S385_1

Unsolved problems in cosmology

Contents

Introduction

Welcome to this free course, *Unsolved problems in cosmology*. If you have read any other books or articles about cosmology, or seen TV programmes on the subject, you will probably be aware that cosmological theory has been very successful in explaining a wide range of observational evidence about the Universe. This includes phenomena such as the detailed properties of the cosmic microwave background (CMB) radiation, the distributions of galaxies, and the abundances of the light elements in astrophysical environments. However, you may have also become aware that there are several major unknowns in cosmological theory. These knowledge gaps are frustrating for cosmologists, students, and members of the public who would like to understand the nature of the Universe.

This course will discuss the three most crucial unresolved problems in modern cosmology:

- the nature of **dark matter**, which is suggested to explain the motion of stars in the outer regions of galaxies and the motion of galaxies in clusters
- the nature of **dark energy**, which is postulated as an explanation for the present-day acceleration of the expansion of the Universe
- the need for an early period of **inflation**, soon after the big bang.

You will explore these ideas by examining a set of articles and videos. In the case of the articles, you only need to read at a depth that allows you to follow the arguments being presented. There is no need, for example, to undertake secondary reading of any sources that are cited within the article (unless you are particularly interested in reading more about the subject in question).

There are exercises associated with the articles and videos, with questions to help guide you through the main ideas presented. You should be able to answer each exercise in a few sentences or, at most, a short paragraph. Discussions are provided to accompany the exercises, but you should try to note down your own answers before accessing these. There is also a short quiz at the end of the course.

This OpenLearn course is an adapted extract from the Open University course S385 *Cosmology and the distant Universe*.

Learning outcomes

After studying this course, you should be able to:

- understand the basis of modern cosmology based on the hot big bang
- summarise the main candidate dark matter particles, and the prospects for directly detecting them
- compare models to explain the late-time acceleration of the Universe's expansion, i.e. dark energy and the cosmological constant
- discuss observational prospects for understanding the nature of dark energy
- explain the theoretical problems that led to the theory of inflation, how an early inflationary period solves them and how researchers are investigating inflation.

1 Cosmology today

Modern cosmology is based on the understanding that the Universe began in an event called the hot **big bang**, in which time and space were created, about 13.8 billion years ago. The Universe has cooled and space has expanded as time has progressed since then. The evidence for this is chiefly based on three sets of observations, which will be discussed in the following sections. A schematic timeline for the main events in the history of the Universe is shown in Figure 1.

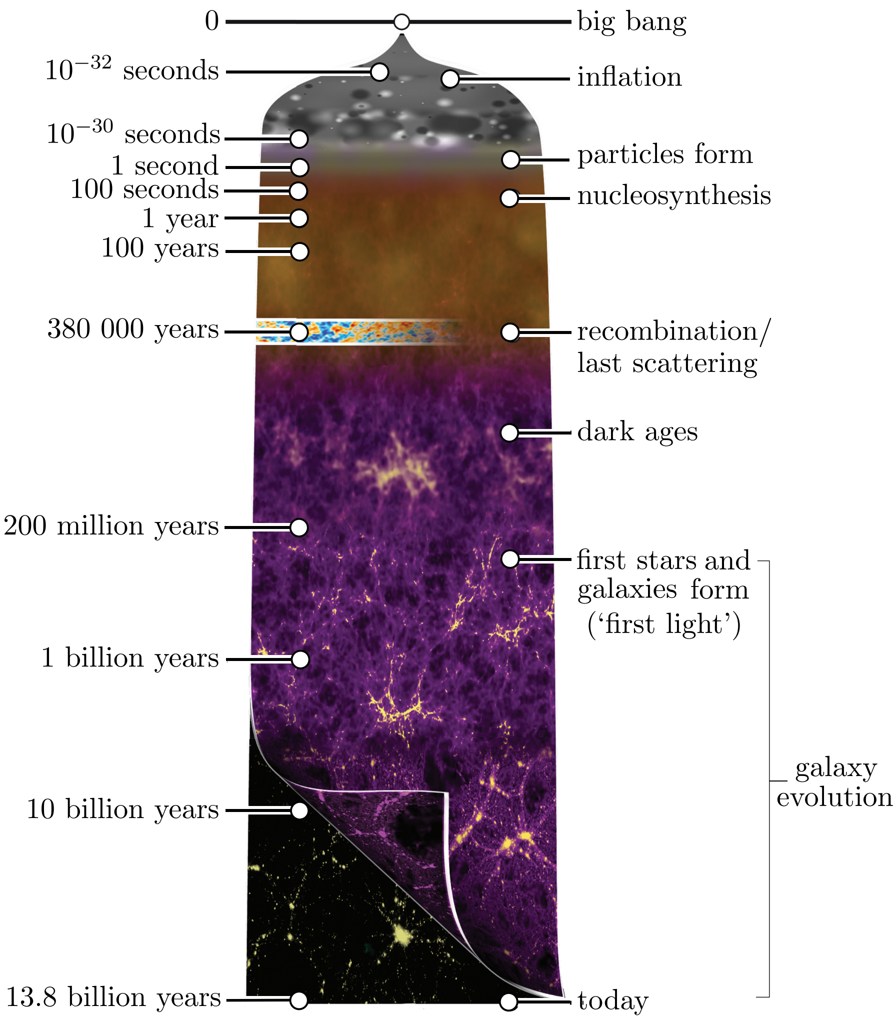

Figure 1 A simple timeline of the main stages in the history of the Universe

1.1 The expanding Universe

The first piece of evidence for the hot big bang was the discovery in 1927 by Edwin Hubble and (independently) by Georges Lemaître that the further away galaxies are, the faster they appear to be receding from us. The apparent speed of recession of a galaxy can be measured by observing the shift to longer wavelengths (i.e. the **redshift**, represented by the dimensionless quantity z) of absorption or emission lines in the object's spectrum. The redshift is calculated as the shift in wavelength of a given spectral line divided by the laboratory (or rest) wavelength of that line, and the apparent recession speed is then equal to the redshift multiplied by the speed of light. The distance to a galaxy can be measured by any of a number of methods that rely on comparing an object's observed brightness to its known luminosity.

This discovery by Hubble and Lemaître is understood as implying that space itself is expanding and the cosmological redshift is therefore the result of an expansion of the intervening space between us and distant galaxies: it is *not* the result of those galaxies moving through space, as in a conventional Doppler shift. The result can be expressed by the linear relationship known as the **Hubble-Lemaître law**:

$$z = \frac{H_0 D}{c} \qquad (1)$$

where D is the galaxy distance, typically measured in units of Mpc (where 1 Mpc = 3.1×10^{19} km), c is the speed of light (which is about 300,000 km s^{-1}) and H_0 is a quantity known as the **Hubble constant**, which has a value of 67.7 km s^{-1} Mpc^{-1}. A modern Hubble diagram showing how galaxies' redshifts vary with distance is shown in Figure 2.

