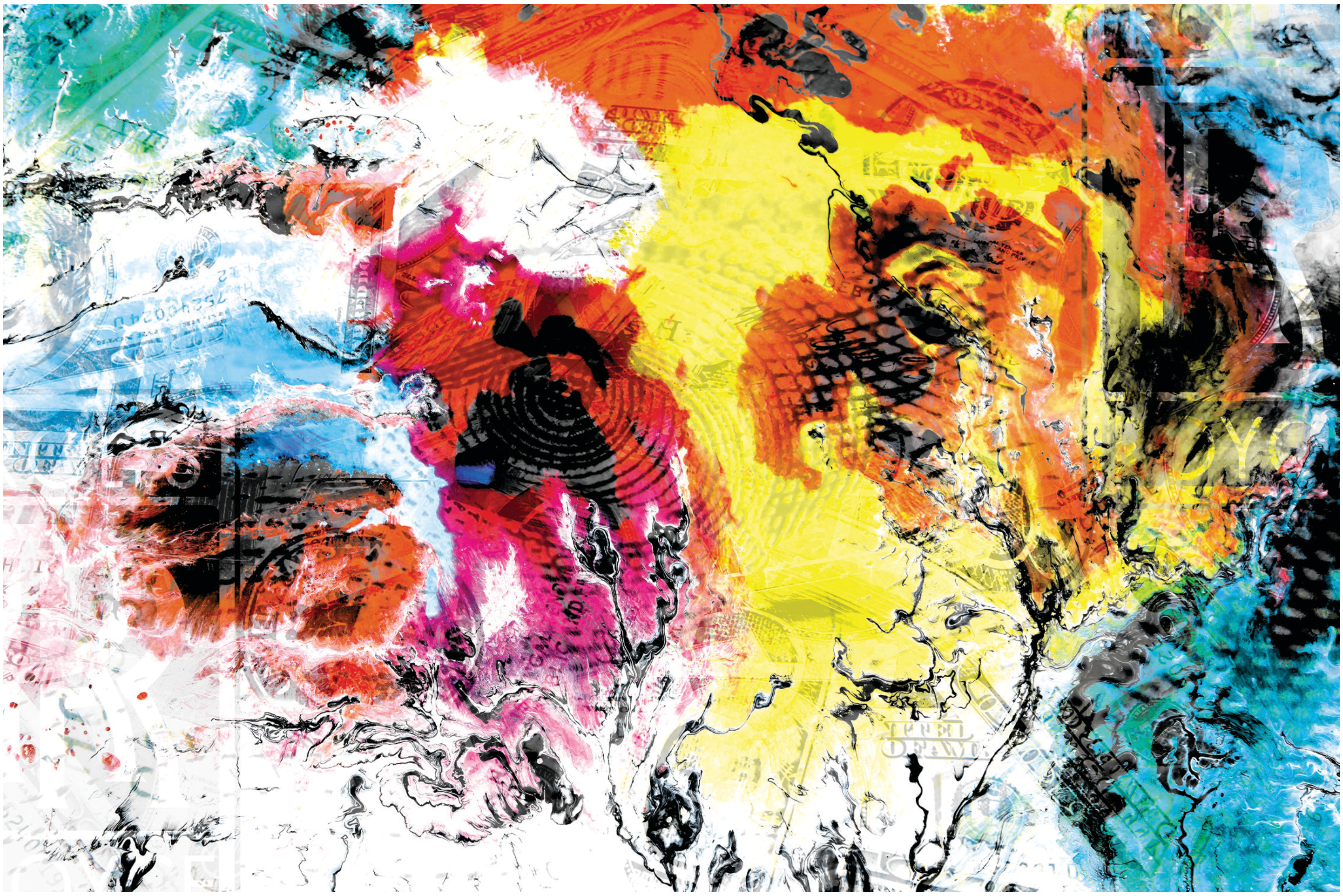
The formation of exoplanets



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Contents

Introduction

The idea that our Solar System may not be unique, and that there might be planets orbiting other stars (or **exoplanets**), has been around for a long time. Important principles that underpin exoplanet research today were foretold by key discoveries in the eighteenth and nineteenth centuries. In 1783, an unseen companion was presented as an explanation of the peculiar periodic dimming observed for the bright star Algol, and in 1844 the observation of a periodic change in position of the bright stars Sirius and Procyon uncovered their two unseen companions. The concept of detecting and exploring an unseen object by studying its influence on a nearby astronomical source has also been applied to exoplanets and their host stars.

However, exoplanets were expected to be extremely hard to observe in practice. Using the orbits and size of planets in our Solar System as a guide, the influence of an exoplanet on its host star was predicted to be very small. Indeed, it took until the late twentieth century for technological advancements to enable the first exoplanet to be identified, and the result was surprising: 51 Pegasi b (named after its Sun-like host star 51 Pegasi in the Pegasus constellation) was unlike anything in our Solar System. 51 Pegasi b is a **hot Jupiter**: a planet with a similar mass to Jupiter, but with an incredibly high surface temperature as its orbit takes it very close to its host star. The fact that a planet like 51 Pegasi b exists triggered what became a radical overhaul of theories of planet formation and evolution. But the discovery of a hot Jupiter was also encouraging, as it showed that exoplanet detection was perhaps not quite as impossible a challenge as many had assumed. All astronomers needed to do was start looking for something that is different from the planets of the Solar System.

Since the discovery of 51 Pegasi b, thousands more exoplanets have been discovered by a variety of techniques. The main outcome of exoplanet searches and characterisation studies carried out using these techniques is a known exoplanet population with a wide variety of physical and orbital characteristics. This course explores the key astrophysical concepts involved in planet formation and how they can be used to explain the great diversity of configurations observed.

Section 1 focuses on the birthplaces of planets: the **protoplanetary discs**. Section 2 describes the **core-accretion scenario** for planet formation. This was initially developed to explain the existence of Jupiter, but has, over the years, become a more general model of planet formation for its ability to account for a large diversity of planetary outcomes, from Earth-sized planetary cores that form first, to ice and gas giants that evolve later. Lastly, Section 3 focuses on the final stages of planet formation, and explores the core-accretion scenario further, as well as an alternative **disc-instability scenario**, where the formation of massive objects occurs first from the collapse of gas into clumps by self-gravity.

This course material will refer to masses of stars in terms of the mass of the Sun (represented by $M_{\odot} = 1.99 \times 10^{30}$ kg), and masses of planets in terms of the mass of the Earth (represented by $M_{\oplus} = 5.97 \times 10^{24}$ kg) and the mass of Jupiter (represented by $M_{Jup} = 1.90 \times 10^{27}$ kg). Orbital distances from stars will be expressed in terms of the distance of the Earth from the Sun; this is 1 astronomical unit (1 au = 1.496×10^{11} m).

This OpenLearn course is an adapted extract from the Open University course S384 *Astrophysics of stars and exoplanets*.

Learning outcomes

After studying this course, you should be able to:

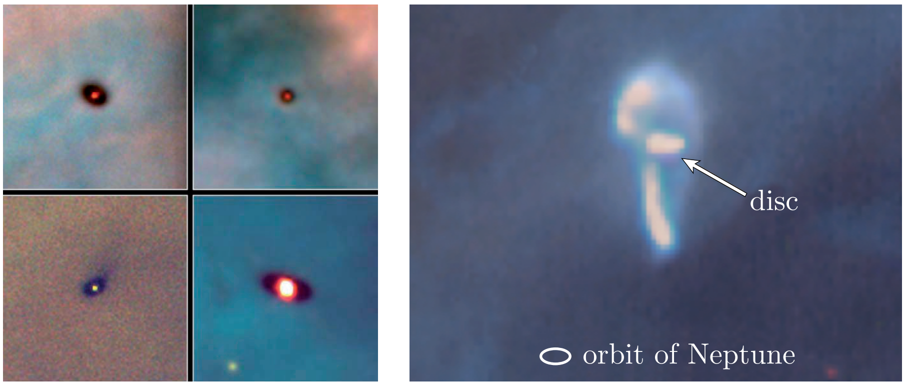
- use mathematical models to calculate properties of protoplanetary discs
- understand the core-accretion scenario for the growth of planetary cores from smaller components
- understand the disc-instability scenario for the formation of planets from a circumstellar disc
- describe how planets migrate and interact after forming
- appreciate how planet formation models can explain the observed exoplanet population.

1 Protoplanetary discs

The idea that planets form from initially microscopic solid material within protoplanetary discs made predominantly of gas dates back to the Enlightenment, possibly starting with the idea that the planets of the Solar System formed out of a nebula surrounding the Sun, which featured in Kant's *Universal Natural History and Theory of the Heavens*. Today, our understanding of protoplanetary discs stems from experimental observations and theoretical models of the behaviour of gases in the gravitational fields of stars.

1.1 Observations of protoplanetary discs

The presence of discs around newborn stars is a natural consequence of the collapse of the **molecular cloud** from which they form, as a mechanism to conserve **angular momentum**. The first evidence of the existence of protoplanetary discs came thanks to the Hubble Space Telescope (HST) in the mid-1990s, which was more or less at the same time as the first exoplanet discoveries. Figure 1 shows the HST images of the protoplanetary discs around four young stars in the Orion nebula, 1500 light-years from the Sun, compared with the much more detailed view of a protoplanetary disc in the same region obtained with the James Webb Space Telescope (JWST) in 2022. Protoplanetary discs have been observed mainly around young stars (with ages of about one to ten million years) that are close to the final stages of formation. This means that most of the material from the disc has been accreted by the central star, so the mass of the disc (M_{disc}) is much lower than the mass of the star (M_*).



(a) (b)

Figure 1 (a) Some of the first images of protoplanetary discs, in the Orion nebula, taken in 1993 with HST. (b) Protoplanetary disc in Orion, imaged with JWST in 2022. The orbit of Neptune is shown for scale.

The presence of planets in protoplanetary discs is strongly supported by observations, which have been supported by the development of ground-based instruments such as SPHERE.

Box 1 SPHERE

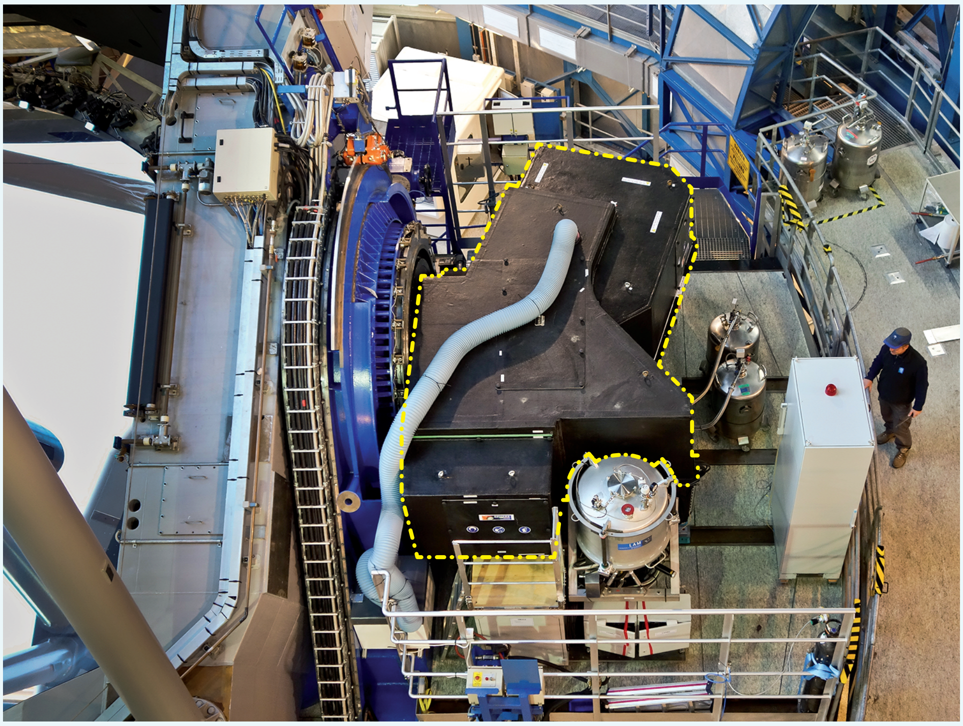


Figure 2 The SPHERE instrument (dashed–dot outline) mounted on the side of one of the four telescopes that form the VLT complex in Chile.

SPHERE (Spectro-Polarimetric High-contrast Exoplanet REsearch) is an instrument operating at near-infrared and visible wavelengths, installed on one of the four telescopes comprising the European Southern Observatory's (ESO) Very Large Telescope (VLT) site in Paranal (Chile). SPHERE is one of the first dedicated direct-imaging instruments and its primary science goal is to directly detect and characterise young exoplanets and the discs in which they form.

Recent observations using VLT/SPHERE include the direct detection of two forming planets in the disc around the young T Tauri star PDS 70 (which gets its name from the *Pico dos Dias Survey* for young stellar objects). One of the directly imaged planets is shown in Figure 3.

