Elementary Mathematics

TESS India

Mathematical stories: word problems

Teacher Education through School-based Support in India
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TESS-India (Teacher Education through School-based Support) aims to improve the classroom practices of elementary and secondary teachers in India through the provision of Open Educational Resources (OERs) to support teachers in developing student-centred, participatory approaches. The TESS-India OERs provide teachers with a companion to the school textbook. They offer activities for teachers to try out in their classrooms with their students, together with case studies showing how other teachers have taught the topic and linked resources to support teachers in developing their lesson plans and subject knowledge.

TESS-India OERs have been collaboratively written by Indian and international authors to address Indian curriculum and contexts and are available for online and print use (http://www.tess-india.edu.in/). The OERs are available in several versions, appropriate for each participating Indian state and users are invited to adapt and localise the OERs further to meet local needs and contexts.

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Video resources

Some of the activities in this unit are accompanied by the following icon:  *. This indicates that you will find it helpful to view the TESS-India video resources for the specified pedagogic theme.*

The TESS-India video resources illustrate key pedagogic techniques in a range of classroom contexts in India. We hope they will inspire you to experiment with similar practices. They are intended to complement and enhance your experience of working through the text-based units, but are not integral to them should you be unable to access them.

TESS-India video resources may be viewed online or downloaded from the TESS-India website, http://www.tess-india.edu.in/). Alternatively, you may have access to these videos on a CD or memory card.

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All India - English

What this unit is about

Word problems are often seen as a way to bridge the divide between real life and the mathematics classroom. However, students across the world often perform poorly in tests involving word problems. Even when students have mastered the technical competencies of doing mathematical operations such as addition, subtraction, multiplication or division, they can still find it hard to work out solutions to word problems that involve applying of those mastered techniques (Morales et al., 1985).

This unit will focus on helping students to make sense of this subject by:

- rephrasing word problems
- asking the students to construct word problems themselves by creating stories.

What you can learn in this unit

- How to help your students interpret word problems more effectively.
- Some ideas to guide your students in using storytelling as a tool for understanding word problems.
- How to help your students in representing mathematical statements by creating stories.

This unit links to the teaching requirements of the NCF (2005) and NCFTE (2009) outlined in Resource 1.

1 Word problems seen as stories

Pause for thought

Thinking about your own classroom, how are word problems perceived by your students? Do they like them? Do they struggle with them? Why do you think this is?

Think back about your experiences as a mathematics learner, how did you perceive word problems? What helped you to understand how to approach them?

Word problems can play a significant role in making school mathematics meaningful and contextual for students. Along with connecting everyday reasoning with classroom context, they can also connect school mathematics with everyday situations and everyday problems, and vice versa. It is therefore very important that students are exposed not only to solving word problems, but also to constructing them themselves.

When working with word problems, difficulties can occur as students try to make sense of the context and come across words and expressions that are not familiar to them, or when they cannot visualise the context of the word problem.

An effective way to help students is by considering word problems as stories. Students tend to like stories and are familiar with them. Stories often catch the interest and attention of the students, who might even be well versed in creating stories themselves. They know that stories can be completely fictitious – but equally they can take place in contexts that are familiar to the students.

Research shows that asking students to develop a story or narrative as part of their learning activities can help understanding. Bruner (1986), an influential educationalist, argues this is the case because 'human beings are essentially narrative beings, telling stories to themselves and others as a way of making sense of the world' (Mason and Johnson-Wilder, 2004, p. 68).

Making drawings or using hands-on teaching aids (manipulatives or props) to depict the story or word problem can also help students to understand the problem and physically see the relationships between different variables.

The first case study describes how Mrs Chadha used stories to introduce the mathematical concept of addition to her students.

Case Study 1: The story of Aditi

I am Mrs Chadha, a teacher of Class I.

I planned to start teaching addition to my students. I believe that for mathematics to make sense to the students, they need to place mathematical concepts in a context; hence, I try to give plenty of concrete experiences whenever I start with any new mathematical topic. So when starting my lessons on addition, I told a short story about a girl named Aditi who loved collecting marbles. I had a box of marbles on my desk.

