Girls' Access to Education
Girls' Education Challenge

Sierra Leone

Maths Unit 2

Name of Learning Assistant ______________________________

School ______________________________

Tutor ______________________________

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The Open University

Maths Unit 2: Week 1

Fractions 1 The basics

Fractions are used frequently in everyday life – for example, when talking about sharing (you can have half of that) or talking about time (it is quarter to nine; it is half past seven). From a young age children learn to understand the concept of sharing, which is working with fractions. How did you feel when you studied fractions at school? What do you remember about fractions?

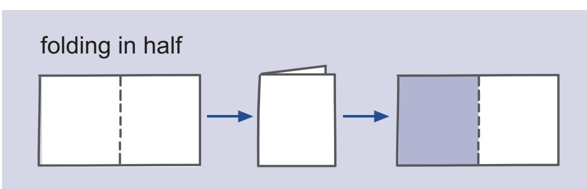
Although all learners have some knowledge of fractions, when the topic is studied in school, learners can see fractions as a very difficult topic to understand. It often triggers negative emotional responses. This unit will develop your understanding of fractions by focusing on visualising what fractions are.

Making fractions

A fraction describes how a whole is divided equally into parts and how many parts you have.

Example 1 Folding fractions

You can see how this happens by doing the following. Take a piece of paper and fold it in half and crease it along the fold. Open it up and shade the left-hand side.

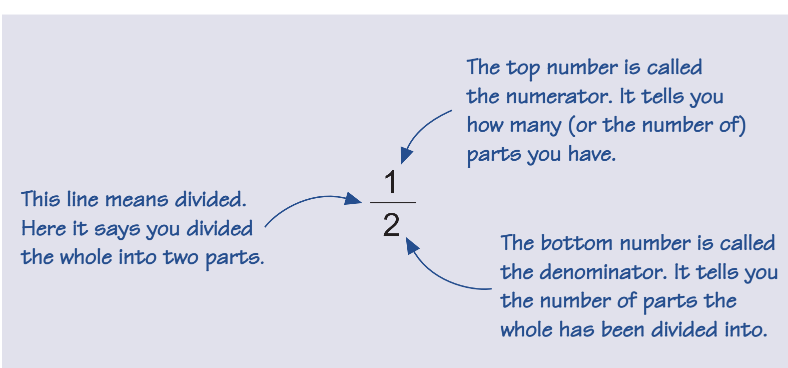


This one piece of paper has been divided into two identical parts. There are now two parts that make up the one piece of paper. Each part is half of the original. This fraction is written as $\frac{1}{2}$ and is read as ‘one-half’.

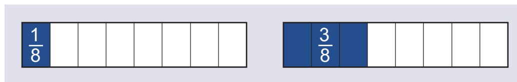
(Please keep this piece of paper because you will need it again later.)

Writing a fraction

Fractions are written like this:



The top number is called the **numerator**. The numerator tells you the number of parts you have to consider. Fractions with a numerator of 1 are called **unitary fractions**. The bigger the numerator, the more parts you have. For example the numerator in the fraction $\frac{1}{8}$ tells you have 1 part of the size of an eighth. The numerator in the fraction $\frac{3}{8}$ tells you have 3 parts of the size of an eighth.



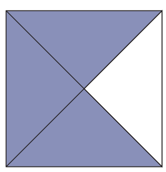
The bottom number is called the **denominator**. It tells you something about the size of the fraction parts you are dealing with: the number of parts the whole has been divided into and also how many parts you need to make a whole.

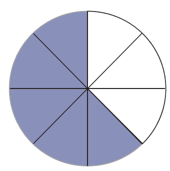
For example, in $\frac{1}{2}$ the denominator is 2. This tells you the whole has been divided into 2 equal parts. It also tells you 2 parts are needed to make a whole.

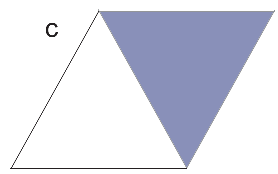
The bigger the denominator, the smaller the parts will be. For example, imagine you cut a cake into 8 equal parts and you are given $\frac{1}{8}$ of the cake; this will be a smaller slice than when you are given $\frac{1}{2}$ of a cake of the same size.

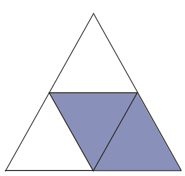
Exercise 1 Writing fractions

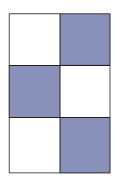
Say which fraction of each shape is shaded.

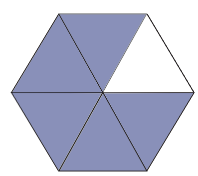
a 

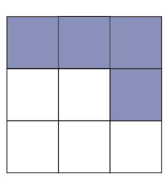
b 

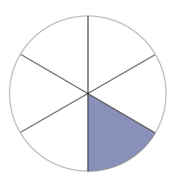
c 

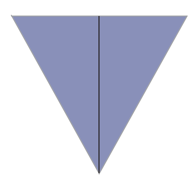
d 

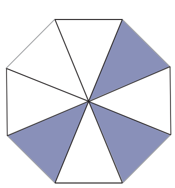
e 

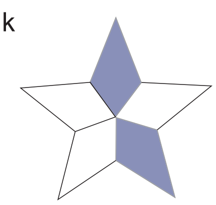
f 

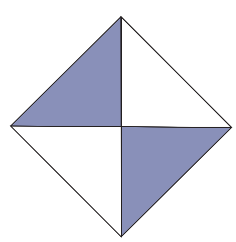
g 

h 

i 

j 

k 

l 

Exercise 2 Ordering unitary fractions

Unitary fractions are fractions that have a numerator of 1.

Put the following unitary fractions in order, beginning with the smallest:

$\frac{1}{3}$ $\frac{1}{67}$ $\frac{1}{34}$ $\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{23}$

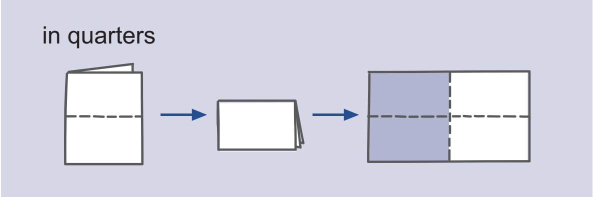
Equivalent fractions

Fractions can look very different, but actually be the same and have the same value. We call these **equivalent fractions**. To see this for yourself, take the piece of paper you folded and shaded in Example 1 and do as described next.