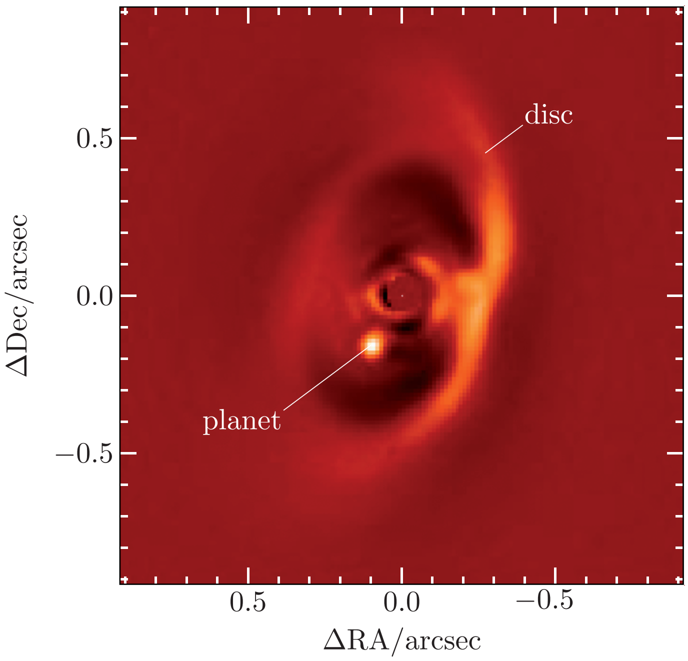


Figure 3 VLT/SPHERE image of PDS 70 with a planet in a gap in the disc. The other planet in the system is obscured by the bright region of the disc to the right of the central star.

1.2 Modelling protoplanetary discs

Material in a protoplanetary disc will be in orbit around a central star (or protostar). A first approximation to the motion of the material is that it is in so-called **Keplerian** motion, that is it obeys **Kepler's laws** of planetary motion. In particular **Kepler's third law**, which is a consequence of Newton's law of gravity, states that the square of the orbital period is proportional to the cube of the orbital radius (assuming circular orbits, which is generally the case), i.e. $P^2 \propto a^3$. As long as the orbiting particle has a mass that is small compared to that of the star, this may be expressed in the following equation:

$$P^2 = \frac{4\pi^2 a^3}{GM_*}$$

where G is the gravitational constant (6.674×10^{-11} N m^2 kg^{-2}) and M_* is the mass of the star. The **Keplerian orbital speed** is therefore the distance travelled in an orbit (the circumference of the orbit, $2\pi a$) divided by the orbital period, P. This is therefore

$$v_K = \left(\frac{GM_*}{a}\right)^{1/2} \quad \text{(Equation 1)}$$

Figure 4 shows a schematic view of a protoplanetary disc, comprised mostly of molecular hydrogen. The rest of this section will aim to derive and solve the differential equations that govern the vertical (out-of-disc plane) gas-density profile as well as the radial dependence of the orbital (in-disc plane) velocity, which turns out to differ from the pure Keplerian motion. The following two subsections therefore will look in turn at how these two properties may be quantified.

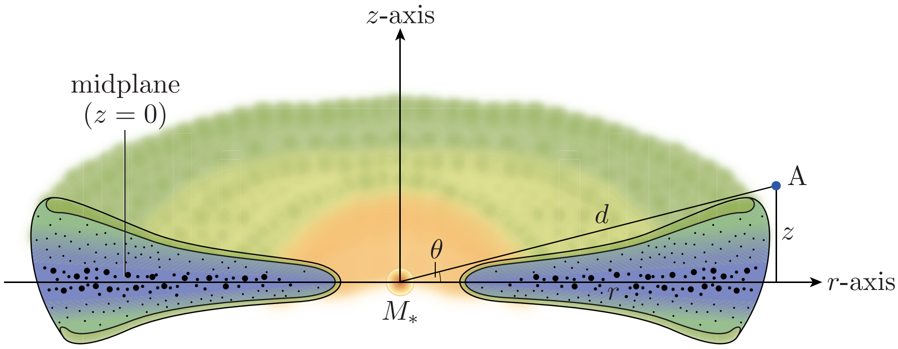


Figure 4 Schematic illustration of a protoplanetary disc.

1.3 Vertical gas-density profile

In the direction perpendicular to the disc plane (vertically, corresponding to the *z*-axis in Figure 4), the profile of the gas density ρ_{gas} is such that the vertical pressure gradient $\mathrm{d}P_{\text{gas}}/\mathrm{d}z$ and the vertical component of the stellar gravity g_z are in **hydrostatic equilibrium**. Since, as mentioned earlier, $M_{\text{disc}} \ll M_*$, any disc contribution to the gravitational force can be ignored. Therefore we may write:

$$\frac{\mathrm{d}P_{\text{gas}}(z)}{\mathrm{d}z} = -\rho_{\text{gas}}(z) g_z.$$

Referring to Figure 4, the vertical component of the stellar gravity at a point A located a distance *d* from the star is $g_z = (GM_*/d^2)\sin\theta$, where *G* is the gravitational constant.

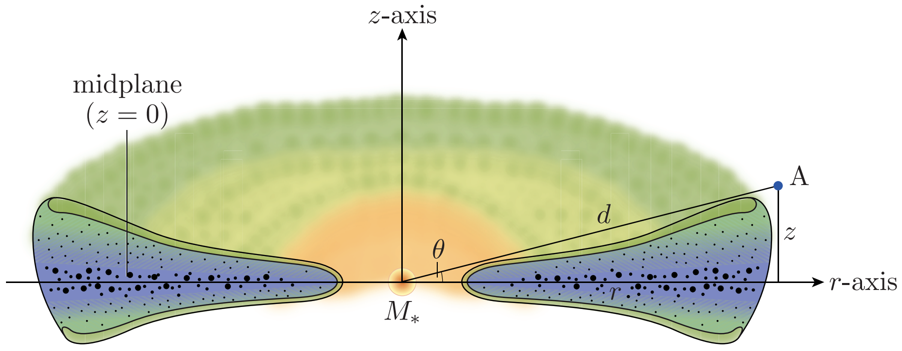


Figure 4 (repeated) Schematic illustration of a protoplanetary disc.

The angle θ is such that $\sin\theta = z/d$, hence $g_z = GM_* z/d^3$. Now, the distance *d* is given by $d^2 = r^2 + z^2$ but for geometrically thin discs, $z \ll r$, so we have simply $d \approx r$, and therefore

$$g_z \approx \frac{GM_* z}{r^3}.$$

Note that the **Keplerian angular speed** ω_K at this same point in the disc is

$$\omega_K = \left(\frac{GM_*}{r^3}\right)^{1/2} \qquad \text{(Equation 2)}$$

so $g_z = z\omega_K^2$ and we may write

$$\frac{dP_{gas}(z)}{dz} = -z\rho_{gas}(z)\omega_K^2.$$

This equation can be simplified by recognising that, for an ideal gas, the pressure P_{gas} and density ρ_{gas} are related by the **sound speed** c_s such that

$$c_{\mathrm{s}}^{2}=\frac{k_{\mathrm{B}}T}{\bar{m}}=\frac{P_{\mathrm{gas}}}{\rho_{\mathrm{gas}}},$$ (Equation 3)

where k_{B} is the Boltzmann constant, T is the temperature of the disc and $\bar{m}$ is the mean molecular mass. The sound speed may be assumed to be constant for a given disc. Hence, $\mathrm{d}P_{\mathrm{gas}}=c_{\mathrm{s}}^{2}\,\mathrm{d}\rho_{\mathrm{gas}}$ and the expression of hydrostatic equilibrium becomes

$$\frac{\mathrm{d}\rho_{\mathrm{gas}}(z)}{\mathrm{d}z}=-z\rho_{\mathrm{gas}}(z)\left(\frac{\omega_{\mathrm{K}}}{c_{\mathrm{s}}}\right)^{2}.$$ (Equation 4)

This differential equation has the following solution, which gives the density in terms of the **disc scale height** H and the density at the midplane ρ_0:

$$\rho_{\mathrm{gas}}(z)=\rho_{0}\exp\left(-\frac{z^{2}}{2H^{2}}\right),$$ (Equation 5)

where

$$H=\frac{c_{\mathrm{s}}}{\omega_{\mathrm{K}}}$$ (Equation 6)

and

$$\rho_{0}=\frac{1}{\sqrt{2\pi}}\frac{\Sigma}{H}.$$ (Equation 7)

Here, $\Sigma=\int\rho_{\mathrm{gas}}(z)\,\mathrm{d}z$ is the **surface density** of the disc (i.e. its mass per unit surface area).

- What is the gas density at a height of $z = H$?
- The gas density is $\rho_{\mathrm{gas}}(H)=\rho_{0}\exp(-1/2)=\rho_{0}/\mathrm{e}^{1/2}\approx 0.607\rho_{0}$.

The shape of a disc can be described by its **aspect ratio** $H/r=c_{\mathrm{s}}/v_{\mathrm{K}}$, where $v_{\mathrm{K}}=r\omega_{\mathrm{K}}$ is the Keplerian speed at a radius r. Normally, for protoplanetary discs,

$$\frac{H}{r}\propto r^{1/4}.$$ (Equation 8)

This is because the speed of sound $c_{\mathrm{s}}\propto T^{1/2}$ (Equation 3) and the temperature profile of the disc is driven by the stellar irradiation, so that usually $T\propto r^{-1/2}$. (The reason for this latter dependence is that, from the Stefan-Boltzmann law, the temperature of the disc $T\propto F_{*}^{1/4}$ where the flux received from the star $F_{*}\propto 1/r^{2}$.) Hence, $c_{\mathrm{s}}\propto r^{-1/4}$ and this result, combined with the fact that $\omega_{\mathrm{K}}\propto r^{-3/2}$ (Equation 2), leads to Equation 8.

- How does Equation 8 explain the shape of the disc shown in Figure 4?
- The aspect ratio of the disc increases with *r*, so the disc is expected to be thicker on the edge than in the centre, like the one in Figure 4.

1.4 Radial dependence of the orbital velocity

Having considered the vertical density profile of the disc, we now turn to the radial dependence of the orbital velocity $v_{\mathrm{orb}}(r)$. In the radial direction, in addition to the gravitational force, there is also a force due to the pressure gradient. Hence, the net centripetal acceleration of a small volume of gas in the disc on an assumed circular orbit at radius r is

$$\frac{v_{\mathrm{orb}}^2(r)}{r} = g(r) + \frac{1}{\rho_{\mathrm{gas}}(r)} \frac{\mathrm{d}P_{\mathrm{gas}}(r)}{\mathrm{d}r}.$$

Here, $g(r) \approx GM_*/r^2$ since M_* is much greater than the total mass of the disc inside the orbital radius. Therefore, the orbital speed v_{orb} of the gas in the disc has two components: one due to the Keplerian speed $v_{\mathrm{K}}(r) = (GM_*/r)^{1/2}$, and one due to this extra pressure gradient. It is given by:

$$v_{\mathrm{orb}}^2(r) = \frac{GM_*}{r} + \frac{r}{\rho_{\mathrm{gas}}(r)} \frac{\mathrm{d}P_{\mathrm{gas}}(r)}{\mathrm{d}r}. \qquad \text{(Equation 9)}$$

Usually, $\mathrm{d}P_{\mathrm{gas}}(r)/\mathrm{d}r < 0$ in the disc, so the gas will behave as if it was feeling a slightly lower gravitational pull from the star, and its orbital speed will be sub-Keplerian, that is, $v_{\mathrm{orb}}(r) < v_{\mathrm{K}}(r)$. The following activity shows how to quantify the magnitude of the deviation between the actual orbital speed of the gas and its Keplerian speed.

Activity 1

Using Equation 9 and approximating the pressure gradient as $\mathrm{d}P_{\mathrm{gas}}(r)/\mathrm{d}r = -nP_{\mathrm{gas}}(r)/r$, where n is a dimensionless constant:

a. Write an approximate expression for $v_{\mathrm{orb}}(r)$ as a function of the radius, scale height and Keplerian speed, $v_{\mathrm{K}}(r) = r\omega_{\mathrm{K}}(r)$.

b. Calculate the difference between the orbital and Keplerian speeds, $\Delta v = v_{\mathrm{K}} - v_{\mathrm{orb}}$, at a radius of 1 au from a star of the same mass as the Sun, for a disc of constant aspect ratio H/r = 0.05 and n = 3. (You may assume 1 au = 1.496×10^{11} m, 1 $M_{\odot}$ = 1.99×10^{30} kg, and G = 6.674×10^{-11} N m^2 kg^{-2}.)