One fine day Aditi was playing in the garden and saw some marbles lying on the ground. She was very happy and decided to collect them. She found three marbles at first. (Now I ask Varun, a student, to count three marbles loudly and take them out of my collection of marbles.)

I continued with the story: as Aditi moved around and looked for more, she found four more marbles. (Now Varun takes out four more marbles.)

I then asked the students: how many marbles did Aditi find in total?

Varun raised his hand to answer. I asked Varun to share with the whole group how he found the answer. Varun explains how he counted to find out the total number of marbles.

Continuing the story, I said that Aditi kept moving as she thought she should check the whole garden. As she neared a bench she saw that there were some more marbles lying under it. She found two more marbles. I then asked the students to count and tell me how many marbles Aditi would now have. I added two more similar steps.

I then shared similar short stories with my students and asked them to find out the total number of things, such as buttons, pencils, pebbles, etc.

After this I started asking how many biscuits there will be in total if one student has three biscuits and another one has two biscuits, and so on. For each problem, I first drew the objects [see Figure 1].

Figure 1 Three biscuits and two biscuits.

Then I wrote the numerical representation on the blackboard as I spoke:

> Three biscuits and two biscuits together become five biscuits.
>
> 3 biscuits + 2 biscuits gives 5 biscuits

At this point, I introduced the symbol '+' for addition and then I introduced the symbol '=' for equivalence [Figure 2].

Figure 2 Three biscuits and two biscuits, with '+' and '=' symbols added.

Then I wrote the expression '3 + 2 = 5'.

I then reminded the students of the story of Aditi and the marbles, and asked them how I should draw this. On their instructions I drew the marbles on the blackboard and wrote the mathematical expression. Together we drew many more 'adding' stories on the blackboard using the '+' and '=' symbols.

Video: Storytelling, songs, role play and drama

http://tinyurl.com/video-ssrpd

In Case Study 1, Mrs Chadha is building links between the mathematical concept of addition and a real-life context that is familiar to the students. At the same time she lets the students be active participants in the narration of the story.

Bruner (1966), an influential educationalist, suggested that learning for understanding happens by going through three modes or stages of representation: enactive (activity-based), iconic (image-based) and symbolic (symbol- or language-based). He argues that these modes of representation are the ways in which information or knowledge is stored and encoded in memory (McLeod, 2008).

Mrs Chadha first provides actual marbles so that the students can physically count and add marbles to find the answers. Later, she represents the same on the blackboard with the help of images of the objects (biscuits), and then moves to write what he says first in words and then in symbols.

At the same time, Mrs Chadha links these three representations by constantly talking about them. For example, she introduces the words 'add', 'altogether' and 'plus' gradually, and associates these with the action of addition. This gives the students an opportunity to encounter the vocabulary many times in different contexts.

Pause for thought

- Can you think of an example in your own teaching practice where you could use a similar approach to that of Mrs Chadha?
- How might Mrs Chadha have adapted these activities to make help all of the students remain fully engaged throughout the lesson?

2 Constructing stories to help make sense of mathematical concepts

Traditionally, word problems appear in textbooks or in classroom teaching at the end of a chapter. Often, little time and attention is spent on making sense of these word problems. Letting students create their own stories, or word problems, to narrate a mathematical sentence like 3 + 4 = 7 can help to build an understanding of the mathematical ideas and lead to greater problem solving skills. It can help students overcome the difficulties of making sense of the context of the word problems, because they will construct their own context and focus on making the story fit the mathematics. In that way it also helps them with identifying which mathematical representation to use.

Before attempting to use the activities in this unit with your students, it would be a good idea to complete all, or at least part, of the activities yourself. It would be even better if you could try them out with a colleague, as that will help you when you reflect on the experience. Trying for yourself will mean you get insights into a learner's experiences that can, in turn, influence your teaching and your experiences as a teacher. When you are ready, use the activities with your students and once again, reflect on the way the activity went and the learning that happened. This will help you to develop a more learner-focused teaching environment.

The next two activities give ideas to help your students create their own stories for mathematical number sentences.

