas, for example if you use a spreadsheet on a computer at home or at work. This section looks at the process of devising a formula in more detail.

Part of a spreadsheet that has been constructed to record monthly income and expenditure is shown below. It is similar to a balance sheet that you might draw up by hand and includes the monthly income and outgoings, the totals and the overall balance. It is used to keep track of expenditure, particularly to try and ensure that the balance remains positive. However the spreadsheet has been stored on a computer rather than on paper and is updated regularly. One of the advantages of using a computer spreadsheet is that you can insert formulas into the spreadsheet to carry out the necessary calculations automatically, without using your calculator.

A spreadsheet is made up of rows and columns of cells. The columns are identified by letters and the rows by numbers. This enables you to identify each cell in the spreadsheet. For example the number 110.56 is in column B, row 11, so this cell is B11.



	A	B
1		Amount/£
2	Monthly Income	
3	Salary	850.28
4	Other Income	25.67
5	Total Income	875.95
6		
7	Monthly Outgoings	
8	Rent	409.91
9	Food	116.57
10	Transport	87.20
11	Regular Bills	110.56
12	Other	68.35
13	Total Outgoings	792.59
14		
15	Balance	83.36

Figure 11

Cells can be found from their reference, by looking down the column and across the row. So cell A3 can be found by looking down column A and across row 3. This cell contains the word 'Salary'. Notice that cells can contain either text or numbers.

Activity 10: Spreadsheet cells

- a. What is contained in the following cells?
- A5
 - A12
 - B15
 - B1
- b. What is the reference for the cells that contain the following?
- The number 792.59
 - The word 'Food'
- a.
- The cell that is in both column A and row 5 contains the words 'Total Income'.
 - Looking down column A and across row 12, cell A12 contains the word 'Other'.
 - The cell that is in column B, row 15 contains the number 83.36.
 - The cell that is in column B, row 1 contains the heading 'Amount /£'. This indicates that the entries in this column are amounts of money and that they are measured in £. This heading could also have been written as 'Amount in £' or 'Amount (£)'.
- b.
- 792.59 is in column B and row 13, so its reference is B13.
 - 'Food' is in column A and row 9, so its reference is A9.

Now have another look at the spreadsheet and see if you can work out what information it shows. For example, if you look at row 3, this shows that the monthly salary is £850.28. Although cell B3 only contains the number 850.28, you know that this is measured in pounds from the heading in cell B1. Overall, the spreadsheet shows the different items that make up the monthly income and where money has been spent over the month as well. If you were keeping these records by hand, you would then need to calculate the total income, the total outgoings and the balance.

For example, to find the total income for the month, you would need to add the salary (£850.28) to the other income (£25.67). This gives the total income of £875.95. In other words, to calculate the value in cell B5, you need to add the values in cells B3 and B4 together. This can be written as the following formula:

value in B5 = value in B3 + value in B4

Activity 11: Finding the balance

Formulas have also been used to calculate the total monthly outgoings and the balance. If you were working these calculations out by hand, what would you do? Check by comparing your answers with the values in cell B13 and B15. Try to write down the formulas for these cells in the form 'value in B13 = ...'.

To calculate the total monthly outgoings, you need to add up the individual outgoings on 'Rent', 'Food', 'Transport', 'Regular Bills' and 'Other'. So the formula will be:

value in B13 = value in B8 + value in B9 + value in B10 + value in B11 + value in B12

To find the balance, you need to take the outgoings away from the total income. So the formula will be:

value in B15 = value in B5 – value in B13

To put these formulas in the spreadsheet, you can type the formula directly into the relevant cell, starting with an equals sign to show that you are entering a formula rather than a word.

	A	B
1		Amount/£
2	Monthly Income	
3	Salary	850.28
4	Other Income	25.67
5	Total Income	=B3+B4
6		
7	Monthly Outgoings	
8	Rent	409.91
9	Food	116.57
10	Transport	87.2
11	Regular Bills	110.56
12	Other	68.35
13	Total Outgoings	=SUM(B8:B12)
14		
15	Balance	=B5 - B13

Figure 12

Notice that in cell B13, a shorthand form for the sum has been entered. You could have typed in = B8+B9+B10+B11+B12, but it is also acceptable to use the shorthand form, =

SUM(B8:B12), which is shown here. This instruction tells the computer to add together the values in all the cells from B8 to B12.

One of the advantages of using a spreadsheet is that if you change some of the numbers, all the calculations that use that particular number will be automatically updated to reflect the change. For example, in this budget the amounts for the salary, rent and the regular bills are likely to remain the same from one month to the next and may only be updated once or twice a year. However, the amounts for food, transport, other bills and other income probably will change from month to month. These values can be changed easily on the spreadsheet and the revised balance produced immediately.

Activity 12: Spreadsheet formulas

The formulas used in a spreadsheet are shown in the diagram below. Note that * is the notation for multiplication and / is the notation for division.

	A	B	C	D
1	Item	Cost/£		Total/ £
2	Radio	64.99	=B2*0.175	=B2+C2
3	Kettle	16.49	=B3*0.175	=B3+C3
4	Fan	12.99	=B4*0.175	=B4+C4
5			Total	

Figure 13

(a) The values in columns C and D are displayed to two decimal places as they represent an amount of money. What values will be displayed in cell C3 and cell D3?

(b) What do you think is being calculated in the cells in column C? Can you suggest a suitable heading for this column, to be entered into C1?

(c) Cell D5 should contain the total of the values in D2, D3 and D4. Write down a formula that could be entered in cell D5. What does this value represent?

(a) The formula in C3 is:

value in C3 = value in B3 \times 0.175.

So the value in C3 = $16.49 \times 0.175 = 2.88575$.

Although the full value is kept in the cell, it will only be displayed to 2 d.p. as 2.89.

