Senior secondary

Maths: Revision units

Scholar study workbook

Forum for African Women Educationalists in Malawi (FAWEMA)

"Supporting Girls and Women to Acquire Education for Development"

The Open University

UKaid from the British people

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For more information about the TESSA programme see:
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'Keeping Girls in School' Scholarship Programme

MSCE Resources: 2014–15

Contents

Revision M1: Numeracy and probability

Key points to remember

1 Two quantities are **directly proportional** to each other if they increase at the same rate. As one quantity increases so does the other.

2 Two quantities are **inversely proportional** to each other if as one quantity increases the other *decreases* at the same rate.

3 An **Arithmetic Progression (AP)** is a list of numbers whose consecutive terms have a common *difference*.

4 A **Geometric Progression (GP)** is a list of numbers whose consecutive terms have a common *ratio*.

5 You can use a **tree diagram or sample space table** to help you work out **probabilities** of events. You may be asked to draw a tree diagram or draw a sample space table.

Exam-type questions with solutions

1 Given that $p \propto q$ and $q = 10$ when $p = 4$, find the value of p when $q = 15$

Solution $p = kq$

$4 = k \times 10$

$k = 0.4$

Write in the form of an equation where k is a constant to be found.

now $p = 0.4q$

$p = 0.4 \times 15$

$p = 6$

Now you can find p when q = 15.

2 Find the sum of the first 10 terms of the arithmetic progression

–1, 2, 5, ...

Solution Sum of n terms $= \frac{n}{2} = [2a + (n-1)d]$

Sum of 10 terms $= \frac{10}{2} = [2(-1) + 9 \times 3]$

$= 125$

You can find the first term and the common difference by looking at the first three terms.

Or use: nth term = a + (n-1)d where a = –1, d = 5–2 = 3

So: $T_{10} = -1 + (10-1)3$

$= -1 + 9(3)$

$= -1 + 27$

$= 26$

Then $S_n = \frac{n}{2}(a + l)$

$S_{10} = \frac{10}{2}(-1 + 26)$

$S_{10} = 5(25)$

$S_{10} = 125$

You may need to use this formula for the sum of an arithmetic progression:

a = the first term
l = last term and
d = the common difference

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Revision M2: Basic algebra and logarithms

Key points to remember

1 You will need to know facts about powers, such as:

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$a^{-m} = \frac{1}{a^m}$

and:

$\sqrt[m]{a} = a^{\frac{1}{m}}$

$a^0 = 1$

$a^1 = a$

2 You will need to manipulate and simplify numbers and expressions using:

$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

3 To rationalise the denominator:

for fractions in the form $\frac{1}{\sqrt{a}}$, multiply the top and bottom by $\sqrt{a}$

for fractions in the form $\frac{1}{a + \sqrt{b}}$, multiply the top and bottom by $a - \sqrt{b}$

for fractions in the form $\frac{1}{a - \sqrt{b}}$, multiply the top and bottom by $a + \sqrt{b}$

4 Work with logarithms using:

If $a^x = b$ then $x = \log_a b$ when $a > 0$

$\log_a xy = \log_a x + \log_a y$

$\log_a \frac{x}{y} = \log_a x - \log_a y$

$\log_a a = 1$ when $a \neq 1$

$\log_a x^k = k\log_a x$

$\log_a 1 = 0$

Exam-type questions with solutions

1 Simplify $\frac{\sqrt{2}}{3 - \sqrt{2}}$ leaving your answer with a rational denominator.

Solution $\frac{\sqrt{2}\,(3 + \sqrt{2})}{(3 - \sqrt{2})\,(3 + \sqrt{2})} = \frac{3\sqrt{2} + 2}{9 - 2} = \frac{3\sqrt{2} - 2}{7}$

Multiply the top and bottom of the fraction as shown.

2 Solve the equation $\log_2 x + \log_2 10 = 3$

Use the laws of logarithms above to rewrite the left-hand side of the equation.

Solution $\log_2 x + \log_2 10 = 3$

$\log_2 10x = 3$

$10x = 3^8$

$x = \frac{8}{10}$

Re-write the equation using powers.

Or: $\log_2 x + \log_2 10 = \log_2 2^2$ (logarithms are powers)

$\log_2 (x \times 10) = \log_2 2^3$

Taking antilog on both sides

$10x = 2^3$

$10x = 8$

$x = \frac{8}{10}$

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Revision M3: Algebra 2

Key points to remember

1 You will need to be able to solve quadratic equations by factorising, completing the square or using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 You will need to be able to solve quadratic equations when solving simultaneous equations where one equation is linear and the other none linear.