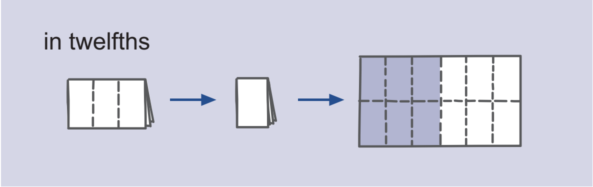
Example 2 Folding equivalent fractions

Now fold the paper back along the original crease and then in half again along the long side.

If you open up the paper, you should see four pieces of the same size, with two of them shaded. So the paper is now divided into quarters, and the fraction of the paper shaded is $\frac{2}{4}$.



Since you haven't altered the shading in any way, this shows that one-half is the same as two-quarters, or $\frac{1}{2} = \frac{2}{4}$. Now fold the paper back into quarters along the crease lines, then fold into three equal pieces or thirds along the long side. If you now open the paper up, you can see that there are 12 equal pieces. These pieces are 'twelfths', of which six are shaded, so $\frac{6}{12}$ of the paper is shaded. This shaded fraction is the same as $\frac{1}{2}$.



The fractions that represent the shaded part are all equivalent to each other. This can be written as:

$\frac{1}{2} = \frac{2}{4} = \frac{6}{12}$

You can carry on folding the paper into smaller and smaller pieces, and each time you open up the paper, it will be divided into smaller fractions, but half of it will still be shaded.

Working out equivalent fractions

When you folded the original piece of paper in half the second time, you doubled the total number of pieces on the paper (to four) and also doubled the number of shaded pieces (to two). This is the same as multiplying both the numerator and denominator of the fraction by two.

If you multiply the top and the bottom of any fraction by the same number (not zero!), you will get a fraction that is equivalent to the original one:

$$\begin{tikzcd} \frac{1}{2} \arrow[r, phantom, "="] \arrow[r, bend left, "\times 2"] \arrow[r, bend right, "\times 2"'] & \frac{2}{4} \arrow[r, phantom, "="] \arrow[r, bend left, "\times 3"] \arrow[r, bend right, "\times 3"'] & \frac{6}{12} \arrow[r, phantom, "="] \arrow[r, bend left, "\times 10"] \arrow[r, bend right, "\times 10"'] & \frac{60}{120} \end{tikzcd}$$

You can also generate equivalent fractions by dividing the top and the bottom of the fraction by the same number (but not zero). This process is known as cancelling.

$$\begin{tikzcd} \frac{40}{60} \arrow[r, phantom, "="] \arrow[r, bend left, "\div 10"] \arrow[r, bend right, "\div 10"'] & \frac{4}{6} \arrow[r, phantom, "="] \arrow[r, bend left, "\div 2"] \arrow[r, bend right, "\div 2"'] & \frac{2}{3} \end{tikzcd}$$

If there is no whole number that can be divided into both the top and the bottom, the fraction is said to be in its **lowest form**. Answers are usually left in this form, as they are easier to visualise and understand.

Exercise 3 Equivalent fractions

1. Fill in the missing numbers in the following.

a. $\frac{2}{3} = \frac{?}{18}$

b. $\frac{4}{5} = \frac{?}{20}$

2. Which of the following fractions are equivalent to each other?

a. $\frac{7}{8}$

b. $\frac{49}{64}$

c. $\frac{35}{40}$

d. $\frac{8}{9}$

3. Cancel the following fractions into their lowest terms.

a. $\frac{2}{8}$

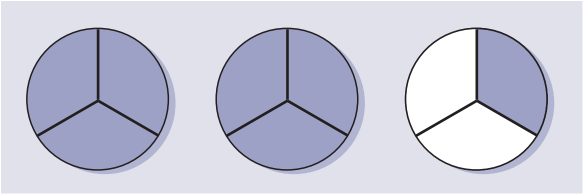
b. $\frac{10}{30}$

c. $\frac{27}{81}$

d. $\frac{60}{12}$

Mixed numbers and improper fractions

An improper or top heavy fraction has a numerator that is greater than its denominator – for example, $\frac{3}{2}$ and $\frac{89}{25}$. Mixed numbers such as $2\frac{1}{3}$ can also be written as improper fractions in which the whole number part is turned into the same type of fraction as the fractional part, in this case thirds. You can imagine this in terms of cakes – if you have two and a third cakes, and cut the two whole cakes into thirds, how many thirds will you have?



In this case the total number of thirds will be (2 × 3) + 1 = 7, so

$$2\frac{1}{3} = \frac{(2 \times 3) + 1}{3} = \frac{6 + 1}{3} = \frac{7}{3}$$

You can change improper fractions back into mixed numbers as well. For example, for $\frac{17}{8}$, imagine that you have some cakes cut into eighths and that you have 17 eighths or pieces. You know that eight eighths make one whole, and that two lots of eight is 16. This leaves one-eighth over. So $\frac{17}{8} = 2\frac{1}{8}$

Exercise 4 Improper and mixed fractions.

1. Change the following mixed numbers into improper (top-heavy) fractions.

a. $3\frac{1}{4}$

b. $2\frac{4}{5}$

c. $7\frac{3}{8}$

2. Change the following improper fractions into mixed numbers.

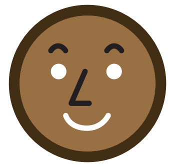
a. $\frac{23}{5}$

b. $\frac{15}{7}$

c. $\frac{17}{4}$

Well done for completing all the exercises! Try to compare your answers with another Learning Assistant – did you get the same answers? Are there any questions that you find difficult? Mark these questions and remember to ask your tutor about them at the next Maths tutorial.

How am I doing?

	 **Easy** (Tick this box if you feel confident that you **understand this section well**)	 **Fine** (Tick this box if you still **need a little work** on this section)	 **Difficult** (Tick this box if you still **need a lot of work** on this section)
I know what a denominator is			
I know what a numerator is			
I can visualise a fraction by drawing a picture			
I can write a fraction in fraction notation from looking at a fraction picture			
I can order unitary fractions by size			
I can write equivalent fractions			
I can cancel down fractions			
I know what an improper fraction is			
I know what a mixed fraction is			
I can turn mixed fractions into improper fractions and vice versa			

Notes on what to do next:

Learning Assistant signature:

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Tutor signature:

Date:

Maths Unit 2: Week 2

Fractions 2 The four rules

In the previous section, you learned about fractions and writing fractions. In this section you are going to learn how to add, subtract, multiply and divide fractions. These different types of sums are called the four rules, or the four operations. It is very useful to be able to do these calculations without using a calculator; you will do these calculations regularly in the school classroom.

