Zimbabwe
Ministry of Primary and Secondary Education

NUMERACY

IGATE
Module 5
Multiplication and division – part 2

iGATE Improving Gender Attitudes, Transition and Education Outcomes
care
World Vision
The Open University
Girls' Education Challenge
UKaid from the British people

For information about the IGATE project see:
www.wvi.org/education-and-life-skills/igate-improving-girls-access-through-transforming-education

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Revised Module 5 (MoPSE)

Foundational numeracy

Module 5: Multiplication and division – part 2

Module contents

About these modules

This is the fifth of six Teacher Professional Development (TPD) modules for all teachers working with learners whose attainment in numeracy is below their Grade or Form level. The modules are also appropriate for Initial Teacher Education (ITE) – particularly during school placements or practicum.

Module 1: What is a number?

Module 2: Early addition and subtraction

Module 3: Addition and subtraction of bigger numbers

Module 4: Multiplication and division – part 1

Module 5: Multiplication and division – part 2

Module 6: Fractions and decimals

The modules were collaboratively developed for the Ministry of Primary and Secondary Education (MoPSE) by the Open University, World Vision and CARE International. The modules have been tried and tested in hundreds of primary and secondary schools across Zimbabwe, strengthening the teaching of foundation skills and improving learning outcomes. Our thanks to everyone who contributed – especially teachers, school heads and schools' inspectors.

MoPSE's highest priority is to empower ALL learners through strong foundations in literacy and numeracy.

Whatever their Grade or Form, all learners need strong foundations in literacy and numeracy to succeed in other learning areas. Learners must **learn to read and use number** so they can **read and use number to learn**.

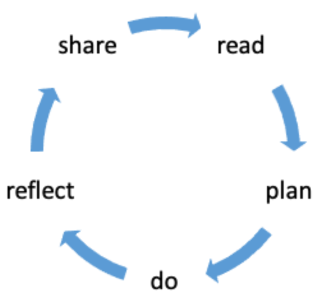
Using the modules

Teachers will benefit most by using the modules within reflective-practice cycles in their schools, as shown below.

Read an activity.
Plan how you will use the activity.
Do the activity with your learners.
Reflect on what learners learned from doing the activity.

- What worked well?
- What would you change next time?

Share your experiences with your colleagues.



- The modules can be used by:
  - individual teachers
  - pairs or groups of teachers
  - whole schools
  - cluster meetings or district workshops.
- The modules provide classroom activities and guidance for effective use.

Learner attainment

We describe learners who can do an activity confidently and successfully as 'higher attaining' and learners who cannot do an activity well as 'lower attaining'.

No one knows what a learner will be able to do given the chance. Every learner has the potential for growth. Teachers have often been surprised when they found a learner who was 'higher attaining' for one activity was then 'lower attaining' for another – and vice versa. So, we don't label learners with words like 'fast' or 'slow'.

A learner may have different levels of attainment in different learning areas, or in different aspects of one learning area. That's why assessment is a big part of the activities. It is important to find out, as often as possible, what learners know and can or can't do. Then they can be given activities at a level that will help them progress.

Working in groups

Learning takes place as a result of **doing** an activity, **thinking** about it, and **understanding** the ideas it contains.

In order to make sure that all learners are **doing**, activities are designed so learners work together in pairs or small groups for most of the lesson. Pairs, or groups of four to six learners, work best because everyone can take part. Sometimes the teacher will need to demonstrate the activity first.

There are several ways in which learners can be put into groups. Teachers should choose the one that works best for their learners.

- **Learners choose their own groups:** Sometimes this can result in friends working (or not working!) together, while other learners are left out.
- **Learners at a similar level of attainment work together:** This can work well, as learners are working at their preferred pace, but learners who need help have to find it from outside the group.
- **Learners at mixed levels of attainment work together:** This type of grouping has the advantage that higher-attaining learners can help lower-attaining ones. This gives lower-attaining learners personal and prompt support, and higher-attaining learners a chance to talk about what they have learned, which helps to deepen their understanding.

Collecting and storing resources

Many of the activities in these modules rely on learners using physical resources. Some activities need large quantities of resources so that learners can work through the activities in small groups. How can you make sure you have enough?

Here are some suggestions from teachers who trialled these materials.

<u>Sticks</u>

- Ask each learner to bring in at least ten small sticks the length of their middle finger.
- Ask learners to hunt for small sticks in the school grounds during break times. (This worked well in rural communities, but not so well in city schools.)
- As an alternative to sticks, ask learners to bring in at least ten toothpicks or drinking straws.
- Bundle up the sticks into ‘tens’ using elastic bands (if you have them, as these work better for subtraction) or short lengths of wool or string.

<u>Counters</u>

- Ask each learner to bring in a collection of counters. In rural communities, learners usually brought in small stones, beans or seeds. In city schools, learners often brought in plastic beads or bottle tops.

<u>Place value counters</u>

- Ask local bottle stores, lodges and hotels to collect tops from drink bottles. Write the value of the counter on the top with a permanent marker.
- Cut up manila or paper into 2-cm squares and write the value of the counter on each piece.

<u>Making resources together</u>

- Some teachers organised a weekly after-school resource-making meeting to make sure all classes had the equipment they needed. This was especially helpful when making number cards or arrow cards!

<u>Sharing resources</u>

- Share a set of resources between two classes. So, for example, while one class has a literacy lesson, the other class does numeracy and vice versa. The teachers quickly swop resources between lessons!
- Split the class into two halves. Work with one half of the class on activities that use resources, while the other half work in their books.

<u>Storing resources</u>

- Teachers often reused the same resources many times, for lots of different activities. They realised that they would save a lot of time by storing resources carefully, either in their classroom or a common store-cupboard.

Unit 1: Building fluency

Contents

Introduction

In this unit, learners play two games in which they practise multiplication facts. They find out how to use a multiplication square to look up multiplication and division facts that they have not yet memorised.

Key words and phrases

- **derive** – work out from what you know already
- **recall** – remember

1.1: Switch and freeze

Aim

This activity helps learners to recall or derive multiplication facts accurately and efficiently.

What the learners will do

The caller tells learners which ‘times table’ they are going to practise. Learners throw a ball around the circle saying the next multiplication fact as they throw the ball. When the caller calls *‘Switch!’* they also call a new ‘times table’. The next learner to throw the ball has to say the multiplication fact for the new ‘times table’. When the caller calls *‘Freeze!’* the person who is holding the ball says as many multiplication and division facts as they can about the fact they have just said or are about to say.

Resources

You will need:

- a large multiplication square (**Resource A Multiplication square**, page 35).

Each group of six to eight learners will need:

- something to throw to each other, for example, a ball or screwed up paper in a plastic bag taped together with sticky tape.

Activity

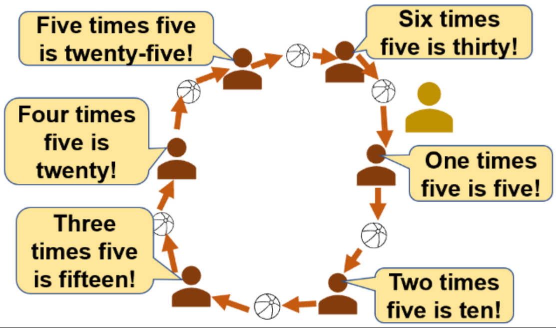
Make sure all learners can see the multiplication square.

Ask learners to stand in groups of six to eight with a ball in each group.

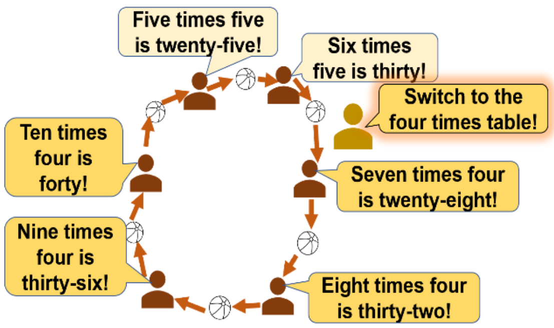
Demonstrate the activity by acting as the caller to begin with, calling *‘Switch!’* and *‘Freeze!’* a few times, then ask each group to choose a caller for their group.

1. The caller tells the group which ‘times table’ facts they are going to practise.	

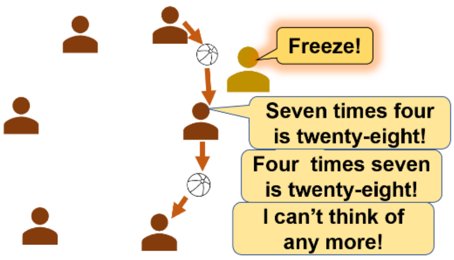
2. Learners throw the ball around the group, saying the next multiplication fact in the 'times table' as they throw.



