Input-Output analysis and modelling with MARIO

Hands-on 4 – The Supply and Use Framework

Please, be aware that all the supporting materials required for this hands-on session is available on Zenodo at the following link: https://doi.org/10.5281/zenodo.8308515

Learning outcomes

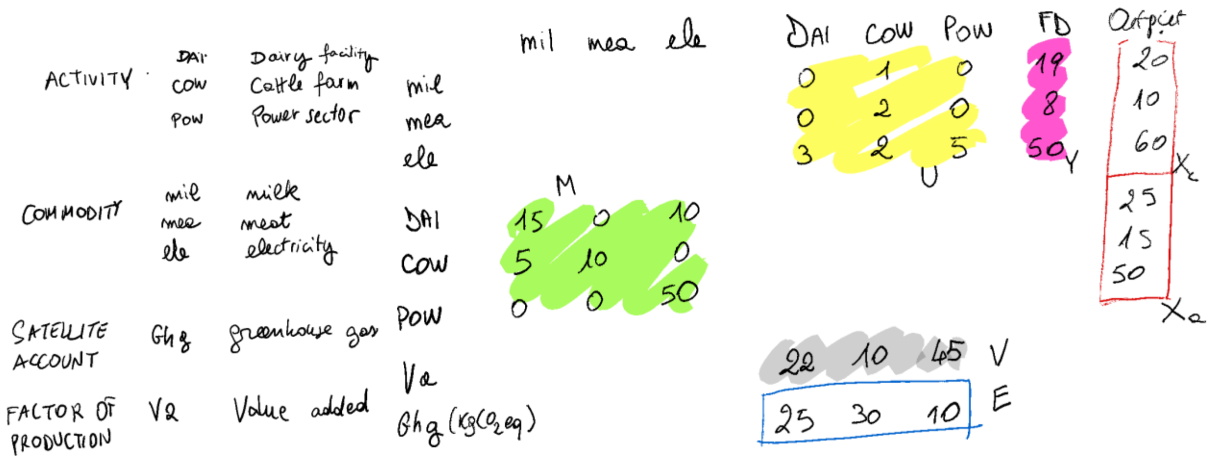
By the end of this exercise, you will learn how to:

1) Understand the significance of matrices in the supply and use framework.
2) Apply the Industry Based Technology Assumption model.
3) Apply the Product Based Technology Assumption model.

Important requirement

It is suggested to compute calculation for this exercise using just pen and paper as shown in the solution. Then, you can use Excel to double check and get more familiar with linear algebra and Excel.

The following supply and use system (measured in economic flows) is provided.



As you can notice the system is balanced.

1) Compute the matrix of technical and environmental coefficients.
2) Compute the market share matrix assuming an industry-based technology assumption.
3) Now compute the same assuming a product-based technology assumption.

SOLUTION

1) Compute the matrix of technical and environmental coefficients.

First of all, we want to obtain u, matrix of use coefficients: $u = U \hat{X}_a^{-1}$

The same can be applied to get the matrix of environmental transaction coefficients

$$v = V \hat{X}_a^{-1} \quad \text{and} \quad e = E \hat{X}_a^{-1}$$

$$u = \begin{vmatrix} \emptyset & 1 & \emptyset \\ \emptyset & 2 & \emptyset \\ 3 & 2 & 5 \end{vmatrix} \begin{vmatrix} 1/25 & \emptyset & \emptyset \\ \emptyset & 1/15 & \emptyset \\ \emptyset & \emptyset & 1/50 \end{vmatrix} = \begin{vmatrix} \emptyset & 1/15 & \emptyset \\ \emptyset & 2/15 & \emptyset \\ 3/25 & 2/15 & 1/10 \end{vmatrix}$$

$$= \begin{vmatrix} \emptyset & 0.07 & \emptyset \\ \emptyset & 0.13 & \emptyset \\ 0.12 & 0.13 & 0.10 \end{vmatrix}$$

$$v = \underset{[1\times3]}{\begin{vmatrix} 22 & 10 & 45 \end{vmatrix}} \underset{[3\times3]}{\begin{vmatrix} 1/25 & \emptyset & \emptyset \\ \emptyset & 1/15 & \emptyset \\ \emptyset & \emptyset & 1/50 \end{vmatrix}} = \underset{[1\times3]}{\begin{vmatrix} 22/25 & 2/3 & 9/10 \end{vmatrix}} = \begin{vmatrix} 0.88 & 0.67 & 0.90 \end{vmatrix}$$

$$e = \begin{vmatrix} 25 & 30 & 10 \end{vmatrix} \begin{vmatrix} 1/25 & \emptyset & \emptyset \\ \emptyset & 1/15 & \emptyset \\ \emptyset & \emptyset & 1/50 \end{vmatrix} = \begin{vmatrix} 1 & 2 & \underset{0.20}{1/5} \end{vmatrix}$$

Now we have all the "use-side" coefficients.
Still nothing has been said on the "make-side".

2) Compute the market share matrix assuming an industry-based technology assumption.

Are we assuming that everytime a commodity is demanded the activities who produce that commodity produce them - using their "use-structure" with some proportions based on each activity's market share?
In this way we would assume an industry-based assumption. $M = M_I = M \hat{X}_c^{-1}$

$$M_I = \begin{vmatrix} 15 & \emptyset & 10 \\ 5 & 10 & \emptyset \\ \emptyset & \emptyset & 50 \end{vmatrix} \begin{vmatrix} \frac{1}{20} & \emptyset & \emptyset \\ \emptyset & 1/10 & \emptyset \\ \emptyset & \emptyset & 1/60 \end{vmatrix} = \begin{vmatrix} 3/4 & \emptyset & 1/6 \\ 1/4 & 1 & \emptyset \\ \emptyset & \emptyset & 5/6 \end{vmatrix} = \begin{vmatrix} .75 & \emptyset & .17 \\ .25 & 1 & \emptyset \\ \emptyset & \emptyset & .83 \end{vmatrix}$$

This gives us a complete description of the economic system that we can use as a model.