3 You may be required to work with algebraic fractions.

4 You may need to change the subject of a formula.

Exam-type questions with solutions

1 Solve the quadratic equation $2x^2 - 7x - 7 = 0$ giving your answers correct to 3 significant figures.

As this question tells you to give your answer to 3 significant figures you have a clue that this quadratic equation cannot be solved by factorising.

Solution $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-7)}}{2(2)}$$

It is a good idea to write out the values of a, b and c so you are less likely to make an error when substituting in to the formula.

$$x = \frac{7 \pm \sqrt{105}}{4}$$

$$x = \frac{7 + \sqrt{105}}{4} \approx 4.31 \quad \text{or} \quad x = \frac{7 - \sqrt{105}}{4} \approx -0.812$$

Here a = 2, b = –7 and c = –7.

2 Solve the simultaneous equations

$y - 2x = 5$

$y2 - xy - x^2 = 11$

Make y the subject of the linear equation.

Solution $y = 2x + 5$

$(2x + 5)^2 - x(2x+ 5) - x^2 = 11$ Substitute 2x+5 into the none linear equation.

$4x^2 + 20x + 25 - 2x^2 - 5x - x^2 - 11 = 0$ Expand and simplify.

$x^2 + 15x + 14 = 0$

$(x + 14)(x + 1) = 0$ Solve to find values of x.

$x = -14$ or $x = -1$

$y = 2(-14)+5$ $\quad y = 2(-1)+5$ Don't forget to find corresponding values of y.

$y= -23$ $\quad y = 3$

Non-linear equations, in this case quadratic equations, should have one variable (unknown).

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Revision M4: Three dimensional shapes and solids

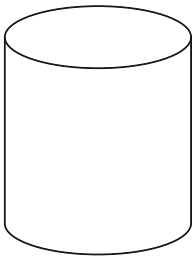
Key points to remember

1. You need to be able to identify the 3D shapes: cubes cuboids, prisms, pyramids, cylinders, cones and spheres.
2. You need to be able to work out the surface areas and volumes of cubes, cuboids and prisms.
3. You may be given a formula to find the volume or surface area of a 3D shape.
4. Remember to use Pythagoras' Theorem to find lengths of sides in right-angled triangles. $c^2 = \sqrt{a^2 + b^2}$ where c is a side opposite to the right angle and a and b are the other remaining sides of the triangle.
5. Use trigonometric ratios to find angles in right-angled triangles.

   $\sin \Theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos \Theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan \Theta = \frac{\text{opposite}}{\text{adjacent}}$

Exam-type questions with solutions

1 A cylindrical water tank with a diameter of 1 metre contains 1m3 of water. Calculate to the nearest cm the height of water in the tank. Take $\pi = 3.142$.



Solution

Volume of water in the tank = $\pi r2h$

$$1 = 3.142 \times 0.5^2 h$$

$$h = \frac{1}{3.142 \times 0.5^2}$$

$$h = 1.27 \text{ m}$$

We need to find the value of *h*.

$$h = 1.27 \text{ m}$$

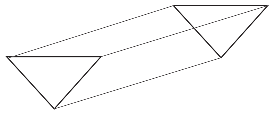
$$h = 1.27 \times 100$$

$$h = 127 \text{ cm}$$

Remember the radius is half the diameter.

Don't forget to round off to the nearest cm.

2 A metal trough is a triangular prism with length 2.5 m and the ends are isosceles right-angled triangles where the two shorter sides are both 20 cm.



(a) Calculate the volume of the trough.

(b) Calculate the area of metal needed to make the trough.

Solution

(a) Volume = area of cross-section × length

$= 0.5 \times 0.2 \times 0.2 \times 2.5 \text{ m}^3$

$= 0.05 \text{ m}^3$

(b) Area of rectangular faces = $2.5 \times 0.2 = 0.5 \text{ m}^2$

Area of triangular faces = $0.5 \times 0.2^2 = 0.02 \text{ m}^2$

Total area = $0.5 \times 2 + 0.02 \times 2 = 1.04 \text{ m}^2$

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Revision M5: Statistics

Key points to remember

1. A **pie chart** shows proportions clearly. You may be asked to interpret or draw a pie chart.
2. **Bar graphs, histograms** and **frequency polygons** are used to represent data when the frequency of each item, value or group of values are known. Frequency is always shown on the vertical axis.
3. You need to be able to calculate the **mean** of a set of data. If you are given the information in a frequency table, use the formula

   $\bar{x} = \dfrac{\Sigma fd}{\Sigma f}$
4. You may be asked to calculate the **variance** or **standard deviation** of a set of data where:

   Variance $= \dfrac{\Sigma d^2}{n}$ or $\dfrac{\Sigma fd^2}{\Sigma f}$ where $d = x - \bar{x}$

   and the standard deviation is the square root of the variance, i.e. standard deviation = √variance

Exam-type questions with solutions

1 A class of 30 children were asked what their favourite fruit was. 12 children said mango, 10 said banana and the rest said oranges. Draw a pie chart to show this information.