3. The caller sometimes calls *'Switch!'* and a new 'times table' to practise. The next learner says the multiplication fact for the new 'table'.



4. Sometimes the caller calls *'Freeze!'* The learner who is holding the ball tells the group other multiplication and division facts that are related to the one they have just said.

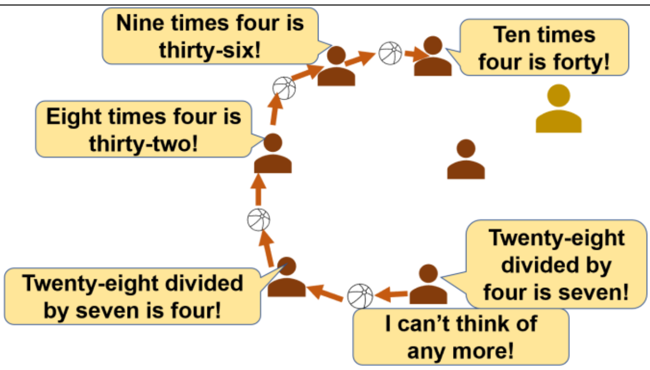


5. If they cannot think of all three related facts, the ball is passed to the next learner to have a go. When all the facts have been found, the next learner carries on with the 'times table'.

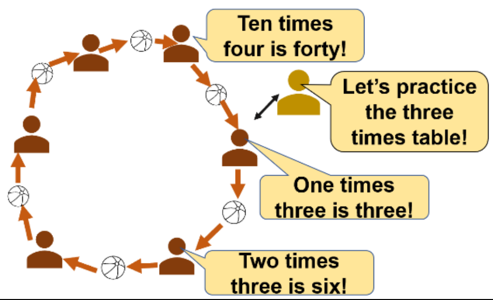
   (For example:

   7 x 4 = 28, 4 x 7 = 28,

   28 ÷4 = 7, 28 ÷ 7 = 4)



6. Learners stop when they reach 10 x __. Another learner becomes the caller and chooses a new 'times table'. The next learner starts at 1 x [the new 'times table'].



Assessment

Notice and note learners who:

- need to look at the multiplication square, even when practising two, five or ten 'times tables' – they may need more practice with earlier activities
- quickly recall the ‘times table’ facts when you are doing the activity with the whole class – make a new group with these learners and ask them to practise more challenging ‘times tables’ such as sevens and eights
- are very slow to say the multiplication facts or need help from others – if there are many of these learners, make a new group and help them to use the multiplication square as a prompt.

In practice

Miss Dube realised that, after learners had understood a new idea, they needed lots of practice to become fluent. She found it difficult to find interesting ways for them to practise.

When her class was learning multiplication facts, she used this activity for just a few minutes several times a week. She liked the fact that every learner had to take part and didn’t always know what the next question would be. They had to do a lot of thinking!

As learners became better at recalling facts, they played the game more quickly and she set them the challenge of seeing how many throws and answers they could fit into one minute.

When higher-attaining learners needed a change of activity, she told them to throw the ball to any learner in the group which made the activity even more challenging!

1.2: Multiplication square quiz

Aim

This activity helps learners connect multiplication and division facts. It also helps them remember the different words that may be used for a multiplication or a division calculation.

What the learners will do

Learners work in pairs. One learner has a multiplication square. This learner asks their partner to tell them the answer to a multiplication or a division question and checks the answer on their multiplication square. Learners take turns to ask and answer questions. They keep a score of their correct answers.

Resources

You will need:

- a large multiplication square (**Resource A Multiplication square**, page 35) where everyone can see it
- a supply of counters.

Each pair of learners will need:

- a multiplication square (**Resource A Multiplication square**, page 35) – they will already have written one in their books if they have completed Module 4
- a supply of counters for those who need them.

Activity

Steps 1 to 5 are teacher led. You will demonstrate on the class multiplication square and learners will copy you.

This will help learners to understand how to find multiplication and division facts on their own multiplication squares. Check that all learners point to the numbers on their squares in this part of the session.

For most of the session, learners work in pairs asking each other questions and checking the answers for themselves (steps 6 and 7).

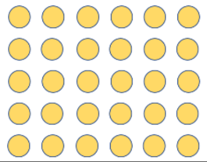
1. Ask learners to copy everything you do on your multiplication square on their own square.
Say that first you are all going to find the answer to 5 x 6. Place a finger on one hand on the 5 row and a finger on the other hand on the 6 column.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

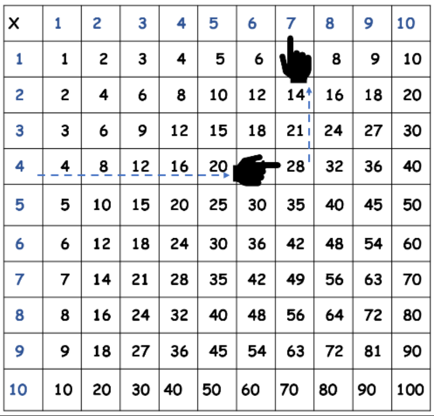
2. Slide the two fingers across the row and down the column until they meet. What is the number on the square where the two numbers meet?

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	[illegible]	28	32	36	40
5	5	10	15	20	[illegible]	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

3. Ask a volunteer to check the answer using a 5 x 6 array of counters.

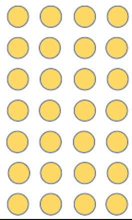


4. Now say you are going to find the answer to 28 ÷ 4. Slide your finger across the 4 row until you find 28. With the other hand, slide your finger up to the top of the column to find the answer.



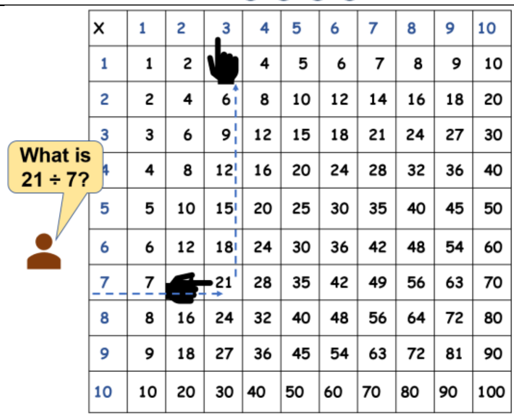
x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

5. Ask a volunteer to check the answer by making rows of 4 with the counters until there are 28 counters. Ask how many rows they have made.



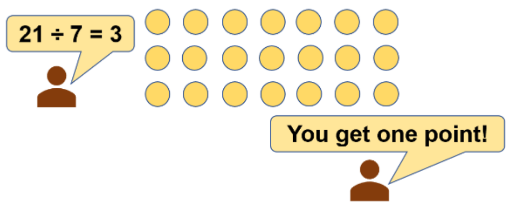
6. In pairs, learners take turns to ask and answer multiplication and division questions.

   One learner asks the question and checks their partner's answer on the multiplication square.



x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

7. The other learner answers the question. If they can answer without using counters, they get 2 points. If they use counters and find the correct answer, they get one point. The player with the most points at the end of the session wins.



Assessment

While learners are asking and answering questions, move around the room. Ask them to think of other ways to ask multiplication and division questions. For example, they could say:

- *‘What is 27 shared between 9?’*
- *‘How many equal groups can you make from 15 children?’*
- *‘How many lots of 7 make 35?’*
- *‘How many ears do 7 children have?’*

Learners who use many different words and are able to think of real-life situations are developing a really good sense of number.

In practice

Mr Muleya found that this activity took quite a long time to demonstrate, but it was time well spent.

Some learners did not have a multiplication square in their books as they had been absent for those lessons. He asked some higher-attaining learners to make copies of the multiplication square for these learners to stick in their books. This was also a good opportunity for the older learners to reinforce their own knowledge!

Mr Muleya knew that many learners find it difficult to remember all the facts about multiplication and division. He was very happy for learners to have the multiplication square in their books, so that they could look up and check multiplication and division facts when they needed to. This way, even if they could not remember their ‘times tables’, they could still solve multiplication and division problems.

1.3: Multiplication and division Bingo

Aim

This activity helps learners improve recall of multiplication and division facts.

What the learners will do

Learners work in pairs. They choose six numbers to write in their 2 x 3 grid. When the caller asks a multiplication or division fact for which they have the answer on their grid, they cross the number off. When they have crossed off all their numbers they call, *‘BINGO!’* The caller checks they have crossed the correct numbers off.

Resources

You will need:

- to prepare a list of multiplication and division facts to call out. Try to include a variety of different formats and vocabulary. Make sure the total does not fall outside of the range of numbers you have asked learners to choose from.

Each pair of learners will need:

- to draw a 3 x 2 grid (see step 1 below). This can be in their books, on the ground with a stick, or on the classroom floor with chalk.

