Girls' Access to Education
Girls' Education Challenge

Sierra Leone

Maths Unit 3

Name of Learning Assistant ____________________

School ____________________

Tutor ____________________

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HANDICAP INTERNATIONAL

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The Open University

Maths Unit 3: Week 1

Algebraic Expressions

In this unit, you are going to learn about algebra. This is where we use letters in place of numbers. Algebra has its own language. You will not be able to see how this relates to everyday situations until you have studied more algebra. There are many problems that can be solved using algebra but at the moment we will concentrate on the basics. For example, if we had a certain length of fencing, we could use algebra to work out how best we could use it to surround the largest area of land for our sheep.

When you learn a new language, you begin by learning the vocabulary, the words, and you also have to learn the rules for putting the words together to make sentences. Algebra has its own words and rules too. We begin looking at these in this section.

In this section, there is something to read followed by some examples to follow – read the examples carefully several times. Then try the sums in the tasks.

Using letters for numbers

$x + 1$ is an example of an **expression**.

It is made up of two parts x and $+1$. These are called **terms**.

The letter x stands for an unknown number.

We do not know what the number is. It could be 3 or 12 or even 100. The number it stands for can vary or change so we call x the **variable**.

$4x$ means four lots of x $(x + x + x + x)$.

x and $4x$ are like terms because they both have xs in them.

This means that we can collect them together.

So $x + 4x = 5x$ $(x + x + x + x + x)$.

This is called **simplifying** and it can make expressions shorter.

Note that in algebra we do not need to write in the 1 for one x ($1x$) so it is written as just x.

Collecting like terms

Only **like** terms can be collected together.

In the first expression, we wrote $x + 1$. The x and the $+1$ could not be collected together because they are **unlike** terms.

The following are all examples of unlike terms.

$4x$ y 7 $2xy$ $2x^2$

Here are some examples of like terms.

$3y$ y $10y$ $-5y$ $7y$

Example

Simplify the following expression

$7a + 2b + b + 3a + a$

There are two variables in this expression *a*s and *b*s.

We collect each variable separately.

$7a + 3a + a = 11a$ (Remember a means $1a$.)

$2b + b = 3b$ (Remember b means $1b$.)

So we have

$7a + 2b + b + 3a + a = 11a + 3b$

Terms can be negative so you need to remember the rules for negative numbers. (Think temperatures from Unit1!)

Example

Simplify the following expression

$3x + 5y – 4x – y + 2x – 2y$

The sign before a term stays with the term when you collect the like terms

$3x – 4x + 2x = x$ $\quad$ $3 – 4 + 2 = 1$

$5y – y – 2y = 2y$ $\quad$ $5 – 1 – 2 = 2$

So $\quad 3x + 5y – 4x – y + 2x – 2y = x + 2y$

In both of these examples, we worked in stages.

When you are feeling more confident, you will be able to write down just the answer.

For example:

$3x + 5y – 4x – y + 2x – 2y = x + 2y$

These questions could include more than two variables or two variables and a number.

They are longer but no more difficult. You just need to work carefully.

Example

Simplify

$7v – 3v^2 + 9v^2 – 10 – 8v – 2 + 3w + 10w + 7$

There are 3 different variables and a number, so simplify each of them in turn.

$7v – 8v = –v$

$–3v^2 + 9v^2 = 6v^2$

$+3w + 10w = 13w$

$–10 – 2 + 7 = 5$

So $\quad 7v – 3v^2 + 9v^2 – 10 – 8v – 2 + 3w + 10w + 7 = –v + 6v^2 + 13w + 5$

You may have written these simplified terms in a different order. It is still correct as long as you have the same signs in front of each of the terms.

Task: These are for you to try. Write the answers in the spaces.

1. Simplify these expressions.

   a. $8a + 4a + a$ ....

   b. $-b + 2b$ ....

   c. $6x - 4x - x$ ....

   d. $-3y + 2y + y$ ....

   e. $10f - 13f + f + 5f$ ....

   f. $9d - d + 5d - 2d$ ....

   g. $2z + 7z + -3z - 3z$ ....

   h. $8g - g - 4g - 5g$ ....

2. Simplify these expressions.

   a. $a + 2b - 3a - 4$ ....

   b. $5x + 3y - x - y$ ....

   c. $-3r + 10s - 5r + 3s$ ....

   d. $z + 6y + 2z - 4y$ ....

   e. $2x^2 + 2x - 7x - x^2$ ....

   f. $3ab + 5ab - 6 - 7ab + 9$ ....

   g. $5m^2 - 2mn - 6m^2 + 3mn$ ....

   h. $-4 + 3jk + 2jk + 7 - jk$ ....

3. Write down an expression of 4 terms which simplifies to $6x + 5y$.

....

4. Write down an expression of 4 terms which simplifies to $2a - 3b$.

....

5. Moses is asked to simplify $8pq - 3qp$. He says that it cannot be done. Is he correct?

   Explain your answer.

....

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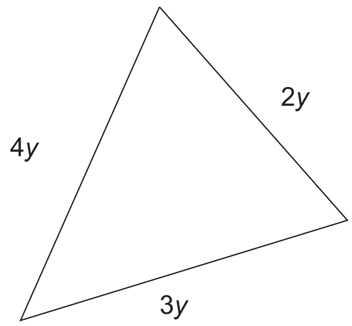
....