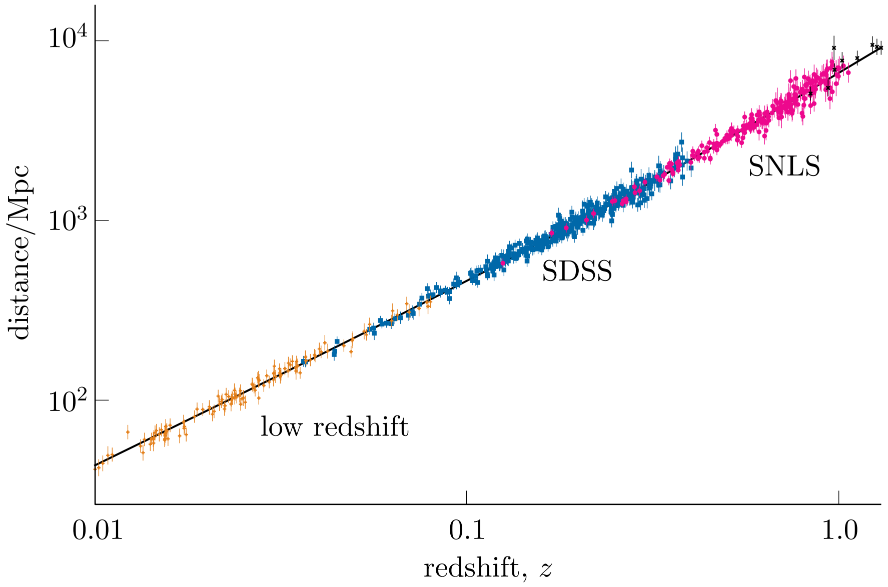

Figure 2 The relationship between distance and redshift from a compilation of supernova measurements made by different surveys

The Hubble constant therefore measures the rate of expansion over a fixed distance: observers at any location at the current time will measure that, over a distance of 1 Mpc, the Universe expands at a rate of 67.7 km s^{-1}. This implies that, for every megaparsec further away galaxies are situated, they appear to recede 67.7 km s^{-1} faster.

Mathematically the Hubble constant can be expressed as

$$H_0 = \frac{\dot{a}}{a} \qquad (2a)$$

where a is referred to as the **scale factor** of the Universe and $\dot{a}$ is its rate of change with time. In fact it is now apparent that the expansion rate of the Universe is not constant in time, so we can refer to the time-varying **Hubble parameter** $H(t)$, which is related to time-varying value of the scale factor by

$$H(t) = \frac{\dot{a}(t)}{a(t)} \tag{2b}$$

For many years it was believed that the expansion rate of the Universe was decelerating (slowing down) but in the 1990s, observations of distant **type Ia supernovae** showed that the expansion rate of the Universe is currently accelerating (speeding up). This is indicated by the fact that the trend-line shown in Figure 2 is *not* a straight line, but bends upwards at large distances and redshifts. Note that, by looking at objects further away in the Universe, we are also looking further back in time. This is because the light from these distant objects was emitted by them when the Universe was much younger than it is now, and it has taken the intervening time for the light to reach us. Therefore an alternative way of describing how far away galaxies are is by referring to their **lookback time**.

Type Ia supernovae occur when a **white dwarf** star accretes enough material from a companion star to exceed the **Chandrasekhar limit** of about 1.4 solar masses and no longer has enough internal pressure to support itself against collapse. The white dwarf explodes in a cataclysmic explosion called a supernova. Because the explosions are so violent, they release a huge amount of energy and can be observed at very large distances away; and because they all result from the explosion of a white dwarf at the same mass, they all emit the same amount of energy or have the same luminosity. Measuring their observed brightness and comparing this to their known luminosity allows the distance to the supernova to be calculated. The redshifts of the supernovae host galaxies can also be measured from their spectra. By observing type Ia supernovae at a range of redshifts, it was discovered that, although the initial expansion rate of the Universe was indeed *decelerating*, at a lookback time of around 6 billion years ago (when galaxies are now observed with a redshift around $z \sim 0.5$) the expansion rate of the Universe instead began *accelerating*.

1.2 The cooling Universe

The second piece of evidence for the hot big bang was the discovery in the 1960s by Arno Penzias and Robert Wilson of the **cosmic microwave background** (CMB) radiation. When observing the sky at microwave wavelengths, a nearly uniform glow can be observed in all directions. The distribution of the radiation corresponds to a blackbody spectrum at a temperature of about 2.7 K (i.e. nearly 3 degrees above absolute zero) and it represents the fading glow of the heat of the big bang. As the Universe expanded and cooled, about 380,000 years after the big bang (known as the **time of last scattering**), electrons were able to combine with protons for the first time, forming hydrogen atoms. This so-called **recombination** event happened when the temperature of the Universe was around 3000 K. As photons did not subsequently interact with these electrically neutral atoms, they began to travel freely through space, resulting in the decoupling of matter and radiation. It is this radiation that is observed today as the CMB, redshifted by a factor of about 1100 from the infrared into the microwave part of the spectrum.

Although the CMB is nearly uniform across the whole sky, tiny fluctuations in its temperature and intensity were discovered in the 1990s – see Figure 3. These so-called ‘ripples in the fabric of spacetime’ represent the tiny fluctuations in density in the early Universe from which the galaxies and clusters of galaxies later grew. More recently, detailed observations of the fluctations in the CMB at different angular scales have led to conclusions about the large-scale geometry of the Universe.

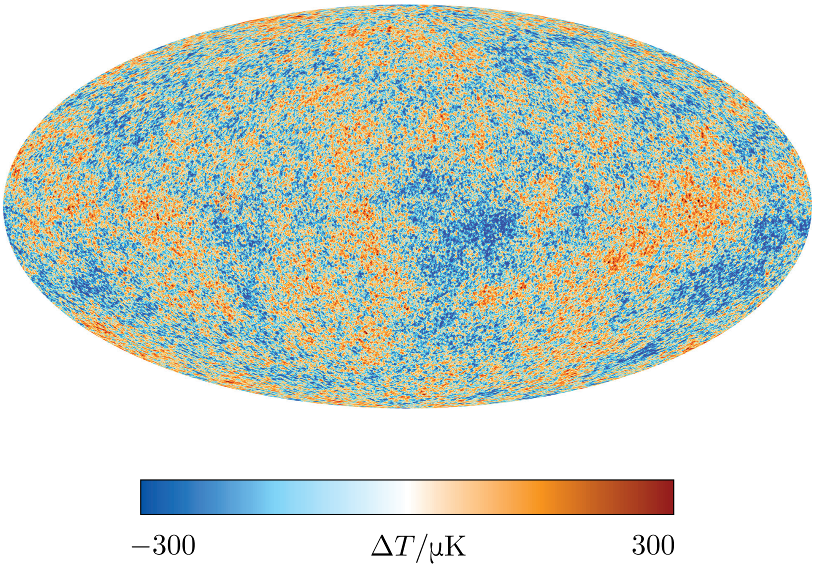

Figure 3 An all-sky map of the CMB radiation, as mapped by ESA's Planck mission. Colour indicates the deviation of temperature from the mean, ΔT, at each position on the sky

The behaviour of space and time, described by Einstein's **theory of general relativity**, can be expressed by the **Friedmann equation**. This equation describes how the scale factor of the Universe changes with time and crucially depends on two parameters: the overall **matter-energy density** of the Universe (represented by ρ) and the **curvature** of space (represented by k) which characterises its overall geometry. Broadly speaking there are three possibilities for the Universe's large-scale geometry: space may either be **flat**, **positively curved** or **negatively curved** (see Figure 4). In flat space, parallel lines remain parallel and the internal angles of a triangle add up to 180 degrees. In positively curved space, parallel lines eventually converge and the internal angles of a triangle add up to more than 180 degrees (as on the two-dimensional surface of a sphere). In negatively curved space, parallel lines eventually diverge and the internal angles of a triangle add up to less than 180 degrees (as on the two-dimensional surface of a saddle). Just which type of geometry the Universe has is determined by the overall matter-energy density of the Universe – if the matter-energy density is higher than some critical value, space is positively curved; if the matter-energy density is lower than this critical value, space is negatively curved.