The value in D3 is obtained by adding together the values in B3 and C3.

So, value in D3 = value in B3 + value in C3 = $16.49 + 2.88575$ which is 19.37575. This will be displayed to 2 d.p. as 19.38.

(b) The numbers in column C are 0.175 or 17.5% of the values in column B. You may know that 17.5 % is the main UK VAT rate, so it looks as if column C represents the amount of VAT paid on the different items described in column A. A suitable heading might be VAT/£ or VAT in £ or VAT (£).

(c) The total can be found by adding together the values in cells D2, D3 and D4. So the formula '= D2+D3+D4' could be entered into cell D5. Alternatively, you

could use `'=SUM(D2:D4)'`. This represents the total cost (including VAT) of the radio, kettle and fan.

With these changes, the spreadsheet will look like the following.

	A	B	C	D
1	Item	Cost/£	VAT/£	Total/ £
2	Radio	64.99	11.37	76.36
3	Kettle	16.49	2.89	19.38
4	Fan	12.99	2.27	15.26
5			Total	111.00

Figure 14

5 Your own formulas

5.1 Number tricks

In the last section, you worked out some formulas that could be used in a spreadsheet. Section 5 gives you some more practice in deriving formulas both by looking at some number tricks and rearranging some existing formulas.

Activity 13: Think of a number

Try this number puzzle. (If you use your calculator, remember to press the equals sign after each instruction.)


'Think of a number. Add 5. Double it. Subtract 8. Divide by 2. Take away the number you first thought of. Add 4.'

Now if 1 represents the letter 'A'; 2 represents the letter 'B'; 3 represents the letter 'C'; and so on, work out the letter represented by your answer and write down the name of an animal beginning with this letter.

If the number 3 were chosen initially, the instructions work out as follows.

Add 5:	$3 + 5 = 8$
Double it:	$8 \times 2 = 16$
Subtract 8:	$16 - 8 = 8$
Divide by 2:	$8 \div 2 = 4$
Take away number:	$4 - 3 = 1$
Add 4:	$1 + 4 = 5$

You will find that you always get 5, whatever number you start with. This gives the letter E. Did you choose an elephant?

To see how this trick works, read through the instructions below. Because we do not know what number you have thought of, we have replaced the number by a thought bubble like this: 

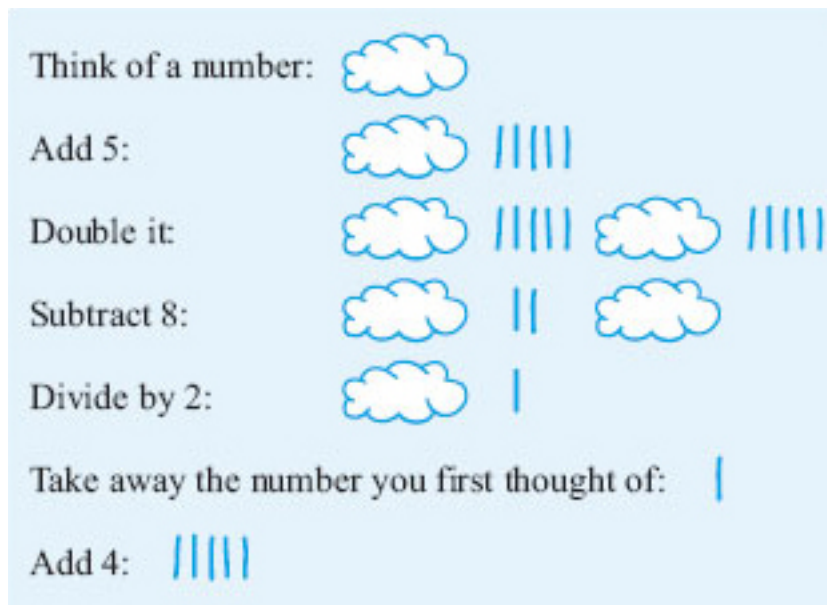


Figure 15

This shows that the numerical answer is always five – it does not depend on which number was chosen first of all. So the letter of the alphabet chosen is E. There are not many animals with names beginning with E and most people do think of an elephant first, but you might be unlucky if someone chooses an emu or an eel!

Rather than using a cloud to represent the number and explaining the trick pictorially, you could write down expressions for the answer at each stage and use either a word (for example, number) or a letter (for example, N) to represent the number. The trick can then be written as follows.

Instruction	Expression
Think of a number	n
Add 5	$n + 5$
Double it	$n + 5 + n + 5 = 2 \times n + 10$
Subtract 8	$2 \times n + 10 - 8 = 2 \times n + 2$
Divide by 2	$(2 \times n \div 2) + (2 \div 2) = n + 1$
Take away the number you first thought of	$n + 1 - n = 1$
Add 4	$1 + 4 = 5$

Activity 15: Another number trick

Try the following trick several times. Think of a number between 1 and 10. This will work with numbers greater than 10, but the restriction is to keep the arithmetic manageable. Multiply by 4. Add 6. Divide by 2. Subtract 3. Divide by 2, and your answer is?

Write down the number you first thought of and your answer. What do you notice? Can you explain why this happens, either by using a diagram or by writing down the expressions for the answer at each stage?

You should find that this time the answer is always the number you chose at the start. The expressions for the answer at each stage are shown below.

Instruction	Expression
Think of a number between 1 and 10	n
Multiply by 4	$4 \times n$
Add 6	$4 \times n + 6$
Divide by 2	$(4 \times n \div 2) + (6 \div 2) = 2 \times n + 3$
Subtract 3	$2 \times n + 3 - 3 = 2 \times n$
Divide by 2	$2 \times n \div 2 = n$

Try making up your own number tricks. What makes a good trick?

5.2 Doing and undoing

Now try the following.