6 Write down an expression for the perimeter of these shapes.

To find the perimeter, we add up all of the lengths of the sides of the shape. This gives the total distance around the outside of the shape.

a.



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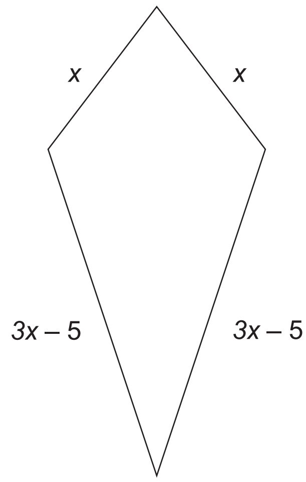

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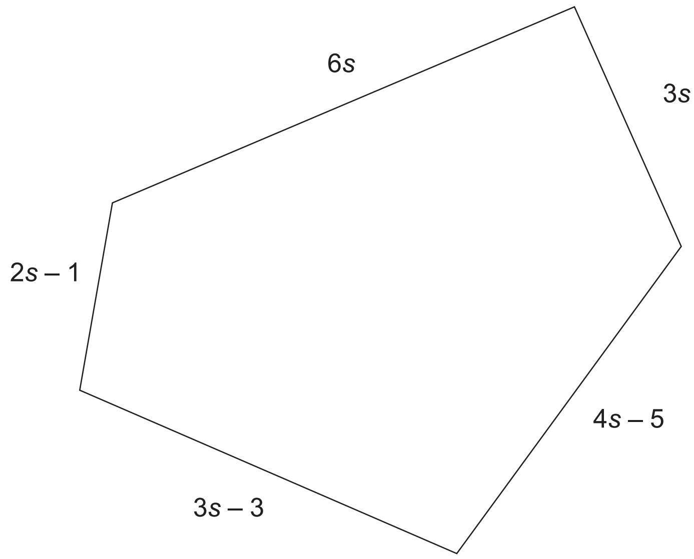

d. Write down the names of each of the above shapes.

Shape a is ....

Shape b is ....

Shape c is ....

7. Alpha simplifies $2x + y - 4 + 4y - 6 - 7x$.

His answer is $5x + 5y + 2$.

He has made two mistakes.

Write down the correct answer and explain his mistakes.

....

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Well done for completing these! Try to compare your answers with another Learning Assistant – did you get the same answers? Are there any questions that you find difficult? Mark these questions and remember to ask your tutor about them at the next Maths tutorial.

Exam question

Here is an exam question for you to try. Take your answers to the next Maths tutorial.

1. Choose the correct answer from the answers below.

Simplify: $3a + 4b - a + 2b$

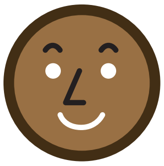
a. $2a + 6b$ b. $4a + 2b$ c. $7ab$ d. $2ab$

....

....

....

How am I doing?

	 **Easy** (Tick this box if you feel confident that you **understand this section well**)	 **Fine** (Tick this box if you still **need a little work** on this section)	 **Difficult** (Tick this box if you still **need a lot of work** on this section)
I can tell the difference between like and unlike terms			
I can simplify algebraic expressions by collecting like terms together.			

Notes on what to do next:

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Learning Assistant signature: ....

Date: ....

Tutor signature: ....

Date: ....

Maths Unit 3: Week 2

Expanding and Factorising Expressions

This section is all about brackets in algebraic expressions. You are going to learn how to remove brackets and how to put them back in. It is very important that you know how to work with brackets as they often tell us which parts of a calculation to do first. This is called **expanding** and **factorising**.

These are both skills that you need to know when doing algebra and again you will not see the real benefit of learning these skills until you get further on in algebra. They are however tested in your Teachers' College Entrance Examination. There are two parts to this section. In each part, there is something to read followed by some examples to follow – read the examples carefully several times. Then try the sums in the tasks.

Expanding brackets

You will remember from Unit 1 that when we have an expression concerning brackets, BIDMAS told us that we needed to deal with any brackets first and then multiply out.

Example

Work out $3(6 - 4)$.

$3(6 - 4) = 3(2)$ (Simplify the brackets first.)

$= 3 \times 2$ (This means 3 times 2.)

$= 6$

Some expressions contain brackets such as

$3(x + 1)$, $5y(y - 7)$ and $z(2z + 1) - 3z$

Now look at the following example.

Example

Expand $3(x + 1)$.

Expand means multiply out the bracket

The expression in the bracket has already been simplified. There are no like terms to collect together.

The 3 is outside the bracket so it means 3 times the bracket: $3 \times (x + 1)$.

The number outside the bracket has to be multiplied by everything inside the bracket.

You may find the grid method useful here. You used this in Unit 1 Section 2 when you were multiplying numbers together. The terms are put in separate boxes and multiplied together.

×	x	+1
3	$3x$	+3

$3 \times x = 3x$

$3 \times +1 = +3$

The unshaded squares show the answer.

$3(x + 1) = 3x + 3$

We now look at another example with xs in it.

Example

Multiply out $7x(x + 4)$.

We use the grid method again.

×	x	+4
$7x$	$7x^2$	$+28x$

$7x \times x = 7x^2$ because $x \times x = x^2$

$7x \times +4 = +28x$

So $7x(x + 4) = 7x^2 + 28x$

The next example has two brackets in it, and we just work with each bracket in turn.

Example

Expand and simplify $2(5a – 6) – 5(3a – 4)$.