Answer

a. From Equation 3, the pressure of the gas P_{gas} is linked to the sound speed c_s such that $P_{\text{gas}} = \rho_{\text{gas}} c_{\text{s}}^2$. Using this, the expression for the pressure gradient becomes:

$$\frac{\mathrm{d}P_{\text{gas}}(r)}{\mathrm{d}r} = -n\frac{P_{\text{gas}}(r)}{r} = -n\frac{c_{\text{s}}^2\rho_{\text{gas}}(r)}{r},$$

then substituting this into Equation 9 gives

$$v_{\text{orb}}^2(r) = \frac{GM_*}{r} - nc_{\text{s}}^2 = v_{\text{K}}^2(r) - nc_{\text{s}}^2.$$

Finally, using the fact that

$$\frac{H}{r} = \frac{c_{\text{s}}}{r\omega_{\text{K}}(r)} = \frac{c_{\text{s}}}{v_{\text{K}}(r)},$$

the requested expression is obtained as:

$$v_{\text{orb}}(r) = v_{\text{K}}(r)\left[1 - n\left(\frac{H}{r}\right)^2\right]^{1/2}. \qquad \text{(Equation 10)}$$

b. From Equation 10, the difference in velocities is

$$\Delta v(r) = v_{\text{K}}(r) - v_{\text{orb}}(r)$$

$$\Delta v(r) = \left[1 - \sqrt{1 - n\left(\frac{H}{r}\right)^2}\right] v_{\text{K}}(r)$$

$$\Delta v(r) = \left[1 - \sqrt{1 - 3 \times 0.05^2}\right] v_{\text{K}}(r)$$

$$\Delta v(r) = 0.00376 v_{\text{K}}(r)$$

So the difference is only about 0.4% of the Keplerian velocity.
To evaluate Δv at r = 1 au we need to calculate the Keplerian velocity v_K at 1 au:

$$v_{\text{K}} = \sqrt{\frac{GM_*}{r}} = \sqrt{\frac{G \times 1\,\text{M}_\odot}{1\,\text{au}}}$$

$$v_{\mathrm{K}} = \sqrt{\frac{6.674 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}} \times 1.99 \times 10^{30}\ \mathrm{kg}}{1.496 \times 10^{11}\ \mathrm{m}}}$$

$$v_{\mathrm{K}} = 29.8 \times 10^{3}\ \mathrm{m\ s^{-1}}$$

which gives $\Delta v = 0.00376 v_{\mathrm{K}} = 112\,\mathrm{m\,s^{-1}}$. So the difference between the orbital and Keplerian speeds at this radius is about 100 m s^{-1}.

2 Rising from the dust

The basis of the **core-accretion scenario** is that planets form by accumulation of solids into a core, on which an atmosphere is accreted once a critical value of the core mass is achieved. This section explores the various stages of this process, going from sub-micron-sized dust particles to metre-sized rocks that grow into kilometre-sized **planetesimals** and finally into Mercury-sized **planetary embryos**, spanning roughly 12 orders of magnitude in size. The embryos then accumulate further matter, becoming the cores of fully formed planets.

2.1 From dust grains to rocks

Consider again the protoplanetary disc from Activity 1. The fact that the velocity of the gas in a protoplanetary disc is usually sub-Keplerian has important consequences for the evolution of solid particles embedded in it. A consequence of Equation 10 is that for geometrically thin discs ($H/r \ll 1$) the radial pressure gradient makes a negligible contribution to the orbital speed of the gas. However, as seen in Activity 1, the difference in speed can be of the order of Δv ~ 100 m s^{-1} at ~1 au from the star and this turns out to be important in determining how the particles in a disc behave. In particular, a finite Δv can cause particles in the disc to slow down and drift inwards towards the star.

One of the most important parameters that determines how a particle of mass *m* interacts with the gas surrounding it is the **stopping time**, defined as

$$\tau_{\text{stop}} = \frac{m\Delta v}{F_{\text{drag}}}, \quad \text{(Equation 11)}$$

where F_{drag} is the magnitude of the drag force that acts in the opposite direction to Δv. This stopping time may be related to the Keplerian orbital speed by

$$\tau_{\text{S}} = \tau_{\text{stop}}\omega_{\text{K}}, \quad \text{(Equation 12)}$$

where τ_{S} is the **Stokes number**, which characterises how well particles follow fluid streamlines. Large particles will generally have large Stokes numbers ($\tau_{\text{S}} \gg 1$), and small particles will generally have small Stokes numbers ($\tau_{\text{S}} \ll 1$).

Small particles of radius *s* will be coupled with the gas; that is, they will move at almost the same speed as the gas. Such particles experience a drag force whose magnitude is given by

$$F_{\text{drag}} = \frac{4\pi}{3}\rho_{\text{gas}}s^2 v_{\text{th}}\Delta v, \quad \text{(Equation 13)}$$

where the thermal speed of the gas, v_{th}, is roughly the same as its sound speed, c_{s}. For spherical particles, the material density is $\rho_{\text{m}} = 3m/(4\pi s^3)$. So, by combining this with Equations 11 and 13, we have

$$\tau_{\text{stop}} = \frac{\rho_{\text{m}}}{\rho_{\text{gas}}}\frac{s}{c_{\text{s}}}. \quad \text{(Equation 14)}$$

We can now define the **radial drift speed**, v_{rad}, as the speed with which particles in the disc move radially through it, drifting inwards towards the star. To derive a general expression for the radial drift speed we write the orbital speed as

$$v_{orb} = v_K(1-\eta)^{1/2}, \quad \text{(Equation 15)}$$

where, from Equation 10, we already know that $\eta = n(H/r)^2$. Remember that n is a dimensionless constant relating the pressure gradient to P_{gas}/r. After much algebra concerning the equations of motion and making appropriate approximations (the details of which are not important here), a general expression for the radial drift speed is found as:

$$v_{rad} = -v_K \frac{\eta}{\tau_S + \tau_S^{-1}}. \quad \text{(Equation 16)}$$

The radial drift speed will reach its *maximum* value when the Stokes number is $\tau_S = 1$. This corresponds to the situation when the stopping time and Keplerian orbital speed are related by $\tau_{stop} = 1/\omega_K$. In this case, Equation 16 shows that the radial drift speed is $v_{rad}(\max) = -\eta v_K/2$.

- Since η will be a small number for geometrically thin discs, make use of the approximation that $(1-\eta)^{1/2} \approx 1-(\eta/2)$ to obtain a simple relation between Δv and v_K. Hence write the maximum radial drift speed in terms of Δv.
- From Equation 10, the difference between the Keplerian speed and the orbital speed is

  $$\Delta v = v_K - v_{orb} = v_K\left[1-(1-\eta)^{1/2}\right].$$

  Therefore, using the approximation for small values of η, we have

  $$\Delta v \approx v_K\left[1-1+(\eta/2)\right] \approx \eta v_K/2$$

  and so $v_K \approx 2\Delta v/\eta$. Hence, the maximum radial drift speed is

  $$v_{rad}(\max) \approx -(\eta/2)\times 2\Delta v/\eta \approx -\Delta v.$$

So the maximum radial drift speed is simply the difference between the Keplerian and orbital speeds.

Activity 2

a. For large particles, with $s > 1$ m, the Stokes number is very large, $\tau_S \gg 1$.

   Obtain an expression for the radial drift speed in this case, in terms of τ_S and Δv only.

b. For small particles, with $s < 1$ cm, the Stokes number is very small, $\tau_S \ll 1$.

   Obtain an expression for the radial drift speed in this case, in terms of τ_S and Δv only.

Discussion

a. For large particles $\tau_S \gg 1$ so Equation 16 becomes

$$v_{rad} \approx -\eta v_K / \tau_S.$$

Then, substituting for v_K using the approximation $v_K \approx 2\Delta v/\eta$, we have

$$v_{rad} \approx -2\Delta v / \tau_S.$$

b. For small particles $\tau_S \ll 1$ so Equation 16 becomes

$$v_{rad} \approx -\eta v_K \tau_S.$$

Then, substituting for v_K using the approximation $v_K \approx 2\Delta v/\eta$, we have

$$v_{rad} \approx -2\Delta v \tau_S.$$

Activity 2 showed that, in the limits of large or small particles, both values of the radial drift speed are independent of η. In each case, the radial drift speed is a small fraction of Δ*v*.

2.2 Assembling the planetesimals

Thanks to the coupling with the disc, small (sub-micron-sized) particles will collide with each other gently enough that they will always stick together. Therefore, they will efficiently form millimetre-sized aggregates, in a process referred to as **coagulation**, that tend to settle on the midplane of the disc.

The simplest assumption for the formation of a planetesimal is that this process continues up to the kilometre-sized scale. As the particles grow, however, so does their speed, and the outcome of a collision no longer necessarily leads to bigger objects, as energetic impacts can be neutral (so the particles will bounce off each other) or even destructive (so the particles fragment and are broken apart once more).

However, there is also another problem that occurs around the metre-sized scale, as illustrated by the following activity. Metre-sized particles, referred to as 'rocks' will have $\tau_S \sim 1$, and so move with the maximum radial drift speed.

Activity 3

a. Consider a thin protoplanetary disc with aspect ratio H/r = 0.05 and dimensionless pressure constant n = 3. Calculate the radial drift speed for a particle with τ_S = 1 at 1 au from a star of mass 1 $M_\odot$. *Hint:* the Keplerian speed at 1 au from a 1 $M_\odot$ star as calculated in Activity 1 is $v_K = 29.8 \times 10^3$ m s^{-1}.

b. Moving at the constant speed from part (a), how long would the particle take to travel a distance of 1 au? Express your answer in terms of the number of orbital periods at 1 au.

Discussion

a. In this case,

$$\eta = n(H/r)^2 = 3 \times 0.05^2 = 7.5 \times 10^{-3},$$

so with τ_S = 1, the radial drift speed is

$$v_{\text{rad}} = -\eta v_K/2$$

$$v_{\text{rad}} = -7.5 \times 10^{-3} \times 29.8 \times 10^3 \text{ m s}^{-1}/2$$

$$v_{\text{rad}} = -112 \text{ m s}^{-1}.$$

(Note that this is equal to Δv, calculated in Activity 1, which is as expected according to the expression for the maximum radial drift speed derived earlier for the case τ_S = 1.)

b. The time *t* to travel a distance of 1 au radially at the speed from part (a) is

$$t = \frac{1\text{ au}}{v_{\text{rad}}} = \frac{1.496 \times 10^{11}\text{ m}}{112 \text{ m s}^{-1}}$$

$$t = 1.34 \times 10^9 \text{ s}.$$

Since the orbital period at a distance of 1 au from a 1 $M_\odot$ star is 1 y = 3.16×10^7 s, this timescale is only about 40 orbital periods.

Activity 3 showed that the radial drift for metre-sized particles can be very rapid. Radial drift therefore reduces the abundance of rocks in the outer regions of the disc, while potentially increasing it in the regions closer to the star. The fact that rocks move through the disc very rapidly gives rise to the ‘metre-sized barrier problem’ in explaining how planetesimals form: the growth to kilometre-sized planetesimals must happen fast enough to be complete before the medium-sized particles drift toward the centre, but also occur via a mechanism that avoids fragmentation.

Figure 5 shows a schematic view of the current picture of planetesimal formation. Once smaller fragments have formed, they settle vertically into the disc: see Figure 5(a). Next, the fragments drift radially towards the centre, leading to a possible build-up of solids in the inner disc: see Figure 5(b). Over-densities of solid material form through **streaming instabilities** and then lead to the formation of planetesimals through gravitational collapse: see Figure 5(c).

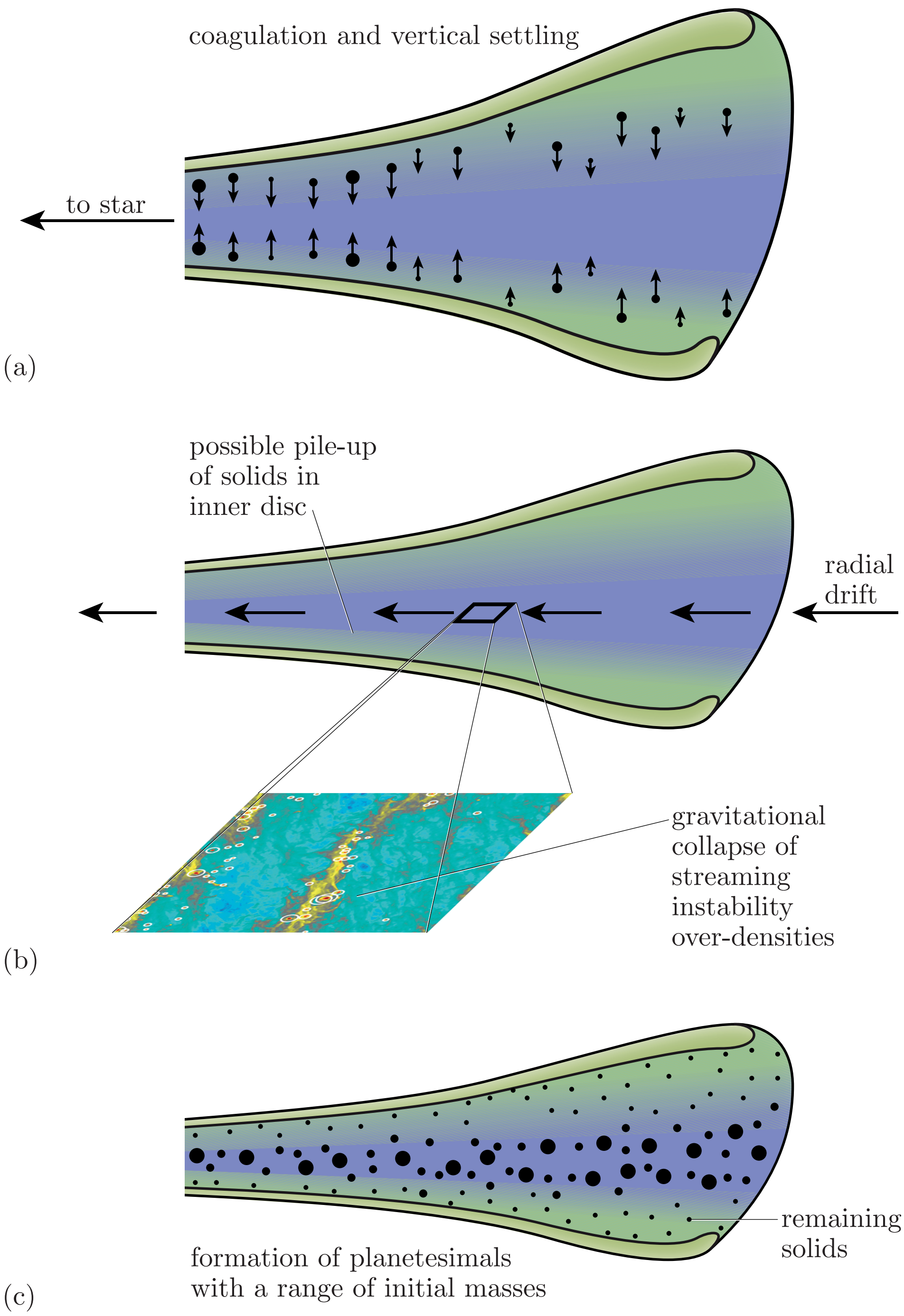


Figure 5 Schematic illustration of the formation of planetesimals.