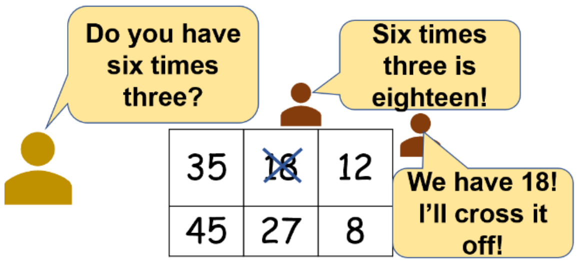
Activity

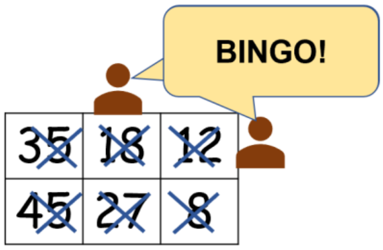
This is a teacher-led activity, with learners working in pairs. As they decide whether they can cross off a number, it is important to allow learners to talk to each other. Talking helps them to learn.

You might like to model the activity by asking for two volunteers to choose numbers to write in a grid on the board. As you read out multiplication or division questions that fit the numbers they have written, they cross off the numbers.

Make sure learners can see a multiplication square so that they can find the answer if they don't know or cannot remember it.

<table>
<tr>
<td>1. Ask learners, in pairs, to choose six numbers from a range that you write on the board (for example, 1–50) and to write each number in one of the boxes on their grid.</td>
<td>
<table>
<tr><td>35</td><td>18</td><td>12</td></tr>
<tr><td>45</td><td>27</td><td>8</td></tr>
</table>
</td>
</tr>
<tr>
<td>2. Read one of the multiplication questions that you have prepared. Learners decide the answer together and, if they have it, take turns to cross the number off.

Note: Keep a record of which questions you have read out so you can check that learners have crossed off the correct numbers!</td>
<td>
</td>
</tr>
<tr>
<td>3. When a pair of learners have crossed off all their numbers, they call, 'BINGO!' If they have crossed off the correct numbers, they are the winners.

You may like to carry on the game until you have several winners.</td>
<td>
</td>
</tr>
<tr>
<td colspan="2">Repeat the game, this time asking division questions. (For division, the range of numbers from which learners can choose must be reduced to 1–10). After this, if time allows, learners could play the game in groups of six, with a higher-attaining learner acting as caller. The caller could find the multiplication or division fact using a multiplication square. Alternatively, you could prepare multiplication and division sheets for the caller.</td>
</tr>
</table>

Assessment

As they play Bingo, notice the different ways that learners use to find the answer. Do they:

- ‘just know’ the answer
- look at the multiplication square
- draw dots to make an array then count the dots?

Notice the numbers that learners choose for their grid. Do they:

- choose random numbers
- choose bigger numbers for the multiplication game and smaller numbers for the division game
- choose numbers that have more factors?

Noticing and noting these things will tell you a lot about how well learners understand the way that numbers work together.

In practice

Mrs Phiri's class enjoyed this activity so much that they asked if they could play the game at home. Mrs Phiri liked this idea because it would give learners lots of practice with their number facts.

She decided to set up a Numeracy Library. Learners could borrow the resources they needed for games, but they had to sign them out in her Library Notebook and had to agree a date on which they would return them. Of course, to begin with her library was empty. To start with, she prepared one or two bags of multiplication facts where the answer was 50 or less, so that learners could play multiplication Bingo.

Learners were keen to borrow these, and to tell her what other games they would like to see in the Numeracy Library.

Unit 2: Multiplying bigger numbers

Contents

Introduction

In this unit, learners are introduced to the grid method of multiplication, supported by place value counters. When their understanding of this method is secure, they progress to multiplying larger numbers without counters. It is important that the activities are not rushed; learners will need plenty of practice with each activity before moving on to the next one.

Key words and phrases

- **product** – the answer when two numbers are multiplied

2.1: Using the 'grid method' and place value counters

Aim

This activity introduces learners to multiplication of a two-digit number by a single-digit number using a grid. It helps them understand that they are working with tens and ones when multiplying bigger numbers.

What the learners will do

Learners work in small groups (ideally pairs, if you have enough place value counters). They use place value counters and a grid to multiply bigger numbers.

Resources

You will need:

- at least 12 large place value counters to represent 'ones' and at least 20 to represent 'tens'
- something with which to stick counters on the board
- a collection of 'tens' and 'ones' place value counters for learners to use if they do not have enough.

Each small group or pair of learners will need:

- place value counters to represent 'ones' and 'tens', about 20 of each
- somewhere to draw a multiplication grid – this could be on the classroom floor with chalk, outside in the dust or on a large sheet of paper. (**Resource B Multiplication grids**, page 36)
- four sets of 1–6 number cards, mixed in a pile (**Resource C Number cards**, page 37)
- one number card with '0' written on (**Resource C Number cards**, page 37).

Each learner will also need:

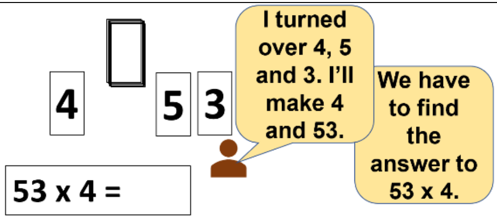
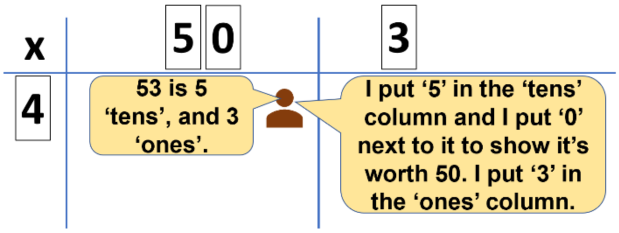
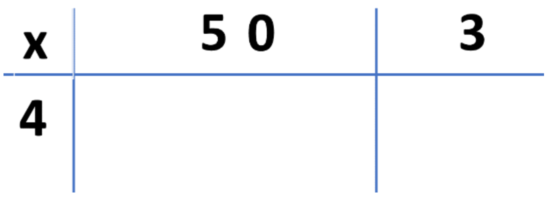
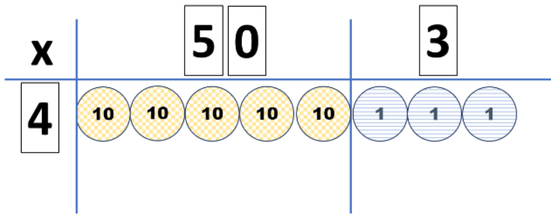
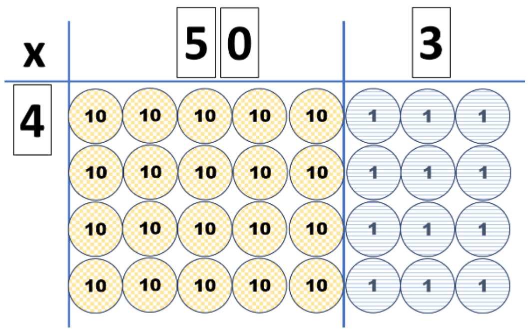
- an exercise book and pen.

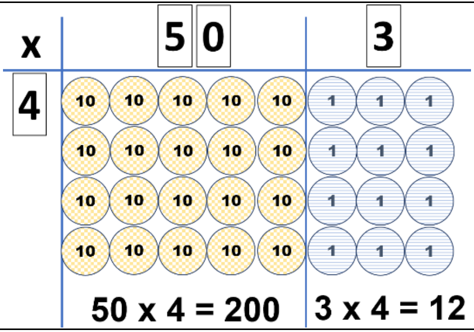
Activity

Before learners begin to work independently, show them how to multiply using the 'grid method'. Draw a grid on the board and stick the counters on it so everyone can see what you are doing.

As you talk through a second example, ask learners to copy you using their own place value counters and grid.

Learners then try some examples for themselves in their small groups or pairs.

1. Learner 1 turns over three cards. They use these to make a single-digit number and a two-digit number. Both learners write the calculation in their books.	
2. Learner 1 places the single number card in the row below the 'x' in their grid. They put the 'tens' number card in the column next to the 'x' on the grid and add their '0' to show the value of that part of the number. They put the 'ones' number card in the next column.	
3. Both learners draw a grid in their books and write the numbers in the correct places.	
4. Learner 2 makes the two-digit number on the grid using place value counters. (If they do not have enough place value counters, they borrow some from the pile at the front of the classroom.)	
5. Learner 2 repeats step 4 according to the number by which the two-digit number needs to be multiplied.	