Perhaps reassuringly, observations of the CMB fluctuations indicate that the large-scale geometry of the Universe is actually flat, as it appears to be locally. This implies that the **curvature parameter** of the Universe, $k = 0$ and that the overall matter-energy density of the Universe is exactly equal to the **critical density**, $\rho_\mathrm{c} = 8.6 \times 10^{-27}$ kg m^{-3}. The **density parameter** of the Universe is equal to the ratio of the actual density to the critical density, $\Omega = \rho/\rho_\mathrm{c}$. In a Universe with flat geometry therefore, $\Omega = 1$.

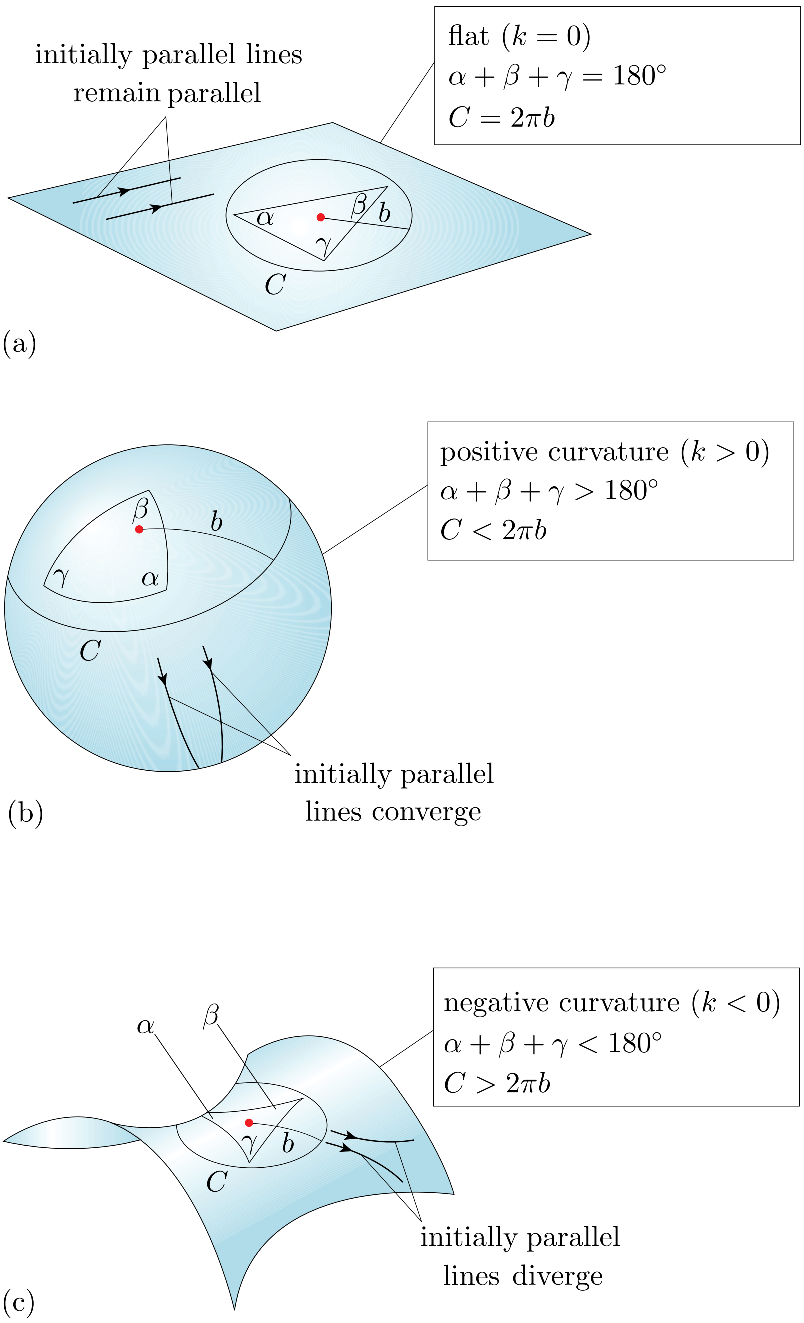

Figure 4 The geometry of the Universe is determined by whether its overall matter-energy density is greater than, less than or equal to the critical density. (a) A universe with zero curvature has $k = 0$ and $\Omega = 1$. (b) A universe with positive curvature has $k > 0$ and $\Omega > 1$. (c) A universe with negative curvature has $k < 0$ and $\Omega < 1$

1.3 Matter in the Universe

The final line of evidence for the hot big bang is the observed relative abundances of the light elements. Calculations of the conditions in the early Universe predict that the Universe should contain about 75% hydrogen, 25% helium-4, about 0.01% deuterium and helium-3, and trace amounts of lithium. This is indeed what is observed in the Universe at large. Further helium and other elements, such as carbon, oxygen and others are made in the cores of stars during stellar nucleosynthesis, then dispersed into the wider Universe when stars die.

The material in the Universe that we can actually *see*, or detect directly by virtue of the electromagnetic radiation that it emits or absorbs, comprises components such as galaxies, stars, planets, gas and dust. These visible components are composed of so-called **baryonic matter**, built from the familiar atoms, such as hydrogen and helium, which are composed of protons, neutrons and electrons. However, baryonic matter comprises only around 5% of the total mass-energy density of the Universe implied by the critical density; we can characterise it by the baryonic matter density parameter, $\Omega_\mathrm{b} = 0.05$.

Non-baryonic matter is referred to as **dark matter**, and it is *not* composed of these familiar constituents. Although dark matter does not interact with electromagnetic radiation, it *does* possess mass, so it interacts via the force of gravity, and appears to comprise a further 25% of the total mass-energy density of the Universe. It may be characterised by the dark matter density parameter, $\Omega_\mathrm{d} = 0.25$. The combined matter density parameter may be written as $\Omega_\mathrm{m} = \Omega_\mathrm{b} + \Omega_\mathrm{d} = 0.30$.

The astute reader will have noted that the 5% of baryonic matter and the 25% of non-baryonic dark matter still leaves a large amount of the Universe's critical density unaccounted for, if the geometry of the Universe really is flat. **Dark energy** is the name given to this missing component of the Universe's critical density budget. It accounts for around 70% of the current total mass-energy density of the Universe and may be characterised in terms of the **cosmological constant**, represented by the symbol Λ. This plays a key role in the evolution of the Universe, as you will see in Section 4 of this course. The main observational evidence for dark energy is the observed accelerating expansion of the Universe referred to earlier. The dark energy density parameter associated with the cosmological constant drives this acceleration, and we may write $\Omega_\Lambda = 0.70$.