Draw separate grids for each expansion.

Expand $2(5a – 6)$ to give $10a – 12$

×	$5a$	–6
2	$10a$	–12

Expand $–5(3a – 4)$ to give $–15a + 20$

×	$3a$	–4
–5	$–15a$	+20

Remember the rules for signs

(–) × (+) = (–) and (–) × (–) = (+)

Now put the two answers together and collect like terms

$$2(5a - 6) - 5(3a - 4) = 10a - 12 - 15a + 20$$
$$= -5a + 8$$

Once you get used to doing these you may want to do these without drawing grid boxes.

Drawing in curved lines may help. They remind you which terms you need to multiply.

$$2(5a - 6) - 5(3a - 4) = 10a - 12 - 15a + 20$$
$$= -5a + 8$$

Task: These are for you to try. Write the answers here.

1. Multiply out these brackets

a. $7(x + 2)$ ....

b. $4(4 - d)$ ....

c. $2(3y + 5)$ ....

d. $-2(3 + 5c)$ ....

e. $-3(5 - z)$ ....

f. $-4(3 + 2f)$ ....

g. $-(8c + 5)$ ....

h. $\frac{1}{2}(4x + 12)$ ....

2. Expand

a. $a(a + 3)$ ....

b. $y(2 + y)$ ....

c. $f(f - 1)$ ....

d. $z(4z + 3)$ ....

e. $2x(3x + 1)$ ....

f. $3b(6 - 5b)$ ....

g. $-x(8 - x)$ ....

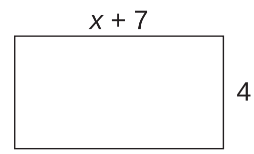
h. $rs(5 + 3t)$ ....

3. To find the area of a rectangle, we multiply the length by the width.

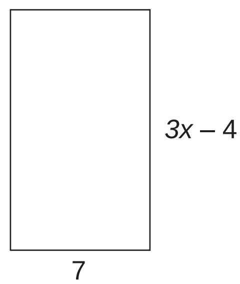
Do this for each of the rectangles below.

The answer you get will be an expression for the area of the rectangle.

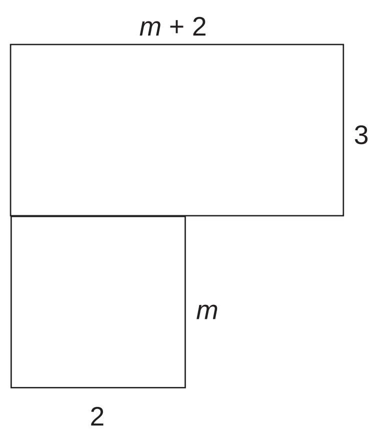
a.



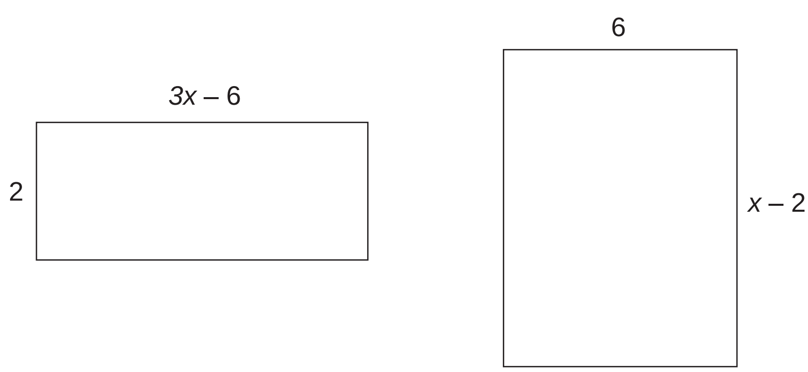
b.



c.



4. Show that the following rectangles have equal areas.



5. Expand and simplify the following expressions.

   Remember to expand each bracket and then collect the like terms together.

   If you find these difficult, look back at the last example

   a. $4(x + 3) + 2(x + 1)$ ....

   b. $3(y+3) – 2(y + 4)$ ....

   c. $2(z – 1) + 3 (z – 2)$ ....

   d. $7(w + 3) – 2(w – 4)$ ....

   e. $–4(x + 2) + 3 (x + 3)$ ....

   f. $–3(y+1) – 2(y + 1)$ ....

   g. $–2z(3z – 4) + 5$ ....

   h. $3w(3w – 2) + 2w(3w + 2)$ ....

6. Find the missing numbers to make these expressions equal.

   a. $3(x + 2) + 2(x – 2)$ and $?x + 2$

   ....

   b. $4(x + 7) + ?(x + 2)$ and $7x – 34$

   ....

   c. $?(x – 1) + 3(x + 1)$ and $2(4x – 1)$

   ....

   d. $5(x – 4) + 4(x – ?)$ and $3(3x – 8)$

   ....

   e. $6(x + 3) – 4(x + 4)$ and $?(x + 1)$

   ....

Factorising brackets

Factorising is the opposite of expanding.

The expression $3x + 3$ could be **factorised** into $3(x + 1)$.

We take out a **common factor** and put the brackets back in.

In this case we have a common factor of 3.

This is because 3 will divide into both terms: $3x$ and $+3$.

Example

Factorise $5x – 15$.

We are trying to put the brackets back in.

Look for the common factor first.

The common factor is 5 because it will divide into both $5x$ and $–15$ exactly.