Adding and subtracting fractions

Adding and subtracting fractions can look daunting, but it is easier when you first try to imagine what the sum is about.

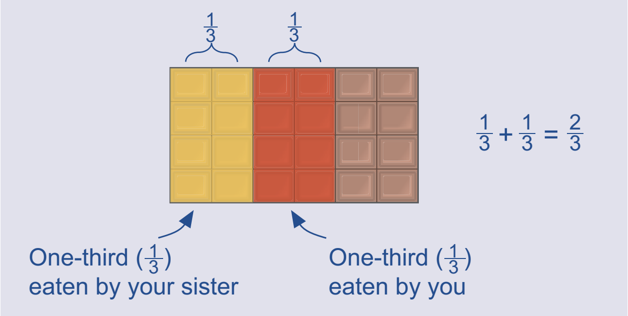
For example, suppose you are asked to work out $\frac{1}{3} + \frac{1}{3}$.

Look first at the operation sign: it tells you that you have to do addition (+). Then look at the denominators (the numbers at the bottom of the fraction). You can see that they are the same, which makes it easy.

Now imagine you had a large bar of chocolate like this one.

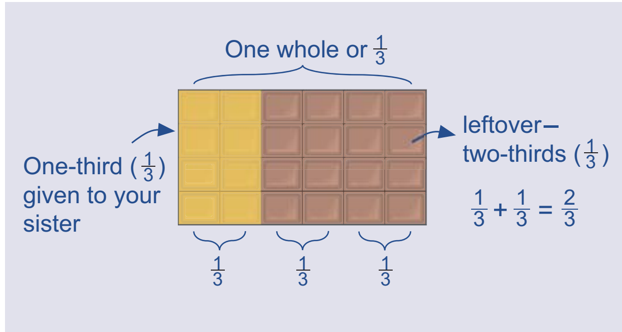
This bar has 24 chocolate pieces. So each piece is $\frac{1}{24}$ of the bar, and the whole bar is $\frac{24}{24}$ or 1. There are four identical rows, so each row is $\frac{1}{4}$ of the bar. There are also six identical columns, so each column is $\frac{1}{6}$ of the bar, and two columns are $\frac{2}{6}$ or $\frac{1}{3}$ of the bar.

Imagine you gave your sister $\frac{1}{3}$ of the bar of chocolate to eat and you ate $\frac{1}{3}$ of the bar of chocolate as well. The fraction of the bar of chocolate that would have been eaten would be: $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$



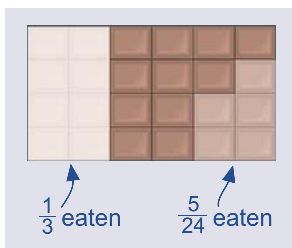
Suppose you are now asked to work out the sum $\frac{3}{3} - \frac{1}{3}$.

To picture this sum you could think of another large chocolate bar and divide it into thirds. You know that the whole bar equals three thirds, or $\frac{3}{3}$. You take away, or subtract, $\frac{1}{3}$ of the chocolate bar and give it to your sister. The fraction of the bar of chocolate that is left is: $\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$.

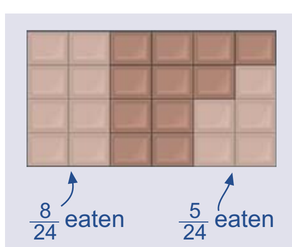


Doing addition and subtraction with fractions that do not have the same denominator is slightly more complicated. The reason for this is that the denominator tells you the kind of fraction you are dealing with (such as thirds, quarters, halves). To add and to subtract fractions the fractions first have to be written with a denominator in common.

For example, suppose you have another large bar of chocolate with 24 pieces. Now imagine that you eat a third ($\frac{1}{3}$) of the bar and, wanting more later, you decide to eat a further five pieces ($\frac{5}{24}$). What fraction of the bar have you eaten? What fraction is left?



The fraction eaten is $\frac{1}{3} + \frac{5}{24}$, but it is quite difficult to imagine what fraction this is as two different kinds of fractions (thirds and twenty-fourths) are involved. It is easier, if you change the fractions into the same type by thinking of how you could change thirds into twenty-fourths. Using what you learned about equivalent fractions you can write $\frac{1}{3}$ as $\frac{8}{24}$. So the fraction eaten is $\frac{8}{24} + \frac{5}{24} = \frac{13}{24}$. This is shown below.



The fraction that is left is $1 - \frac{13}{24}$. So, thinking of the whole bar as $\frac{24}{24}$, the calculation becomes: $\frac{24}{24} - \frac{13}{24} = \frac{11}{24}$.

This method of changing the fractions into the same kind using equivalent fractions can be used for adding and subtracting fractions in general. If you need to add or subtract fractions, the first question that you need to ask yourself is: 'Are these the same kind of fractions?' If the answer is no, then you need to convert the fractions into equivalent fractions that are of the same type.

Exercise 1

Work out the following sums.

1. $\frac{5}{18} + \frac{7}{18}$
2. $\frac{2}{5} + \frac{2}{5}$
3. $\frac{2}{4} + \frac{1}{3}$
4. $\frac{1}{8} + \frac{1}{4}$
5. $\frac{3}{4} - \frac{1}{4}$
6. $\frac{8}{25} - \frac{3}{25}$
7. $\frac{19}{21} - \frac{3}{21}$
8. $\frac{5}{6} + \frac{3}{4}$
9. $\frac{19}{24} - \frac{3}{8}$
10. $\frac{12}{4} - \frac{1}{4}$
11. $\frac{8}{9} + \frac{3}{9}$

If you get stuck, make a drawing to help you picture the question. Can you think of more examples to take to your tutorial?

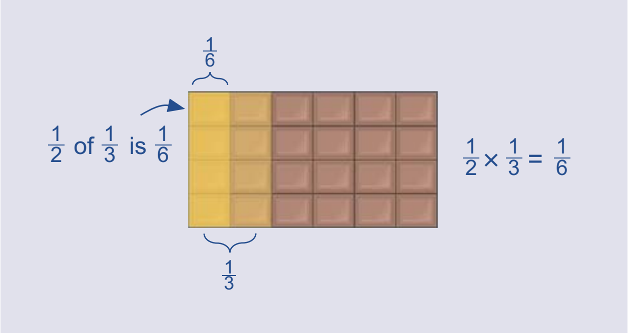
Multiplying fractions

Sometimes the way mathematics is written can be difficult to understand, and multiplication and division of fractions is a good example of this. Again, trying to picture what the question is asking you to do will help to make it clearer.