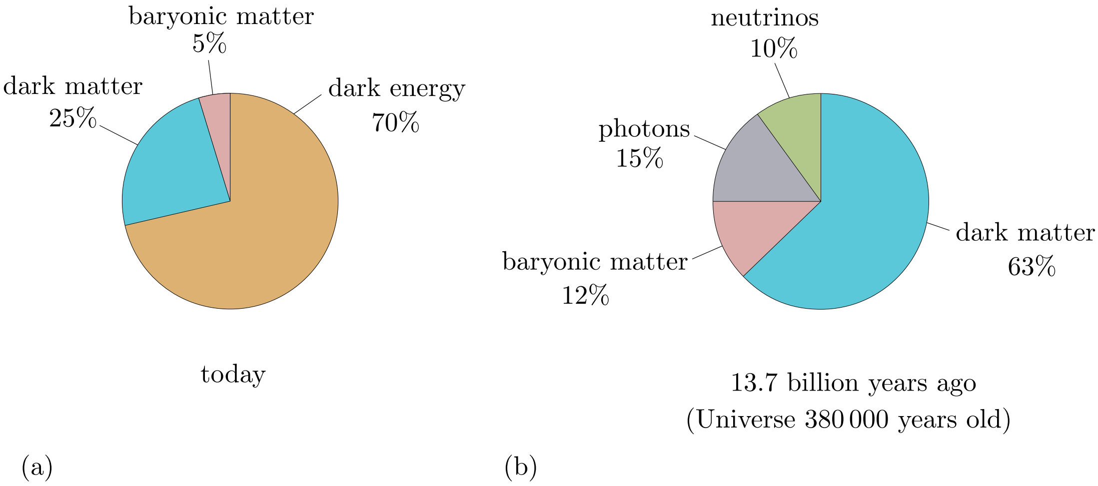

Figure 5 The matter and energy content of (a) the present day Universe, and (b) the early Universe, soon after the big bang

As Figure 5 shows, the proportions of baryonic matter, dark matter and dark energy in the Universe were quite different soon after the big bang at the time when the Cosmic Microwave Background was produced. Although neutrinos and photons are still present in vast numbers in the Universe today, they contribute a negligible amount to the overall matter-energy density of the Universe because their densities have become significantly diluted as the Universe has expanded. In contrast, the contribution of dark energy was initially negligible, but today is the dominant component of the Universe: unlike any form of matter or radiation, it does *not* become more dilute as space expands.

2 Is modern cosmology fundamentally wrong?

As a scientist it is essential to question assumptions made by yourself or others. This might mean taking a step back to ask whether the consensus explanations for unsolved problems are right, or whether our underlying models may be fundamentally wrong or incomplete. The gaps in modern cosmological theory, such as the apparent need for dark matter and dark energy, make it especially important to ask such things. The exercise below asks you to read a short article considering these questions.

Exercise 1

Read the article 'Dark energy, paradigm shifts, and the role of evidence' by Lahav and Massimi (2014), and answer the following questions.

a. What do the authors think can be learned from the predictions of Bessel, Le Verrier and Adams in the nineteenth century?
b. What do the paper's authors think can be learned from the history of particle physics?
c. Do the authors think there is strong evidence that a paradigm shift to a new cosmological model is needed?

Provide your answer...

Discussion

a. It was observed that the orbit of Uranus appeared inconsistent with Newtonian gravity. Bessel postulated that a change in the theory of gravity was needed, whereas Le Verrier and Adams both predicted the existence of an unknown planet. The latter explanation was proved correct with the discovery of Neptune. In contrast, when Le Verrier used similar arguments to postulate the existence of another planet to account for unexpected behaviour in Mercury's orbit, it transpired that this time the correct explanation *was* a modification of the theory of gravity, e.g. general relativistic effects. Hence it can require both improvements in observational evidence and in theory to distinguish between different types of explanation for an unexplained phenomenon.
b. Lahav and Massimi argue that there have been several occasions in the past where new particles were required by theory, and then were eventually observed. The discovery of the neutrino is one key example.
c. No – the authors conclude that general relativity and current cosmological paradigm have a large amount of predictive power, and so considerably more evidence would be needed to rule out this model (and/or rule in a better one).

3 The nature of dark matter

Key evidence for the existence of dark matter includes: the rotation curves of spiral galaxies (i.e. the speeds with which stars move as a function of their distance from the centre of a galaxy); the galaxy velocities and gas properties of galaxy clusters; gravitational lensing by galaxy clusters; and the distribution of the intensity of the CMB radiation at different angular scales across the sky.

The cold dark matter (CDM) model of structure formation assumes that dark matter is some form of comparatively massive (and hence slow-moving) particle. An in-depth exploration of the physics of candidate dark matter particles requires knowledge of advanced particle physics and quantum field theory, which is not covered in this course. The following exercise explores some of the many candidates for a dark matter particle, and discusses some observational and experimental prospects for detecting them.

Exercise 2

Read Section 2 of the article 'Dark matter, dark energy and alternate models: a review' by Arun *et al.* (2017), and answer the following questions.

a. What do the authors suggest are the most likely candidate dark matter particles? What reasons do they give for this?
b. How do direct detection experiments work, and why are they difficult to undertake?
c. What methods might enable the detection of axions?

Provide your answer...

Discussion

a. The two currently favoured dark matter candidates are **weakly interacting massive particles** (WIMPs) and **axions**. WIMPs could result from theories of supersymmetry that extend the standard model of particle physics. Axions are a popular candidate because – in addition to having the potential to act as dark matter – their existence would solve a problem in particle physics in which a symmetry of the strong force known as charge-parity symmetry is observed. Finally, primordial black holes are possible, but this theory may require more fine-tuning.

   Other candidates remain possible, but, as explained at the start of Section 2.4.4 of the paper, the three explanations outlined above are the mainstream options (i.e. they are considered the most likely).
b. Direct detection experiments for WIMPs look for very rare interactions of these particles with a large volume medium, such as liquid xenon or argon. Such experiments are difficult to undertake because, by their nature, dark matter particles are only expected to interact extremely rarely with ordinary matter, hence the need for very large volumes of target material. They must also be shielded from other particles that could produce spurious signals, so are typically located deep underground.
c. It is postulated that axions are converted to and from photons in the presence of strong magnetic fields. Laboratory and telescope experiments involving

strong magnets aim to induce this conversion process and detect the resulting photons.

A particle model for dark matter is the current scientific consensus. However, it is important not to dismiss the possibility of alternative theories.

As with particle models, a full exploration of modified gravity theories requires physics beyond the scope of this course. However, the next exercise asks you to read two popular-science level discussions of such theories.

Exercise 3

Read the two short articles below, which present contrasting views of modified gravity theories.

- 'The case against dark matter' (Wolchover, 2016)
- 'Why modifying gravity doesn't add up' (Siegel, 2022)

Do you think Modified Newtonian Dynamics (MOND) is a better explanation for galactic orbits than dark matter? Which theory involves the least amount of 'new physics' that we don't yet understand?

4 The nature of dark energy

The origin of the acceleration of the expansion of the Universe is one of the biggest questions in modern astrophysics, and it underpins many ongoing theoretical and observational research programmes. To explore these ideas further, watch Video 1 in which a cosmologist is interviewed about the nature of dark energy and the prospects of its detection, and then answer the questions that follow.

View at: youtube:IJYDcXZMe9c

Video 1 Dark energy and the 'big rip' (34 minutes)

Exercise 4

Based on Video 1, answer the following questions:

a. In what way does dark energy behave like 'anti-gravity'?
b. What is Professor Copeland's reason for rejecting an explanation for the accelerated expansion in which the cosmological principle is incorrect (i.e. the Universe is not homogeneous)?
c. What is a 'scalar field'?
d. What is special about 'chameleon fields'?
e. Will the Universe end in a 'big rip'?

Provide your answer...