Activity 1: Making stories

Preparation

Read Case Study 2. Adapt the mathematics in the questions to fit the level of learning of your students. Think about how you will organise your students when they work on the activity. You may wish to have a look at the key resource 'Using groupwork' (http://tinyurl.com/kr-usinggroupwork).

The activity

Tell your students to choose a problem from Table 1 and use their imagination to create a story around the given problem.

Table 1 Maths problems and the first line of stories.

Maths problem	The first line of a story
4 + 7 = …	A girl was playing 'Snakes and Ladders' with her brother …
A box has three white balls and six red balls. How many balls are there in all?	Shyam is very fond of collecting balls …
9 – 7 = …	My aunt lives a few houses away from my home. Her house is …
If 5 is subtracted from 8, what is the answer?	Our dog …
2 × 4 = …	A group of friends were playing cards …

Then ask the students to get into pairs, tell each other their stories and comment on them.

- Some more complex examples:
  - $4 + 7 = 3 + 8$
  - $2(3 + 1) = 2 \times 4$
  - $2(3 + 1) = 6 + 2$
- Make up some more of your own. At least one should be an easy one to work out, and at least one should be a difficult one. Remember you have to be able to work out the answers yourself as well!

Activity 2: Making many stories for the same number sentence

Tell your students the following.

Consider this number sentence:

$3 + 4 = 7$

This number sentence could be represent by several mathematical relationships, such as:

- adding 3 and 4 together makes 7
- 4 more added to 3 gives 7
- the total number of things is $3 + 4 = 7$
- 4 less from something leaves 3.

Now ask your students to formulate a story or word problem for each of these relationships. Encourage them to use their imagination! For example, for the first relationship, the story or word problem could be something like this:

> Mohini and Rohini were playing together and making balls from clay. Mohini made three balls from the clay and Rohini made four balls. They wanted to know how many balls they had made in all. They kept them together in a box. Can you help them to find out how many balls they made in total?

To link in with Bruner's modes of representation, you could also ask the students to make a drawing depicting their stories.

Case Study 2: Mrs Meganathan reflects on using Activities 1 and 2

This is the account of a teacher who tried Activities 1 and 2 with his elementary students.

For both activities, I asked my students to work in the groups of three or four because I thought that would give them more ideas to share. They could also support each other if one of them was stuck. We had a go at the first three questions of Activity 1 as a whole class because my students never had done something like this before. I think this helped them to understand what I wanted them to do. It also opened up their imagination to think of all kinds of examples. Some involved monsters, stars, going to the market or being in a Bollywood movie. I then ask them to come up with their own group examples and that they were not to use the examples we had already mentioned as a whole class. I did change some of the harder questions because my students have not come across brackets in mathematical sentences yet.

The students did not find the second activity that easy to start with. They could understand the differences in mathematical relationships when I read them out, but found it hard to come up with stories that would fit these relationships. I decided to write it up on the blackboard instead of just reading it out

to them, and then asked one of the students to read out loud what I had written. That seemed to help them realise that there were subtle differences.

When each group had come up with something for every equation, we shared them with the whole class. I asked the students whether they agreed with each of the examples. This helped to clear up some misconceptions.

I then asked them 'Which one was most difficult and why?' This meant that the students had to think about how they thought about their mathematics – that is called 'metacognition', I think. Asking them to take this overview also meant I became more aware of what they found difficult, and therefore where more practice would be needed.

Reflecting on your teaching practice

When you do such an exercise with your class, reflect afterwards on what went well and what went less well. Consider the questions that led to the students being interested and being able to progress, and those you needed to clarify. Such reflection always helps with finding a 'script' that helps you engage the students to find mathematics interesting and enjoyable. If they do not understand and cannot do something, they are less likely to become involved. Use this reflective exercise every time you undertake the activities, noting as Mrs Meganathan did some quite small things that made a difference.

Pause for thought

- How did it go with your class?
- What responses from students were unexpected? Why?
- What questions did you use to probe your students' understanding?
- Did you modify the task in any way? If so, what was your reasoning for doing so?
- What did you notice about your students understanding of the mathematics?
- Which of your students are less confident about the concept of addition?