When multiplying fractions it is important to remember that the words 'of', 'times', 'multiplied by' and the symbol '×' all have the same meaning in the calculations. Use the wording that works best for you.

For example, $\frac{1}{2} \times \frac{1}{3}$ can be read as 'one-half times a third' or 'one-half of a third' or 'one-half multiplied by a third'. Imagining 'one-half multiplied by a third' can be difficult to imagine, but perhaps easier to work out if you know the rules. 'A half of a third' may be easier to picture.

Using the image of the large bar of chocolate, $\frac{1}{2} \times \frac{1}{3}$ read as 'a half of a third', would look like this:



You can see that a half of one third is the same as $\frac{1}{6}$. You can check this by counting 4 pieces of chocolate, which is $\frac{4}{24}$. This can be cancelled down to $\frac{1}{6}$.

The rule for multiplying fractions is as follows:

1. If the fraction given is a mixed number, first change it to an improper or top heavy fraction.
2. Cancel down the fractions if you can – it is easier to work with smaller numbers (especially without a calculator).
3. For multiplying fractions you do **not** need to have common denominators.
4. Multiply the numerators together.
5. Multiply the denominators together.
6. Cancel down the fraction to its simplest form.
7. If applicable, change the improper fraction into a mixed number.

Here are some examples. Read through them carefully to make sure you understand them. You might like to explain the working to another Learning Assistant.

Example

For example: $\frac{6}{8} \times \frac{2}{3}$

First cancel down if possible

$$\frac{\cancel{6}^{3}}{\cancel{8}_{4}} \times \frac{2}{3} = \frac{3}{4} \times \frac{2}{3}$$

Multiply the numerators together, then multiply the denominators together

$$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12}$$

Cancel down if possible

$$\frac{\cancel{6}^{1}}{\cancel{12}_{2}} = \frac{1}{2}$$

Example

For example: $2\frac{3}{4} \times 1\frac{1}{3}$

Write mixed numbers as improper fractions first, and cancel as far as possible:

$$2\frac{3}{4} \times 1\frac{1}{3} = \frac{11}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{3}$$

Multiply the numerators together, then multiply the denominators together

$$\frac{11}{1} \times \frac{1}{3} = \frac{11 \times 1}{1 \times 3} = \frac{11}{3}$$

Write as a mixed number if appropriate:

$$\frac{11}{3} = 3\frac{2}{3}$$

Exercise 2

Work out the following.

1. $\frac{1}{2} \times \frac{1}{4}$
2. $\frac{3}{4} \times \frac{2}{8}$
3. $1\frac{1}{4} \times \frac{3}{8}$
4. $\frac{1}{2} \times \frac{1}{2}$
5. $1\frac{1}{3} \times 2\frac{2}{3}$
6. $\frac{4}{5} \times 2\frac{1}{3}$
7. $\frac{1}{2} \times \frac{2}{4}$
8. $4\frac{2}{3} \times 1\frac{1}{2}$
9. $\frac{7}{8} \times \frac{3}{6}$
10. $\frac{9}{27} \times \frac{12}{24}$
11. $\frac{4}{3} \times \frac{8}{4}$
12. $\frac{20}{1} \times \frac{1}{20}$
13. $\frac{2}{3} \times \frac{10}{90}$
14. $1\frac{1}{5} \times \frac{2}{3}$

Think of some more examples to take to your tutorial or try them out with other Learning Assistants at your school.

Dividing fractions

To picture dividing fractions is not that easy. The trick is to think about different ways to read division sums.

A sum such as 6 ÷ 2 = 3 can be read as ‘six divided by two is three’ or ‘six shared between two is three’. You could also read it in a different way: ‘How many two’s are there in six? Three.’ This last way helps with picturing dividing fractions.

So a sum such as $\frac{1}{2} \div \frac{1}{4}$ can be read as: ‘How many quarters are there in a half?’ This is the same as asking how many fractions of the size of one quarter fit into a fraction of the size of a half’. The answer is 2. This is because two fractions of the size of one-quarter will be needed to cover a fraction the size of one-half. You might like to look at Example 2 in week 1’s work to remind yourself.

Exercise 3 has some questions for you to work out. Remember, if you get stuck, it can help to draw a picture or discuss it with other Learning Assistants at your school.

Exercise 3

Work out the following.

1. $\frac{2}{2} \div \frac{1}{2}$
2. $\frac{1}{4} \div \frac{1}{8}$
3. $\frac{2}{3} \div \frac{1}{3}$
4. $\frac{3}{6} \div \frac{1}{6}$
5. $\frac{1}{3} \div \frac{1}{6}$

The rule for dividing fractions is as follows:

1. If the fraction given is a mixed number, change it to an improper or top heavy fraction.
2. Cancel down the fractions if you can – it is easier to work with smaller numbers (especially without a calculator).
3. For dividing fractions you do **not** need to have a common denominator.
4. Turn the fraction after the ÷ sign upside down **and** change the ÷ to ×. The sum is now a multiplication sum.
5. Multiply the numerators together.
6. Multiply the denominators together.
7. Cancel down the fraction to its simplest form.
8. If applicable, change the improper fraction into a mixed number.

Here are some examples.

Example

For example: $\frac{4}{16} \div \frac{2}{3}$

Cancel down if possible:

$$\frac{\cancel{4}^{1}}{\cancel{16}_{4}} \div \frac{2}{3} = \frac{1}{4} \div \frac{2}{3}$$

Turn the fraction after the ÷ sign upside down and change the ÷ to ×:

$$\frac{1}{4} \times \frac{3}{2}$$

Multiply out:

$$\frac{1}{4} \times \frac{3}{2} = \frac{1 \times 3}{4 \times 2} = \frac{3}{8}$$

Example

For example: $2\frac{4}{9} \div 1\frac{1}{3}$

Write as improper fractions first:

$$2\frac{4}{9} \div 1\frac{1}{3} = \frac{22}{9} \div \frac{4}{3}$$

Then turn the fraction after the $\div$ sign upside down, and change the $\div$ to $\times$:

$$\frac{22}{9} \div \frac{4}{3} = \frac{22}{9} \times \frac{3}{4}$$

Cancel if possible, and multiply out:

$$\frac{\cancel{22}^{11}}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{\cancel{4}_{2}} = \frac{11}{6}$$

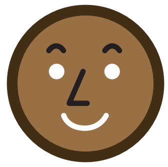
Exercise 4

Work out the following.