You could use the grid method to do these.

Your expanded value is $5x - 15$ so you are trying to work backwards to find a missing numbers in the shaded part of the table

×	?	?
?	$5x$	–15

The grid becomes

×	x	–3
5	$5x$	–15

Now $5 \times ? = 5x$, so $? = x$,

and $5 \times ? = -15$, so $? = -3$.

So $5x - 15 = 5(x - 3)$

Example

Factorise $7x + 28$.

7 will divide into both $7x$ and $+28$ exactly, so 7 is a common factor

×	?	?
7	$7x$	+28

This gives

×	x	+4
7	$7x$	+28

So $7x + 28 = 7(x + 4)$

The common factors in the above two examples are numbers.

Here is another example where the common factor is a number.

Example

Factorise 18b + 27d.

9 divides into both 18b and 27d exactly.

9 is the common factor.

×	?	?
9	$18b$	$27d$

We cannot take out a letter as the letters are different.

This gives

$\times$	$2b$	$+3d$
9	$18b$	$27d$

So $18b + 27d = 9(2b + 3d)$

Sometimes the common factors can be letters or both letters and numbers

Example

Factorise $2x^2 - 6x$.

2 will divide into both $2x^2$ and $-6x$ exactly.

x will divide into both $2x^2$ and $-6x$ exactly.

So $2x$ is the common factor

$\times$	x	-3
$2x$	$2x^2$	$-6x$

So $2x^2 - 6x = 2x(x - 3)$

Example

Factorise $20pq - 15q$.

The common factor is $5q$

$\times$	$4p$	-3
$5q$	$20pq$	$-15q$

So $20pq - 15q = 5q\,(4p - 3)$

Remember that you can check that you have factorised correctly by multiplying the brackets back out again to see if you can get back to the original expression.

In this question you would multiply $5q$ by $4p - 3$ and see if you got back to $20pq - 15q$.

Task: These are for you to try. Write the answers here.

1. Factorise the following expressions.
   a. $2a - 16$
   b. $6x + 18$
   c. $10 - 25c$
   d. $24y + 3$
   e. $99z - 22$
   f. $64 + 32s$
   g. $16x + 24xy$
   h. $40st - 12t$
2. Factorise the following expressions.
   a. $y^2 + 3y$
   b. $x^2 - x$
   c. $7y^2 - 2y$
   d. $14x - 7x^2$
   e. $b^2 + 4ab$
   f. $3gh - h^2$
   g. $6p^2 - 21p$
   h. $5p^2qr - 8pq^2r$
3. In these questions, the area of each of the rectangles is shown. One other side is also shown.

   We are trying to find the expression for the missing side.

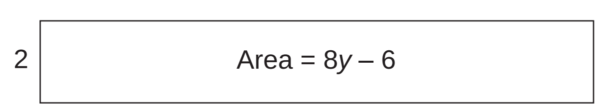
   As we know the area, we must work backwards to find the missing side by factorising the area.

   For example, in the first question we know that $8y - 6$ has a common factor of 2.

   So $8y - 6 = 2($...........................$)$.

   Whatever is in the bracket is the missing side.

   a.



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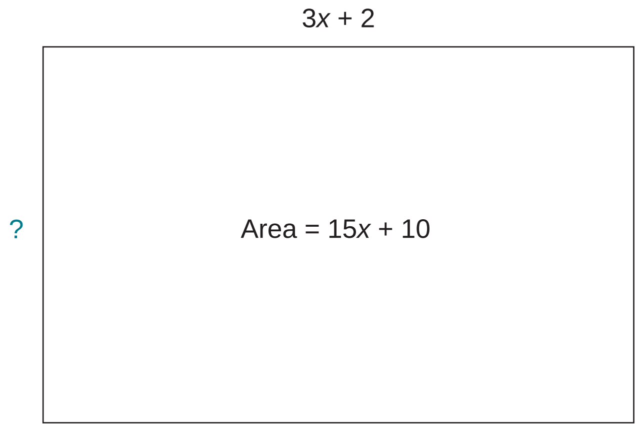
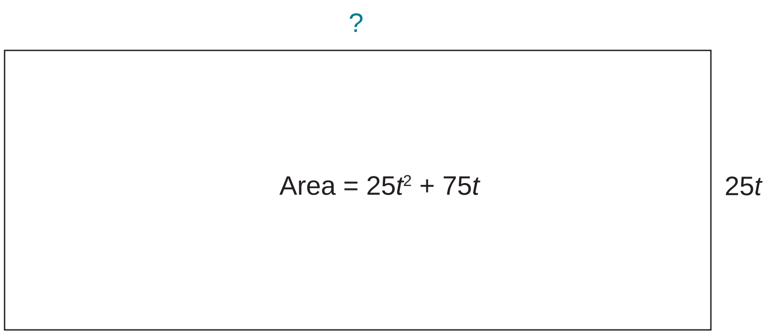


c.



4. Explain why the expression $3x + 7$ cannot be factorised.

5. In each part of the question, there are three expressions. Which one is the odd one out?

   (Try factorising them.)

   a. $18z^2$ $\quad$ $3z^2 + 6z$ $\quad$ $3z^2 + 6y^2$

   b. $4x + 2$ $\quad$ $6x^2 + 2$ $\quad$ $14x + 7$

   c. $5p + 5q$ $\quad$ $-2p + 2q$ $\quad$ $2p^2q + 2pq^2$

6. Morie and Samura factorised this expression: $60xy^2z - 36xy^2$

   They both got different answers.