Think of a number. Add 4. If my answer is 11, can you work out what number I was thinking of?

You might have said 'What number do I have to add on to 4 to get 11?' or perhaps 'If I take away 4 from 11 what number do I get?' In both cases you should have arrived at the answer 7.

In the second method 'subtracting 4' undoes the 'adding 4' in the original instructions. This process can be illustrated by a 'doing-undoing diagram'.

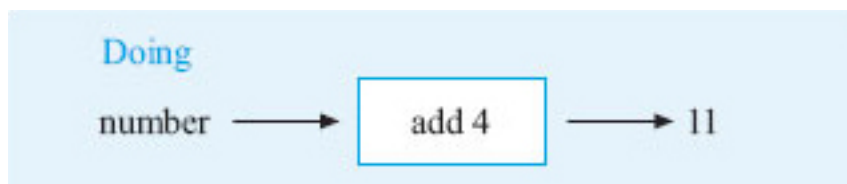


Figure 16

In the doing part of the diagram, start with the number and write down the operations applied in turn until you get the answer. Here there is just one operation 'add 4'.

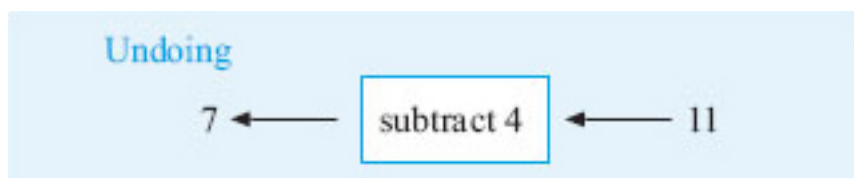


Figure 17

For the undoing part of the diagram, start on the **right** with the answer, in this case 11. Then work back towards the **left**, undoing each operation in turn until you find the starting

number. In this case, 'subtract 4' undoes 'add 4' and $11 - 4 = 7$. So, the number first thought of was 7. Notice how the arrows indicate the direction to read the diagram.

The next activity gives you some practice with doing and undoing. You will find these techniques helpful for rearranging formulas. Part (d) involves two stages.

Activity 16: Doing and undoing

Try to work out what number I was thinking of in the following problems.

You may find it helpful to use some doing–undoing diagrams.

- Think of a number, subtract 3 and the answer is 2.
- Think of a number, multiply by 5 and the answer is 35.
- Think of a positive number, square it and the answer is 81.
- Think of a number, add 4, double it and the answer is 14.

(a)

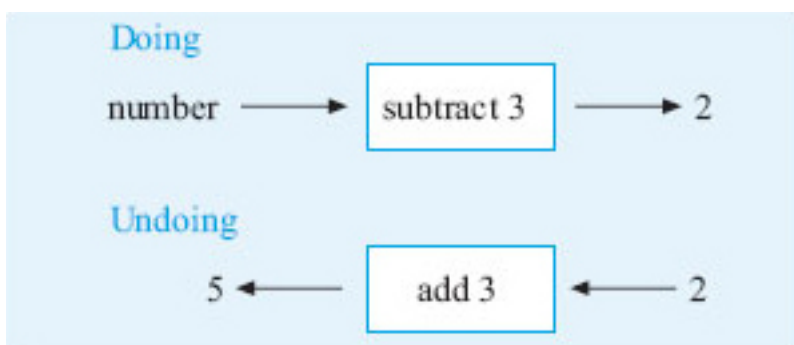


Figure 18

So the number I thought of was 5.

Remember you can check your answer by using it in the trick: $5 - 3 = 2$. ✓

(b)

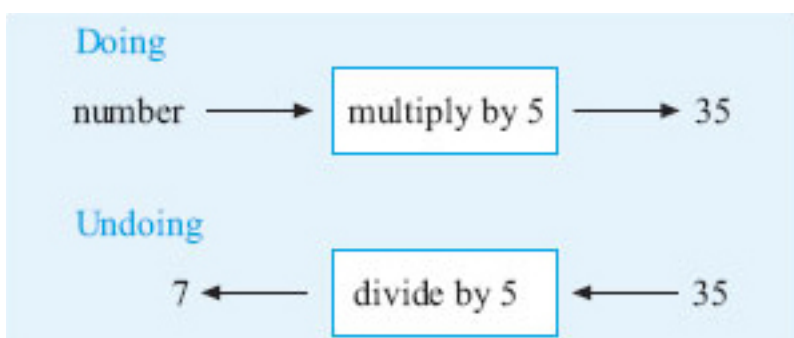


Figure 19

So the number I thought of was 7. (Check: $7 \times 5 = 35$. ✓)

(c)

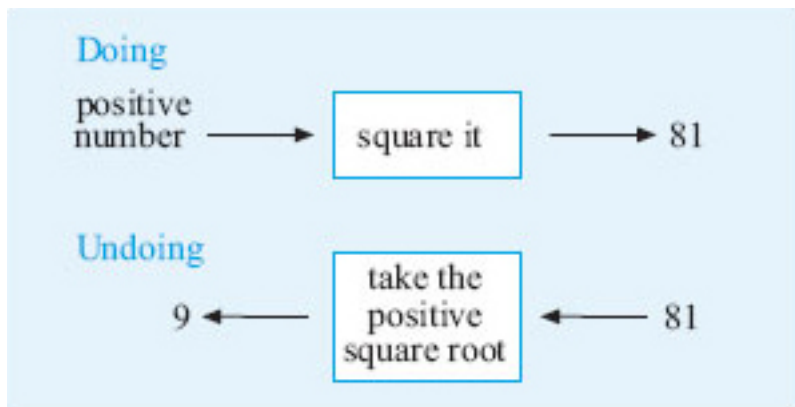


Figure 20

So the number I thought of was 9. (Check: $9^2 = 81$. ✓)