1. $\frac{4}{6} \div \frac{2}{3}$
2. $\frac{1}{5} \div \frac{2}{5}$
3. $1\frac{1}{3} \div \frac{3}{4}$
4. $2\frac{1}{2} \div \frac{4}{6}$
5. $\frac{12}{5} \div \frac{2}{3}$
6. $\frac{4}{7} \div \frac{2}{7}$
7. $1\frac{3}{4} \div 1\frac{2}{3}$
8. $\frac{18}{27} \div 1\frac{7}{14}$

Well done for completing all the exercises! Try to compare your answers with another Learning Assistant – did you get the same answers? Are there any questions that you find difficult? Mark these questions and remember to ask your tutor about them at the next Maths tutorial.

How am I doing?

	 **Easy** (Tick this box if you feel confident that you **understand this section well**)	 **Fine** (Tick this box if you still **need a little work** on this section)	 **Difficult** (Tick this box if you still **need a lot of work** on this section)
I can picture adding two fractions			
I can work out adding any two fractions			
I can picture subtracting two fractions			
I can work out subtracting any two fractions			
I can picture multiplying two fractions			
I can work out multiplying any two fractions using the rule			
I can picture dividing two fractions			
I can work out dividing any two fractions using the rule			

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Maths Unit 2: Week 3

Percentages

Percentages are used everywhere, in newspapers, on the radio and in shops. It is also an important part of the school syllabus. It is important to have a good understanding of percentages so you can teach it well to your students. In this section you will learn what a percentage is, and how to change fractions, decimals and percentages into each other.

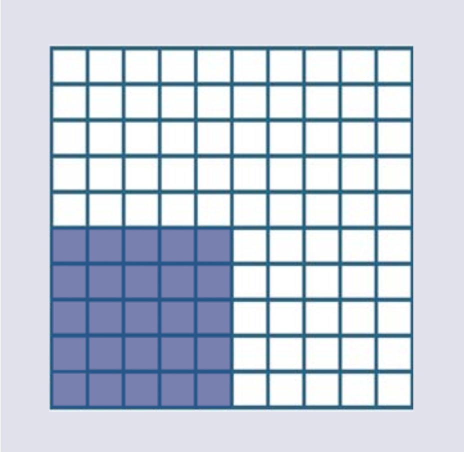
What are percentages?

One special type of fraction that is used a lot in everyday life is fractions with a denominator of 100 such as $\frac{25}{100}$ or $\frac{50}{100}$. The word percent comes from the Latin *per centum* meaning 'for each hundred'. Because these fractions are so useful they have their own notation, %, which is called 'percentage'. You take the '$\frac{}{100}$' part and replace it with %.

So $\frac{25}{100}$ is written as 25%, which is read as 'twenty-five per cent' and means '25 out of a hundred'.

You could model percentages by drawing a square that has been cut, or divided, in 100 equal parts.

If we colour in 25 of them, we say '25 out of a hundred' or 25 per cent' are coloured in. We can write this as $\frac{25}{100}$ = 25%. From what you learned earlier about equivalent fractions you can see this is the same as $\frac{1}{4}$. So we can write $\frac{1}{4} = \frac{25}{100}$ = 25%.



One of the reasons why percentages are useful is that you can compare and order them easily. For example, comparing 42%, 35% and 23% is easier than comparing the corresponding fractions $\frac{21}{50}$, $\frac{7}{20}$ and $\frac{23}{100}$.

In fact, fractions, decimals and percentages are all interchangeable, so you can choose to use whichever is most appropriate for your situation.

Writing percentages as fractions

As long as you remember that percentage means 'out of a hundred' you can easily write percentages as a fraction with a denominator of 100. If possible, cancel down the fraction to its simplest form.

For example, 73% = $\frac{73}{100}$ and 80% = $\frac{80}{100} = \frac{8}{10} = \frac{4}{5}$

The method can be summarised as follows:

> To change a percentage to a fraction, write the percentage as a fraction with a denominator of 100 and then simplify.

Exercise 1

Write the following percentages as fractions. They may need simplifying.

1. 10%
2. 40%
3. 8%
4. 50%
5. 125%
6. 2.5%

Writing fractions as percentages

You may remember from your work on equivalent fractions that if you multiply the top and the bottom of a fraction by the same number (not zero), then its value remains the same.

To change a fraction into a percentage you need to write the fraction you have been given as an equivalent fraction with a denominator of 100, and then re-write it in percentage notation.

The method for changing a fraction to a percentage can be summarised as follows:

> 1. Write down the fraction. If the fraction is a mixed number (such as $2\frac{1}{4}$) re-write it first as an improper or top heavy fraction ($\frac{9}{4}$).
> 2. Look at the denominator and decide what you have to multiply (or divide) it by to get an equivalent fraction with a denominator of 100.
> 3. Write down the equivalent fraction with a denominator of 100.
> 4. Re-write this as a percentage by replacing the '$\frac{1}{100}$' part by %.

Here are some examples.

$\times 50$

$$\frac{1}{2} = \frac{?}{100} = \frac{50}{100} = 50\%$$

$\times 50$

$\times 25$

$$\frac{1}{4} = \frac{?}{100} = \frac{25}{100} = 25\%$$

$\times 25$

$\times 20$

$$\frac{3}{5} = \frac{?}{100} = \frac{60}{100} = 60\%$$

$\times 20$

$\times 10$

$$1\frac{2}{10} = \frac{12}{10} = \frac{?}{100} = \frac{120}{100} = 120\%$$

$\times 10$

Exercise 2

Write the following fractions as percentages:

1. $\frac{1}{5}$
2. $\frac{2}{10}$
3. $\frac{4}{8}$
4. $2\frac{2}{20}$
5. $3\frac{7}{20}$

Writing decimals as percentages

Writing decimals as percentages and percentages as decimals is very straightforward. You need to remember place value in the decimal system you learned about earlier.

The following diagram shows place value in the decimal system.

Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
				2	6	4 •	3	5	0	9		
							$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	$\frac{1}{100000}$	$\frac{1}{1000000}$
1 000 000	100 000	10 000	1000	100	10	1	0.1	0.01	0.001	0.0001	0.00001	0.000001

When you look at the chart above you can see that the columns after the decimal point are called 'tenths', 'hundredths', 'thousandths' of a unit, and so on.

This means that decimals can be re-written as:

0.5 has 5 tenths, and can be re-written as $\frac{5}{10}$.

The equivalent fraction with a denominator of 100 is $\frac{50}{100} = 50\%$

0.43 has 4 tenths and 3 hundreds and can be re-written as $\frac{4}{10} + \frac{3}{100}$.

Working this out you get $\frac{40}{100} + \frac{3}{100} = \frac{43}{100} = 43\%$

An easier way is to write a given decimal as a decimal with 2 decimal places, so re-write 0.5 as 0.50; 0.7 as 0.70 (0.25 is fine as it is). Write these decimals as hundredths, then as percentages.

Here is the rule for changing decimals into percentages:

Write the decimal to 2 decimal places.

Re-write it as a fraction of hundredths.

Remove the numerator and add the % sign.

Here are some examples:

$0.2 = 0.20 = \frac{20}{100} = 20\%$

$0.3 = 0.30 = \frac{30}{100} = 30\%$

$0.15 = \frac{15}{100} = 15\%$

$0.08 = \frac{8}{100} = 8\%$

Re-writing percentages as decimals follows a similar method:

Re-write the percentage as a fraction with a denominator of 100.

Write it in decimal form with 2 decimal places.

Here is an example: 51% can be written as $\frac{51}{100}$ which is 0.51.

Exercise 3

1. Write the following decimals as percentages.

a. 0.50

b. 0.5

c. 0.18

d. 0.75

e. 0.8

1. Write the following percentages as decimals.

a. 8%

b. 13%

c. 70%

d. 12.5%

Interchanging percentages, fractions and decimals

Percentages, fractions and decimals are all interchangeable, so you can choose to use whichever is most appropriate for your situation. If you are sharing food, it is probably easier to use fractions. If you need to compare some quantities such as two-thirds, five-eighths and 0.64 percentages are better.

The next exercise gives you some practice in changing fractions, decimals and percentages into each other.

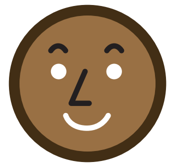
Exercise 4

Fill in the missing gaps. The first row has been completed for you.

Fraction	**Fraction with a denominator of 100**	**Percentage**	**Decimal**
$\frac{1}{4}$	$\frac{25}{100}$	25%	0.25
		50%	
	$\frac{14}{100}$		
			0.38
$\frac{7}{20}$			
	$\frac{100}{100}$		
			0.4
			0.01

Well done for completing all the exercises! Try to compare your answers with another Learning Assistant – did you get the same answers? Are there any questions that you find difficult? Mark these questions and remember to ask your tutor about them at the next Maths tutorial.

How am I doing?

	 **Easy** (Tick this box if you feel confident that you **understand this section well**)	 **Fine** (Tick this box if you still **need a little work** on this section)	 **Difficult** (Tick this box if you still **need a lot of work** on this section)
I know what a percentage is			
I can visualise a fraction by drawing a picture			
I can write a fraction as a percentage			
I can write a percentage as a fraction			
I can write a decimal as a percentage			
I can write a percentage as a decimal			
I can interchange decimals, fractions and percentages			

Notes on what to do next:

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Maths Unit 2: Week 4

Ratio

Ratio is another way to describe how quantities or sizes relate to each other. It is often used in recipes. This week you will learn what a ratio is, how to write ratios, simplify ratios and solve problems involving ratio.

What is ratio?

Earlier, you saw how a fraction such as $\frac{3}{4}$ can be thought of as '3 parts out of 4 parts'. There are often reports in the media that describe proportions in this form, for example: 'Three out of four people in Sierra Leone are below the age of 35 years.' This is the same as saying '$\frac{3}{4}$ or 75% of the population is below the age of 35 years.'

You could also say: 'For every one person who is over 35 years old, there are three people who are under 35 years old.' Mathematically, we say: 'The ratio of those people who are over 35 years old to those who are under 35 years old is one to three.' This ratio is written as 1:3. The ':' is read as 'to'.

When you use fractions to express the relationship between quantities you are comparing a part with the whole lot (e.g. 3 out of 4). When you are using ratio you are comparing parts with parts.

This becomes clearer when using ratio in recipes.

Writing ratio

A recipe for making biscuits says to mix:

- 6 cups of flour
- 2 cups of sugar
- 4 cups of butter

There are thus three ingredients or parts needed to make the dough, that is:

flour; sugar; butter

To compare the quantities needed we can express them as a ratio, that is the ratio of flour to sugar to butter and write this as:

flour : sugar : butter

6 : 2 : 4

This means that the ratio of flour to sugar to butter is 6:2:4. The total number of parts (in this case the number of cups) in this mixture is 6 (the flour) + 2 (the sugar) + 4 (the butter) = 12.

Ratios can be cancelled down like fractions. So dividing by 2, this ratio can be expressed more simply as 3:1:2, or 3 parts flour:1 part sugar:2 parts butter. This ratio cannot be cancelled down further if you want to keep using whole numbers. This is called **writing the ratio in its simplest form**. The total number of parts now in the whole mixture would be 3 (the flour) + 1 (the sugar) + 2 (the butter) = 6 parts.

You would write it like this:

flour : sugar : butter

Total number of parts

divide all by 2 ($\div 2$)

6 : 2 : 4 $\qquad$ $6 + 2 + 4 = 12$

3 : 1 : 2 $\qquad$ $3 + 1 + 2 = 6$

Please note it is very important to use good notation when working with ratio. Many mistakes can be avoided by writing down the name of the parts (such as, in the example, flour:sugar:butter) and showing your working.

Exercise 1

Write the following as a ratio in its simplest form:

1. I mix 200ml of water and 50ml of milk.
2. There are 20 girls and 10 boys in my sister's class.
3. A batch of concrete is made of 40kg of sand, 8kg of cement, 20kg of water and 8kg of gravel.

Calculating with ratio

Ratio problems are of two types. The first one asks you to work out the quantities required if one of the parts or ingredients changes; for example, you can make more biscuits because you want to use up the sugar in your cupboard.