   Morie's answer was $12xy^2(5z - 3)$.

   Samura's answer was $6y^2(10xz - 6x)$.

   Which answer is right?

   Explain your answer.

....

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Well done for completing these! Try to compare your answers with another Learning Assistant – did you get the same answers? Are there any questions that you find difficult? Mark these questions and remember to ask your tutor about them at the next Maths tutorial.

Exam question

Here is an exam question for you to try. Take your answers to the next Maths tutorial.

1. Factorise the following.

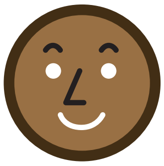
   a. $6x^2 + 9x$

....

   b. $12a - 8b$

....

How am I doing?

	 **Easy** (Tick this box if you feel confident that you **understand this section well**)	 **Fine** (Tick this box if you still **need a little work** on this section)	 **Difficult** (Tick this box if you still **need a lot of work** on this section)
I understand what is meant by expanding and factorising			
I can expand a single bracket			
I can factorise simple expressions			

Notes on what to do next:

Learning Assistant signature:

Date:

Tutor signature:

Date:

Maths Unit 3: Week 3

Solving Algebraic Equations

This section is all about solving equations. You are going to learn to recognise an algebraic equation – this is a special sort of equation which has an unknown as part of the equation. You will learn how to find the value of the unknown of the equation. This is called **solving the equation.** You will use what you have learned in the previous two units to help you do this.

Solving equations is a very important part of doing algebra. It is also tested in your Teachers' College Entrance Examination. There are two parts to this section. In each part, there is something to read followed by some examples to follow – read the examples carefully several times. Then try the sums in the tasks.

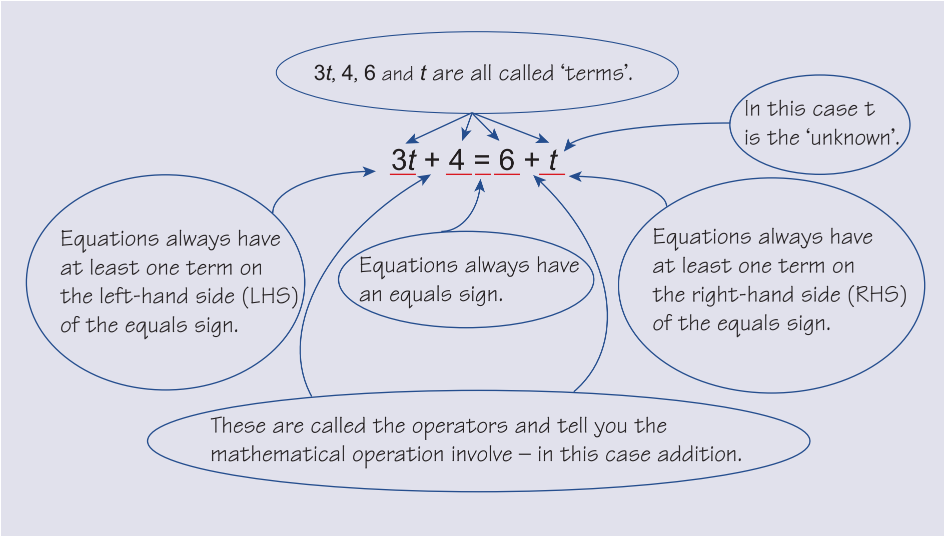
The algebraic equation and its parts

It is useful to know names of the parts of an equation so that you can talk about it instead of saying 'those bits there'. It will also help you to know what to look for when solving the equation.

This is an example of an algebraic equation and its parts.

Remember that $3t$ is the same as $3 \times t$ and t is the same as $1 \times t$.

Solving an equation means finding the value of the unknown. In the following example it is t.



The number in front of the unknown is called the coefficient. In this example, there is an unknown on the LHS with a coefficient of 3 ($3t$), and on the RHS of 1 ($1t$, usually written as t).

An equation says that whatever is on the LHS of the equals sign is equal to whatever is on the RHS of the equals sign. In this example it says '$3t + 4$ equals $6 + t$'.

Task 1: These are for you to try. Fill in the spaces.

Some answers have already been worked out to give you some more examples.

Is this an algebraic equation?	What is the unknown?	Terms on the LHS of the equation	Terms on the RHS of the equation	Operators on LHS	Operators on RHS
a. $x + 3 = 7$					
Yes. It has an • equals sign • 2 terms on the LHS • 1 term on the RHS • an unknown	x	x, 3	7	+	none
b. $7 – 9 = –2$					
No, it does not have an unknown	n/a	n/a	n/a	n/a	n/a
c. $8 – t =$					
No, it has no terms on the RHS	n/a	n/a	n/a	n/a	n/a
d. $14 – s = 18 + 3s$					
	s			–	+, ×
e. $3(p + 4) = 4p – 1$					
		$3(p + 4)$ 3 is the coefficient of $(p + 4)$	$4p$, 1	×, + and also brackets	×, –
f. $17.3g – 9 = 6.45$					
g. $1 = 14(w – 3)$					
		1			
h. $34.938 – 3847 =$					
i. $9f + 3 = 4f – 4$					
	f				

Rules to remember when solving an algebraic equation

Solving an equation means finding the value of the unknown of the equation.