Discussion

a. Ordinary matter and radiation are both sources of gravitational attraction, and in the context of the Universe's expansion they act to slow it down. Dark energy has a negative pressure, and so its gravitational effect is to push spacetime apart, rather than pull it together, and so it effectively acts in the opposite direction to gravity.
b. At around 09:50 in the video, Professor Copeland argues that the distribution of structure in the CMB (i.e. the uniformity and lack of large scale inhomogeneity) strongly supports the cosmological principle. The CMB structure suggests that the probability of us being located in a region with atypical acceleration (e.g. caused by local effects) must be very small.
c. A scalar field describes a function that takes a single (scalar) value at every point in space. The field has a potential energy and kinetic energy associated with each location, and these may evolve with time.
d. Chameleon fields are one possible model for dark energy, in which the energy associated with the field depends on environmental conditions. This enables it to behave differently on different size scales, which makes it a potentially useful dark energy candidate.
e. Continuing acceleration of the Universe's expansion would be expected to lead to a big rip. However, Professor Copeland argues that plausible theories for dark energy are likely to involve decay of the energy in the scalar field at some

> point in the future, which could result in a return to a matter-dominated Universe with different long-term evolution.

Video 1 discussed three main explanations for the acceleration of the Universe. Further information about each of these explanations is provided in the following sections.

4.1 A cosmological constant

The cosmological constant Λ arises as an adjustment to Einstein's field equations of general relativity, and therefore the Friedmann equations, to describe the evolution of the Universe. Λ has a gravitational effect in the opposite direction to that of matter and radiation, and therefore could in principle enable a static Universe; however, in the favoured cosmological model it must dominate over matter and radiation to enable accelerated expansion.

The equation of state for a cosmological fluid is given as follows:

$$P = w\rho c^2 \tag{3}$$

where P is the pressure of the fluid, ρ is its density, c is the speed of light, and w is known as the **equation of state parameter**. The equation of state parameter is therefore the ratio between the pressure of a fluid and its energy density. Ordinary matter has $w_{\mathrm{m}} = 0$, and radiation has $w_{\mathrm{r}} = 1/3$. The peculiar property of dark energy models is that $w < 0$, so that the fluid has a negative pressure. This is what enables it to act in the opposite direction to the gravitational effect of ordinary matter.

In the case of a cosmological constant, $w_{\Lambda} = -1$. Therefore the pressure and the energy density have the same magnitude but opposite signs, and – as the name suggests – these quantities remain constant with time despite the expansion of the Universe. In this model it is therefore the decrease in the energy density of other components (e.g. $\rho_{\mathrm{m}} c^2$ or $\rho_{\mathrm{r}} c^2$) with time that causes Ω_{Λ} to dominate the evolution of the Universe at late times.

4.2 Quintessence models

Another family of dark energy models are those in which the equation of state evolves, i.e. w changes with time. These are known as **quintessence** models, and provide a slightly different explanation for why Ω_{Λ} is unimportant in the early Universe, but dominates at late times.

In this model, w can be written as a function of the scale factor, a, as follows:

$$w(a) = w_0 + (1 - a)w_{\mathrm{a}} \tag{4}$$

where w_0 is a constant and w_{a} is a coefficient that determines how w changes with a. The scale factor itself is simply a mathematical quantity that describes the changing separation of two points in space as the Universe expands.

Measuring the parameters in Equation 4 is a key aim of many observational surveys to study dark energy. This can be done, for example, by targeting observations of Type Ia supernovae at redshifts corresponding to the epochs when the Universe changed from being matter-dominated to Λ-dominated under different theories, and through observations of the evolution of large-scale structure.

Figure 6 illustrates the constraints on w_0 and w_a obtained by combining the *Planck* (2018) CMB angular power spectrum data with Type Ia supernovae (SNe), weak gravitational lensing (WL), baryon acoustic oscillation (BAO) measurements and an additional method using galaxy statistics called redshift space distortions (RSD).

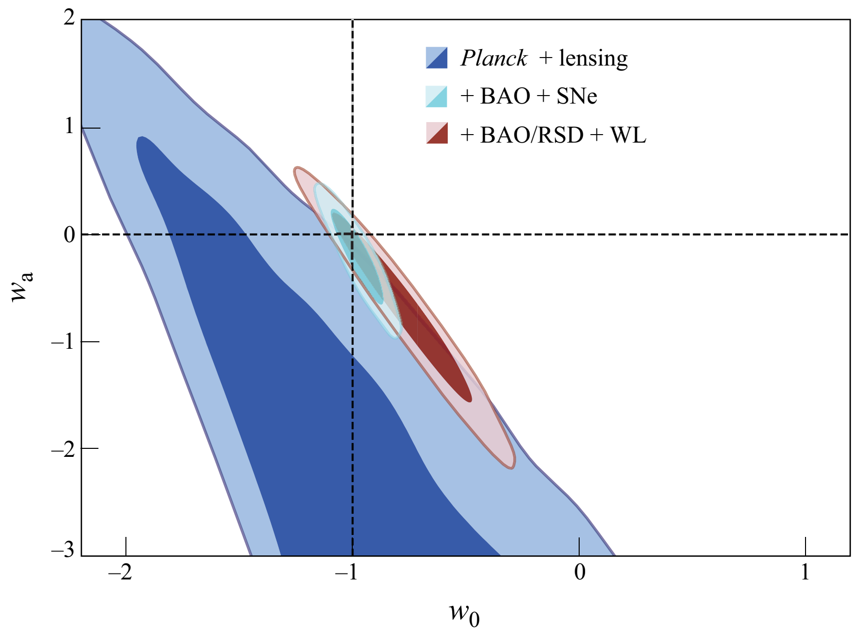

Figure 6 The measured constraints on an evolving dark energy equation of state. The 2018 *Planck* data (blue shaded regions) are further constrained by BAO and SNe measurements (cyan shading) and by BAO, RSD and weak lensing (WL) measurements (red shading). Darker and lighter shading within each data set indicate greater and lesser degrees of certainty, respectively.

- All of the observational constraints agree with the values at the intersection of the dashed lines. Which of the models discussed in this section does that position correspond to?
- The point at the intersection of the dashed lines is $w_0 = -1$ and $w_\mathrm{a} = 0$. This coordinate corresponds to a cosmological constant model.

4.3 Modified gravity

A third alternative explanation for the acceleration of the Universe is modifications to the theory of gravity on the largest scales. As discussed previously, this remains in some ways an attractive possibility, but does not currently provide a single consistent explanation for all of the phenomena attributed to dark matter and dark energy.

5 Inflation

Inflation is a process by which the scale factor a increased by a factor of at least 20–30 orders of magnitude over a tiny fraction of a second, about 10^{-36} seconds after the big bang. The theory was developed in the 1970s and 80s by physicists including Alan Guth, Alexei Starobinsky and Andrei Linde, in order to address several problems with standard cosmological models. It is now well accepted as part of the cosmological model, but it is difficult to test observationally and presents some challenges for particle physics.

The three main problems solved by inflation are the horizon problem, the flatness problem and the monopole problem, which will now be summarised in turn. Then, in the final section of this course, you will learn more about how inflation works, and the state of current theory and observational tests.

5.1 Horizon problem

The **horizon problem** is perhaps the most fundamental problem for cosmology, and arises because of the uniformity of the observed CMB. This uniformity is the result of thermal equilibrium between matter and radiation at the **time of last scattering**, 380,000 years after the big bang. However, in order for such equilibrium to be possible, transfer of energy needs to be possible across the region being considered – in other words, a region in thermal equilibrium cannot have a size that is larger than the distance a signal can travel at the time being considered (i.e. the acoustic scale).