2.3 The growth of planetary cores

Once kilometre-sized planetesimals have formed with a mass of ~ $10^{12} – 10^{13}$ kg, they are massive enough to interact significantly with their neighbours via gravity and modify their velocity, thus becoming prone to collisions.

In the same way as for the smaller particles, collisions of planetesimals with other planetesimals need to happen at sufficiently low speed to lead to accretion. Under this assumption, the rate at which a planetesimal of mass M_p and radius R_p grows with time t can be written as:

$$\frac{\mathrm{d}M_\mathrm{p}}{\mathrm{d}t} = \pi R_\mathrm{p}^2 \omega_\mathrm{K} \Sigma \left(1 + \frac{v_\mathrm{esc}^2}{v_\mathrm{rel}^2}\right) = \pi R_\mathrm{p}^2 \omega_\mathrm{K} \Sigma F_\mathrm{g}. \qquad \text{(Equation 17)}$$

Here, Σ is the surface density of the disc, v_{esc} is the planetesimal's **escape velocity**, v_{rel} is the relative velocity between the two impacting planetesimals and F_g is the **gravitational focusing**, which is a dimensionless parameter that describes how the gravitational attraction between two bodies increases their collision probability.

- How does dM_p/dt change with the disc's surface density Σ?
- The growth rate scales linearly with the disc's surface density, so the growth rate will be higher for discs with a higher mass in planetesimals.

- And how does growth rate scale with the distance from the central star?
- With everything else being equal, growth is slower at large distances where the Keplerian angular speed, ω_K is smaller.

The following activity shows a quantitative example of the impact of the gravitational focusing on the growth rate, considering a planetesimal in an orbit similar to that of Jupiter.

Activity 4

a. Assuming that F_g is constant, starting from Equation 17 write an expression for the planetesimal's radius growth rate dR_p/dt as a function of its density ρ_p.

b. Estimate the value of F_g needed for a planetesimal at the same distance as Jupiter with ω_K = 0.16 y^{-1} to grow to a radius of R_p = 1000 km in 10^5 years. Use a surface density of Σ = 100 kg m^{-2} and a planetesimal density of ρ_p = 3000 kg m^{-3}.

Answer

a. Assuming the planetesimal to be spherical, its mass M_p can be expressed in terms of its radius R_p and density ρ_p as:

$$M_\mathrm{p} = \frac{4}{3}\pi R_\mathrm{p}^3 \rho_\mathrm{p}, \qquad \text{(Equation 18)}$$

then we note that

$$\frac{\mathrm{d}R_\mathrm{p}}{\mathrm{d}t} = \frac{\mathrm{d}M_\mathrm{p}}{\mathrm{d}t} \div \frac{\mathrm{d}M_\mathrm{p}}{\mathrm{d}R_\mathrm{p}}.$$

Since dM_p/dt is given by Equation 17, and $\mathrm{d}M_\mathrm{p}/\mathrm{d}R_\mathrm{p} = 4\pi R_\mathrm{p}^2 \rho_\mathrm{p}$ (by differentiation of Equation 18), this means

$$\frac{dR_p}{dt} = \frac{\pi R_p^2 \omega_K \Sigma F_g}{4\pi R_p^2 \rho_p} = \frac{1}{4}\frac{\omega_K \Sigma}{\rho_p} F_g. \quad \text{(Equation 19)}$$

b. Using the values provided, we obtain:

$$\frac{dR_p}{dt} = \frac{1}{4} \times \frac{0.16\,\mathrm{y^{-1}} \times 100\,\mathrm{kg\,m^{-2}}}{3000\,\mathrm{kg\,m^{-3}}} F_g$$

$$\frac{dR_p}{dt} = 1.33 \times 10^{-3}\, F_g\,\mathrm{m\,y^{-1}}.$$

To reach a radius of 1000 km in 10^5 years, the desired growth rate must be $dR_p/dt = 10^6$ m / 10^5 = 10 m y^{-1}.

Hence, the gravitational focusing needs to be approximately

$$F_g \approx \frac{10\,\mathrm{m\,y^{-1}}}{1.33 \times 10^{-3}\,\mathrm{m\,y^{-1}}} \approx 7500.$$

The expression that involves F_g (Equation 17) depends on the relative velocities between planetesimals, the range of which is characterised by the **velocity dispersion** within the disc. Hence, velocity dispersion plays a crucial role in determining the accretion rates. It turns out that there are two regimes:

1. In cases where gravitational focusing of planetesimals is initially very strong, **runaway growth** occurs. This is an accelerated phase in the growth of planetesimals such that the largest bodies get larger at a rapid and increasing rate, proportional to their mass. This phase is thought to be generally quite short, and ends when the velocity dispersion of the resulting planetary embryos increases to the point where gravitational focusing is ineffective.
2. In cases where the largest planetary embryos grow quickly while the smallest grow slowly, **oligarchic growth** occurs. This leads to a bimodal mass distribution with a number of embryos comparable to the mass of the Moon, Mercury or Mars (~10^{23} kg) embedded in a large population of smaller planetesimals.

2.4 The isolation mass

Once the oligarchic growth phase is over, the resulting embryos are relatively isolated and on initially circular orbits. They continue growing into **planetary cores**, by accreting the nearby leftover planetesimals within a **feeding zone** which extends a distance Δa either side of the planetary core. We may write the radius of this feeding zone as $\Delta a = CR_{Hill}$, where C is a constant and R_{Hill} is the **Hill radius**, defined as:

$$R_{Hill} = \left(\frac{M_p}{3M_*}\right)^{1/3} a. \quad \text{(Equation 20)}$$

Here, M_p is the mass of the planetary core, M_* is the mass of the star and a is the radius of the orbit. The Hill radius is defined as the distance from the planetary core at which its gravitational force dominates over that of the star.

The growth of the cores continues until all the neighbouring planetesimals have been consumed. At this point, the mass of the core reaches the **isolation mass** M_{iso}, defined as the total mass of the planetesimals within the feeding zone.

$$M_{\rm iso} = \frac{8}{\sqrt{3}}(\pi \Sigma C)^{3/2}\frac{a^3}{M_*^{1/2}}. \quad \text{(Equation 21)}$$

The origin of Equation 21 is explored in the next activity.

Activity 5

a. Write down an expression for the mass of planetesimals within the feeding zone in terms of the disc surface density Σ, distance to the central star a and feeding zone width Δa. Hence, derive Equation 21.

b. Use Equation 21 to evaluate M_{iso} in the terrestrial planets region at $a_\oplus$ = 1.0 au and in the Jovian planets region at a_{Jup} = 5.2 au, for M_* = 1 $M_\odot$, Σ = 100 kg m^{-2} and $C = 2\sqrt{3}$.

Answer

a. The mass of planetesimals within the feeding zone is the area of the annulus with width 2Δa at a radius a, multiplied by the surface density. Hence,

$$M_{\rm iso} = 2\pi a \times 2\Delta a \times \Sigma.$$

The width of the feeding zone is defined in terms of the Hill radius of the resulting planetary core (Equation 20) as $\Delta a = CR_{Hill}$. Therefore, once this mass is all contained within a single core, its mass is given by

$$M_{\rm iso} = 4\pi a^2 \Sigma C \left(\frac{M_{\rm iso}}{3M_*}\right)^{1/3}.$$

This simplifies to

$$M_{\rm iso}^{2/3} = \frac{4\pi a^2 \Sigma C}{(3M_*)^{1/3}},$$

which may be rearranged to give the requested expression.

b. In the terrestrial planets region (where $a = a_\oplus$ = 1.0 au), this gives

$$M_{\rm iso} = \frac{8}{\sqrt{3}} \times (\pi \times 100\,{\rm kg\,m^{-2}} \times 2\sqrt{3})^{3/2} \times \frac{(1.496\times 10^{11}\,{\rm m})^3}{(1.99\times 10^{30}\,{\rm kg})^{1/2}}$$

$$M_{\rm iso} = 3.94 \times 10^{23}\,{\rm kg}.$$

(This is about 0.066 times the mass of the Earth, or around the mass of Mercury.)

Similarly, at the distance of Jovian planets (where $a = a_{Jup}$ = 5.2 au), Equation 21 gives an isolation mass that is $(5.2)^3$ larger. Hence,

$$M_{iso} = 5.53 \times 10^{25}\ \mathrm{kg}$$

(This is about 9.3 times the mass of the Earth, which is around half the mass of Neptune.)

As shown in Activity 5, the fact that $M_{iso} \propto a^3$ means that more massive cores up to several Earth masses can form at larger distances from the central star. However, at $a \sim 10$ au the speed of the planetesimals is too high for the collisions to lead to accretion, so it becomes increasingly hard to build massive planetary cores.

Planetary cores formed in this way will have sizes from around that of Mercury (with radius a few thousand kilometres) to several times that of the Earth (with radius a few tens of thousand kilometres).

Activity 6

Summarise the typical sizes of objects involved in the various stages of the core-accretion scenario.

Discussion

Initially, the particles are dust grains with a typical size of a micron or less which coagulate into millimetre-sized aggregates. These accumulate into rocks that are around one metre in size, which grow further into kilometre-sized planetesimals. Gravitational focusing helps these grow into planetary embryos with sizes and masses around that of the Moon, Mercury or Mars. These then become planetary cores by accreting leftover planetesimals within their feeding zone to reach a mass and size of a few times that of the Earth.

3 The final stages of planet formation

This section explores the possible outcomes of the final stages of planet formation, starting with the formation of giant planets via core accretion, then turning to alternative formation routes and, finally, exploring the effect of migration and planet–planet interaction on the final architectures of planetary systems.

3.1 Forming giant planets via core accretion

Once the mass of a planetary core reaches a few Earth masses, it starts to build up a gas envelope. This process can lead to a variety of outcomes because, as seen in Activity 5, the speed at which a core grows and its final mass depend on where in the disc it is found.

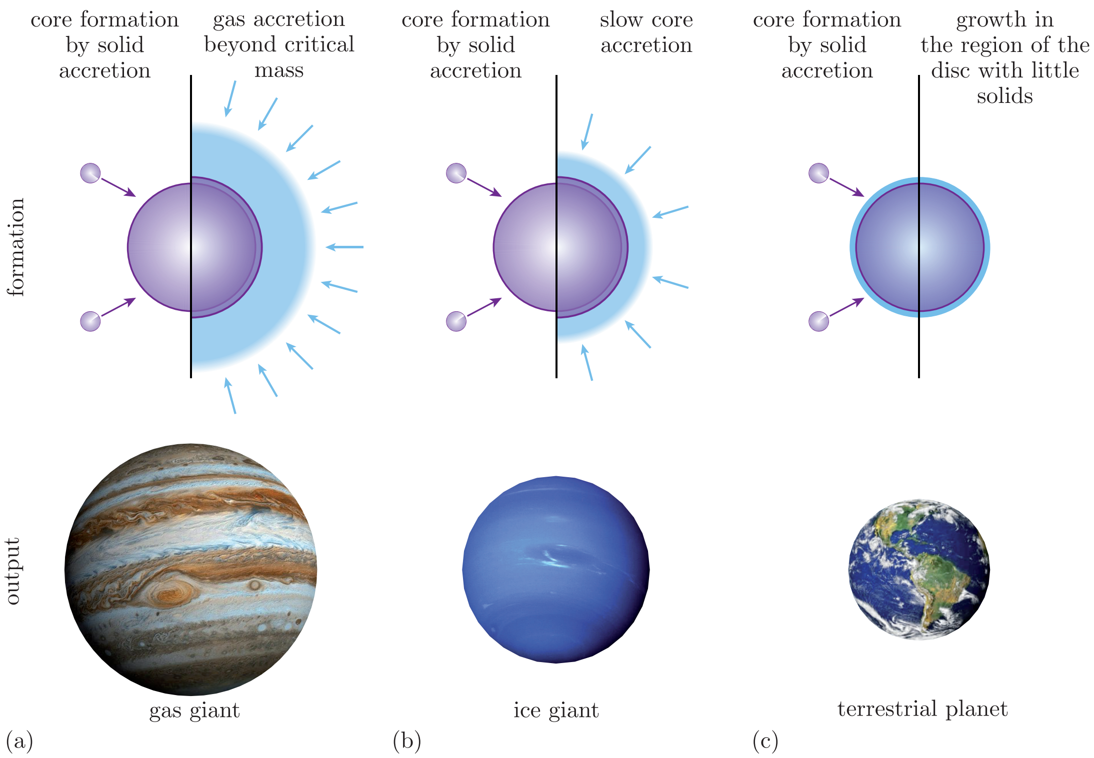


Figure 6 Schematic view of possible outcomes of the core-accretion model.