(d)

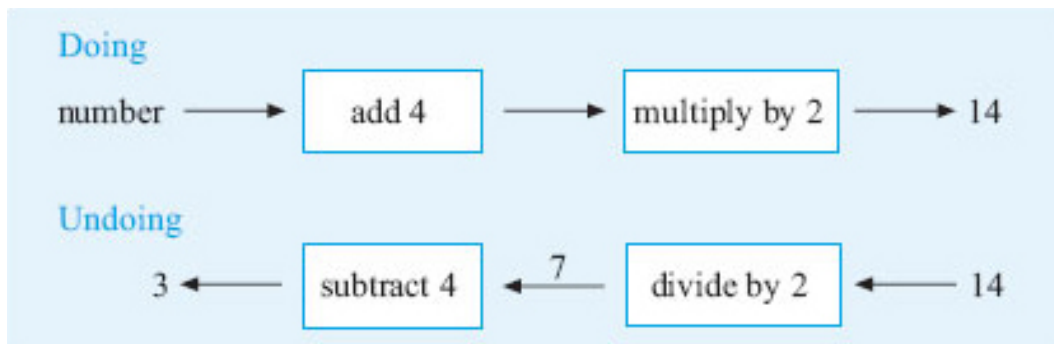


Figure 21

So, working from the right, $14 \div 2 = 7$ and $7 - 4 = 3$.

If you like, you can write the output from each box on the arrow as shown above.

Hence, the number I thought of was 3. (Check: $3 + 4 = 7$; $7 \times 2 = 14$. ✓)

When you have to deal with more than one operation, just take each step in turn. Write down the doing diagram and draw the undoing one underneath, working backwards to undo each operation.

To summarise, the following operations undo each other:

- addition and subtraction
- multiplication and division
- squaring and taking the positive square root.

Activity 17: A multi-step problem

Think of a number. Add 5. Multiply by 3. Subtract 4.

If the answer is 17, can you work out what number I was thinking of?

Here there are several steps, so to find the number it will be necessary to undo each of these steps in turn, starting with the last step. Draw the doing diagram and then the undoing one beneath it.

The diagram below shows the steps taken in this case and then how to undo each one.

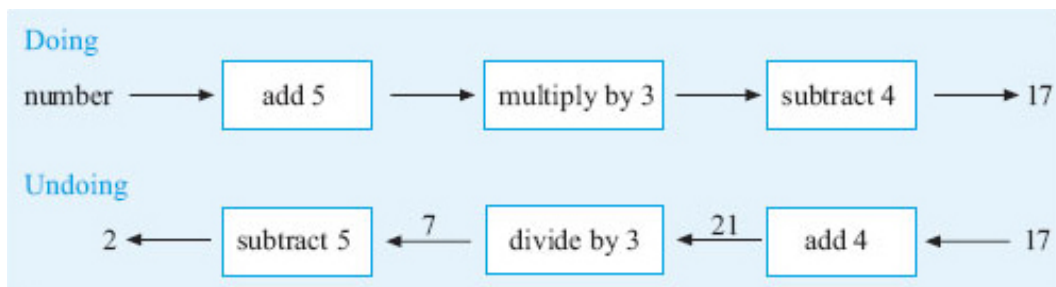


Figure 22

So if the answer is 17, undoing the steps by working from right to left gives:

add 4: $17 + 4 = 21$

divide by 3: $21 \div 3 = 7$

subtract 5: $7 - 5 = 2$

So the number I thought of was 2. (Check: $2 + 5 = 7$; $7 \times 3 = 21$; $21 - 4 = 17$.)

The same sort of technique can be used to change formulas around. For example, earlier in the course, you found a formula to convert pounds into euros:

$$\text{number of euros} = \text{number of pounds} \times 1.48.$$

However, what if you wished to convert euros into pounds, say while you were shopping on holiday?

Then you would need a formula for the number of pounds based on the number of euros. We can tackle this by drawing the 'doing and undoing' diagrams for this situation:

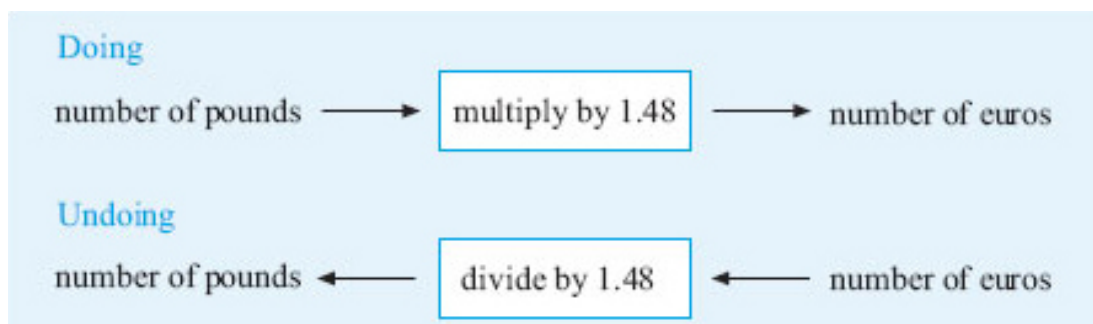


Figure 23

So, starting at the right of the undoing diagram, the formula for converting euros into pounds is:

$$\text{number of pounds} = \text{number of euros} \div 1.48.$$

Activity 18: Miles and kilometers

To change kilometres into miles, you used the formula:

$$\text{distance in miles} = \text{distance in kilometres} \times 0.6214.$$

Starting with 'distance in kilometres', draw a doing diagram to get the distance in miles. Draw the undoing diagram and hence write down the formula for changing miles into kilometres.

Your formula should start 'distance in kilometres = '.

The doing and undoing diagrams are shown below.