At a redshift of $z \sim 1100$ (corresponding to the epoch noted above), the acoustic scale corresponds to an angular separation of $\sim 1^{\circ}$, and so regions on the sky separated by an angular scale of more than this amount should have been outside each other's horizon distance at the time of last scattering, and therefore not causally connected. Yet we observe that they appear to have been in thermal equilibrium. Postulating a brief period of inflation can solve this problem by rapidly separating regions that were initially in contact and able to come into equilibrium.

5.2 Flatness problem

Observations of the CMB and Type Ia supernovae tell us that the Universe appears very close to spatially flat, that is, the curvature parameter of the Universe is essentially zero, $k \approx 0$. The flatness of the Universe is described by the deviation of Ω (the total matter and energy content) from 1:

$$1 - \Omega(t) = \Omega_k = \frac{-kc^2}{a(t)^2 H(t)^2} \tag{5}$$

Here c is the speed of light, $a(t)$ is the scale factor and $H(t)$ is the Hubble parameter. The observations constrain this deviation from flatness to be $|1 - \Omega| < 0.005$ at the present time, but although k remains constant, $a(t)$ and $H(t)$ are evolving with time. It can be shown that over the period when first radiation and then matter dominated in the early Universe, the deviation of Ω from 1 should have evolved according to:

$$1 - \Omega(t) = \frac{(1 - \Omega_0)a(t)^2}{\Omega_{\mathrm{r},0} + a(t)\,\Omega_{\mathrm{m},0}} \tag{6}$$

where Ω_0, $\Omega_{\mathrm{r},0}$ and $\Omega_{\mathrm{m},0}$ are the current overall density parameter, current radiation density parameter and current matter density parameter, respectively. This leads to a prediction that the deviation of Ω from 1 must decrease substantially with time, so that at the **Planck time** (about 10^{-43} seconds after the big bang), $|1 - \Omega|$ is predicted to be $< 2 \times 10^{-62}$.

Although everyday experience of the world around us leads us to think that a flat spatial geometry is perhaps most 'natural', this flatness has historically been understood to present a fine-tuning problem for cosmological theory, known as the **flatness problem**. It is unclear what physics contrives to ensure the matter and energy content of the Universe at the Planck time was precisely that needed to exactly match the critical density ρ_c.

Inflation removes this worry, because during an inflationary era Equation 6 no longer applies, and instead the denominator on the right-hand side in Equation 5 grows exponentially. Therefore instead of growing, the deviation from flatness rapidly drops very close to zero in this scenario, making it possible to start the radiation-dominated era from a point close enough to $\Omega = 1$ to avoid the flatness problem.

5.3 Monopole problem

The final problem that inflation was designed to solve is the absence of observed magnetic monopoles (isolated sources of magnetic field), which is a prediction of some grand unified theories. This absence is known as the **monopole problem**.

Monopoles have never been observed, and it is not known whether they exist. It is possible that the theories that predict them are not correct. However, if they are created at the extremely high energies present in the very early Universe, then – by expanding the scale of the Universe by a factor of, say, 20–30 orders of magnitude – their space density becomes so diluted that they become sufficiently rare for it to be highly improbable that we would have detected one.

5.4 Inflationary theories and how to study them

To explore the idea of inflation further, watch Video 2 in which a cosmologist is interviewed about the theory of inflation and how it might be tested. Then answer the questions in the exercise that follows.

View at: youtube:m7C9TjdziPE

Video 2 Inflation and the Universe in a grapefruit (24 minutes)

Exercise 5

Based on Video 2, answer the following questions:

a. Why is reheating needed after inflation, and what happens as a result?
b. Does inflation happen before or after the big bang?
c. What alternatives are there to inflation?
d. What is the best current evidence for inflation?
e. Are dark energy and inflation related, and if so, how?

Provide your answer...

Discussion

a. Inflation involves a very large increase in volume (and therefore decrease in density) in a short time. Particles present prior to this stage will now be very diluted, not consistent with the hot big bang model. Instead, it is the energy of the inflation field that is thought to be converted into ordinary particles (the cosmic soup of radiation, quarks and leptons) in a stage known as reheating, which begins the evolution of ordinary matter.
b. Professor Copeland argues that it is more logical to consider the big bang as the point at which inflation ends (which is the point towards which the current expansion of the Universe can be extrapolated back). However, he then concedes that some form of starting point, and release of energy, must have been the trigger for inflation. Hence, views differ on whether to label the start or end of inflation as the big bang.
c. A possible alternative (discussed from ~17:40 in the video) involves cyclic Universes, which occur in string-theory models. These expand and contract, so that our assumed big bang was not the ultimate starting point for the Universe.
d. The best tests of inflationary theory come from precision measurements of the angular power spectrum of the CMB, which are generally in extremely good agreement with inflation predictions. However, there are some subtle effects that can be explored further to learn more, or to potentially refute inflation theories.
e. The favoured models for both dark energy and inflation involve scalar fields. In many theories the two scalar fields are not related – the energies involved are very different. However, one theory mentioned in the video is quintessential inflation, in which the same field that drives inflation later evolves to be the origin of dark energy.

6 Quiz

Answer the following questions in order to test your understanding of the key ideas that you have been learning about.

Question 1

The galaxy NGC1234 has a redshift of z = 0.0113. Assuming the Hubble constant is 67.7 km s^{-1} Mpc^{-1}, what is the distance to NGC1234 (to 2 significant figures)?

- 0.23 Mpc
- 5.0 Mpc
- 21 Mpc
- 34 Mpc
- 50 Mpc
- 68 Mpc

Answer

The Hubble constant implies that the apparent recession speed of galaxies increases by 67.7 km s^{-1} for every increase in distance by 1 Mpc. Using Equation (1), $D = zc/H_0 = (0.0113 \times 300,000\ \mathrm{km\ s^{-1}})/67.7\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$, so D = 50 Mpc.

Question 2

Which of the following is the correct description for the expansion of the Universe?

- The expansion rate of the Universe has been constant since soon after the big bang.
- The expansion rate of the Universe has constantly decelerated since soon after the big bang.
- The expansion rate of the Universe has constantly accelerated since soon after the big bang.
- The expansion rate of the Universe initially decelerated soon after the big bang but has accelerated more recently.
- The expansion rate of the Universe initially accelerated soon after the big bang but has decelerated more recently.
- The expansion rate of the Universe cannot be measured.

Answer

Soon after the big bang, the expansion rate of the Universe decelerated because of the dominance of matter. About 6 billion years ago, dark energy came to dominate the Universe causing the expansion rate to accelerate.

Question 3

Which one of the following statements about the CMB is *true*?

○ The CMB radiation currently has a spectrum that is characteristic of a temperature of about 3000 K.

False

○ The CMB radiation is perfectly uniform in temperature and intensity across the entire sky.

False

○ The CMB radiation is today detected mainly in the infrared part of the electromagnetic spectrum.

False

○ The fluctuations in the CMB radiation across the sky imply that the geometry of the Universe has positive curvature.

False

○ The fluctuations in the CMB radiation across the sky imply that the geometry of the Universe has negative curvature.

False

○ The CMB radiation is a relic of the decoupling of photons from atoms that happened 380,000 years after the big bang.

True

Answer

Only the last statement is true.

Question 4

Which of the following statements about baryonic and non-baryonic matter are *true*?

- ☐ Baryonic matter makes up about 5% of the matter-energy density of the Universe.

  True
- ☐ Non-baryonic matter makes up about 25% of the matter-energy density of the Universe.

  True
- ☐ Baryonic matter comprises the familiar protons, neutrons and electrons.