6. Together, learners find the value of each column.	
7. Both learners write the calculations for each column in the correct boxes in their books.	x | 5 0 | 3 4 | 50 x 4 = 200 | 3 x 4 = 12
8. Finally, learners find the total of the two columns. They write the answer to the calculation in their books.	200 + 12 212 53 x 4 = 212
Learners put the number cards at the bottom of the pile and repeat steps 1–8. If learners are working in pairs, they change roles. If they are working in small groups, two new learners take the role of Learner 1 and Learner 2.	

Assessment

Notice and note learners who:

- count the place value counters in ones – they may need more practice with the activities in Unit 1 to help them with recalling ‘times tables’ facts
- grasp the concept very quickly. Gather these learners together and ask them to try using the grid method without using the place value counters. Make sure they know that if this is too difficult, they can continue to use the place value counters for a bit longer.

In practice

Mrs Sithole already had a large collection of place value counters that she had prepared for previous activities. She had stored them in the 'Numeracy Toolkit' she was building up for numeracy teaching.

Mrs Sithole found that learners understood the process of multiplication much better when the calculation was written in a grid. When doing the calculation 53 x 4, the grid helped them to remember that it was 50 being multiplied and not 5. Using the counters enabled learners to ‘see’ the result of each part of the calculation. Writing down a separate addition calculation to find the total made sense to them.

When Mrs Sithole's learners had understood this method of multiplication, she gave them just one calculation to do at the beginning of every lesson the following week. Learners were happy to draw place value counters in their books, and soon they were telling her that they could do multiplication calculations without them.

2.2: Multiplying bigger numbers without counters

Aim

Learners connect the grid method with formal vertical methods for multiplication of bigger numbers.

What the learners will do

Learners work in pairs. They take turns to ask each other questions as they use the grid method to answer multiplication questions. They record their answers in a vertical calculation format.

Resources

Each pair of learners will need:

- two sets of 1–9 number cards, mixed in a pile (**Resource C Number cards**, page 37).

Each learner will also need:

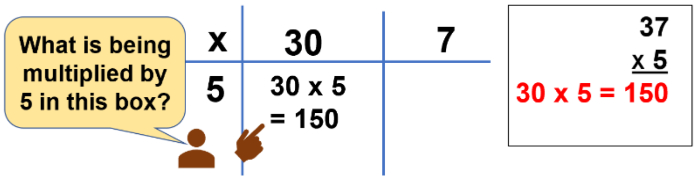
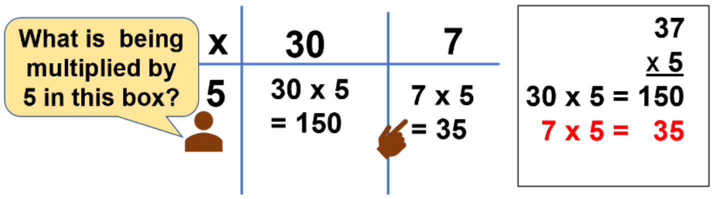
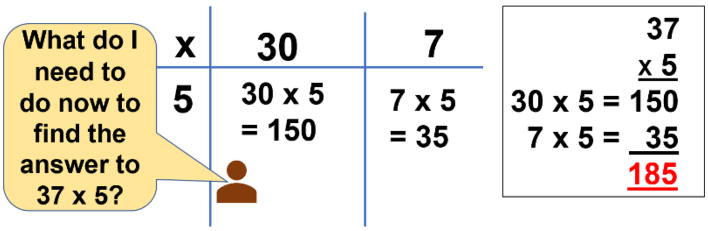
- an exercise book and pen.

Activity

Show learners how to do this activity. After you have done it once, ask for two volunteers to demonstrate a second example, before asking learners to work in pairs.

<table>
<tr>
<td>1. Learner 1 turns over three cards and makes a single-digit and a two-digit number. Both learners write the multiplication as a vertical calculation and draw a multiplication grid.</td>
<td>
<table>
<tr><td>x</td><td>30</td><td>7</td></tr>
<tr><td>5</td><td></td><td></td></tr>
</table>

37
x 5
</td>
</tr>
</table>

2. Learner 2 asks, *'What is being multiplied by __ in this box?'* (pointing to the first column) 3. Both learners write the multiplication and the answer in the box. They record what they have done under the vertical calculation.	
4. Learner 2 asks, *'What is being multiplied by __ in this box?'* (pointing to the second column) 5. Both learners write the multiplication and the answer in the box. They record what they have done under the vertical calculation.	
6. Learner 2 asks, *'What do I need to do now to find the answer to __ x __?'* 7. Both learners find the total of the multiplication of the 'tens' and the 'ones' and write the answer.	
8. Learners change roles and repeat steps 1–4 as often as time allows.	

Assessment

Notice and note learners who:

- forget that digits in the 'tens' column have a different value to digits in the 'ones' column and therefore have answers which are far too small – these learners should continue to use place value counters until they have a good understanding of the value of each digit
- can tell you roughly what the answer will be. Ask them how they know. Being able to make a sensible estimate is a sign that these learners have a good understanding of multiplication of bigger numbers. Ask them to see if they can use the grid method to multiply a single-digit number by a three-digit number.

In practice

Miss Marufu read this activity through carefully before she used it with her class. She knew it was important that she understood the purpose of all the steps, so that she could explain the activity clearly to her class. She realised that she needed to use exactly the same diagrams as in the activity. If she changed steps, or left any steps out, it would be more difficult for learners to understand what they were doing.

She made sure that learners were talking to each other during the activity to practise their mathematical language, so that later on they could read and understand problems set in real-life contexts.

The classroom was noisy. Learners were all talking about how to divide. This pleased her very much because she knew her class was learning.

2.3: Multiplying two-digit by two-digit numbers

Aim

When learners have a good understanding of previous activities, they extend their understanding of the grid method and formal methods to multiplication of two-digit by two-digit numbers. They develop an awareness of the size of the answer when multiplying bigger numbers.

What the learners will do

Learners work in groups of up to six, taking turns to choose cards and put place value counters in their grid. They use the grid to help them multiply two two-digit numbers, writing the vertical calculation in their books as they do so. When they feel confident to do so, they stop using the place value counters but continue to use the grid to help them with the calculations.

Resources

You will need:

- at least 72 large place value counters to represent ‘ones’ and at least 60 to represent ‘tens’
- something with which to stick counters on the board
- a collection of ‘tens’ and ‘ones’ place value counters for learners to use if they do not have enough.

Each group of up to six learners will need:

- place value counters to represent ‘ones’ and ‘tens’ (ask each learner to take ten ‘ones’ and ten ‘tens’)

- somewhere to draw a grid – this could be on the classroom floor with chalk, outside in the dust or on a large sheet of paper (**Resource B Multiplication grids**, page 36)
- four sets of 2–6 number cards, mixed together in a pile (**Resource C Number cards**, page 37)
- two number cards with '0' written on (**Resource C Number cards**, page 37).

Each learner will also need:

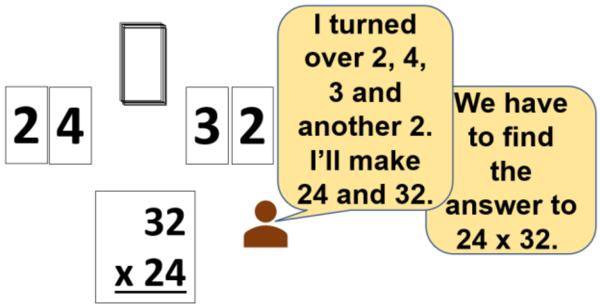
- an exercise book and pen.

Activity

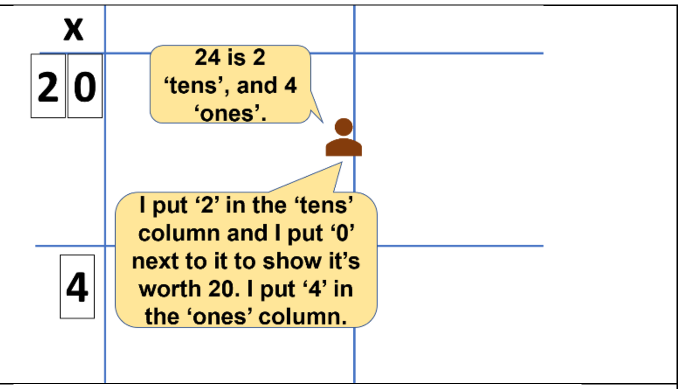
Learners should only progress to multiplying two-digit by two-digit numbers when you are sure they are confident and accurate multiplying single-digit numbers by two-digit and three-digit numbers.

Ask learners to copy you with their place value counters and grid as you talk through the example below. Then provide all learners with the calculation 35 x 15. Ask them to work through it in their groups. When all groups have completed the calculation using place value counters and written what they have done, check their working by doing the calculation together on the board.