Figure 6(a) shows that for Jupiter-like gas giants to form, the core needs to reach a **critical mass**, high enough that the gas envelope cannot maintain hydrostatic equilibrium and start contracting. (Usually, M_{crit} is in the approximate range 5 – 20 Earth masses.) Exceeding the critical mass triggers a phase of rapid accretion, which continues until either the gas is dispersed or the planet opens a gap in the disc and the rate of gas accretion slows down. (In the core-accretion scenario, the gas-dispersal timescale is one of the factors that governs the lifespan of the disc and hence the final mass of gas giants.)

Figure 6(b) shows that if the core grows in a region of the disc where accretion is slower than in (a), there will be less gas in the vicinity of the planetary core by the time the critical mass is reached. This will typically occur further out than (a). Therefore, the final mass of the gas envelope will be smaller than it is for gas giants, and the resulting planet will be a core-dominated ice giant, like Uranus and Neptune.

Finally, Figure 6(c) shows that if the timescale for the core growth is much smaller than the gas-dispersal timescale, or if the core grows much closer to the star than gas or ice giants (where very little solid material is available) the planet will only develop a thin hydrogen-dominated atmosphere, like that of the primordial Earth.

Many observed protoplanetary discs show gaps, bright rings, asymmetries, spirals and other structures. Figure 7 provides a stunning example of the variety of configurations that can arise from the interactions of forming planets with the disc.

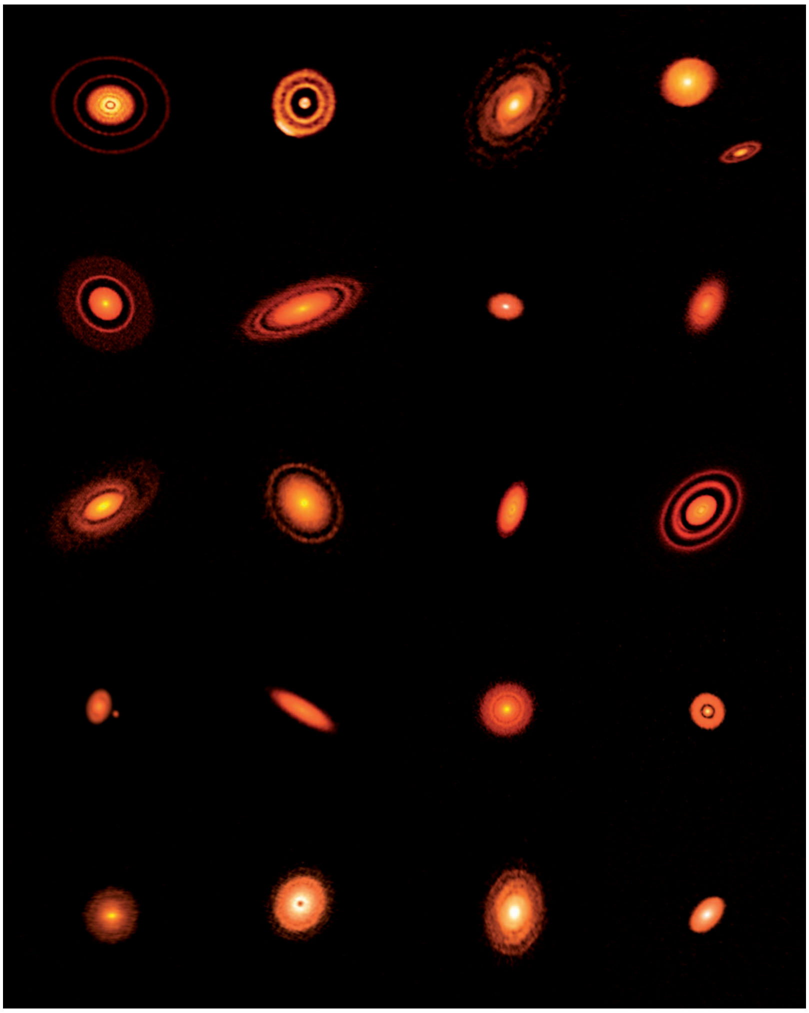


Figure 7 Gallery of protoplanetary disc images obtained with the Atacama Large Millimeter Array (ALMA).

3.2 Super-Jupiter exoplanets

The core-accretion scenario discussed so far has its origin in the search for an explanation for our Solar System which, until the discovery of 51 Pegasi b, was the source of all available observational data used to constrain planet-formation theories. While, in its modern form, the core-accretion scenario succeeds in explaining some of the characteristics of the observed exoplanet population, there are still some aspects of planet formation that the model struggles to describe. One such aspect is connected with the formation of giant planets and, in particular, of directly imaged planets in wide orbits.

As briefly mentioned in Section 3.1, the main problem with the growth of giant planets via core accretion is connected with the lifetime of the discs, which appear to have gas-dissipation timescales that are too short to allow for the formation of many of the observed gas-giant exoplanet systems. This is true for Jupiter-sized exoplanets, but becomes even more crucial when considering the directly imaged super-Jupiters (exoplanets with masses several times that of Jupiter), like the one orbiting the young solar-type star YSES 2 (named after the Young Suns Exoplanet Survey). The YSES 2 planetary system, which is part of the Scorpius–Centaurus association, is shown in Figure 8. The giant exoplanet YSES 2 b, which is visible as a bright dot indicated with the arrow in the figure, has a mass of about 6 Jupiter masses and a semimajor axis of around 110 au, and is one of the few directly imaged planets around a solar-type star.

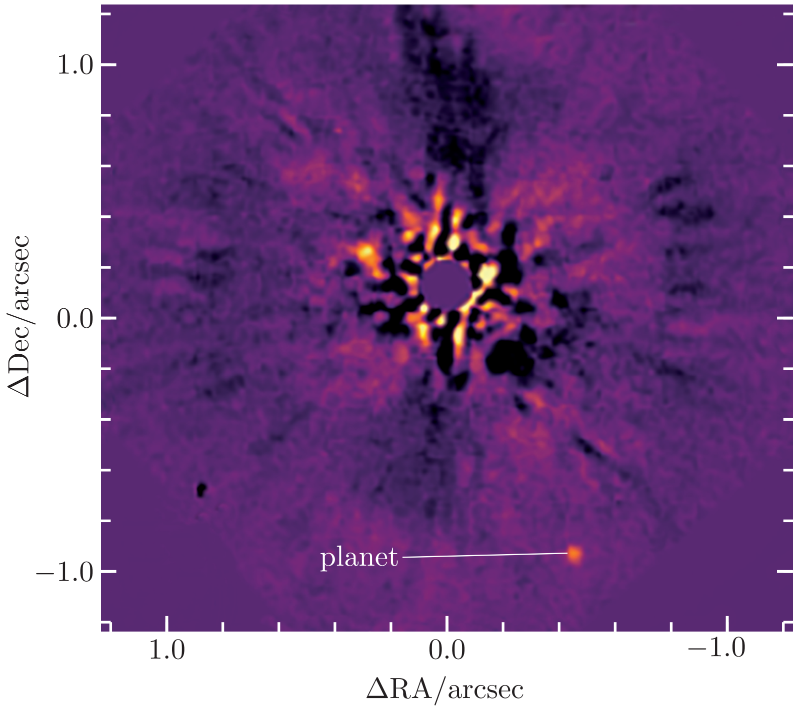


Figure 8 Near-infrared image of the planet around the young star YSES 2, obtained with the VLT/SPHERE instrument.

3.3 The disc-instability scenario

There is, however, an alternative to the core-accretion scenario that succeeds in predicting the formation of giant planets, including those with $M_p > M_{Jup}$ via direct collapse of the gas in the protoplanetary disc. The key idea of this **disc-instability scenario** is that a sufficiently cold and/or massive disc tends to be gravitationally unstable. Thus, the disc can undergo **fragmentation** to form gravitationally bound clumps that evolve into giant planets. For fragmentation to occur, the local surface density in the disc needs to be high enough that the self-gravity of the gas and its differential rotation (both drivers of gravitational collapse) are higher than the thermal pressure (which counteracts collapse). These competing effects are nicely summarised by the **Toomre criterion**, which states that, for a disc to fragment, its **Toomre *Q* parameter** must satisfy:

$$Q = \frac{\omega_K c_s}{\pi G \Sigma} < 1. \qquad \text{(Equation 22)}$$

Here, c_s is the sound speed (Equation 3), ω_K is the Keplerian angular speed (Equation 2) and Σ is the gas surface density.

To understand the significance of the Toomre criterion for exoplanet systems, it is instructive to consider it in the context of a protoplanetary disc similar to the one that formed our Solar System. To that end, it is useful to introduce the concept of the **minimum-mass solar nebula**. This is a hypothetical protoplanetary disc with a surface density profile $\Sigma_{sn}(r)$ defined as the minimum value of the surface density that a disc would need to have (as a function of radius r from a Sun-like star) to form our Solar System. The composition of this model nebula is derived from the observed mass of heavy elements in the Solar System planets, plus enough hydrogen and helium to mimic the solar composition. The distribution of this model nebula is determined by spreading the mass needed for each planet over an annulus extending over the distance between them. The accepted surface density distribution for the minimum-mass solar nebula is:

$$\Sigma_{\text{sn}}(r) = 1.7 \times 10^4 \left(\frac{r}{1\,\text{au}}\right)^{-3/2} \text{kg m}^{-2}.$$

The following activity shows how to express Q as a function of the stellar mass, disc mass and the disc aspect ratio, and compares the minimum value of Σ required for fragmentation to that of the minimum-mass solar nebula.

Activity 7

a. Starting from the definition of scale height $H = c_s/\omega_K$ (Equation 6), demonstrate that for a protoplanetary disc with uniform surface density Σ, the Toomre Q parameter can be written as

$$Q = \frac{M_*}{M_{\text{disc}}} \frac{H}{r}, \qquad \text{(Equation 23)}$$

where r is the distance from the star, H is the scale height, M_* is the mass of the central star and M_{disc} is the mass of the disc.

b. Show that for the Toomre criterion to be satisfied, the surface density must obey:

$$\Sigma > 1.4 \times 10^6\,\text{kg m}^{-2} \left(\frac{H/r}{0.05}\right) \left(\frac{M_*}{\text{M}_\odot}\right) \left(\frac{r}{1\,\text{au}}\right)^{-2}.$$

c. Consider a disc with aspect ratio H/r = 0.05 around a solar-type star (M_* = 1 $\text{M}_\odot$). Show that the minimum surface density at r = 1 au for the disc to fragment is roughly two orders of magnitude greater than that of the minimum-mass solar nebula at the same radius.

Discussion

a. Using the scale height definition, Equation 22 becomes:

$$Q = \frac{\omega_K^2 H}{\pi G \Sigma}.$$

Using $\omega_{\mathrm{K}} = (GM_*/r^3)^{1/2}$ (Equation 2), and noting that

$varSigma = M_{\mathrm{disc}}/(\pi r^2)$ for a disc with uniform surface density, then Q

becomes

$$Q = \frac{GM_*}{r^3}\frac{H}{\pi G}\frac{\pi r^2}{M_{\mathrm{disc}}} = \frac{M_*}{M_{\mathrm{disc}}}\frac{H}{r},$$

as required.

b. Starting from Equation 23 and multiplying by a factor of $M_{\mathrm{disc}}/(\pi r^2 \varSigma) = 1$ and the relevant normalised quantities gives

$$Q = \frac{M_*}{M_{\mathrm{disc}}}\frac{H}{r} \times \frac{0.05}{0.05}\left(\frac{1.99 \times 10^{30}\,\mathrm{kg}}{\mathrm{M}_\odot}\right)\left(\frac{M_{\mathrm{disc}}}{\pi r^2 \varSigma}\right)\left(\frac{1.496 \times 10^{11}\,\mathrm{m}}{1\,\mathrm{au}}\right)^{-2}$$

$$Q = 1.4 \times 10^6\,\mathrm{kg\,m^{-2}} \times \frac{1}{\varSigma}\left(\frac{H/r}{0.05}\right)\left(\frac{M_*}{\mathrm{M}_\odot}\right)\left(\frac{r}{1\,\mathrm{au}}\right)^{-2}.$$

For the Toomre criterion to be satisfied, we need $Q < 1$, therefore rearranging the equation above:

$$\varSigma > 1.4 \times 10^6\,\mathrm{kg\,m^{-2}}\left(\frac{H/r}{0.05}\right)\left(\frac{M_*}{\mathrm{M}_\odot}\right)\left(\frac{r}{1\,\mathrm{au}}\right)^{-2}.$$

c. Using the result from part (b), for a solar mass star with a disc whose aspect ratio is 0.05, fragmentation occurs at r = 1 au when $\varSigma > 1.4 \times 10^6$ kg m^{-2}. Comparing this to the surface density of the minimum-mass solar nebula at r = 1 au gives

$$\frac{\varSigma}{\varSigma_{\mathrm{sn}}} \approx \frac{1.4 \times 10^6\,\mathrm{kg\,m^{-2}}}{1.7 \times 10^4\,\mathrm{kg\,m^{-2}}} \approx 80 \sim 10^2.$$

Activity 7(c) showed that the minimum-mass solar nebula does *not* meet the Toomre criterion for fragmentation at r = 1 au. In fact, the Toomre criterion would only have been met at distances greater than several thousand astronomical units in the case of the minimum-mass solar nebula. So the Solar System planets probably did *not* form via disc instability.