Solution

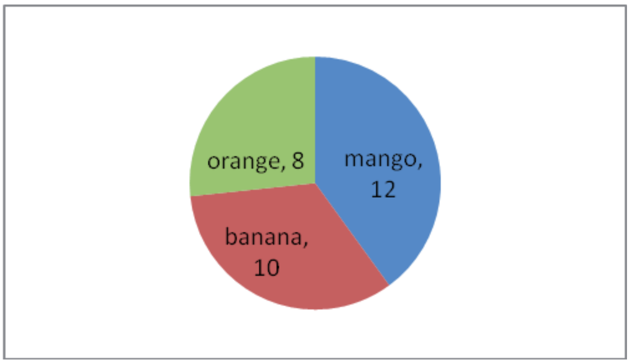
$\dfrac{12}{30} \times 360° = 144°$

$\dfrac{10}{30} \times 360° = 120°$

$\dfrac{8}{30} \times 360° = 96°$

Work out the angle related to each fruit.

Measure the angles carefully with a protractor.



2 Find the mean and standard deviation of the following values:
1, 3, 4, 5, 6, 8, 9, 12.

Give your answer correct to 3 decimal places.

Solution mean = (1 + 3 + 4 + 5 + 6 + 8 + 9 + 12)/8 = 6

Work out the mean.

x	1	3	4	5	6	8	9	12
$d = x - \bar{x}$	–5	–3	–2	–1	0	2	3	6
$d^2 = (x - \bar{x})^2$	25	9	4	1	0	4	9	36

Standard deviation = $\sqrt{\dfrac{\Sigma d^2}{n}} = \sqrt{\dfrac{88}{8}} \approx 3.317$

Find the difference between each value and the mean and square it.

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Revision M6: Angles and circles

Key points to remember

1 You need to be able to use angle properties of straight lines, parallel lines and triangles.

2 You must know the five circle theorems.

3 You must understand what a tangent to a circle is and the two tangent properties.

4 You may be asked to construct the following:

a perpendicular line to a straight line

a perpendicular bisector

angles of 30°, 45° and 60°

a tangent to a point on a circle

a tangent to a circle from a given external point.

Exam-type questions with solutions

1

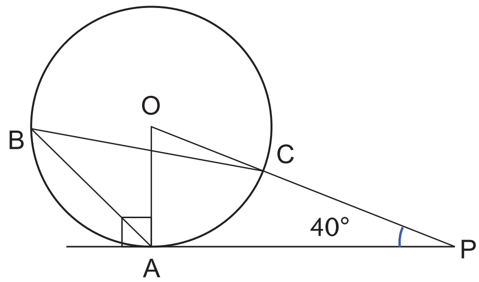

Figure 1

This figure shows a circle ABC with centre O. OCP is a straight line and AP is a tangent to the circle at A. If angle OPA = 40°, calculate the value of angle ABC.

Solution

Angle OAP = 90°	Tangent perpendicular to the radius.
Angle AOP = 180° – (90° + 40°)	Angles in a triangle add up to 180°.
= 50°	
Angle ABC = $\frac{50°}{2}$	
= 25°	Angle at the centre is twice the angle at the circumference.

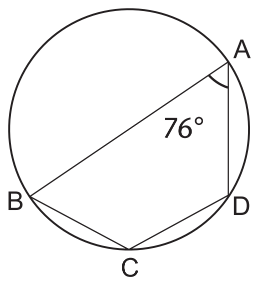

Figure 2

The cyclic quadrilateral ABCD has two parallel sides AB and CD. If angle BAD = 76º, calculate the size of the other 3 angles.

Solution

	The opposite angles of a cyclic quadrilateral are supplementary.
Angle BCD = 180º – 76º = 104º	
Angle ADC = 180º – 76º = 104º	Allied angles are supplementary.
Angle ABC = 180º – 104º = 76º	The opposite angles of a cyclic quadrilateral are supplementary. *or* Allied angles are supplementary.

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