The second type asks you to work out the different ratios when the total number of parts is changed. Both problems can be solved easily if you stick to using good notation and layout and remembering that you change the parts by multiplying or dividing **all** of the parts by the same number. Here is the method:

Step 1	Write down the ratio.
Step 2	Write down the ratio in its simplest form.
Step 3	Write the changed part underneath the original part and work out by what number you would have to divide or multiply the original part to get the new part.
Step 4	Multiply or divide all parts by that number.
Step 5	Write down your answer in a sentence.

Example One of the parts changes

The recipe for making biscuits, used earlier, says to mix

- 6 cups of flour
- 2 cups of sugar
- 4 cups of butter

How much flour and butter will you need if you have 3 cups of sugar?

Here is how to set out your working clearly:

	flour : sugar : butter	Total number of parts
divide all by 2 (÷ 2)	6 : 2 : 4	6 + 2 + 4 = 12
	3 : 1 : 2	3 + 1 + 2 = 6
× 3	× 3	
	? : 3 : ?	
	9 : 3 : 6	9 + 3 + 6 = 18

With 3 cups of sugar, you will need 9 cups of flour and 6 cups of butter to make the biscuit mixture.

Example The total number of parts changes

Using the same biscuit recipe example from earlier:

The recipe for making biscuits says to mix:

- 6 cups of flour
- 2 cups of sugar
- 4 cups of butter

How much flour, sugar and butter will you need use if the total weight of the biscuit mixture is 24 cups?

We set out the working in a similar way to the previous example:

	flour : sugar : butter	Total number of parts
divide all by 2 (÷ 2)	6 : 2 : 4	6 + 2 + 4 = 12
	3 : 1 : 2	3 + 1 + 2 = 6
× 4		× 4
	? : ? : ?	24
	12 : 4 : 8	24 = 12 + 4 + 8

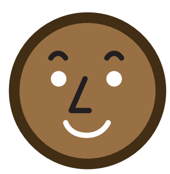
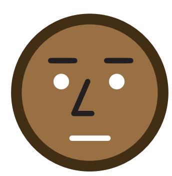
You will need 12 cups of flour, 4 cups of sugar and 8 cups of butter to make 24 cups of biscuit mixture.

Exercise 2

1. Many household chemicals are supplied in a concentrated form and need to be diluted before use. For example, the directions on a bottle of bleach state that 1 part of bleach should be mixed with 9 parts water. The ratio of bleach to water is therefore 1:9. Suppose that there is 40 ml of bleach left in the bottle. How much water should be mixed with it?
2. An alloy of gold and silver contains 3 parts of gold to 5 parts of silver by mass. How much of each metal does 16g of the alloy contain?
3. A mother has a bag with 60 marbles and divides this in the ratio 4:3:3 among her daughter and her two sons. How many marbles does each child get?

Well done for completing all the exercises! Try to compare your answers with another Learning Assistant – did you get the same answers? Are there any questions that you find difficult? Mark these questions and remember to ask your tutor about them at the next Maths tutorial.

How am I doing?

	 **Easy** (Tick this box if you feel confident that you **understand this section well**)	 **Fine** (Tick this box if you still **need a little work** on this section)	 **Difficult** (Tick this box if you still **need a lot of work** on this section)
I know what a ratio is			
I can write a ratio for a given problem			
I can work out a ratio in its simplest form			
I can do calculations with ratios			

Notes on what to do next:

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Maths Unit 2: Week 5

The unitary method

In this unit you have learned what fractions, percentages and ratio are, how to write them, and how to do simple calculations with them.

This week you will learn how to solve harder problems involving proportion (ratio), fractions and percentages. We shall use one method, the **unitary method**. The method is called the **unit**-ary method because it asks you to work out how much one **unit** would be.

The great thing about this method is that it is based on logical thinking – so there are no ‘tricks’ to learn that do not make sense. The other great advantage of this method is that you can use it for any problem that asks you about change, such as a percentage increase, or about calculating parts of an amount, such as working out the fraction of an amount.

The structure of the unitary method and what to look out for

The unitary method is used to find out the value of a number of items by first working out the value of one item.

The method consists of three steps, neatly written underneath each other. (In some countries the method is called **the three-step rule** because it consists of three steps.)

The ‘neatly written’ part is very important because it helps you to see and keep track of the logical thinking. Here are the steps.

Step 1	Write down what is given in the problem. To avoid problems, write what is asked for on the right-hand side (RHS) of the equals sign (it will still work it you do not, but it gets a bit messier).
Step 2	Then work out how much one unit would be. Use only multiplication or division, not addition or subtraction.
Step 3	Finally, write down the quantity that is asked for in the question and work out the solution based on what you worked out in Step 2.

Suppose, you are asked to solve the following problem:

If 12 large eggs cost Le 12,000, how much would 9 large eggs cost?

Using the unitary method, the working would be set out like this:

Step 1 Write down what is given in the question.

Step 2 Work out how much one unit will be.

Step 3 Work out the solution for the quantity that is asked for (here it is 9 eggs).

12 eggs = Le 12000

$\downarrow \div 12 \downarrow$

To get from 12 to 1 divide by 12.

1 egg = Le 1000

$\downarrow \times 9 \downarrow$

To get from 1 to 9 multiply by 9.

9 eggs = Le 9000

The cost of 9 eggs is asked for so write the cost on the right-hand side of the equals sign.

So the answer is 9 eggs will cost Le 9000.

Here is another example:

Twenty building bricks weigh 5kg or 5000g. How much do 7 building bricks weigh?

Using the unitary method, the working would look like this:

Step 1 The problem asks for the weight, so put the weight on the RHS of the equals sign.

Step 2 To get from 20 to 1 divide by 20.

Step 3 To get from 1 to 7 multiply by 7.

20 bricks = 5000g

$\downarrow \div 20 \downarrow$

1 brick = 250g

$\downarrow \times 7 \downarrow$

7 bricks = 1750g = 1.75kg.

It is easier to use grams rather than kilograms if you do not want to end up with decimals.

Exercise 1

1. Three oranges cost Le 600. How much do 5 oranges cost?
2. Five dogs have 20 legs altogether. How many legs do 12 dogs have?
3. Thirty papayas fit exactly into 5 boxes. How many papayas would fit into 7 boxes? (Hint: remember to put what is asked for on the right-hand side of the equation sign.)

Can you think of some more examples to share with the Learning Assistants in your school?

Using the unitary method to solve fraction problems

The unitary method is often useful for finding fractions of an amount.

Suppose a water tank is $\frac{3}{4}$ full and contains 600 litres of water. How much water will the water tank contain when it is completely full?