Computational simulations show that as a disc becomes unstable, due to Q falling below 1, shock waves are generated within the disc. These shock waves follow a spiral pattern and heat up the disc. Since $Q \propto c_{\mathrm{s}} \propto T^{1/2}$, the net effect of the shocks is for Q to increase again so the disc stabilises. This effect is known as **self-regulation** and because of this the disc temperature and surface density tend to reach values for which $Q \sim 1$. Therefore, an additional condition is necessary for fragmentation: the cooling needs to be fast enough to prevent self-regulation. This is known as the **cooling criterion** and it is satisfied if the **cooling time** obeys:

$$\tau_{\text{cool}} \lesssim \frac{1}{3\omega_{\text{K}}}. \qquad \text{(Equation 24)}$$

The fact that both the conditions in Equation 22 and Equation 24 need to be satisfied for fragmentation effectively limits the mass and semimajor axis of the planets that can form via the disc-instability scenario, as shown in Figure 9.

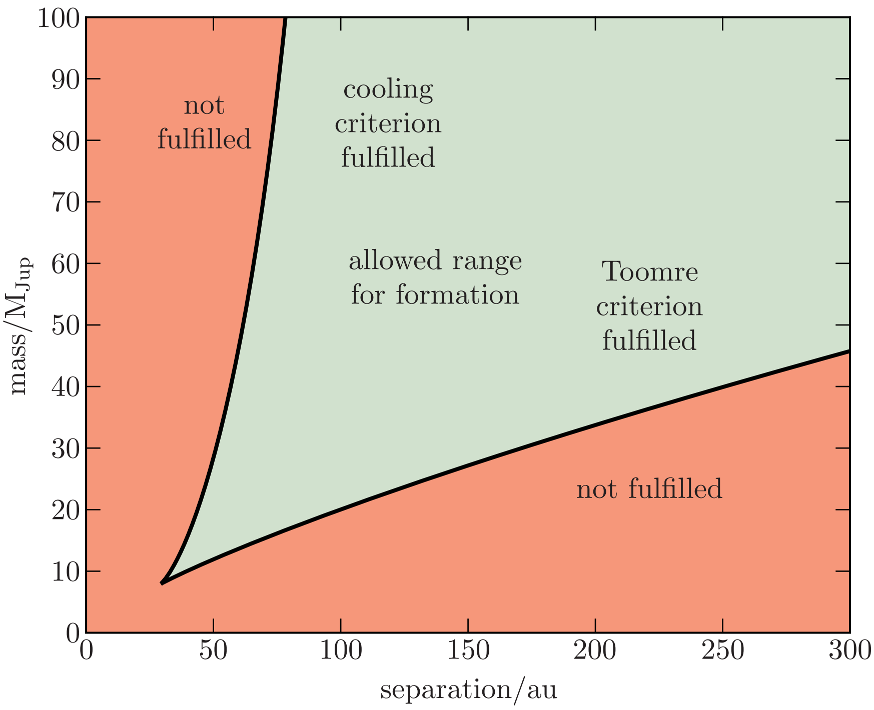


Figure 9 Mass and separation (semimajor axis) of possible planets forming around a solar-type star satisfying both the cooling and Toomre criteria, for a typical disc aspect ratio.

3.4 The Jeans mass for fragmentation

The typical mass of a planet formed through fragmentation can be estimated starting from the definition of its **Jeans mass**. The Jeans criterion states that a gas cloud will collapse if the cloud's kinetic energy is less than the magnitude of its gravitational energy; the minimum mass of a gas cloud for which the Jeans criterion is met is known as the Jeans mass. For the geometry of a protoplanetary disc, the Jeans mass in terms of the surface density is

$$M_{\text{Jeans}} = \frac{1}{\Sigma}\left(\frac{2k_{\text{B}}T}{G\bar{m}}\right)^2,$$

where T and $\bar{m}$ are the temperature and mean molecular mass of the gas. Noting that the sound speed is given by $c_{\text{s}}^2 = k_{\text{B}}T/\bar{m}$ (Equation 3), so the Jeans mass may be written as

$$M_{\text{Jeans}} = \frac{4c_{\text{s}}^4}{G^2\,\Sigma}.$$

Using Q = 1 to express the surface density Σ for a disc that is just becoming unstable (Equation 22) this becomes:

$$M_{\text{Jeans}} = \frac{4c_s^4}{G^2}\frac{\pi G}{c_s \omega_K} = \frac{4\pi c_s^3}{G\omega_K}.$$

Then, recognising that $H = c_s/\omega_K$ (Equation 6) and $\omega_K^2 = GM_*/r^3$ (Equation 2), we have

$$M_{\text{Jeans}} = \frac{4\pi}{G\omega_K} \times (H\omega_K)^3$$

$$M_{\text{Jeans}} = \frac{4\pi}{G} \times H^3 \times \frac{GM_*}{r^3}.$$

Therefore, this gives the final result:

$$M_{\text{Jeans}} = 4\pi M_* \left(\frac{H}{r}\right)^3. \qquad \text{(Equation 25)}$$

- Estimate the Jeans mass for a typical disc with *H*/*r* = 0.05, centred on a Sun-like star.
- Inserting values into Equation 25:

  $$M_{\text{Jeans}} = 4\pi \times 1.99 \times 10^{30}\,\text{kg} \times 0.05^3$$

  $$M_{\text{Jeans}} = 3.1 \times 10^{27}\,\text{kg}.$$

  (This is about 1.6 Jupiter masses.)

3.5 Migration and planet interaction

Regardless of the mechanism involved, once planets have managed to form, they tend to interact with each other and with the remaining gas and planetesimals in the disc. Therefore, planets often end up in a different mass and orbital configuration than the one they had at formation. There are several ways this can happen.

Interaction with remaining gas in the disc

The angular momentum exchange between a planet and the remaining gas in the disc causes the planet to migrate. **Migration** affects both Earth-sized rocky planets and giant planets, and is one possible mechanism for the formation of hot Jupiters like 51 Pegasi b, which almost certainly formed much further out and migrated inwards.

Interaction with the remaining planetesimals

Giant planets can interact with the leftover planetesimals in the disc. The resulting exchange in angular momentum can cause the planetesimals to be ejected from the system.

Planet–planet interaction

There is no guarantee that newly formed planets will be on stable orbits. Instabilities can cause the planets' orbits to cross, and the net effect of this is usually the ejection of the smaller-mass body involved in the interaction, leaving the surviving planet on a highly eccentric orbit. This effect could explain the high eccentricities seen in many exoplanetary systems.

Interaction with additional stellar companions

If a distant stellar companion is present and a planet is formed on an orbit that is misaligned with that of the binary companion star, the planet's eccentricity will change due to the **Kozai-Lidov effect**. This is a dynamical phenomenon that affects systems where two bodies are orbiting each other in the presence of a third, more distant companion. The presence of the third body causes the position of the inner pair's orbital periapsis to oscillate, leading to periodic exchanges between the planet's orbital eccentricity and inclination on a timescale of many orbital periods. This is another possible explanation for the formation of hot Jupiters.

3.6 Comparing theory and observation

Figure 10 shows a snapshot of the exoplanet population (as of early 2023). While being a result of the observational biases connected to the various detection techniques, the figure highlights some interesting trends, including the existence of types of planet that are not present in our Solar System. While core accretion successfully explains the bulk of the giant planet population discovered through transit, radial-velocity and microlensing techniques (orange diamonds, red squares and green triangles, respectively, in Figure 10), and disc instability may explain some of the planets discovered by direct imaging (blue triangles in Figure 10), neither scenario is able to easily explain the characteristics of *all* planets shown here.

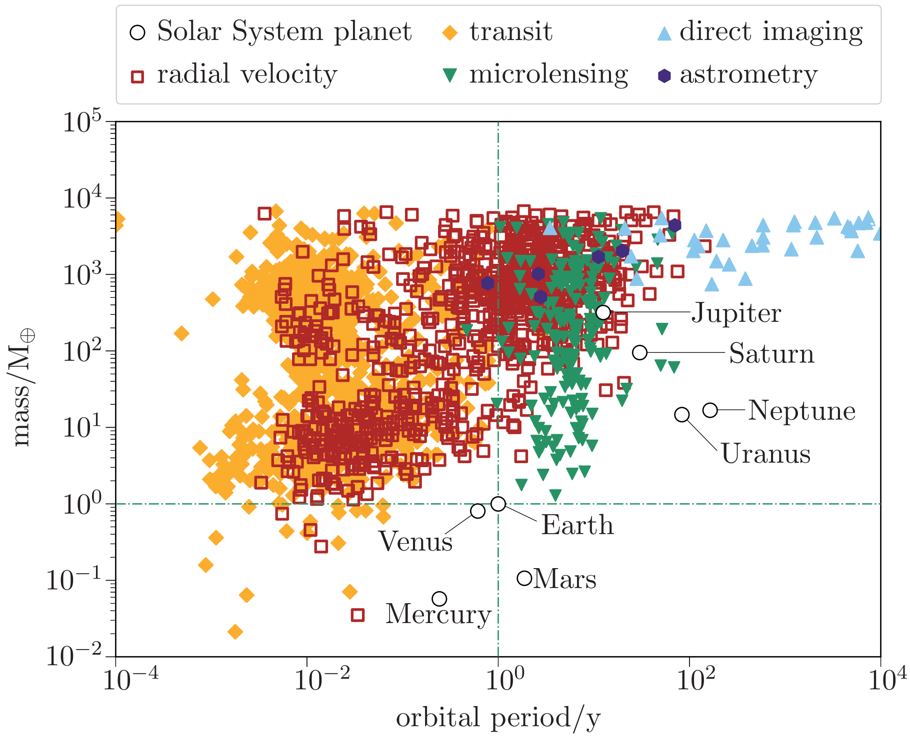


Figure 10 Masses of the known exoplanets (in units of Earth mass) plotted against their orbital period in years. The different colours and shapes represent the different detection methods. Solar System planets are plotted as open circles. The dashed-dotted line marks the position of the Earth.

In particular, the core-accretion scenario struggles to explain the formation of giant planets at orbital distances larger than a few astronomical units (corresponding to orbital periods longer than a few years), because of the extended time needed to form big enough cores at these distances. On the other hand, discs are unlikely to fragment at small orbital distances from the central star, because the stellar irradiation tends to stabilise the disc by maintaining high temperature (hence high sound speed and high Q), so the Toomre and cooling criteria cannot be satisfied simultaneously. Therefore, unless planets formed by the core-accretion scenario can migrate or be scattered outward to large distances, then the directly imaged giant planets at large orbital distances must have formed on their current orbits via another mechanism, such as the disc-instability scenario. This suggests that giant-planet formation could be bimodal, with different mechanisms dominating depending on the distance from the central star.

Activity 8

A current catalogue of known exoplanets is maintained at the website exoplanet.eu. Visit the website now, and you will see two buttons labelled ‘The catalog’ and ‘The plots’. The first of these allows you to explore the current catalogue of known exoplanets, while the second allows you to plot various planet parameters against each other. Try to produce an updated version of Figure 10 by plotting the masses of known exoplanets against their orbital periods. You can click on the axis labels to change the units in which the quantities are displayed.

4 Quiz

Answer the following questions in order to test your understanding of the key ideas that you have been learning about.

Question 1

Consider four protoplanetary discs, with the same temperature, around stars of similar mass. One is composed entirely of molecular hydrogen, one is a mixture of molecular hydrogen and helium, one is composed entirely of helium, and one is a mixture of hydrogen, helium and other heavier elements giving a mean molecular mass of 2.3*u* (where *u* is the atomic mass unit, 1.66×10^{-27} kg). Which disc will have the *largest* scale height at a given radius?

- The disc composed of molecular hydrogen only.
- The disc composed of helium only.
- The disc composed of molecular hydrogen and helium.
- The disc composed of molecular hydrogen, helium and heavier elements.
- All four discs will have the same scale height.

Answer

The scale height is given by $H = c_\mathrm{s}/\omega_\mathrm{K}$. The Keplerian angular speed at a given radius will be the same for all four discs since it depends only on the stellar mass and the radius, which are the same in all four cases.