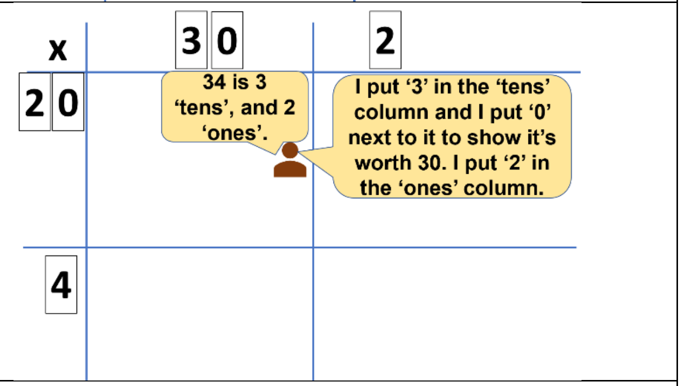
After this, learners should have a good concept of the size of the product when two two-digit numbers are multiplied. Tell them that if they feel confident to do so, they can stop using the place value counters for the group work and just record the steps in a grid and as a vertical calculation in their books.

1. Learner 1 turns over four cards. They use these to make two two-digit numbers. All learners write the calculation in their books as a vertical multiplication, so the 'tens' of each number are in the same column below each other and the 'ones' of each number are in the same column below each other, lining up the two numbers carefully beneath each other.	

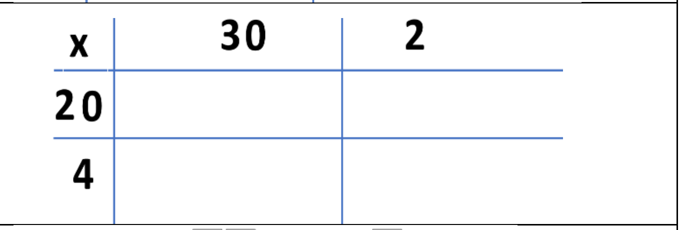
2. Learner 2 places the card for the 'tens' digit of the first number in the row below the 'x' in their grid. They put one of the '0' number cards next to it to show the value of that part of the number. In the same column, they put the 'ones' card in the box below.



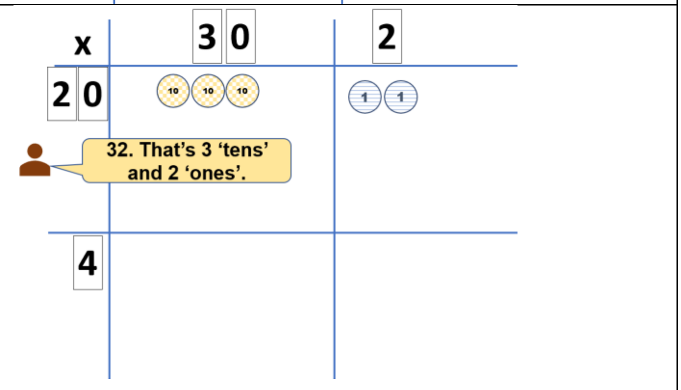
3. They put the 'tens' number card of the second number in the column to the right of the 'x' on the grid and put '0' next to it to show the value of that part of the number. They put the 'ones' number card in the next column.



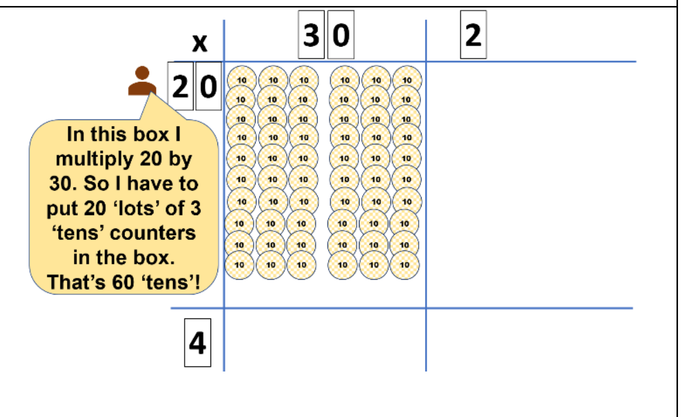
4. All learners draw a grid in their books and write the numbers in the correct places.

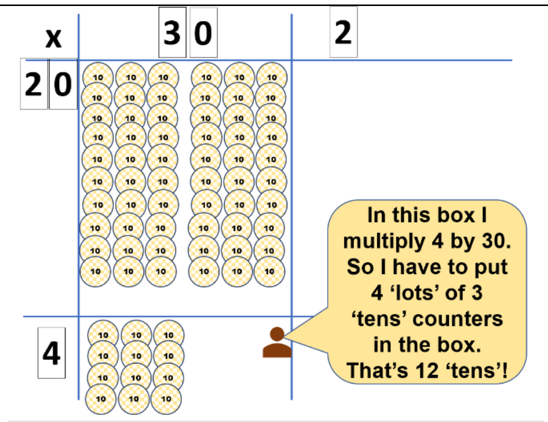


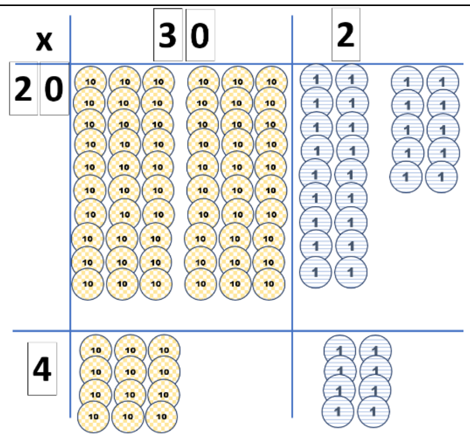
5. Learner 3 puts 'tens' and 'ones' counters below the numbers across the top of the grid to represent the number that is to be multiplied.



6. Learner 4 points to the top left-hand box and says, *'In this box I multiply __ by __.'* They put the correct number of counters in the box.



<table>
<tr>
<td>7. All learners write the calculation in their books.</td>
<td>
<table>
<tr><td>x</td><td>30</td><td>2</td></tr>
<tr><td>20</td><td>20 x 30 = 600</td><td></td></tr>
<tr><td>4</td><td></td><td></td></tr>
</table>
32
x 24
20 x 30 = 600
</td>
</tr>
<tr>
<td>8. Learner 5 points to the box below and says, 'In this box I multiply __ by __.' They put the correct number of counters in the box.</td>
<td>


</td>
</tr>
<tr>
<td>9. All learners write the calculation in their books.</td>
<td>
<table>
<tr><td>x</td><td>30</td><td>2</td></tr>
<tr><td>20</td><td>20 x 30 = 600</td><td></td></tr>
<tr><td>4</td><td>4 x 30 = 120</td><td></td></tr>
</table>
32
x 24
20 x 30 = 600
4 x 30 = 120
</td>
</tr>
<tr>
<td>10. Learners continue to take turns to fill the boxes with the correct number of counters.</td>
<td>


</td>
</tr>
<tr>
<td>11. Learners work through all the boxes, writing in their books at each stage.</td>
<td>
<table>
<tr><td>x</td><td>30</td><td>2</td></tr>
<tr><td>20</td><td>20 x 30 = 600</td><td>20 x 2 = 40</td></tr>
<tr><td>4</td><td>4 x 30 = 120</td><td>4 x 2 = 8</td></tr>
</table>
32
x 24
20 x 30 = 600
4 x 30 = 120
20 x 2 = 40
4 x 2 = 8
</td>
</tr>
</table>

12. Learners all find the total and write the answer between the two lines at the bottom of the vertical calculation in their books.	32 x 24 20 x 30 = 600 4 x 30 = 120 20 x 2 = 40 4 x 2 = 8 768

Assessment

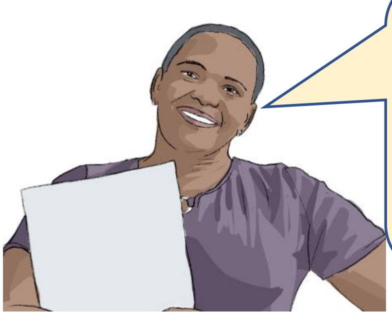
Learners should not attempt this activity until they have a very good understanding of multiplying two-digit or three-digit numbers by single digits. When you notice some learners are ready, you could use the activity with small groups while the rest of the class continues to work with earlier activities.

Watch carefully as learners use the place value counters when working through the examples. Give extra support to learners who find multiplying the ‘tens’ digits difficult (they may not understand that the answer is likely to be in the hundreds). Learners who seem to have understood the ‘grid method’ should be encouraged to work without counters.

In practice

Mr Khumalo found that, when he taught learners the formal method for multiplying two two-digit numbers, learners got in a muddle. They had so many steps to remember that they didn’t always remember them correctly. Although there were many steps in this activity, learners understood each one. They were able to work out what to do next without having to memorise all the steps.