So in the example:

$2p - 3 = 6$

you have to find out what p is equal to. To do this you have to get p on its own on one side of the equation like this:

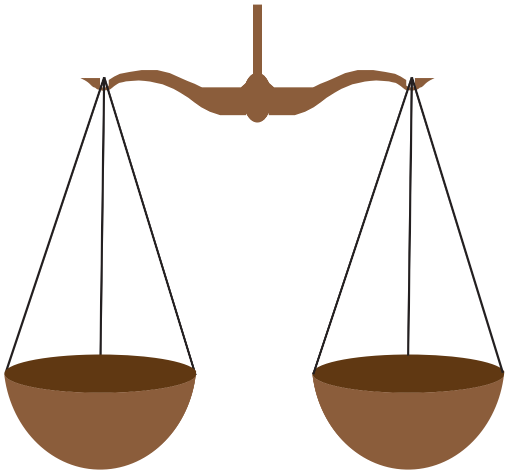
$p = \dots$ or $\dots = p$

There are some rules you have to obey when solving equations:

Remembering the role of the equals sign

The equals sign represents quantitative sameness – in other words, what is on the left-hand side of the equals sign represents the same quantity as what is on the right-hand side. The equals sign can be read as ‘is the same as’ or ‘is equivalent to’ or ‘is of equal value’.

You can think of this as weighing scales hanging in balance.



To keep the scales in balance, it means that whatever you do to one side of the equation, you have to do to the other side of the equation. So if you add 3 to one side, you would need to add 3 to the other side as well.

Inverse operations

When solving equations you will have to remove (get rid of) terms from the LHS or RHS of the equation. To do this, you will have to ‘undo’ what is there. For example, if you have to remove –3, you can do this by adding 3, because –3 + 3 = 0. So you have to use the inverse of the operation that is given.

You will know this already, but here is a list of the operations and their inverses:

Operation	**Inverse operation**
+ (add)	– (subtract)×
– (subtract)	+ (add)
× (multiply)	÷ (divide)
÷ (divide)	× (multiply)
$\dots^2$ (to the power of 2)	√ (square root)

BIDMAS and BIDMAS in reverse

You will remember from Unit 1 that when more than one mathematical operation needs to be done, for example in a sum, there is an order in which to do these. You learned to use BIDMAS to help you remember this order.

This is what it said in Unit 1:

Ordering operations

Sometimes sums involve more than one operation.

$3 + 5 \times 2$ involves two operations: addition and multiplication.

If you add first, you will get

$3 + 5 \times 2 = 8 \times 2 = 16$

If you multiply first, you will get

$3 + 5 \times 2 = 3 + 10 = 13$

One of these answers is wrong. Which do you think it is?

The correct answer is 13. We have to do the multiplication first.

Mathematicians have decided on a rule for this, known as BIDMAS.

This tells us the order to do the operations in.

B	Brackets	Do these first
I	Indices (or powers; squares and cubes ...)	Then any indices
D	Division	Division and multiplication should be
M	Multiplication	be worked together from left to right
A	Addition	Addition and subtraction should be
S	Subtraction	worked together from left to right

You could see this order as brackets are the most powerful, they overrule all other rules, then indices, etc. and addition and subtraction are the weakest operations.

When solving equations you are actually trying to undo operations. That means that the weakest operation will go first and the strongest will stay till last. What you have to do when solving equations and deciding which operation to undo first is to use BIDMAS in reverse.

When solving equations use BIDMAS in reverse

B	Brackets	Do these first
I	Indices (or powers; squares and cubes ...)	Then any indices
D	Division	Division and multiplication should be
M	Multiplication	be worked together from left to right
A	Addition	Addition and subtraction should be
S	Subtraction	worked together from left to right

This means that addition and subtraction are undone first, then multiplication and division, then indices, and finally brackets.

How to solve an algebraic equation

Example 1: The unknown is on the LHS

Look at the following equation:

$4f - 7 = 1$

You know that:

- the unknown is *f*
- it has to be put in the form of *f* = ...
- the –, the 7, the 4 and the (hidden) × sign have to be removed from the LHS of the equation
- the inverse of – is +
- the inverse of × is ÷
- the order to do this is first –, then ×, because of BIDMAS rules in reverse
- whatever is done to the LHS has to be done to the RHS.

Here is the solution to the equation:

$4f \cancel{-7} = 1$
$\quad \cancel{+7} \quad +7$

Add 7 to both sides
to remove the –7

$\cancel{4}f = 8$
$\cancel{\div 4} \quad \div 4$

Divide both sides
by 4 to remove the 4

This normally gets written
as a fraction because otherwise
it can look really messy
and confusing.

$\frac{4f}{4} = \frac{8}{4}$

$\frac{\cancel{4}^{1}f}{\cancel{4}_{1}} = \frac{\cancel{8}^{2}}{\cancel{4}_{1}}$

This can be cancelled down.
$\frac{1}{1} = 1$ and $\frac{2}{1} = 2$

So $f = 2$

This is the answer!

Example 2: the unknown is on the RHS

Look at the following equation:

$8 = 7m - 6$

You know that:

- the unknown is *m*
- it has to be put in the form of ... = *m* (easier than the form *m* = …, because *m* is already on the RHS
- the 7, the (hidden) × sign, the – and the 6 have to be removed from the RHS of the equation

- the inverse of – is +
- the inverse of × is ÷
- the order to do this is first –, then ×, because of BIDMAS rules in reverse
- whatever is done to the RHS has to be done to the LHS.