The sound speed is inversely proportional to the mean molecular mass of the gas in the disc. For the disc composed entirely of molecular hydrogen, $\bar{m} = 2u$ and for the disc composed entirely of helium, $\bar{m} = 4u$. The disc composed of a mixture of molecular hydrogen and helium will have a mean molecular mass that is somewhere between 2*u* and 4*u*, and as noted in the question, the disc composed of molecular hydrogen, helium and other heavier elements has a mean molecular mass of 2.3*u*.

The largest scale height will be for the disc with the largest sound speed, and this will be for the disc with the smallest mean molecular mass. Therefore the protoplanetary disc composed entirely of molecular hydrogen will have the largest scale height at a given radius. (Conversely, the disc composed entirely of helium will have the smallest scale height.)

Question 2

Consider the same four protoplanetary discs as in Question 1. Which disc will have the *smallest* difference between the orbital and Keplerian speeds at a given radius?

- The disc composed of molecular hydrogen only.
- The disc composed of helium only.
- The disc composed of molecular hydrogen and helium.
- The disc composed of molecular hydrogen, helium and heavier elements.
- All four discs will have the same difference in speeds.

Answer

The difference between the orbital and Keplerian speeds at a given radius is given by

$$\Delta v(r) = \left[1 - \sqrt{1 - n\left(\frac{H}{r}\right)^2}\right] v_{\mathrm{K}}(r)$$

As noted in the answer to Question 1, the Keplerian angular speed at a given radius will be the same for all four discs since it depends only on the stellar mass and the radius, which are the same in all four cases. So the difference in speeds will be smallest when the term under the square root is largest. This term will be largest when H/r is smallest. From the information in Question 1, this will be for the disc composed entirely of helium.

Question 3

Consider again the same four protoplanetary discs as in Question 1. Which disc will have the *largest* radial drift speed at a given radius for particles with a Stokes parameter of $\tau_S = 1$?

- ○ The disc composed of molecular hydrogen only.
- ○ The disc composed of helium only.
- ○ The disc composed of molecular hydrogen and helium.
- ○ The disc composed of molecular hydrogen, helium and heavier elements.
- ○ All four discs will have the same radial drift speed.

Answer

For particles with a Stokes parameter of $\tau_S = 1$, the radial drift speed from Equation 16 is $v_{\mathrm{rad}} = -v_{\mathrm{K}}\eta/2$. As noted in the answer to Question 1, the Keplerian angular speed at a given radius will be the same for all four discs since it depends only on the stellar mass and the radius, which are the same in all four cases. So the radial drift speed will be largest for the disc with the largest value of η. Since this is given by $\eta = n(H/r)^2$, and the disc aspect ratio is largest for the disc with the largest scale height, this corresponds to the disc composed entirely of hydrogen, as revealed in Question 1.

Question 4

Which of the following statements about the isolation mass involved in the growth of planetesimals are *true*?

- □ The isolation mass increases as the surface density of the protoplanetary disc increases, for a given star at a given orbital radius.
- □ The isolation mass increases with orbital distance from the central star, for a given star and a given disc surface density.
- □ The isolation mass decreases as the mass of the star increases, for a given orbital distance and a given disc surface density.

- ☐ For the situation described in Activity 5, the isolation mass at 0.1 au is 3.94×10^{20} kg.
- ☐ For the situation described in Activity 5, an isolation mass of 3.94×10^{26} kg corresponds to a distance of 10 au.
- ☐ The isolation mass can never be larger than the mass of the Earth.

Answer

The isolation mass is given by Equation 21:

$$M_{\mathrm{iso}} = \frac{8}{\sqrt{3}} (\pi \Sigma C)^{3/2} \frac{a^3}{M_*^{1/2}}.$$

Therefore the first three statements are true, since $M_{\mathrm{iso}} \propto \Sigma$, $M_{\mathrm{iso}} \propto a^3$ and $M_{\mathrm{iso}} \propto 1/M_*^{1/2}$. Furthermore, since the isolation mass in Activity 5 at 1.0 au is 3.94×10^{23} kg, the corresponding masses at distances 10× smaller and 10× larger are 1000× smaller and 1000× larger respectively, therefore the next two statements are also true. Hence all statements are true except the last one.

Question 5

Match the following core-accretion scenarios to the type of planet that results.

- Core formation by solid accretion followed by gas accretion beyond the critical mass.
- Core formation by solid accretion followed by slow core accretion.
- Core formation by solid accretion followed by growth in the region of the disc with little solids.

Match each of the items above to an item below.

- Gas giant planet
- Ice giant planet
- Terrestrial planet

Answer

See Figure 6 for details.

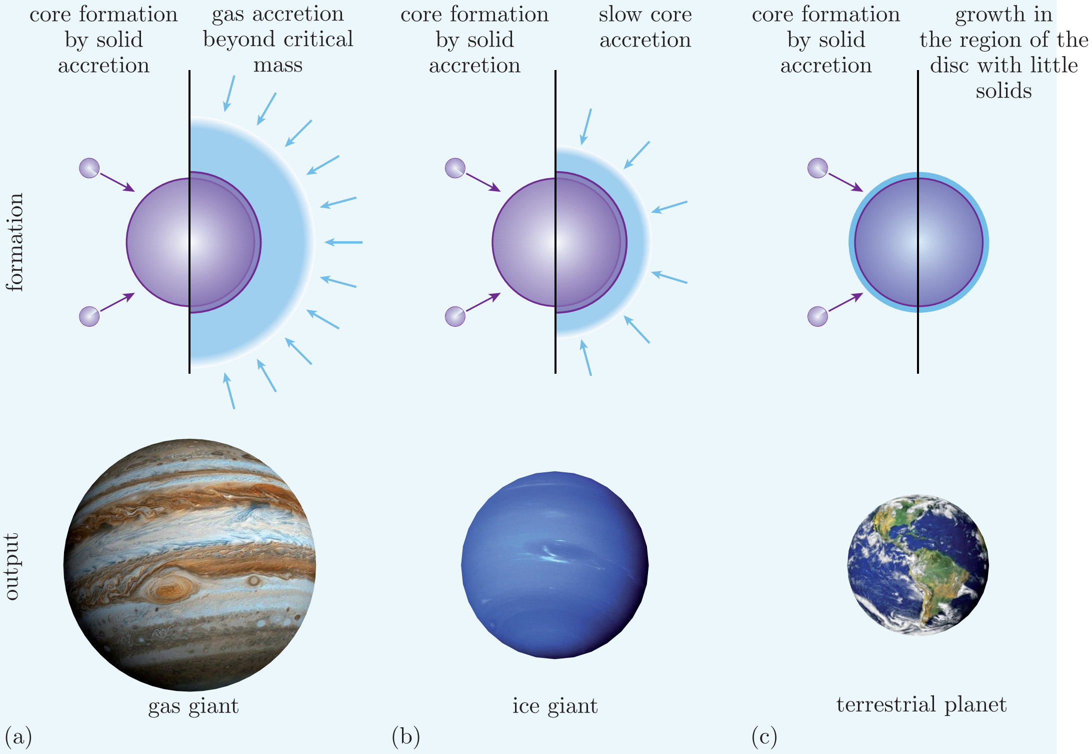


Figure 6 (repeated) Schematic view of possible outcomes of the core-accretion model.

Question 6

A protoplanetary disc around a star of mass M_* = 0.75 $M_\odot$ has a surface density of Σ = 4700 kg m^{-2} at a radius of r = 3.3 au. If the disc aspect ratio is H/r = 0.065, determine whether the disc satisfies the Toomre criterion for fragmentation.

- ○ The Toomre parameter is less than 1 and so the disc does meet the Toomre criterion.
- ○ The Toomre parameter is greater than 1 and so the disc does not meet the Toomre criterion.

Answer

The Toomre criterion for fragmentation is

$$Q = \frac{\omega_K c_s}{\pi G \Sigma} < 1.$$

The sound speed may be written $c_s = H\,\omega_K$, where H is the scale height, so this becomes

$$Q = \frac{\omega_K^2 H}{\pi G \Sigma} < 1.$$

Then we note that the Keplerian angular speed is $\omega_K = (GM_*/r^3)^{1/2}$ so this now becomes

$$Q = \frac{GM_*}{r^3}\frac{H}{\pi G\Sigma} = \frac{M_*}{\pi r^2 \Sigma}\frac{H}{r} < 1.$$

So, calculating in this case

$$Q = \frac{0.75 \times 1.99 \times 10^{30}\ \mathrm{kg}}{\pi \times (3.3 \times 1.496 \times 10^{11}\ \mathrm{m})^2 \times 4700\ \mathrm{kg\,m^{-2}}} \times 0.065$$

$$Q = 27\ (2\ \mathrm{s.f.})$$

Since this is greater than 1, the Toomre condition is *not* satisfied and the disc will *not* fragment.

Question 7

In a protoplanetary disc around a star of mass $M_* = 0.45\ M_\odot$, what is the minimum value of the disc aspect ratio H/r to ensure that the Jeans mass exceeds the mass of Jupiter?

- ○ $H/r > 0.0055$
- ○ $H/r > 0.055$
- ○ $H/r > 0.55$
- ○ $H/r > 5.5$
- ○ $H/r > 55$

Answer

The Jeans mass is given by Equation 25 as

$$M_{\mathrm{Jeans}} = 4\pi M_* \left(\frac{H}{r}\right)^3$$

So, if the Jeans mass exceeds the mass of Jupiter, we have

$$4\pi M_* \left(\frac{H}{r}\right)^3 > M_{\mathrm{Jup}}$$

$$H/r > \left(\frac{M_{\mathrm{Jup}}}{4\pi M_*}\right)^{1/3}$$

In this case

$$H/r > \left(\frac{1.90 \times 10^{27}\mathrm{kg}}{4\pi \times 0.45 \times 1.99 \times 10^{30}\mathrm{kg}}\right)^{1/3}$$

So the disc aspect ratio must be greater than 0.055 (2 s.f.).

Question 8

The formation of which of the following types of planets *cannot* be explained by the core-accretion scenario?

- ○ Hot Jupiter planets at very small orbital distances.
- ○ Giant planets at orbital distances larger than a few au.
- ○ Terrestrial planets in Earth-like orbits around their stars.
- ○ Mini-Neptune planets at orbital periods of a few months.
- ○ Super-Earth sized planets at orbital periods of less than a year.

Answer

The core-accretion scenario can explain the formation of most types of exoplanets. However, it struggles to explain the formation of giant planets at orbital distances larger than a few astronomical units (corresponding to orbital periods longer than a few years), because of the extended time needed to form big enough cores at these distances. These planets probably formed by the disc-instability scenario.

5 Conclusion

The focus of this course has been on how planets form around stars from the material in protoplanetary discs. These were some of the key learning points:

1. **Protoplanetary discs** comprised of gas and solid material are believed to be the birthplaces of planets. In **hydrostatic equilibrium**, the density profile $\rho_{gas}(z)$ of the gas in a disc as a function of vertical height z can be expressed as:

   $$\rho_{\text{gas}}(z) = \rho_0 \exp\left(-\frac{z^2}{2H^2}\right). \qquad \text{(Equation 5)}$$

   Here, $H = c_\text{s}/\omega_\text{K}$ (Equation 6) is the **disc scale height**, ρ_0 is the density at the midplane (z = 0), $c_\text{s} = (P_\text{gas}/\rho_\text{gas})^{1/2}$ (Equation 3) is the **sound speed** in the gas and $\omega_\text{K} = (GM_*/r^3)^{1/2}$ (Equation 2) is the **Keplerian angular speed** for an orbit at a distance r from a star of mass M_*.
2. In the radial direction, in addition to the gravitational force, there is also a force due to the pressure gradient of the gas $\text{d}P_{gas}/\text{d}r$. Therefore, the orbital speed $v_{orb}(r)$ of the gas in the disc has two components: one due to the **Keplerian speed**, $v_\text{K}(r) = (GM_*/r)^{1/2}$ (Equation 1), and one due to this extra pressure gradient, given by

   $$v_{\text{orb}}^2(r) = \frac{GM_*}{r} + \frac{r}{\rho_{\text{gas}}(r)} \frac{\text{d}P_{\text{gas}}(r)}{\text{d}r}. \qquad \text{(Equation 9)}$$

   Usually, $\text{d}P_{gas}/\text{d}r < 0$, so the orbital speed is sub-Keplerian, $v_{orb}(r) < v_K(r)$. The difference between the Keplerian speed and the orbital speed is $\Delta v = v_\text{K} - v_\text{orb}$ and is typically ~ 100 m s^{-1} at 1 au from a 1 $M_\odot$ star.
3. The **core-accretion scenario** predicts that planets form by accumulation of initially sub-micron-sized dust grains to form metre-sized rocks, then kilometre-sized **planetesimals** and Mercury-sized **planetary embryos**, and eventually **planetary cores** up to several times the size of the Earth.
4. The relation between the orbital speed and Keplerian speed of particles in a protoplanetary disc can be expressed as $v_\text{orb} = v_\text{K}(1-\eta)^{1/2}$ (Equation 15) where $\eta = n(H/r)^2$ with n a numerical constant. Particles in the disc experience a radial drift inwards with a speed:

   $$v_\text{rad} = -v_\text{K} \frac{\eta}{\tau_\text{S} + \tau_\text{S}^{-1}},$$

   where $\tau_\text{S} = \tau_\text{stop}\omega_\text{K}$ (Equation 12) is called the **Stokes number**. The Stokes number is related to the **stopping time**

   $$\tau_\text{stop} = \frac{\rho_\text{m}}{\rho_\text{gas}} \frac{s}{c_\text{s}} \qquad \text{(Equation 14)}$$

where ρ_m is the material density of the particles and *s* is their radius. The maximum radial drift speed occurs when $\tau_S = 1$ which corresponds to roughly metre-sized rocks. In this case, $v_{\rm rad}(\max) = -\eta v_{\rm K}/2 \approx -\Delta v$.