3 Rephrasing word problems

Word problems have been around for a very long time. Consider these two specific examples:

- 'It takes three men six hours to dig a ditch. How long does it take two men to dig the same ditch?' (Traditional)
- 'Suppose a scribe says to thee, four overseers have drawn one hundred great quadruple hekat of grain, their gangs consisting, respectively, of twelve, eight, six and four men. How much does each overseer receive?' (Problem 68, Rhind Mathematical Papyrus, c. 1700 BC)

You probably found the second problem more difficult to understand because the context is less familiar. Many students find difficulties with this.

Difficulties with word problems arise because:

- the students are not yet fluent readers
- the language of instruction is not their mother tongue
- they do not understand the language used because the vocabulary is not familiar to them.

This can also mean that the students cannot imagine the context of the word problem (Nunes, 1993). Word problems are often essentially mathematical problems dressed up in everyday language. They can help students understand that mathematics can model the real world and they themselves become

mathematicians when they do this. This is why students need to realise that the power of mathematics in real-world problems lies in its ability to model complex situations, from which they must extract the essential elements in order to solve these problems.

Focusing on the process of making sense of a complex situation and modelling it mathematically can also help students focus on making sense of word problems. Activity 3 looks at how to help students find out what they need to know more independently by rephrasing the word problems.

Activity 3: Making sense of the context and the mathematics in a word problem

Adapt these word problems so that they fit the level of learning of your own students.

The activity

Tell your students to read each problem and answer the following questions:

- Mandeep had 21 marbles. Simi had 18 **fewer than** Mandeep. If they wanted to **share** the marbles **equally,** how many would they each have?
- Rasheed's mother baked three **identical circular** cakes for his birthday to share with friends and family. Fourteen adults and 20 children came to his party. The **size of the slice** of the cake the children got was **half that of** an adult size. What **fraction** of a cake was an adult **portion** and what fraction of a cake was a child **portion**?
- Savitri had to **make a model** of a **cuboid kaleidoscope** for her **science project**. She wanted to **use chart paper** to make **the surface of the kaleidoscope**. What **area of chart paper** would she require if she wanted to **make a kaleidoscope** with a **length of 25 cm and a breadth of 4 cm** ?
- Ramesh and Mahesh together can **row a boat** at a **speed of 12 kph**. At that speed it takes them 30 minutes to cross the lake. If they row at 10 kph, **how long would it take them** then to cross the lake?
- Lalita **was awarded a 5 per cent increment** in **her annual salary** because of her **contribution to the water conservation project** for her company. If her **base salary** was **Rs. 3.5 lakh per annum**, what is her **revised monthly salary?**

For each problem, consider:

- Do you know the meaning of each of the words or phrases highlighted in bold? Are there some terms or phrases that are new for you? Do you feel that these will be relevant to solving the problem?
- How would you go about learning what these words or phrases mean, or the mathematical ideas you would need to work with them?
- Rephrase the words and phrases highlighted in bold to simplify the word problem. If a particular term, word or phrase is not required, you may omit it. Which terms did you find difficult to rephrase? Why?

Your students will probably not all be at the same stage in their ability to make sense of the context and the mathematics in a word problem. This activity should provide you with an excellent opportunity to monitor their performance and provide them with constructive feedback. You may wish to have a look at Resource 2, 'Monitoring and giving feedback', to help you prepare for this aspect of the activity.

Video: Monitoring and giving feedback

http://tinyurl.com/video-monitoringandfeedback

Case Study 3: Mrs Chakrakodi reflects on using Activity 3

I am pleased that I used these three problems with my class. I have to say it was hard work getting them to focus at the start – they just seemed to find problems and say that they were stuck. However, I persevered. I told them to work in pairs, which I always think is helpful if the work is unfamiliar and likely to require a lot of thinking. I reminded them to note down anything that they did not know the meaning of and then think about how they could find out about these ideas.