Here is how to solve the equation:

$$8 = 7m - \cancel{6}$$
$$+\,6 \qquad +\cancel{6}$$

Add 6 to both sides to get rid of the –6

$$\frac{14}{7} = \frac{7m}{7}$$

Divide both sides by 7 to get rid of the 7

$$\frac{\cancel{14}^{\,2}}{\cancel{7}_{\,1}} = \frac{\cancel{7}^{\,1}m}{\cancel{7}_{\,1}}$$

$\frac{2}{1} = 2$ and $\frac{1}{1} = m$

So $2 = m$ or $m = 2$

Task 2: These are for you to try.

a. $x + 3 = 8$ ....

b. $x - 3 = 8$ ....

c. $2t = 10$ ....

d. $8s = -16$ ....

e. $8s + 2 = 26$ ....

f. $4f - 7 = 9$ ....

g. $12 = 3q$ ....

h. $14 = 3q + 2$ ....

i. $17 = v - 2.5$ ....

j. $\frac{t}{2} + 4 = 8$ ....

k. $\frac{q}{4} + 4 = 0.5$ ....

l. $-3 + 11y = 30$ ....

Make up some examples of your own and share them with the other Learning Assistants at your next tutorial.

Equations with brackets

At times you may be asked to solve equations that have brackets, for example:

$3(y + 2) = 18$

There are two ways to solve this type of equation.

Either multiply out or expand the brackets (as you did in last week's unit).

In this case you get:

$3(y + 2) = 18$	Expand the bracket
$3y + 6 = 18$	
$3y + \cancel{6} = 18$ $\quad -\cancel{6} \quad -6$	Subtract 6 from both sides to remove the 6
$\frac{3y}{3} = \frac{12}{3}$	Divide both sides by 3
So $y = 4$	

Alternatively, first remove the coefficient in front of the bracket. This would mean you follow the BIDMAS rule in reverse – that is, brackets are removed last.

Remember that $3(y + 2)$ is the same as $3 \times (y + 2)$

In this case you get:

$\frac{3(y + 2)}{3} = \frac{18}{3}$	Divide both sides by 3
$\frac{\cancel{3}^{1}(y + 2)}{\cancel{3}_{1}} = \frac{\cancel{18}^{6}}{\cancel{3}_{1}}$	Cancel
$1(y + 2) = \frac{6}{1}$	$1 \times (y + 2) = (y + 2)$, because the brackets are no longer needed mathematically, you can remove them
$y + \cancel{2} = 6$ $\quad -\cancel{2} \quad -2$	Subtract 2 from both sides
So $y = 4$	

As you can see, you get the same answer! Practise both methods and then decide which one you like better.

Task 3: These are for you to try.

a. $2(t - 2) = 16$

b. $3(x + 4) = 18$

c. $15 = 5(q - 2)$

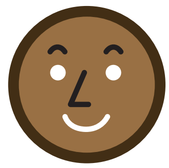
Well done for completing these! Try to compare your answers with another Learning Assistant – did you get the same answers? Are there any questions that you find difficult? Mark these questions and remember to ask your tutor about them at the next Maths tutorial.

Exam question

Here is an exam question for you to try. Take your answers to the next Maths tutorial.

If $2x + 3 = 27$, x represents………………

How am I doing?

	 **Easy** (Tick this box if you feel confident that you **understand this section well**)	 **Fine** (Tick this box if you still **need a little work** on this section)	 **Difficult** (Tick this box if you still **need a lot of work** on this section)
I can recognise an equation			
I know the names of parts of an equation			
I can solve equations with the unknown on the LHS			
I can solve equations with the unknown on the RHS			
I can solve equations with brackets.			

Notes on what to do next:

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Maths Unit 3: Week 4

Substitution into Formulae

In the last section you learned about solving algebraic equations. This section is about working with formulae. You are going to learn how to write a formula and how to use substitution to solve problems that involve formulae.

Formulae are used everywhere, e.g. in science, in medicine, on household bills, in banks. Because formulae are used so much in everyday life, it is important that you and the pupils you will be teaching in schools become skilled in working with formulae and using substitution.

There are two parts to this section. In each part, there is something to read followed by some examples to follow – read the examples carefully several times. Then do the tasks.

Substitution

Substitution is changing or swapping one thing for another. It is a powerful and necessary thinking tool. In real life, substitution is used constantly: for example, deciding which different herbs and spices to use in today's meal to make it different from yesterday's, which mode of transport to use (will you walk, cycle or use the bus?) or which dress to wear. Substitution involves thinking about possibilities and alternatives while also considering limitations; for example, substituting a dress for a calculator will not be valid when the aim is to change your dress. But this substitution could be valid when considering what you are going to spend your money on.

Substitution in mathematics uses the same principles: you swap one value for another or a letter for a number.

Here is an example:

To count the total number of legs of his 25 chickens, a farmer works this out as 25 chickens each with two legs, that is:

25 (the number of chickens) × 2 (2 legs each) = 50 legs

He sells some of his chickens and is left with 20 chickens. To work out the total number of chicken legs now, he replaces the number 25 in his calculation by 20. In mathematical language this is called **substituting 20 for 25**. He then gets:

20 (the number of chickens) × 2 (2 legs each) = 40 legs

When you do substitution it really helps if you set out your work clearly so you can see the change that is made; for example, like this:

25 (the number of chickens) × 2 (2 legs each) = 50 legs

↓

20 (the number of chickens) × 2 (2 legs each) = 40 legs

Task 1: These are for you to try.