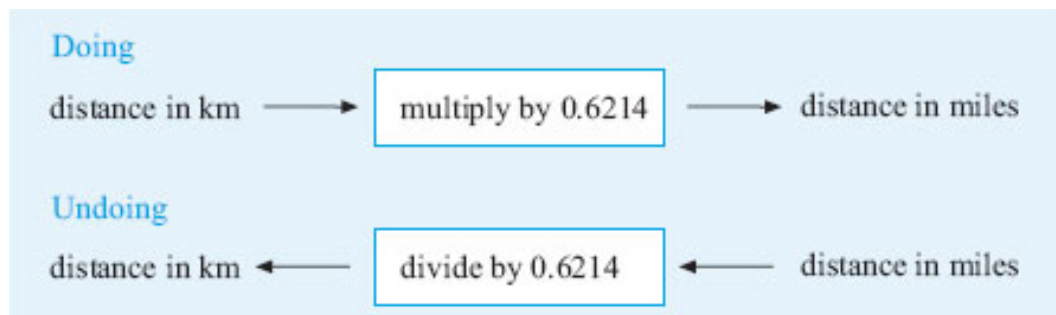


Figure 24

So the formula for converting miles into kilometres is

$$\text{distance in kilometres} = \text{distance in miles} \div 0.6214.$$

Activity 19: How many minutes for chatting?

Recall the formula for the monthly cost in £ of a mobile phone that we used earlier:

$$\text{monthly cost in } \pounds = 15 + (\text{total number of minutes} - 30) \times 0.1.$$

The owner wishes to stick to a monthly budget of £25. Starting with the 'total number of minutes', draw a doing diagram to show the operations to find the 'monthly cost'. Put the monthly cost equal to £25 and then draw an undoing diagram to work out how many minutes of phone calls can be made if the monthly cost is £25.

Here is the doing diagram.

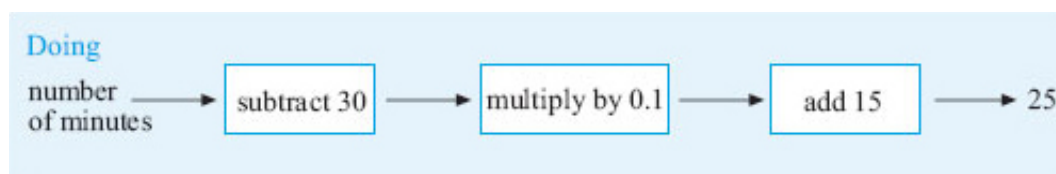


Figure 25

Undoing each step in turn gives the following undoing diagram.

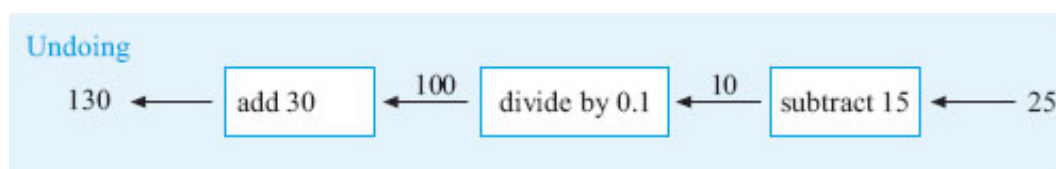


Figure 26

So applying these operations in turn:

subtract 15 gives $25 - 15 = 10$,

divide by 0.1 gives $10 \div 0.1 = 100$,

add 30 gives $100 + 30 = 130$.

Hence, 130 minutes can be used for a monthly budget of £25.

With practice, this process of using a doing–undoing diagram does become second nature. In some cases, it can be useful if you need a formula in a different form. However, for complicated formulas, a different approach (which you will meet if you continue your mathematical studies) is often used.

6 Proportion

6.1 Direct proportion

The first type of proportion relationship is known as **direct proportion**. Two quantities are said to be directly proportional to each other if when one doubles, triples, quadruples, the other also doubles, triples, quadruples. For example, if you buy three times as many items as usual, you would expect to have to pay three times as much money (unless there was some special offer available), because the price is directly proportional to the number of items bought.

Activity 20: Directly proportional relationships

Which of the following quantities are in direct proportion?

- (a) The number of kilometres and the equivalent number of miles in [Activity 18](#).
- (b) The number of euros that are exchanged for pounds in [Activity 3](#).
- (c) The monthly cost of a mobile phone and the minutes used for talking in [Activities 6](#) and [7](#).

Parts (a) and (b) are both examples of direct proportion. If you multiply the number of kilometres by any factor, the number of miles will also change by this factor. For example, if you double the number of kilometres, the number of equivalent miles will also double, and similarly for exchanging currency. If you triple the number of pounds you exchange, you would expect to get three times as many euros. However part (c) is not a directly proportional relationship. If you use 30 minutes, the charge is £15. But if you double the time used to 60 minutes, the charge is £18, and the price has not doubled.

You can tackle problems involving direct proportion in many different ways – using a formula is not always the easiest way, as the following example shows.

Suppose that in a tea club at work, a group of people share a carton of milk each day, but provide their own tea. The carton contains enough milk for 12 cups of tea. If the carton costs 36p, how much should someone who has four cups of tea with milk pay?

This person had $\frac{4}{12} = \frac{1}{3}$ of the milk, so should pay $\frac{1}{3}$ of the cost of the milk which is 12p.

Alternatively, you can work out the cost of milk for one cup of tea as a first step.

Since there is enough milk for 12 cups, one cup will cost $36\text{p} \div 12 = 3\text{p}$.

So four cups will cost $4 \times 3\text{p}$ or 12p.

Note how the problem has been simplified by considering just one cup of tea first!

Activity 21: The more the merrier

A recipe for a vegetable curry for four people requires 300 g of rice. By first working out how much rice one person requires, calculate how much rice would be needed for nine people.