3) Now compute the same assuming a product-based technology assumption.

However, we can assume a different relation between activities and commodities.

We can say that every time an activity produces an output, it will do it supplying a fix proportions of commodities.

Take the supply matrix (S) which is the transpose of the original make matrix ($S = M'$). Taking one unit of output of each activity, how is the value distributed among commodities? $s = S\hat{X}_a^{-1}$

$$s = \begin{vmatrix} 15 & 5 & \emptyset \\ \emptyset & 10 & \emptyset \\ 10 & \emptyset & 50 \end{vmatrix} \begin{vmatrix} 1/25 & \emptyset & \emptyset \\ \emptyset & 1/15 & \emptyset \\ \emptyset & \emptyset & 1/50 \end{vmatrix} = \begin{vmatrix} 3/5 & 1/3 & \emptyset \\ \emptyset & 2/3 & \emptyset \\ 2/5 & \emptyset & 1 \end{vmatrix} = \begin{vmatrix} .60 & .33 & \emptyset \\ \emptyset & .67 & \emptyset \\ .40 & \emptyset & 1 \end{vmatrix}$$

But this is not telling me how commodities should be produced. We are looking for the "inverse" of this information, in other words: how commodities can be provided by activities considering that in some cases some activity output of the main producer can be saved by some other activity which is producing its main commodity as a co-product.

In this case we would be using a product-based assumption: $M = M_p = S^{-1}$

This is a kind of "special" market share matrix.

We need to invert S. We need to compute its determinant (provided) and adj(S) $\rightarrow S^{-1} = \frac{1}{|\det(S)|} \, adj(S)$

$\det(S) = 2/5$

Compute determinant

$$S = \begin{vmatrix} 3/5 & 1/3 & 0 \\ 0 & 2/3 & 0 \\ 2/5 & 0 & 1 \end{vmatrix} \qquad \begin{vmatrix} 3/5 & 0 & 2/5 \\ 1/3 & 2/3 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

All the minors

$$\begin{vmatrix} 2/3 & 0 \\ 0 & 1 \end{vmatrix} = \frac{2}{3} \qquad \begin{vmatrix} 1/3 & 0 \\ 0 & 1 \end{vmatrix} = 1/3 \qquad \begin{vmatrix} 1/3 & 2/3 \\ 0 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & 2/5 \\ 0 & 1 \end{vmatrix} = 0 \qquad \begin{vmatrix} 3/5 & 2/5 \\ 0 & 1 \end{vmatrix} = \frac{3}{5} \qquad \begin{vmatrix} 3/5 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & 2/5 \\ 2/3 & 1 \end{vmatrix} = -\frac{2}{5} \cdot \frac{2}{3} = -\frac{4}{15} \qquad \begin{vmatrix} 3/5 & 2/5 \\ 1/3 & 0 \end{vmatrix} = -\frac{2}{5} \cdot \frac{1}{3} = -2/15 \qquad \begin{vmatrix} 3/5 & 0 \\ 1/3 & 2/3 \end{vmatrix} = \frac{2}{3} \cdot \frac{3}{5} = \frac{6}{15}$$

$$\mathrm{Adj}(s) = \begin{vmatrix} 2/3 & 1/3 & 0 \\ 0 & 1/5 & 0 \\ -4/15 & -\frac{2}{15} & \frac{6}{15} \end{vmatrix} \times \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$= \begin{vmatrix} 2/3 & -1/3 & 0 \\ 0 & 1/5 & 0 \\ -4/15 & 2/15 & 6/15 \end{vmatrix}$$

Now we can obtain the inverse of the starting matrix s.

$$S^{-1} = \frac{1}{\underbrace{|\det(s)|}_{\to\ 2/5}} \cdot \mathrm{adj}(s)$$

$$\mathrm{adj}(s) = \begin{vmatrix} 2/3 & -1/3 & \emptyset \\ \emptyset & 3/5 & \emptyset \\ 4/15 & 2/15 & 6/15 \end{vmatrix}$$

$$S^{-1} = \begin{vmatrix} \frac{2}{3}\cdot\frac{5}{2} & -\frac{1}{3}\cdot\frac{5}{2} & \emptyset\cdot\frac{5}{2} \\ \emptyset\cdot\frac{5}{2} & \frac{3}{5}\cdot\frac{5}{2} & \emptyset\cdot\frac{5}{2} \\ -\frac{4}{18}\cdot\frac{5}{2} & \frac{2}{18}\cdot\frac{5}{2} & \frac{6}{18}\cdot\frac{5}{2} \end{vmatrix}$$

$$= \begin{vmatrix} 5/3 & -5/6 & \emptyset \\ \emptyset & 3/2 & \emptyset \\ -2/3 & 1/3 & \frac{3}{3}=1 \end{vmatrix} = m_p$$

Now we have a complete model of this economic system, with the possibility of using 2 different assumptions

$$\begin{vmatrix} \emptyset & u \\ m & \emptyset \end{vmatrix} \begin{vmatrix} X_c \\ X_a \end{vmatrix} = \begin{vmatrix} Y_c \\ \emptyset \end{vmatrix}$$

m_I $\quad$ $m_p = S^{-1}$