One thing that he found very interesting was that learners no longer offered him answers that were too small e.g. 32 x 24 = 192. They used to get this incorrect answer because they had multiplied 32 by 2 instead of by 20. Not only was their work using the grid method more accurate, but – over time – learners knew what size of answer to expect and were spotting their own mistakes.



I found that I had to explain to my learners that the shape and size of the multiplication grid depended on the size of the number to be multiplied. Some of my learners thought that it always had to be 2 x 1 or 2 x 2, and didn't know what to do with the calculation 321 x 6!

Unit 3: Dividing bigger numbers

Contents

Introduction

In this unit, learners are introduced to formal methods of division, including exchange, with the support of place value counters. Learners are given the opportunity to use multiplication and division to answer real-life problems.

Key words and phrases

- **efficient method** – a method by which learners get to the correct answer quickly and accurately
- **exchange** – converting a number from one place value unit to another (for example exchanging one ‘ten’ for ten ‘ones', or one ‘hundred’ for ten ‘tens’

3.1: Using place value counters and a division chart

Aim

This activity uses place value counters to introduce learners to the formal method for division of bigger numbers, including remainders.

What the learners will do

Learners work in groups of four. They divide two-digit numbers by single-digit numbers by putting place value counters into equal groups. They write the calculation in the form of a division chart.

Resources

You will need:

- place value counters to represent ‘ones’ and ‘tens’
- something with which to stick place value counters on the board
- to write the division calculations in *Figure 1* on the board. (These numbers have been chosen because they do not require exchange from ‘tens’ to ‘ones’.)

$3\overline{)69}$	$5\overline{)58}$
$4\overline{)84}$	$3\overline{)37}$
$2\overline{)68}$	$4\overline{)81}$
$3\overline{)94}$	$3\overline{)92}$
$4\overline{)47}$	$5\overline{)50}$

Figure 1

Each group of four learners will need:

- place value counters to represent ‘ones’ and ‘tens’
- somewhere to draw a division chart (**Resource D Division charts**, page 38). This could be on the classroom floor with chalk, outside in the dust, or on a large sheet of paper.

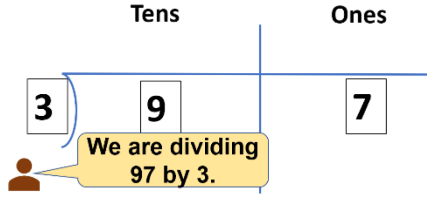
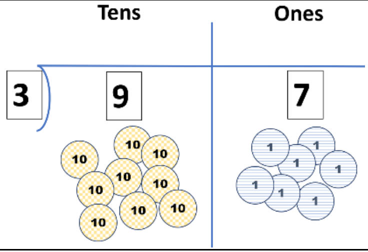
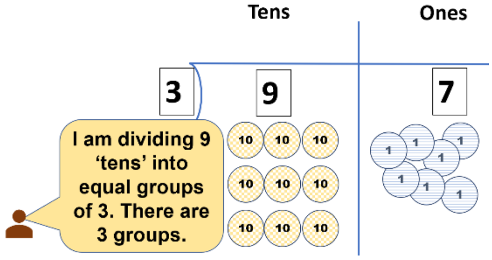
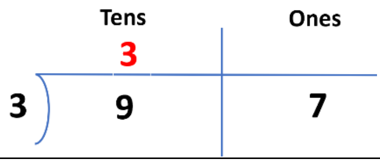
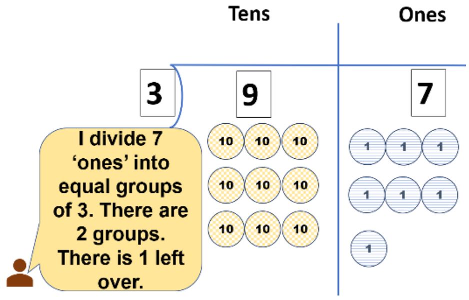
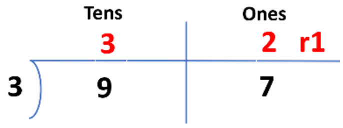
Each learner will also need:

- an exercise book and pen.

Activity

Before learners work independently, draw a division chart (**Resource D Division charts**, page 38) on the board and use the example below to demonstrate the activity. Ask learners to copy each step using their place value counters and division chart.

Next, ask learners to work in groups of four to answer the questions written on the board.

1. Learner 1 chooses a calculation from the board, finds the correct number cards and places them on the division chart as shown. They say, *'We are dividing __ by __.'*	
2. All learners write the division sentence in their books.	$97 \div 3 =$
3. Learner 2 puts the correct number of 'tens' and 'ones' below the number cards on the division chart.	
4. Learner 3 divides the 'tens' into equal groups, saying, *'I am dividing __ 'tens' into equal groups of __. There are __ groups.'*	
5. All learners draw a division chart under the sentence in their books. They write the answer to step 4 above the line in the 'tens' column.	
6. Learner 4 divides the 'ones' into equal groups, saying, '*I divide __ 'ones' into equal groups of __. There are __ groups.'* If there are any left over, they say how many.	
7. All learners write the answer to step 6 above the line in the 'ones' column. They read the number sentence to each other. *'__ divided by __equals __remainder__.'*	$97 \div 3 = 32\ r\ 1$ 
The number cards and counters are taken off the division chart and learners choose another question. Each learner takes a new role.	

Assessment

Listen carefully to learners as they do this activity. This will tell you a lot about how well they understand the concept of division of bigger numbers and remainders. Some learners will need a lot of experience of working with counters before they are able to work without them. Only move on to **Activity 3.2** when you are sure learners are confident with the concepts introduced here.

In practice

Mrs Simango had thought that the formal method for division was straightforward. She usually taught it to her learners without using place value counters. By the end of the lesson, she found that many learners could remember how to do a division calculation, but couldn't explain what they were doing. When she gave her class division word problems, very few learners understood what they needed to do.

After using this activity, Mrs Simango found that most learners understood what division meant, and could explain the meaning of all the numbers in the calculation. She realised that counters helped learners to 'see' the calculation better. When they understood what they were doing, they told her that they didn't need counters any more. However, she stored her counters carefully, as she could see that she would need them again when learners started to work on the division of larger numbers.

3.2: Dividing bigger numbers, including exchange

Aim

This activity builds on **Activity 3.1**. It shows learners what to do when there is an unequal number of groups of ten in the calculation.

What the learners will do

Learners work in groups of five. They divide two-digit numbers by single-digit numbers, exchanging 'tens' for 'ones' when necessary. They write the calculation in the form of a division chart, recording each step as they move place value counters on the division chart.

Resources

You will need:

- place value counters to represent 'ones' and 'tens'
- something with which to stick place value counters on the board.

Each group of learners will need:

- place value counters to represent 'ones' and 'tens'
- two sets of 1–9 number cards (**Resource C Number cards**, page 37)

- somewhere to draw a division chart (**Resource D Division charts**, page 38). This could be on the classroom floor with chalk, outside in the dust, or on a large sheet of paper.

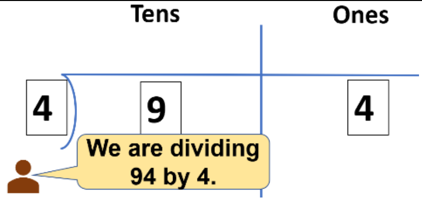
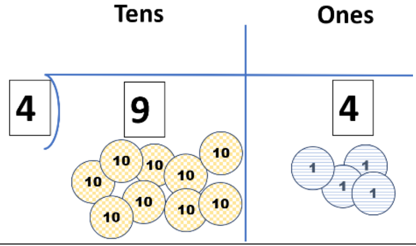
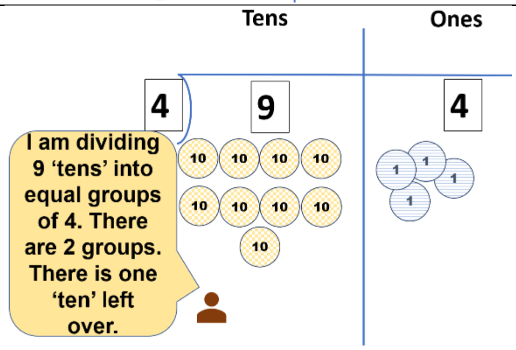
Each learner will also need:

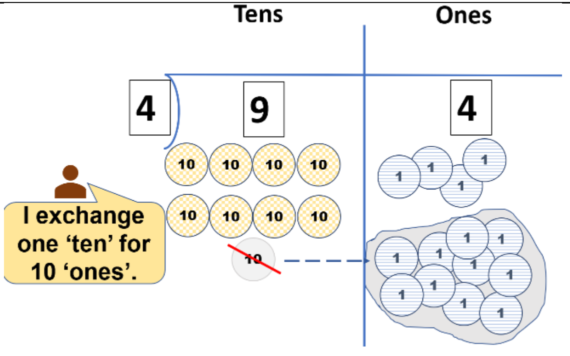
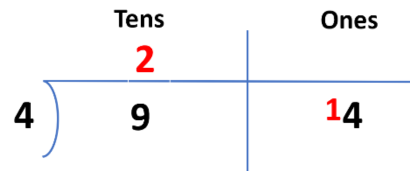
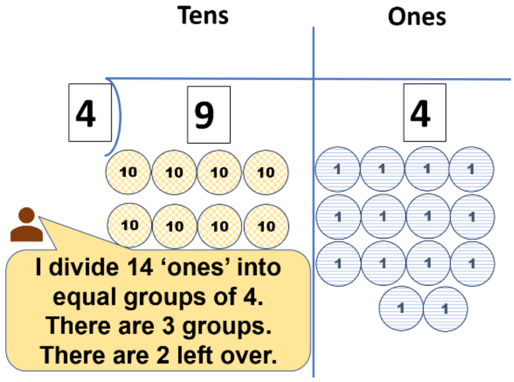
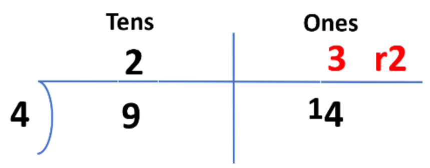
- an exercise book and pen.

Activity

Before learners work independently, draw a division chart (**Resource D Division charts**, page 38) on the board and use the example below to demonstrate the activity. Ask learners to copy each step using their place value counters and division chart.

After this, ask learners to work in groups.

1. Learner 1 turns over three cards. They arrange them to make a two-digit number and a single-digit number. (If possible, they should choose a number between 2 and 5 for the single-digit number.) They arrange the cards on the division chart as shown. They say, *'We are dividing __ by __'.*	
2. All learners write the division sentence in their books.	94 ÷ 4 =
3. Learner 2 puts the correct number of 'tens' and 'ones' below the number cards on the division chart.	
4. Learner 3 divides the 'tens' into equal groups, saying, *'I am dividing __ 'tens' into equal groups of __. There are __ groups. There is/are__ ten/s left over.'*	

5. Learner 4 exchanges each ‘ten’ that is left over for ten ‘ones’. They say, *‘I exchange__ ‘tens’ for __ ‘ones’.’*	
6. All learners draw a division chart under the calculation in their books. They write the number of groups of ‘ten’ above the line in the ‘tens’ column. They write the number of ‘tens’ they have exchanged beside the digit in the ‘ones’ column to show there are now 14 ‘ones’ in the ‘ones’ column.	
7. Learner 5 divides the ‘ones’ into equal groups, saying, *‘I divide __ ‘ones’ into equal groups of __. There are __ groups.’* If there are any left over, they say how many.	
8. In their books, all learners write the answer to step 7 above the line in the ‘ones’ column. They also write the answer to the number sentence. They read the number sentence to each other. *‘__ divided by __equals __remainder__.’*	**94 ÷ 4 = 23 r2** 
9. The number cards and counters are taken off the division chart and learners repeat the process as often as time allows. Each learner takes a new role.	

Assessment

As they work in groups, notice and note learners who:

- begin dividing the 'ones' first – formal division is the only process that begins on the left-hand side and so is confusing to some
- forget to exchange 'tens' for 'ones'
- only exchange one 'ten', even if more than one ten is left over – learners may be remembering exchange in subtraction where only one 'ten' is exchanged
- think they always have to exchange or always have a remainder – these learners are following a process rather than understanding the mathematics; spend some time working with these learners in a small group, encouraging them to explain what they are doing and why
- quickly and accurately predict the answer before they have worked through the question with place value counters. Group these learners together and ask them to answer questions without the place value counters. Some may be ready to tackle three-digit by single-digit division, with and without exchange.

In practice

Miss Sibanda had been using activities from the modules for some time and had begun to think about the way in which they were written. Instructions for demonstrating activities were always very precise. If learners were asked a question, there was often only one correct answer, which was usually provided. This made her feel confident that she was using the activity correctly. Her learners always like getting the answer right.

However, she felt that she could treat learners' 'wrong' answers differently. Instead of searching for the 'right' answer by asking another learner, she began to ask learners who offered incorrect answers questions such as:

- *Can you tell me how you got that answer?*
- *Why do you think that?*
- *What made you decide to do it that way?*

The first time she tried this, she found that it gave her a better understanding of what the learner was thinking. Sometimes the learner had just made a mistake; sometimes it was a misunderstanding that Miss Sibanda could correct. She decided to ask her colleagues to help her to compile and share a bank of questions that would help them to explore 'wrong' answers without making learners feel that they had failed.

3.3: Solving real-life problems

Aim

Learners connect their learning about multiplication and division with real-life experiences.

Resources

You will need:

- a class 'bank' of counters and place value counters
- a multiplication square on the classroom wall (**Resource A Multiplication square**, page 35)
- to prepare enough problem cards so each group of six learners can have four.

Each group of six learners will need:

- four problem cards. These should be a mixture of multiplication and division problems and use a variety of language (**Resource E Multiplication and division problems**, page 39 – the problems are arranged so each row of four problems has a mixture of multiplication and division questions and uses a variety of language).

Each learner will also need:

- an exercise book and pen.

What the learners will do

Groups of six learners read their problem cards and decide whether they should use multiplication or division to solve each of them. They choose one problem to solve and work on it in pairs. When all the pairs in the group have finished, they compare their answers. If the answers are different, they decide which is correct and why.

Some learners may use place value counters and/or charts or grids to help them.

Activity

Make sure that there is at least one confident, accurate reader in each group of six learners, so that difficulties in reading do not stop learners from being able to solve the problems.

Before they begin, tell learners that they can use:

- any method they like to solve the problems
- drawings or jottings in their books or on scrap paper
- the multiplication square in their books or the one on the classroom wall
- the class set of counters and place value counters.

Remind them that the most efficient method might be different for each problem.

1.	As a group, learners read the problem cards one at a time. They decide together which ones can be solved using multiplication, and which ones need division.
2.	They decide together which problem they will work on first.
3.	Learners work in pairs to solve the problem. Each pair may use a different method, or they may all use the same method.
4.	When all three pairs of learners have solved the problem, they compare their work and explain how they found the answer. If the answers are different, they must decide, together, which answer is right and why.
5.	The group picks another card and repeats steps 2–4. If they worked with a division problem first, the next one should be a multiplication problem and vice versa.

Towards the end of the session, ask four pairs of learners who were using effective strategies to show the class how they answered their problems. Draw attention to the wide variety of words that are used for multiplication and division problems.

Assessment

As learners work in pairs, notice and note the resources and strategies that they use when solving different types of problems. If you notice that a number of your learners are finding a particular concept difficult (particularly the comparison multiplication problems, for example '3 times as many'), stop the class and talk through one of the problems together.

In practice

Miss Mudenda didn't use this activity until her learners were efficiently answering multiplication and division questions written in numbers.

She found that some learners were having difficulty reading the problems. She decided to talk to her colleague, Mrs Mlilo, to see whether her experience was the same.

Both teachers agreed that learners' literacy skills were holding back their numeracy learning. Some learners could not read the words in the questions. Others could read the words out loud, but did not know what they meant. Another group could read the problem, but did not know what they had to do to answer it.

They decided that, before doing this activity again, they would use a literacy lesson to explore the words in the questions. They thought they might try:

- looking through all the problems and identifying words that might be difficult, then writing these on the board and explaining what they mean
- putting learners in small, mixed-attainment groups that included at least one higher-attaining reader, to read some of the questions together, until everyone in the group could read the questions aloud
- asking learners, in the same groups, to discuss the meaning of any unfamiliar words in the questions.

They wondered whether other teachers had the same problem. They decided to ask colleagues about this at their next TPD meeting and to share their strategies for overcoming the problem.

Reflection

When you have completed this module, and used the activities in class, reflect on what you have learned from it. You can do this by yourself, but, if you have the opportunity, it is better to do so with other teachers in your school or cluster. Perhaps you can meet after school or set up a WhatsApp group to work with teachers some distance away.

By the end of Module 5, you and your learners will have taken part in many activities that have enabled your learners to understand the nature of numbers and the operations of addition, subtraction, multiplication and division.

Along the way, you will have noticed errors made by learners. Some of these may be simple mistakes in calculation. Others may be misconceptions – errors based on faulty thinking. Misconceptions, if not addressed as they occur, can prevent learners from making further progress with their numeracy learning.