Work out the amount of rice for one person: $300 \text{ g} \div 4 = 75 \text{ g}$.

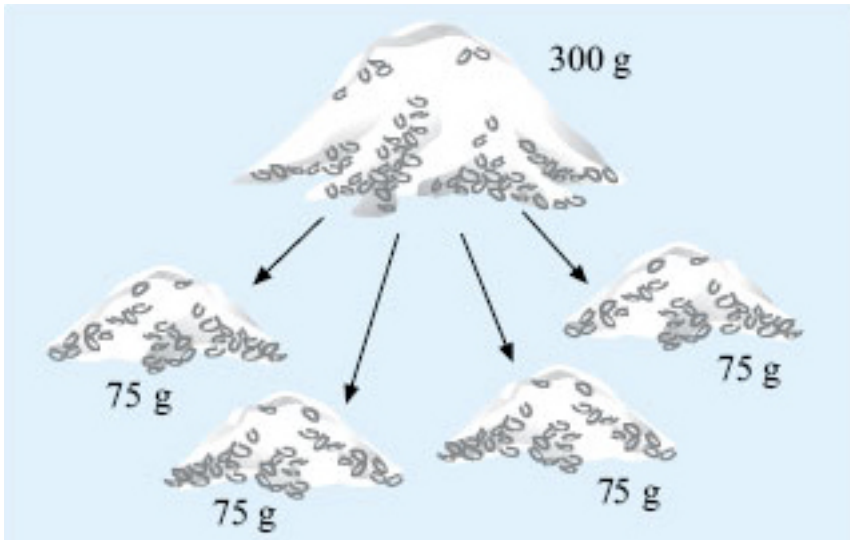


Figure 27

So nine people will need $9 \times 75 \text{ g} = 675 \text{ g}$.
(You can work this out in other ways too!)

Activity 22: How much medicine?

A patient needs to have 600 mg of a drug each day, given in three doses. Each tablet contains 100 mg of the drug. How many tablets are needed for each dose?

One tablet contains 100 mg of the drug.

So six tablets will contain 600 mg of the drug. The patient will need to take six tablets spread over three doses.

So the number of tablets in each dose will be $6 \div 3 = 2$.



Figure 28

You can also use a formula here:

$$\text{total number of tablets} = \frac{\text{amount of drug required}}{\text{amount of drug in one tablet}}$$

Use the formula to see if you get the same answer.

Remember to check that your answer seems reasonable!

6.2 Inverse proportion

The second type of proportional relationship is known as **inverse proportion**.

Suppose you have decided to hire a taxi to take a group of colleagues from work to the railway station. If the taxi firm charges a **set fee for the journey**, then the more people who go in the taxi, the less each person has to pay: if two people go, each pays half the cost; if three people go, each pays a third of the cost and if four people go, each person pays a quarter of the cost. This is an example of inverse proportion. As one quantity (the number of people) doubles or triples, the other quantity (the cost) halves or is reduced to a third of the original value. In other words if you multiply one quantity by a factor, the other quantity is divided by that factor.

The cost per person can be calculated directly or by using the formula:

$$\text{cost per person in } \pounds = \text{taxi fare in } \pounds \div \text{number of people.}$$

For example, if the fare is £7.80 and four people go in the taxi, each person will have to pay $\pounds 7.80 \div 4 = \pounds 1.95$.

Two quantities are said to be inversely proportional if their product is a constant. For example, suppose you go on a journey of 30 km and it takes you an hour to get there. Your average speed for the journey is 30 km per hour. Now if you doubled your speed to 60 km per hour, it would take you half as long (that is, half an hour) to get there; if you halved your speed, it would take twice as long and so on. In each case, the following relationship holds:

$$(\text{speed in km per hour}) \times (\text{time for journey in hours}) = (\text{distance travelled in km}).$$

This shows that for a given distance speed and time are inversely proportional to each other.

This formula can also be written as:

$$\text{speed in km per hour} = \frac{\text{distance travelled in km}}{\text{time for journey in hours}}$$

Activity 23: How fast?

For the journey of 30 km above, what is the average speed if the journey took $1\frac{1}{2}$ hours?

Substituting the distance as 30 and the time as the decimal 1.5 in the formula above gives:

$$\text{average speed in km per hour} = \frac{30}{1.5} = 20.$$

So the average speed is 20 km per hour.

7 Inequalities

In this course, there have been three occasions when checks have been made to see whether the result is greater than or less than some other value. The first case was in calculating the BMI and determining whether the person was overweight or underweight; the second case was in determining whether a person was walking or running from his footprints; and the third case was in checking whether a phone had been used for more than 30 minutes. Checking whether values are greater than or less than some limit happens frequently in everyday problems, particularly in safety limits but elsewhere too. For example, medicines may have to be stored at a temperature of 25°C or less. Child rail tickets can be bought for children who are 5 or more years old but less than 16 years old. Rather than writing out 'greater than' or 'less than', some shorthand notation is often used as shown below.

Inequalities

$>$ greater than

\geq greater than or equal to

$<$ less than

\leq less than or equal to

If you have difficulty interpreting these symbols, you can think of them as arrows which point to the smaller number.

The symbols can be read from left to right. For example, $11 > 9$ is read as '11 is greater than 9' and holiday cost (in £) < 1000 is read as 'holiday cost is less than £1000'.

To use the symbols in your own writing, decide what you want to say first, then use the symbol. For example, since 10 is greater than 5, this could be written as $10 > 5$ or since on the number line -4 lies to the left of -3 , -4 is less than -3 and this could be written as $-4 < -3$.