  True
- ☐ Non-baryonic matter may be composed of WIMPS and/or axions.

  True
- ☐ Baryonic and non-baryonic matter together cannot account for all of the critical density of the Universe.

  True
- ☐ Baryonic and non-baryonic matter exceed the critical density of the Universe.

  False

Answer

All statements are true except the last one.

Question 5

Which is the correct order of contributions to the overall matter-energy density of the Universe from smallest to largest?

- baryonic matter, dark energy, dark matter
- dark matter, dark energy, baryonic matter
- baryonic matter, dark matter, dark energy
- dark matter, baryonic matter, dark energy
- dark energy, baryonic matter, dark matter
- dark energy, dark matter, baryonic matter

Answer

The correct order is baryonic matter (5%), dark matter (25%), dark energy (70%).

Question 6

Which values of the equation of state parameter are associated with each of the following types of fluid?

$w = 0$

$w = +1/3$

$w = -1$

$w = w_0 + (1 - a)w_a$

Match each of the items above to an item below.

baryonic matter

radiation

cosmological constant

quintessence

Answer

The equation of state parameter is zero for matter, minus one for a cosmological constant, +1/3 for radiation, and a function of the scale parameter for quintessence.

Question 7

Which of the following problems in cosmology *cannot* be solved by inflation?

- ◻ The monopole problem
- ◻ The flatness problem
- ◻ The horizon problem
- ◻ The problem of the accelerating expansion of the Universe
- ◻ The problem of dark energy
- ◻ The problem of dark matter

Question 8

What type of curvature does the universe have for each of the following combinations of matter density (Ω_m) and dark energy density (Ω_Λ)? (Assume that *only* matter and dark energy are present in these hypothetical universes.)

flat

negative

positive

Match each of the items above to an item below.

$\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$

$\Omega_m = 0.05$, $\Omega_\Lambda = 0.50$

$\Omega_m = 0.65$, $\Omega_\Lambda = 0.65$

Answer

If $\Omega_m + \Omega_\Lambda > 1$, the universe has positive curvature; if $\Omega_m + \Omega_\Lambda < 1$, the universe has negative curvature; and if $\Omega_m + \Omega_\Lambda = 1$, the universe is flat.

7 Conclusion

The focus of this course has been on what we do, and don't, understand about how the Universe works. These were some of the key learning points:

- Although **dark matter**, **dark energy** and **inflation** require new physics that is not yet fully understood, they each remain the best explanation for a range of observational data at the present time.
- The most popular particle dark matter candidates are **weakly interacting massive particles** (WIMPs) and **axions**, both of which are preferred because they naturally solve other problems in particle physics. Primordial black holes of very low mass may also be a possibility.
- Direct detection experiments have ruled out some mass ranges for both types of particle, but continue to explore parameter space. In the case of WIMPs, experiments involve underground detectors and large volumes of noble gases, while it is hoped axions may be detected via their interaction with magnetic fields.
- The observed acceleration of the Universe may be explained by a **cosmological constant** Λ, evolving dark energy (**quintessence**), or modified theories of gravity.
- Observations aim to measure the **equation of state parameter** for dark energy, w, and its evolution. To date, observational constraints are consistent with a cosmological constant and do not require quintessence models.
- Inflation can explain the
  - **horizon problem**, by enabling now-widely separated regions to have been previously in causal contact
  - **flatness problem**, by enabling the total density parameter to evolve to be extremely close to 1 at early times
  - **monopole problem**, by diluting the density of any magnetic monopoles so that they would now be extremely rare.
- Inflationary models are in good agreement with the observed properties of the CMB, but more precise experiments are needed to test inflation models further.

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Glossary

axions

Hypothetical particles which may be a component of dark matter.

baryon

A term used to describe strongly interacting particles that have half odd-integer spin i.e. spin 1/2, 3/2 etc. Each baryon is a combination of three quarks. The lowest mass baryons are the proton and the neutron. A type of hadron.

baryonic matter

Matter composed of baryons.

big bang

The name given to the current standard cosmological model (see cosmology), in which the Universe began in a very hot, dense state and has been expanding and cooling ever since. The big bang model successfully explains the observed recession of distant galaxies (see Hubble law), the properties of the cosmic microwave background radiation, and the abundances of the light elements in the Universe. As a result of the cosmological principle, the expansion of the Universe can be described in terms of the evolution of a single quantity, the scale factor, which describes the changing physical distance between typical points in the Universe. At the present time the scale factor is increasing with time, giving rise to the observed expansion. The behaviour of the scale factor depends on the amount of matter (and energy) in the Universe, and the ultimate fate of the Universe is determined by whether the gravitational effects of matter are strong enough to overcome the expansion.

black-body spectrum

The spectrum emitted by a black body.

Chandrasekhar limit

The theoretical upper limit to the mass of a white dwarf, about $1.4\,\mathrm{M}_{\odot}$, also called the Chandrasekhar mass.

CMB

See **cosmic microwave background** radiation.

cosmic microwave background

Low-energy black-body radiation seen with almost identical properties in all directions. Its black-body spectrum corresponds to a temperature around 2.7 K. In the big bang cosmological model, the background radiation is a relic of the early stages of the Universe, when the temperatures and densities were much higher, the whole Universe was optically thick, and matter and photons were in thermal equilibrium. Study of the microwave background radiation therefore gives important information about the structure of the young Universe.

cosmological constant

A non-zero value of Λ in the Einstein field equations. Its value is given by $\Lambda = 3\Omega_{\Lambda,0}H_0^2/c^2 = 1.3 \times 10^{-52}\ \mathrm{m}^{-2}$.

cosmological redshift

Redshift arising from the expansion of the Universe. It is related to the scale factor by $1 + z = \frac{a(t_{\mathrm{ob}})}{a(t_{\mathrm{em}})}$ where t_{em} and t_{ob} are the times at which the radiation was emitted and observed respectively.

cosmology

The branch of science that is concerned with the study of the Universe as a whole, including its structure and history.

critical density

With reference to cosmological models, the quantity defined by $\rho c(t) = 3H^2(t)/8\pi G$.

curvature

See **curvature parameter**.

curvature parameter

The quantity *k* that has a value of 0 for a spatially flat geometry, or can take values <0 or >0 for spatial geometries that have positive or negative curvature.

dark energy

A proposed form of energy that affects the Universe on the largest scales. Its primary effect is to drive the accelerating expansion of the Universe.

dark matter

Matter that does not produce radiation, and so can only be detected (at present) by its gravitational effects on other matter. Evidence from the rotation curve of spiral galaxies, the velocity dispersion of clusters of galaxies, gravitational lensing and observations of the cosmic microwave background radiation suggest that there is more dark matter than luminous matter in the Universe by a large factor, and that most of it is non-baryonic (that is, not made primarily of protons and neutrons as normal matter is). The nature of the non-baryonic dark matter is one of the major puzzles of modern astrophysics.

density

Also known as mass density. The ratio of mass to volume for a homogeneous system. It is possible to define the density at a given point in any system by taking a small volume element around that point and evaluating the ratio of mass to volume for that volume element. Contrast with number density.

density parameter

One of the fractional densities defined by: matter density, $\Omega_m(t) = \rho_m(t)/\rho_c(t)$; radiation density, $\Omega_r(t) = \rho_r(t)/\rho_c(t)$; dark energy density, $\Omega_\Lambda(t) = \rho_\Lambda(t)/\rho_c(t)$ where $\rho_c(t)$ is the critical density. The effective energy density associated with curvature is $\Omega_k = 1 - \Omega_m - \Omega_\Lambda - \Omega_r$.

equation of state parameter

The parameter w in the equation of state relating the pressure P and density ρ of a perfect fluid: $P(\rho) = w\rho c^2$. For non-interacting matter (referred to as ‘dust’) $w = 0$, for radiation $w = +1/3$, and for dark energy $w = -1$.

flat

A space that is not curved, i.e. the curvature parameter $k = 0$.

flatness problem

The recognition of the fact that the initial density of the Universe is apparently very finely tuned, such that the density is currently extremely close to the critical value required for a flat universe.