(Hint: remember that ‘completely full’ or ‘the whole tank’ is the same as 1 and can be written as a fraction equivalent to 1; for example, $\frac{4}{4}$, $\frac{6}{6}$, $\frac{700}{700}$, $\frac{34}{34}$ and so on. Pick an equivalent fraction that will make the calculation easy – that is, pick the same size of fraction as the one given in the problem. In this case it is quarters and the whole tank would be $\frac{4}{4}$.)

Here is the working:

Step 1	$\frac{3}{4}$ = 600 litres
	$\downarrow \div 3 \downarrow$
Step 2 To get from $\frac{3}{4}$ to $\frac{1}{4}$ divide by 3.	$\frac{1}{4}$ = 200 litres
	$\downarrow \times 4 \downarrow$
Step 3 To get from $\frac{1}{4}$ to $\frac{4}{4}$ multiply by 4.	$\frac{4}{4}$ = 800 litres

So the water tank will contain 800 litres when full.

Here's another question: find $\frac{3}{5}$ of Le 10,000.

It is useful to remember that Le 10,000 is 'the whole lot'. So what is actually given in the question, the Le 10,000, is $\frac{5}{5}$.

Here is how to work out $\frac{3}{5}$ of Le 10,000:

Step1	$\frac{5}{5}$ = Le 10000
	$\downarrow \div 5 \downarrow$
Step2	$\frac{1}{5}$ = Le 2000
	$\downarrow \times 3 \downarrow$
Step 3	$\frac{3}{5}$ = Le 6000

So $\frac{3}{5}$ of Le 10000 is Le 6000.

Exercise 2

1. Find $\frac{3}{4}$ of 480kg.
2. A full jar of jam contains 330g of jam. If the jar is $\frac{2}{3}$ full, how much jam will it contain?
3. A class has 60 students when everyone is present. Yesterday only $\frac{3}{4}$ of the class was present. How many students were in the class?

Can you make up some more problems to share with other Learning Assistants at your school or to take to your tutorial?

Using the unitary method to solve percentage problems

The unitary method works well for solving percentage problems. What you need to remember is that 'the whole lot' is the same as 100%.

Suppose one loaf of bread cost Le 1000. If the price increases by 8%, how much more will you have to pay?

Step1 $\quad 100\% = \text{Le } 1000$

$\downarrow \div 100 \downarrow$

Step2 $\quad 1\% = \text{Le } 10$

$\downarrow \times 8 \downarrow$

Step 3 $\quad 8\% = \text{Le } 80$

So you will have to pay Le 80 **more** for a loaf of bread.

Here's another example. A manufacturer of diesel engines has produced a new model of engine which he claims will use 10% less diesel. If you need 60 litres of diesel to do a job with the old model, how much less diesel will you now need with the new model?

(Hint: it actually does not matter that much if you do not really understand the technical aspects of a problem like this – here, for example, you can still work out what is asked for without knowing much about engines or their diesel use!)

Step1 $\quad 100\% = 60 \text{ litres}$

$\downarrow \div 100 \downarrow$

Step2 $\quad 1\% = 0.6 \text{ litres}$

$\downarrow \times 10 \downarrow$

Step 3 $\quad 10\% = 6 \text{ litres}$

So the new model will use 6 litres of diesel less to do the job.

When the problem is about percentage increase or decrease, you are often asked about the new price or amount, and not only how much the increase or decrease would be. In that case remember to add or take away the increased or decreased amount from the original amount.

For example, using the loaf of bread problem, a new problem might be:

One loaf of bread cost Le 1000. If the price increases by 8%, what will the new price be?

Step1 $\quad 100\% = \text{Le } 1000$

$\downarrow \div 100 \downarrow$

Step2 $\quad 1\% = \text{Le } 10$

$\downarrow \times 8 \downarrow$

Step 3 $\quad 8\% = \text{Le } 80$

So you will have to pay Le 80 **more** for a loaf of bread.

So the price increase will be Le 80. So the new price for a loaf of bread will be:

$$\underbrace{\text{Le } 1000}_{\text{old price}} + \underbrace{\text{Le } 80}_{\text{price increase}} = \underbrace{\text{Le } 1080}_{\text{new price}}$$

Exercise 3

1. Find 10% of Le 320.
2. Find 20% of 900kg.
3. Interest on a loan is 15% per year. How much interest would be paid on a loan of Le 200,000 after 1 year?
4. A mathematics book costs Le 9000. The shopkeeper is prepared to sell it to you for Le 8100. What percentage discount is this? (Hint: there is more than one way to work this out.)
5. Interest on a loan is 25% per year. Amal borrows Le 100,000 and pays it back together with the interest after one year. How much does he pay back in total?

Can you make up some more problems to try out with other Learning Assistants at your school or to take to your tutorial?

Exam questions (from Teachers Certificate Entrance Examination)

Here are some exam questions for you to try. Take your answers to your next tutorial to show to your Maths Tutor.

1. Mr Sesay sold a bag for Le 4300. If he bought the bag for Le 5000, find his percentage loss.
2. A car travels 8 miles in 12 minutes. What is its speed in miles per hour (mph)?

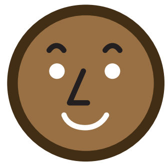
   a. 20 mph b. 96 mph c. 40 mph d. 80 mph
3. Four men took 3 hours to erect a brick wall. How long would it take 6 men to erect the same wall?

   a. 1 hour b. 2 hours c. 3 hours d. 4 hours
4. A man borrowed Le 26,000 from his employer. He had to pay back with an interest of 25%.How much money did he pay back?

   a. Le 19,500 b. Le 28,500 c. Le 32,500 d. Le 33,500
5. There are 80 pupils in a class. If 8 of them are absent, what percentage is present?

   a. 90% b. 80% c. 10% d. 45%

How am I doing?

	 **Easy** (Tick this box if you feel confident that you **understand this section well**)	 **Fine** (Tick this box if you still **need a little work** on this section)	 **Difficult** (Tick this box if you still **need a lot of work** on this section)
I know how the three steps of the unitary method work			
I can use the unitary method to find out the value of a number of items			
I can use the unitary method to work out fractions of an amount			
I can use the unitary method to work out percentages of an amount			
I can use the unitary method to work out percentage changes such as increase and decrease			

Notes on what to do next:

Learning Assistant signature: ....

Date: ....

Tutor signature: ....

Date: ....