After everyone had done some thinking, we shared what we could use to find out about the ideas. They first said, 'Ask the teacher', but I banned that for this exercise and asked them to be more imaginative. One said, 'Use the internet', another 'Look it up in the textbook', so I suggested that they look up what they could in their textbooks and that if they brought me a note of anything they couldn't find, I would be their internet search engine for today! I made sure to be awkward and only give information on what was actually 'entered in the search bar' in order to make them think about what they really needed to know.

Once everyone felt they had the information they needed, they went onto the rephrasing exercise. It seemed that this was easy now because the class were working collaboratively and learning together by this time.

What I had not expected was that the students in my classroom who were multilingual really benefitted from the discussions of what the words meant. I told them to make sure to make a note of the meaning of the words in whatever language they are most comfortable with so they could refer to it later.

Pause for thought

- What questions did you use to probe your students' understanding?
- Did you feel you had to intervene at any point?
- Did you modify the task in any way? If so, what was your reasoning for this?

4 Summary

This unit has explored some of the issues around word problems and their capacity for working on mathematical thinking processes.

Students can find it hard to engage with and solve word problems; however, there are ways to overcome such barriers. Suggested approaches in this unit were:

- linking enactive/iconic/symbolic representations
- using imagination to create contextualised stories around mathematical statements
- rephrasing word problems.

Pause for thought

Identify the techniques or strategies you have learned in this unit that you might use in your classroom, and some ideas that you want to explore further.

Resources

Resource 1: NCF/NCFTE teaching requirements

This unit links to the following teaching requirements of the NCF (2005) and NCFTE (2009) and will help you to meet those requirements:

- View students as active participants in their own learning and not as mere recipients of knowledge; to encourage their capacity to construct knowledge; to ensure that learning shifts away from rote methods.
- Organise student-centred, activity-based and participatory learning experiences for students.
- Help students to see mathematics as something to talk about, to communicate through, to discuss among themselves and to work together on.

Resource 2: Monitoring and giving feedback

Improving students' performance involves constantly monitoring and responding to them, so that they know what is expected of them and they get feedback after completing tasks. They can improve their performance through your constructive feedback.

Monitoring

Effective teachers monitor their students most of the time. Generally, most teachers monitor their students' work by listening and observing what they do in class. Monitoring students' progress is critical because it helps them to:

- achieve higher grades
- be more aware of their performance and more responsible for their learning
- improve their learning
- predict achievement on state and local standardised tests.

It will also help you as a teacher to decide:

- when to ask a question or give a prompt
- when to praise
- whether to challenge
- how to include different groups of students in a task
- what to do about mistakes.

Students improve most when they are given clear and prompt feedback on their progress. Using monitoring will enable you to give regular feedback, letting your students know how they are doing and what else they need to do to advance their learning.

One of the challenges you will face is helping students to set their own learning targets, also known as self-monitoring. Students, especially struggling ones, are not used to having ownership of their own learning. But you can help any student to set their own targets or goals for a project, plan out their work and set deadlines, and self- monitor their progress. Practising the process and mastering the skill of self-monitoring will serve them well in school and throughout their lives.

Listening to and observing students

Most of the time, listening to and observing students is done naturally by teachers; it is a simple monitoring tool. For example, you may:

- listen to your students reading aloud
- listen to discussions in pair or groupwork
- observe students using resources outdoors or in the classroom
- observe the body language of groups as they work.

Make sure that the observations you collect are true evidence of student learning or progress. Only document what you can see, hear, justify or count.

As students work, move around the classroom in order to make brief observation notes. You can use a class list to record which students need more help, and also to note any emerging misunderstandings. You can use these observations and notes to give feedback to the whole class or prompt and encourage groups or individuals.

Giving feedback

Feedback is information that you give to a student about how they have performed in relation to a stated goal or expected outcome. Effective feedback provides the student with:

- information about what happened
- an evaluation of how well the action or task was performed
- guidance as to how their performance can be improved.

When you give feedback to each student, it should help them to know:

- what they can actually do
- what they cannot do yet
- how their work compares with that of others
- how they can improve.