Friedmann equation

The equation relating the scale factor *a* and its derivatives to the density parameters and the curvature parameter *k*: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2}$, where ρ is the density (of matter, radiation and dark energy).

general relativity

See **theory of general relativity**.

horizon problem

The recognition of the fact that objects that are further apart than a certain distance could not have been in causal contact in the past. This poses a problem in understanding how parts of the cosmic microwave background radiation that are more than a few degrees apart ever managed to look so similar.

hot big bang

See **big bang**.

Hubble diagram

A plot of apparent magnitude against redshift.

Hubble law

See **Hubble-Lemaître law**.

Hubble constant

The value of the Hubble parameter at the current time.

Hubble parameter

In terms of the scale factor $a(t)$, the Hubble parameter at any given time can be written $H(t) = \frac{1}{a(t)}\frac{\mathrm{d}a(t)}{\mathrm{d}t} = \frac{\dot{a}}{a}$. The value of the Hubble parameter at the current time is called the Hubble constant.

Hubble-Lemaître law

The linear relationship, discovered independently by Edwin Hubble and Georges Lemaître, between the distance of a galaxy and its cosmological redshift, expressed as an apparent recession speed. The law states $v = H_0 D$ where v is the apparent recession speed in km s^{-1} and D is the distance in megaparsecs. H_0 is the Hubble constant.

inflation

A hypothetical epoch in the very early development of the Universe, when the Universe is supposed to have undergone a brief period of very rapid expansion.

lookback time

The time elapsed between the emission of a photon by a distant astronomical source and its detection by us. For objects at cosmological distances, the lookback time can be a significant fraction of the age of the Universe.

matter-energy density

The equivalent mass per unit volume of a source of matter and/or energy, since matter and energy are related by $E = mc^2$, where c is the speed of light.

monopole problem

Grand unified theories predict about one magnetic monopole per horizon size at the time the Universe was at the critical GUT temperature. Therefore the present-day Universe should have many magnetic monopoles and they would dominate the energy density of the Universe. The fact that we see none is known as the monopole problem.

negatively curved

The situation when the curvature parameter k has a value < 0.

non-baryonic matter

Matter not composed of baryons. See **dark matter**.

Planck time

A fundamental timescale, given by $t_{Pl} = (hG/2\pi c^5)^{1/2} = 5.39 \times 10^{-44} s$.

positively curved

The situation when the curvature parameter k has a value > 0.

quintessence

The name given to a postulated fifth fundamental force (in addition to the established four fundamental forces of nature: electromagnetic interaction, gravitational interaction, strong nuclear interaction and weak nuclear interaction). It is one form of dark energy with a time-varying equation of state parameter.

recombination

The process in which a free electron combines with an ion, releasing energy in the form of a photon; the reverse of ionisation.

redshift

A shift of a spectral line to redder (longer) wavelengths. There are three important types of redshift: (1) Doppler shift - due to the motion of the emitting object away from the observer. (2) Gravitational redshift - due to strong gravity at the surface of the emitting object. (3) Cosmological redshift - due to the expansion of the Universe (see the Hubble constant, big bang). Numerically, the redshift z is defined by $z = \frac{\Delta\lambda}{\lambda_0}$ where λ_0 is the original emitted wavelength (the wavelength that the emission line would have in the laboratory) and $\Delta\lambda$ is the difference between the observed and emitted wavelengths. If the redshift is a small Doppler shift, then $z = v/c$, where v is the speed of recession. For a cosmological redshift, the same formula can be used together with Hubble's law to infer distances, but only if $z < 1$; otherwise more complex results, depending on the geometry of the Universe, must be applied.

rest wavelength

The wavelength of a spectral line in a frame of reference in which the material emitting the line is itself at rest.

scale factor

A numerical quantity used to describe the expansion of the Universe in big bang cosmology; the scale factor gives the relationship between the true distance between two objects and their separation in co-moving coordinates (which do not change with time). If we adopt an Earth-centred co-moving coordinate system, in which r is the radial distance (we are at $r = 0$) then, in the simple case of a spatially flat Universe, the distance d to an object is given by $d = a(t)r$. Because the scale factor describes the expansion of the Universe, the ratio of the scale factors when a photon was emitted and when it is observed give us the redshift: $(1 + z) = a(\text{observed})/a(\text{emitted})$. The usefulness of the scale factor is that (1) the equations describing the expansion of the Universe can easily be written in terms of a and its time derivatives and (2) observable cosmological quantities such as the Hubble constant can be described in the same way. If the scale factor increases with time, the Universe is expanding; if the scale factor decreases with time, the Universe is contracting.

theory of general relativity

The theory published by Albert Einstein in 1915 that generalises the ideas of his earlier special theory of relativity by extending them to non-inertial frames of reference. An important principle of the theory asserts that an accelerating frame of reference is locally equivalent to one that is located in a gravitational field. Consequently, the general theory of relativity is also a theory of gravitation, and as such supersedes Newton's theory of gravity. (The predictions of Newton's theory approximate those of general relativity in situations where the gravitational fields are weak.) According to general relativity, gravity manifests itself in the geometric structure (curvature) of spacetime. Mass and other sources of gravity determine that curvature, and moving bodies respond to that curvature, giving rise to the appearance of a gravitational force.

time of last scattering

The epoch, about 380,000 years after the big bang, at which electrons combined with protons (see **recombination**) to form neutral atoms. After this time, radiation ceased to scatter from matter in the universe. The **cosmic microwave background** radiation is a relic of this epoch.

type Ia supernovae

Type Ia supernovae are thought to occur when accretion onto the surface of a white dwarf in a binary system takes its mass over the Chandrasekhar limit. When this happens, the star can no longer be supported by degeneracy pressure, and so it starts to collapse, igniting runaway thermonuclear reactions between the heavy nuclei in the star. The resulting explosion destroys the star and gives rise to the observed supernova. Type Ia supernovae show no hydrogen lines in their spectra, consistent with an origin in a massive white dwarf; absorption features from heavy elements such as silicon are common. Because all Type Ia supernovae have a mass around the Chandrasekhar mass, they have a very similar peak absolute magnitude and so can be used as standard candles.

weakly interacting massive particles

A possible class of dark matter particles, which includes the neutralino. Often abbreviated to WIMPs.

white dwarf

A stellar-mass compact object, with a mass below the Chandrasekhar mass (1.4 solar masses) supported against gravitational collapse by the degeneracy pressure of electrons. White dwarfs are the final products of the evolution of low-mass stars, after thermonuclear reactions have ceased and the outer regions of the star have been lost in stellar winds or as a planetary nebula. If left isolated they will gradually cool and contract until they become invisible, but since the luminosity is low, the cooling time is long. White dwarfs in binary systems may meet a different fate: when their companion star moves off the main sequence, mass transfer may take the white dwarf over the Chandrasekhar mass. In this case, the white dwarf can end its life as a Type Ia supernova.