Similarly, the instructions for the medicine could be written as:

medicine temperature in $^{\circ}\text{C} \leq 25$

Sometimes, you may find it helpful to draw a number line to visualise this sort of information. For example, the ages which children can claim the child rail fare are from their fifth birthday up to but not including their sixteenth birthday. This means that their age has to be greater than or equal to 5 and less than 16.

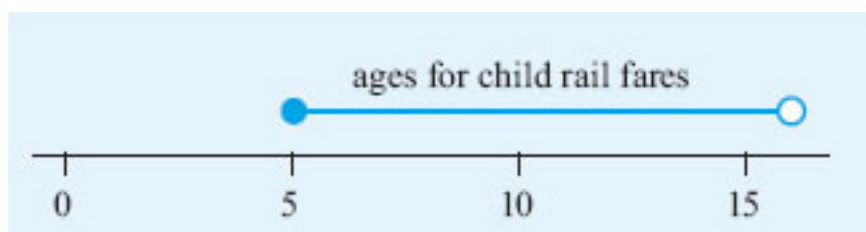


Figure 29

Note how drawing a diagram here helps you to see what is happening.

The empty circle means that this number (16) is not included and the filled-in circle means that this number (5) is included in the interval. This can be written as:

$5 \leq \text{age for child rail fare} < 16$.

Activity 24: Inequalities

- a. Which symbol ($<$ or $>$) should appear in each of the boxes below.
 - i. $4 \square 7$
 - ii. $18 \square 10$
 - iii. $3 \square -2$
 - b. Explain what the following mathematical statements mean.
 - i. Balance in account > 0 .
 - ii. Speed (in mph) on motorway ≤ 70 .
 - iii. $18 \leq \text{age (in years)} \leq 50$.
 - c. Express the following conditions using the inequality symbols.
 - i. The fridge temperature should be less than 4°C .
 - ii. There must be at least five people on the committee.
 - iii. For an ideal weight, a person's BMI should be greater than 20 and less than 25.
-
- a.
 - i. $4 < 7$
 - ii. $18 > 10$
 - iii. $3 > -2$
 - b.
 - i. The balance in the account is greater than zero, or in credit.
 - ii. The speed on the motorway is less than or equal to 70 mph.
 - iii. The age is between 18 and 50 years inclusive.
 - c.

- i.
Fridge temperature (in $^{\circ}\text{C}$) < 4 .
- ii.
Number of people on committee ≥ 5 .
- iii.
 $20 < \text{BMI} < 25$.

Inequalities are also used a lot in computer programs to check whether conditions have been fulfilled. For example, if the balance in your bank account is negative, you may be prevented from withdrawing money from a cash machine.

8 An unsolved problem!

A lot of the mathematics you have looked at so far in the course has been used to help people solve problems for centuries. However there is much more to mathematics than that! Exciting and new developments are being made all the time and there are many problems that mathematicians have not managed to work out. This section takes a brief look at one of these problems.

It was unsolved at the time of writing!

In 1742, a Prussian mathematician, Christian Goldbach, made the following conjecture:

All positive even integers bigger than or equal to 4 can be expressed as the sum of two primes.

A conjecture is a suggestion that has not yet been proved to be true.

This statement contains quite a lot of mathematical terminology, so before going further, here are some definitions.

- An **integer** is a whole number, negative, zero or positive; for example, -6 , 23, 0 and 281 are all integers.
- An **even** integer is one that can be divided exactly by two; for example, -8 , 0, 42, 128 or 1002. The last digit of an even integer is always 0, 2, 4, 6 or 8. Integers that are not even are **odd**.
- A **prime** number is one that can only be divided exactly by 1 and itself. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59.

Now, break down Goldbach's conjecture into steps.

A lot of encryption processes depend on prime numbers.

'All positive even integers bigger than or equal to 4 ...' means the numbers 4, 6, 8, 10, 12, and so on.

'... can be expressed as the sum of two primes' means that you can choose any of the numbers from the prime number list to add together. For example, $8 = 5 + 3$.

Activity 25: Prime numbers

Try to express the following numbers as a sum of two primes.

4 12 28 40 62

For each number, try to find as many combinations of two primes as you can.

Do you think Goldbach's conjecture is true?

It can help if you look at the list of prime numbers systematically, starting with 2. Is there a number in the list that you can add on to 2 to get the number you want? Then look at 3, and so on. You should be able to find the following combinations:

$$4 = 2 + 2;$$

$$12 = 5 + 7;$$

$$28 = 5 + 23 \text{ or } 28 = 11 + 17;$$

$$40 = 3 + 37 \text{ or } 40 = 11 + 29 \text{ or } 40 = 17 + 23;$$

$$62 = 3 + 59 \text{ or } 62 = 19 + 43 \text{ or } 62 = 31 + 31.$$

Are you convinced that Goldbach's conjecture is always true?

Although the conjecture does work for the few numbers we have tried here, that does not necessarily mean that it is going to work for all the even integers greater than or equal to four. The even integers go on for ever, so it is not possible to check them all individually either by hand or computer – some other way of proving the result is needed. Although higher branches of mathematics have been successful in tackling a lot of similar problems, this one is turning out to be particularly tough. So tough in fact that, in March 2000, the publishers Faber and Faber offered a \$1 000 000 prize to anyone who managed to prove the conjecture by March 2002 and no one did! So even though many great mathematicians have attempted a proof, over 250 years later, it still hasn't been sorted out. You might find that quite reassuring the next time you get stuck on a mathematics problem – being stuck really is a natural state for a mathematician! It's often the time when great discoveries are made or when the best learning takes place. There is also a \$1 000 000 prize on offer for each of the seven great unsolved problems of mathematics known as the Millennium Problems.

9 Patterns in nature and elsewhere

If you take a cauliflower and break off one of the florets, the floret appears to be just the same shape as the original cauliflower but on a much smaller scale.