5. Once planetesimals have formed, their mass M_p grows through collisions with other planetesimals at a rate:

$$\frac{\mathrm{d}M_{\rm p}}{\mathrm{d}t} = \pi R_{\rm p}^2 \omega_{\rm K} \Sigma \left(1 + \frac{v_{\rm esc}^2}{v_{\rm rel}^2}\right) = \pi R_{\rm p}^2 \omega_{\rm K} \Sigma F_{\rm g}, \quad \text{(Equation 17)}$$

where R_p is the planetesimal's radius, v_{esc} is its **escape velocity**, v_{rel} is the relative velocity between the two impacting bodies, Σ is the surface density of the disc and F_g is the **gravitational focusing**.

6. Planetary embryos continue growing into planetary cores by accreting leftover planetesimals within a **feeding zone** that extends a distance Δa either side of the core, such that $\Delta a = CR_{\rm Hill}$. Here, *C* is a constant and R_{Hill} is the **Hill radius** that is

defined as the distance from the planetary core at which its gravitational force dominates over the gravitational force of the star of mass M_*, which it orbits at a distance *a*:

$$R_{\rm Hill} = \left(\frac{M_{\rm p}}{3M_*}\right)^{1/3} a. \quad \text{(Equation 20)}$$

7. The total mass of material within the feeding zone is called the **isolation mass** and represents the final mass of the planetary core:

$$M_{\rm iso} = \frac{8}{\sqrt{3}} (\pi \Sigma C)^{3/2} \frac{a^3}{M_*^{1/2}}. \quad \text{(Equation 21)}$$

8. Once the mass of the core reaches a few Earth masses, it starts to build up a gas envelope. This can lead to the formation of gas giant planets, ice giant planets or terrestrial planets, depending on the amount of gas accreted by the time the **critical mass** for hydrostatic equilibrium is reached. Many observed protoplanetary discs show gaps, bright rings, asymmetries, spirals and other structures where planets are forming within them.
9. The **disc-instability scenario** provides an alternative way to form gas giants. In this model, a cold and/or massive disc fragments into clumps due to gravitational instabilities, and these clumps eventually evolve into gas giants. Two conditions need to be satisfied for disc **fragmentation**: the **Toomre criterion**:

$$Q = \frac{\omega_{\rm K} c_{\rm s}}{\pi G \Sigma} < 1, \quad \text{(Equation 22)}$$

where c_s is the speed of sound, ω_K is the Keplerian angular speed and Σ is the gas surface density; and the **cooling criterion**:

$$\tau_{\rm cool} \lesssim \frac{1}{3\omega_{\rm K}}. \quad \text{(Equation 24)}$$

The fact that both conditions need to be satisfied for fragmentation effectively limits the mass and semimajor axis values of the planets forming via the disc-instability scenario.

10. The typical mass of a planet formed via fragmentation can be estimated from the **Jeans mass**, which may be expressed as

$$M_{\rm Jeans} = 4\pi M_* \left(\frac{H}{r}\right)^3. \qquad \text{(Equation 25)}$$

    The Jeans mass is of order 1–2 times the mass of Jupiter for typical discs.
11. Once formed, the planets interact with the disc and with each other, undergoing **migration** in some cases, until the system reaches its final configuration. Factors influencing the final composition and orbital configuration of planets can include interactions with the remaining gas in the disc, interactions with remaining planetesimals, planet–planet interactions and interactions with additional stellar companions.
12. Neither the core-accretion nor disc-instability scenarios can explain all of the observed exoplanet population. Therefore, it is plausible that both scenarios play a role in planet formation, where different mechanisms are at play at different distances from the parent star.

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Glossary

angular momentum

The momentum associated with the rotational motion of a body.

aspect ratio

The ratio of the height H to the radius r for a two-dimensional structure such as a **protoplanetary disc** or an accretion disc. Typically $H/r = c_s/v_K$ where c_s is the **sound speed** and v_K is the **Keplerian speed**.

coagulation

The process by which small (micron-sized) particles in a **protoplanetary disc** collide with each other gently enough that they stick together to form millimetre-sized aggregates.

cooling criterion

The condition necessary for a **protoplanetary disc** to undergo **self-regulation** when forming planets via the **disc-instability scenario**. It is satisfied if the **cooling time** obeys $\tau_{\rm cool} \lesssim 1/(3\omega_K)$ where ω_K is the **Keplerian angular speed**.

cooling time

The characteristic timescale for a system to reduce its temperature to some previous level.

core-accretion scenario

A model for planet formation in which planets form by accumulation of solids into a core, on which an atmosphere is accreted once a critical value of the core mass is achieved. Initially, micron-sized dust grains in a **protoplanetary disc** coagulate to form metre-sized rocks, then kilometre-sized **planetesimals**, Mercury-sized **planetary embryos** and eventually **planetary cores**. Contrast with **disc-instability scenario**.

critical mass

In relation to planet formation, the limiting mass of a **planetary core** above which the gas surrounding it cannot maintain **hydrostatic equilibrium** and starts contracting. Exceeding the critical mass triggers a phase of rapid accretion onto the core until the gas in the **protoplanetary disc** is dispersed.

disc-instability scenario

A model for planet formation in which planets form directly from gravitational instabilities within a **protoplanetary disc**. It may be responsible for the formation of massive planets that lie at large distances from their star. Contrast with **core-accretion scenario**.

disc scale height

The scale height of an accretion disc or **protoplanetary disc**. It is generally given by $H = \dfrac{c_s}{\omega_K}$ where c_s is the **sound speed** and ω_K is the **Keplerian angular speed**.

escape velocity

A quantity that gives the minimum speed required for an object to escape the gravitational influence of a massive body. In Newtonian gravity, the magnitude of the escape velocity is given by $v_{\rm esc} = (2GM/r)^{1/2}$ where G is the universal gravitational constant, M is the mass of the gravitating body and r is the initial distance from its centre.

exoplanet

A planet orbiting a star other than the Sun. According to the International Astronomical Union (IAU), an exoplanet has a mass that is below the limiting mass for nuclear fusion of deuterium (currently calculated to be 13 times the mass of Jupiter for objects with the same isotopic abundance as the Sun) and orbits a star or stellar remnant. This definition takes no account of how the object formed, so it is possible that the definition may include objects that would otherwise be classified as brown dwarfs.

feeding zone

The distance Δa either side of the core from within which further **planetesimals** are accreted during the growth of **planetary cores** in a **protoplanetary disc**. Typically $\Delta a = CR_{\mathrm{Hill}}$ where C is a small constant and R_{Hill} is the **Hill radius**.

fragmentation

The process by which a contracting interstellar cloud breaks up into a number of separate cloudlets as energy is radiated from the cloud and the **Jeans mass** decreases.

gravitational focusing

A dimensionless parameter that describes how the gravitational attraction between two bodies increases their collision probability. It is expressed as $F_{\mathrm{g}} = 1 + \frac{v_{\mathrm{esc}}^2}{v_{\mathrm{rel}}^2}$ where v_{esc} is the **escape velocity** and v_{rel} is the relative velocity between the two impacting bodies.

Hill radius

The radius of the Hill sphere defined by $R_{\mathrm{Hill}} = a\left(\frac{M_{\mathrm{p}}}{3M_*}\right)^{1/3}$ where a is the semimajor axis of the planet's orbit around a star, M_{p} is the mass of the planet and M_* is the mass of the star.

hot Jupiter

A giant **exoplanet** in an extremely close orbit around a star.

hydrostatic equilibrium

A situation in which the forces acting on a fluid (normally gravitational forces) are balanced by the internal pressure of the fluid (including thermal, degeneracy and radiation pressure), so that the fluid neither collapses nor expands.

isolation mass

During the growth of a **planetary core**, this is the total mass of **planetesimals** within the feeding zone.

Jeans mass

In a disc geometry (such as a **protoplanetary disc** undergoing planet formation via the **disc-instability scenario**), the Jeans mass is $M_{\mathrm{Jeans}} = \frac{1}{\Sigma}\left(\frac{2k_{\mathrm{B}}T}{G\bar{m}}\right)^2$ where Σ is the surface density of the disc.

Kepler's first law

One of three laws of planetary motion stated by Johannes Kepler. The first law states that planets orbit stars in elliptical orbits with the star at one focus of the ellipse.

Kepler's laws

Three laws summarising the nature of planetary motion.

Kepler’s second law

One of three laws of planetary motion stated by Johannes Kepler. The second law states that a line joining a planet and its star sweeps out equal areas in equal times. The consequence of this is that planets move fastest when they are closest to their star.

Kepler’s third law

One of three laws of planetary motion stated by Johannes Kepler. The third law states that the square of a planet’s orbital period is proportional to the cube of the semimajor axis of its orbit $P_{\mathrm{orb}}^2 \propto a^3$. More generally: $\dfrac{a^3}{P_{\mathrm{orb}}^2} = \dfrac{GM}{4\pi^2}$ where M is the total mass of the star and planet.

Keplerian

A term used to denote quantities that relate to properties of a (circular) **Keplerian orbit**, e.g. **Keplerian speed**, **Keplerian angular speed**.

Keplerian angular speed

The angular speed of a body in a **Keplerian orbit**, i.e. $\omega_{\mathrm{K}} = (GM/R^3)^{1/2}$ where M is the mass of the central body and R is the orbital radius.

Keplerian orbit

The orbit a point mass executes if it is subject only to the gravitational force from another point-like mass. Quite often this term is used in a stricter sense to denote a circular orbit with constant angular speed that obeys **Kepler’s third law**.

Keplerian orbital speed

The tangential speed of a body in a **Keplerian orbit**, i.e. $v_{\mathrm{K}} = (GM/R)^{1/2}$ where M is the mass of the central body and R is the orbital radius.

Keplerian speed

The tangential speed of a body in a **Keplerian orbit**, i.e. $v_{\mathrm{K}} = (GM/R)^{1/2}$where M is the mass of the central body and R is the orbital radius. Contrast with **Keplerian angular speed**.

Kozai-Lidov effect

Synchronised changes in the eccentricity and inclination of an orbit such that one increases while the other decreases, in a cyclic manner, caused by the presence of a third, more distant companion.

migration

The process by which **protoplanets** move away from their place of formation in a **protoplanetary disc**.

minimum-mass solar nebula

A hypothetical **protoplanetary disc** with a surface density profile defined as the minimum value of the surface density that a **protoplanetary disc** would need to have to form our Solar System.

molecular cloud

A cloud of dense cold gas containing molecules, principally molecular hydrogen ($\mathrm{H_2}$), together with dust. Molecular clouds are generally detected through emission lines of molecular species at radio frequencies; important species include CO, OH and CN.

Because molecular clouds are cold and dense, they are important sites for star formation.

oligarchic growth

In planetary formation, this describes the situation where the largest **planetary embryos** grow quickly while the smallest grow slowly.

planetary core

A solid body resulting from a **planetary embryo** that will accumulate further material to form the core of a planet.

planetary embryo

An object that will likely grow into a planet. Planetary embryos comprise roughly Mercury-sized bodies formed from **planetesimals** and may grow into **planetary cores**.

planetesimal

Solid, roughly kilometre-sized bodies that are intermediate in size between rocks and **planetary embryos** during the growth of planets in **protoplanetary discs**.

protoplanet

A planet growing by a process of accretion in the **protoplanetary disc** of a young star or protostar. Small inhomogeneities in the disc are thought to lead to the growth of protoplanets.

protoplanetary disc

A protoplanetary disc consists of cold gas and dust, and is left over from the material that formed the central protostar. Small inhomogeneities in the disc are thought to lead to the growth of **protoplanets**. Radiation pressure and the solar wind compete against the gravity of the protoplanets and eventually drive off the remaining material of the protoplanetary disc.

radial drift speed

The speed with which particles in a disc move radially through it. It depends on the Stokes number τ_S typically according to $v_\mathrm{rad} = -v_\mathrm{K} \dfrac{\eta}{\tau_\mathrm{S} + \tau_\mathrm{S}^{-1}}$ where v_K is the Keplerian speed and $\eta = n(H/r)^2$ where n is a dimensionless constant and H/r is the aspect ratio of the disc.

runaway growth

An accelerated phase in the growth of **planetesimals**.

self-regulation

In relation to the **disc-instability scenario** for planet formation, the situation where, as a **protoplanetary disc** becomes unstable (due to the **Toomre *Q* parameter** falling below 1), shock waves are generated in the disc. These heat up the disc, so increasing