It is important to remember that effective feedback helps students. You do not want to inhibit learning because your feedback is unclear or unfair. Effective feedback is:

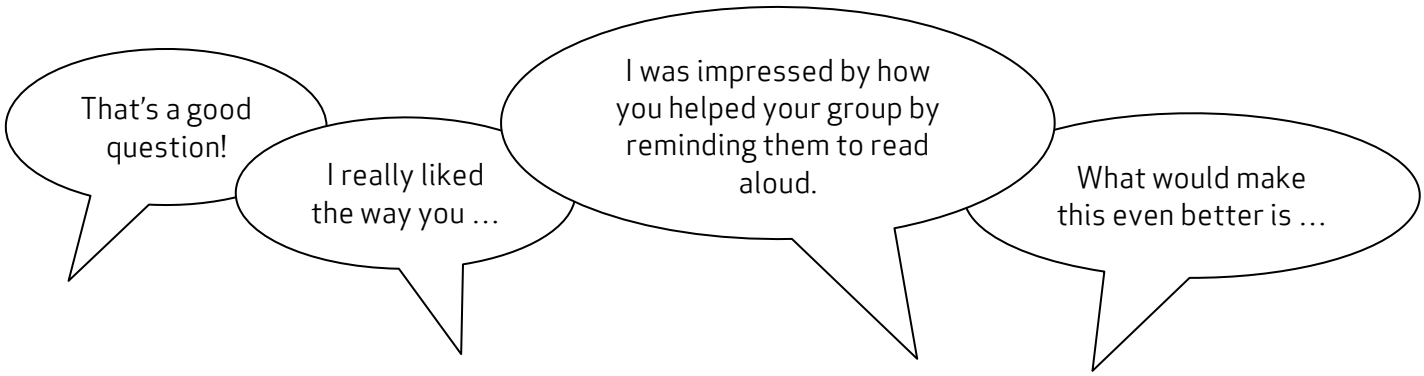
- **focused** on the task being undertaken and the learning that the student needs to do
- **clear and honest**, telling the student what is good about their learning as well as what requires improvement
- **actionable**, telling the student to do something that they are able to do
- given in **appropriate language** that the student can understand
- given at the **right time** – if it's given too soon, the student will think 'I was just going to do that!'; too late, and the student's focus will have moved elsewhere and they will not want to go back and do what is asked.

Whether feedback is spoken or written in the students' workbooks, it becomes more effective if it follows the guidelines given below.

Using praise and positive language

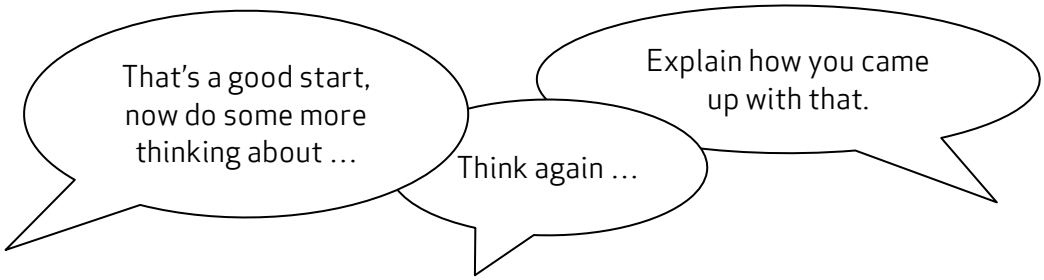
When we are praised and encouraged, we generally feel a great deal better than when we are criticised or corrected. Reinforcement and positive language is motivating for the whole class and for individuals of all ages. Remember that praise must be specific and targeted on the work done rather than about the student

themselves, otherwise it will not help the student progress. 'Well done' is non-specific, so it is better to say one of the following:

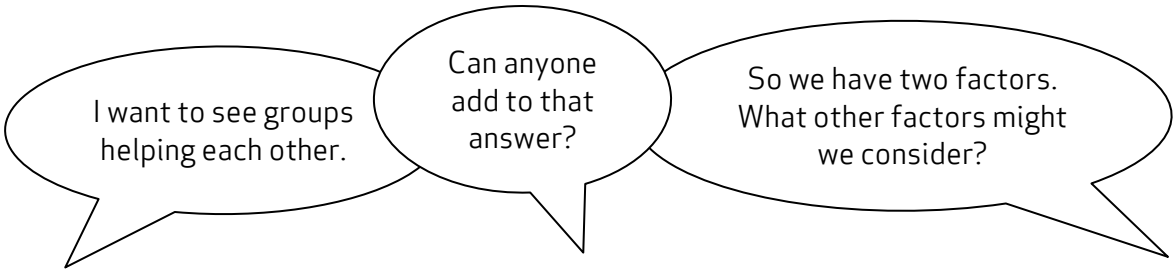


Using prompting as well as correction

The dialogue that you have with your students helps their learning. If you tell them that an answer is incorrect and finish the dialogue there, you miss the opportunity to help them to keep thinking and trying for themselves. If you give students a hint or ask them a further question, you prompt them to think more deeply and encourage them to find answers and take responsibility for their own learning. For example, you can encourage a better answer or prompt a different angle on a problem by saying such things as:



It may be appropriate to encourage other students to help each other. You can do this by opening your questions to the rest of the class with such comments as:



Correcting students with a 'yes' or 'no' might be appropriate to tasks such as spelling or number practice, but even here you can prompt students to look for emerging patterns in their answers, make connections with similar answers or open a discussion about why a certain answer is incorrect.

Self-correction and peer correction is effective and you can encourage this by asking students to check their own and each other's work while doing tasks or assignments in pairs. It is best to focus on one aspect to correct at a time so that there is not too much confusing information.

Additional resources

- A newly developed maths portal by the Karnataka government: http://karnatakaeducation.org.in/KOER/en/index.php/Portal:Mathematics
- National Centre for Excellence in the Teaching of Mathematics: https://www.ncetm.org.uk/
- National STEM Centre: http://www.nationalstemcentre.org.uk/
- National Numeracy: http://www.nationalnumeracy.org.uk/home/index.html
- BBC Bitesize: http://www.bbc.co.uk/bitesize/
- Khan Academy's math section: https://www.khanacademy.org/math
- NRICH: http://nrich.maths.org/frontpage
- Art of Problem Solving's resources page: http://www.artofproblemsolving.com/Resources/index.php
- Teachnology: http://www.teach-nology.com/worksheets/math/
- Math Playground's logic games: http://www.mathplayground.com/logicgames.html
- Maths is Fun: http://www.mathsisfun.com/
- Coolmath4kids.com: http://www.coolmath4kids.com/
- National Council of Educational Research and Training's textbooks for teaching mathematics and for teacher training of mathematics: http://www.ncert.nic.in/ncerts/textbook/textbook.htm
- AMT-01 *Aspects of Teaching Primary School Mathematics*, Block 1 ('Aspects of Teaching Mathematics'), Block 2 ('Numbers (I)'), Block 3 ('Numbers (II)'): http://www.ignou4ublog.com/2013/06/ignou-amt-01-study-materialbooks.html
- LMT-01 *Learning Mathematics*, Block 1 ('Approaches to Learning') Block 2 ('Encouraging Learning in the Classroom'), Block 4 ('On Spatial Learning'), Block 6 ('Thinking Mathematically'): http://www.ignou4ublog.com/2013/06/ignou-lmt-01-study-materialbooks.html
- *Manual of Mathematics Teaching Aids for Primary Schools*, published by NCERT: http://www.arvindguptatoys.com/arvindgupta/pks-primarymanual.pdf
- *Learning Curve* and *At Right Angles*, periodicals about mathematics and its teaching: http://azimpremjifoundation.org/Foundation_Publications
- Textbooks developed by the Eklavya Foundation with activity-based teaching mathematics at the primary level: http://www.eklavya.in/pdfs/Catalouge/Eklavya_Catalogue_2012.pdf
- Central Board of Secondary Education's books and support material (also including *List of Hands-on Activities in Mathematics for Classes III to VIII*) – select 'CBSE publications', then 'Books and support material': http://cbse.nic.in/welcome.htm

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