- Think about three wrong answers that learners have given you recently. Were they mistakes or misconceptions? If misconceptions, how did you convince the learner that their thinking was incorrect?
- If you were supporting a trainee teacher, what advice would you give them on the effective teaching of multiplication and division?
- As a result of reading this module, will you make any changes to the way in which you teach formal methods of multiplication and division? If so, what will you do differently?

Resources

Resource A Multiplication square

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

(Shading not necessary, but may help learners follow the rows and columns.)

Resource B Multiplication grids

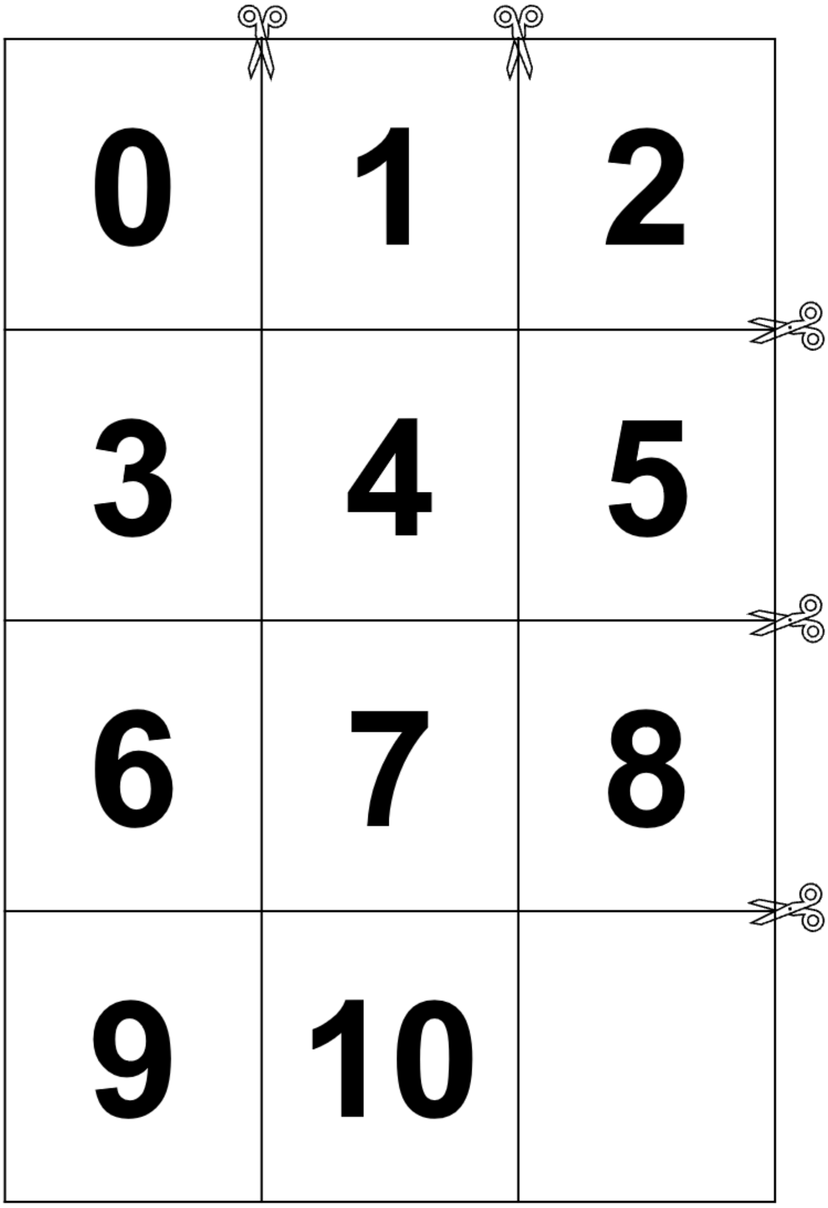
x		

x			

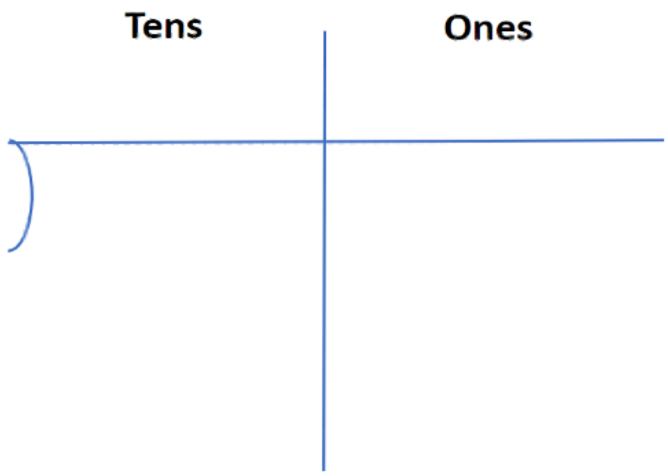
x		

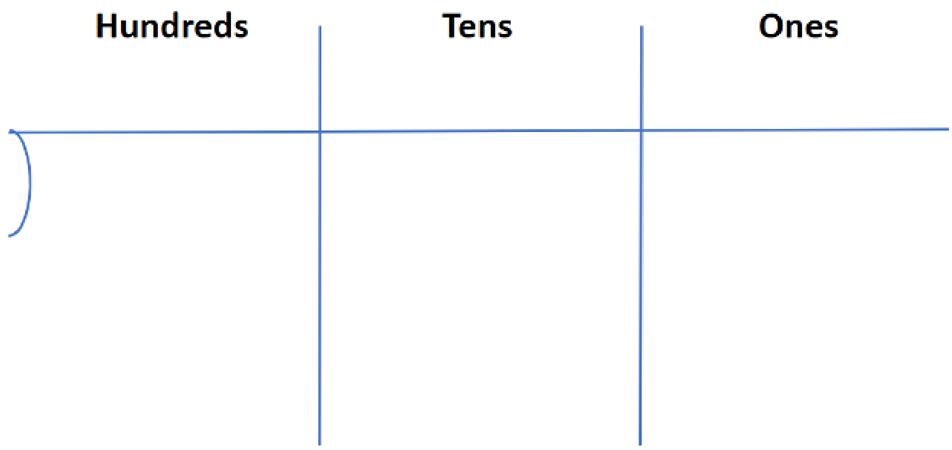
Resource C Number cards

You can make these cards by cutting a sheet of paper as shown and writing the numbers on the pieces, or you can cut up old cardboard boxes and write the numbers on those. Each card should be about 6 cm x 4 cm.



Resource D Division charts





Resource E Multiplication and division problems

Harriet has 28 cookies. She puts them on 4 plates. How many cookies are on each plate?	Charity walks 7 kilometres every day. How many kilometres does she walk in 5 days?	65 pencils are shared between 5 classes. How many pencils does each class get?	Mr Chandra has 7 cows. Mr Ncube has 3 times as many. How many does Mr Ncube have?
A baker packs 42 cakes in boxes of 6. How many boxes does she need?	Mrs Dube asks some children to put chairs into 8 rows of 6. How many chairs do they need?	Simba has 2 sisters. Chipo has 3 times as many sisters as Simba. How many sisters does Chipo have?	7 children share 84 apples. How many apples do they get each?
There are 30 learners in the class. 6 learners sit at each table. How many tables are needed?	Lois walks 5 kilometres to school. Tembi walks 3 times as far as Lois. How far does Tembi walk?	3 chickens lay 27 eggs in a month. How many eggs do they each lay?	Bus tickets cost $9. How much are 6 tickets?
How many legs are there on 32 donkeys?	It costs $8 to catch a bus to town. It costs 3 times more to get a taxi. How much is a taxi?	A teacher cuts a watermelon into 24 pieces. She gives 3 pieces to each learner. How many learners get watermelon?	Grandfather sold a goat for $36 and gave the money to his 6 grandchildren. How much money did they each get?
There are 56 children in each class of a school with 6 classes. How many children are there altogether?	28 learners sit on 7 benches. How many learners are on each bench?	Tinashe brings 6 counters to school. Beautiful brings 4 times as many. How many counters does Beautiful bring?	There are 18 donkeys. 3 donkeys pull each cart. How many carts are there altogether?
Mlindeli has 5 goats. He has 4 times as many chickens. How many chickens does he have?	Onions are sold for $2 per bag. How many bags can I buy with $12?	Cookies are sold in packs of 28. In one day, 7 packs of biscuits are sold. How many cookies are sold in 7 days?	Daniel shares 24 fish equally between 4 ponds. How many fish are in each pond?
Beauty breaks an orange into 15 segments and shares them equally between her 5 friends. How many segments does each friend get?	If a family eats 6 bananas every day, how many bananas do they eat in 7 days?	There are 2 shops in our village. The next village has 3 times more shops than our village. How many shops do they have?	Mercy plants 64 seeds in rows of 8. How many rows will she have altogether?

Each row has a mixture of multiplication and division problems and uses a variety of language. One row can be given to each group of learners.

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