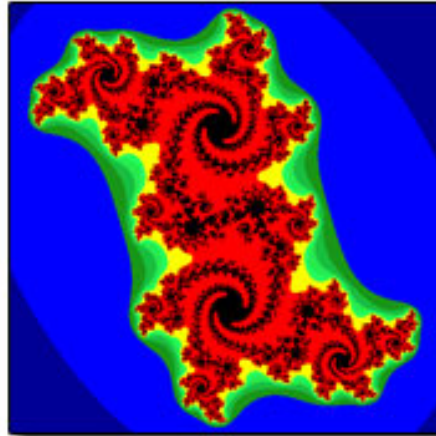
Figure 30 Romanesco cauliflower

This idea of self-similarity occurs throughout nature – the frond of a fern looks like the whole fern, a branch of a tree splits into further branches which look like trees themselves, and so on.

Self-similarity now forms part of an area of mathematical research known as fractal geometry. Beautiful patterns can be generated by fractal formulas, such as the Barnsley fern and the fractal below. Fractal geometry also has many practical applications in the physical sciences, medical research, economics and computing, particularly in compressing images.



Barnsley ferns



Fractal

Figure 31

You can generate part of a fractal yourself, by using Pascal's triangle that you constructed earlier in the course. If you shade in (or cover) all the odd numbers in the triangle, a pattern starts to emerge as shown below.

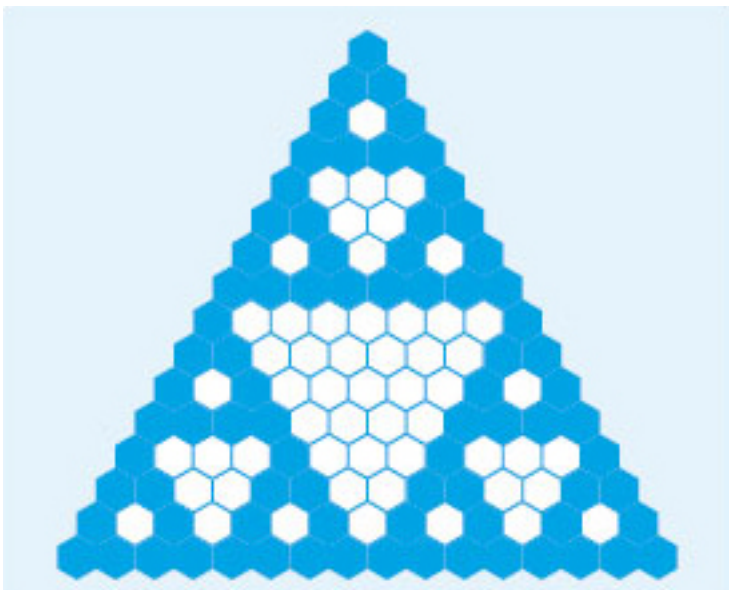


Figure 32

This is part of a fractal known as Sierpinski's triangle (Waclaw Sierpinski (1882–1969) was a Polish mathematician). It can also be generated by starting with a triangle, splitting it into four smaller triangles and removing the middle one. Then split each of the remaining triangles into four smaller triangles and remove the middle one. Carrying on in this way results in something like the sequence below.

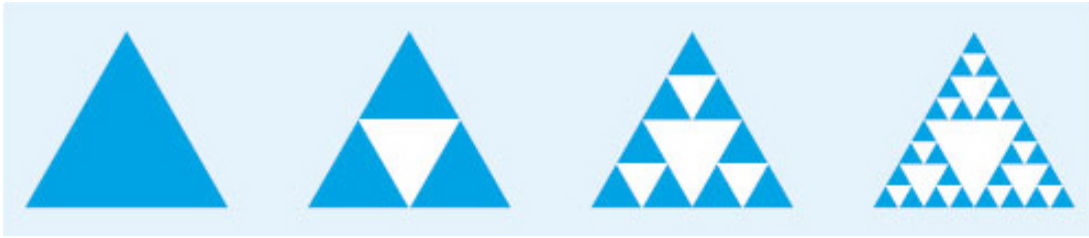


Figure 33 Constructing Sierpinski's triangle

10 Summing up

In this course, you have looked at a variety of problems all of which involved using patterns or formulas and you have also extended some of your strategies for solving problems. One of the first steps in tackling any problem is to check that you understand both the problem and the information you have been given. This step can concentrate on what the question means. However, this can also involve looking up or checking on mathematical terms, notation or definitions as in Goldbach's conjecture. You may be able to sort this out yourself by referring back to your notes or books or, if it is an unfamiliar situation, it may be easier to discuss it with someone else. Understanding the notation is also important. The notation used for inequalities is often used to express ideas concisely and you may find it helpful to add this to your dictionary or to your notes for future use. You have already seen how diagrams can be helpful and that applies to this course as well (for example a number line when dealing with inequalities).

Visualising a problem in a practical way, as in the currency exchange, can help you to work out what to do next as well. You may also like to try working through a problem with a few simple numbers first to get a feel of what's involved. This strategy was used in Goldbach's conjecture. However although we broke Goldbach's conjecture down into steps and found that the conjecture was true for all the examples we tried, this does not prove that the conjecture is **always** true. A pattern isn't proof!

Using formulas is an important skill, but it is important to check that the formula is appropriate for your particular problem (see the [mobile phone problem](#)) and also that any measurements are in the same units as those specified in the formula. It is easy to make mistakes in this way, so checking that the answer you obtain seems reasonable is very important too. You may like to make an estimate first as well. It is also important to check formulas you have derived yourself, particularly if you are using a spreadsheet. If you work out the numerical answer to the calculation first, you can then check that you get the same result when you use the formula in the spreadsheet. Although this doesn't guarantee that you have got the right formula (you may make the same mistake twice after all!), it can help to eliminate careless mistakes.

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