Write up your method and answers neatly. You will need them again for Task 2.

This is the story:

A farmer had 30 cows in January. By July she has managed to buy 5 more cows so she has 35 cows.

Work out, first for January, and then for July:

a. The number of ears all her cows have altogether (each one has 2 ears)

b. The number of legs all her cows have altogether (each one has 4 legs)

c. The number of tails all her cows have altogether (each one has 1 tail)

d. The number of buckets of water the cows drink per day altogether (each one drinks 3 buckets of water per day)

e. How many litres of milk the cows produce altogether every day (each one produces 6 litres of milk per day).

Make up some more stories like this and take them to your next tutorial to share with the other Learning Assistants.

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Writing formulae

When repeating the same kind of calculations again and again it is useful to have a formula rather than going through each step every time. This is particularly useful with more complicated formulae, for example:

The formula to calculate the area of a circle is:

$$A = \pi r^2$$

where A is the area of the circle, π is a constant (3.14) and r is the length of the radius.

To convert metres into inches use the following formula:

$$i = 39.57 \times m$$

where i is the number of inches and m is the number of metres.

To convert degrees Celsius into degrees Fahrenheit, the formula is:

$$F = 1.8C + 32$$

where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

Looking back at the farmer who wanted to know how many legs his chickens have altogether, he could write a formula as follows:

$$l = 2 \times c$$

the number of chicken legs

equals

2

times

The number of chickens

where *l* is the number of chicken legs and *c* is the number of chickens

When writing algebra, we omit the '×' sign for multiplication, so the properly written formula is:

$$l = 2c$$

Task 2: These are for you to try. Write the answers here.

Using your answers to Task 1, write a formula for:

a. The number of ears all her cows have altogether (each one has 2 ears)

b. The number of legs all her cows have altogether (each one has 4 legs)

c. The number of tails all her cows have altogether (each one has 1 tail)

d. The number of buckets of water the cows drink per day altogether (each one drinks 3 buckets of water per day)

e. The number of litres of milk the cows produce altogether every day (each one produces 6 litres of milk per day).

Also write a formula for the stories you made up in Task 1. Take your answers to your next tutorial to share with the other Learning Assistants.

Substitution into a formula

When letters in a formula are replaced by numbers, it is called substituting into a formula.

When you are asked to substitute into a formula, you will be given a formula, and then the values to be substituted into the formula.

Example

For the following formula:

$t = 3w + 7$

work out t when:

a. $w = 2$ ....

b. $w = 4$ ....

c. $w = 200$ ....

d. $w = –1$ ....

e. $w = 0$ ....

The equation $t = 3w + 7$ is the same as $t = 3 \times w + 7$.

a	For $w = 2$, substitute in the equation	$t = 3 \times w + 7$ substitute $t = 3 \times 2 + 7$ $t = 6 + 7$ $t = 13$
b	For $w = 4$, substitute in the equation	$t = 3 \times w + 7$ substitute $t = 3 \times 4 + 7$ $t = 12 + 7$ $t = 19$
c	For $w = 200$, substitute in the equation	$t = 3 \times w + 7$ substitute $t = 3 \times 200 + 7$ $t = 600 + 7$ $t = 607$
d	For $w = –1$, substitute in the equation This means negative 1	$t = 3 \times w + 7$ substitute $t = 3 \times –1 + 7$ $t = –6 + 7$ $t = 1$
e	For $w = 0$, substitute in the equation	$t = 3 \times w + 7$ substitute $t = 3 \times 0 + 7$ $t = 0 + 7$ $t = 7$

Task 3: These are for you to try.

For the following formula:

$$p = 8q - 3$$

work out p when:

a. $q = 2$

....

b. $q = 5$

....

c. $q = 1000$

....

d. $q = -3$

....

e. $q = 0$

....

Well done for completing these tasks! Try to compare your answers with another Learning Assistant – did you get the same answers? Are there any questions that you find difficult? Mark these questions and remember to ask your tutor about them at the next Maths tutorial.

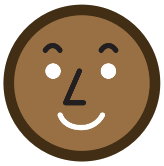
Exam question

Here is an exam question for you to try. Take your answers to the next Maths tutorial.

The number of years in x calendar months is:

a. $12 + x$ b. $x/12$ c. $12x$ d. $12 - x$

How am I doing?

	 **Easy** (Tick this box if you feel confident that you **understand this section well**)	 **Fine** (Tick this box if you still **need a little work** on this section)	 **Difficult** (Tick this box if you still **need a lot of work** on this section)
I can substitute one number for another			
I can write simple formulae			
I can substitute into formulae			

Notes on what to do next:

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Learning Assistant signature: ....

Date: ....

Tutor signature: ....

Date: ....

Maths Unit 3: Week 5

This section is about drawing graphs. You will start by learning to plot and read coordinates of a graph. You will then work out coordinates using the skills of substitution which you developed in the previous section. By the end of this section you will know how to plot straight-line graphs.

Plotting coordinates and drawing graphs are very important skills you need to have when doing algebra. Graphs offer a visual representation which is very useful in analysing and interpreting an equation or formula. You might not see the real benefit of learning these skills until you get further on in algebra. However, you may find constructing these graphs enjoyable!

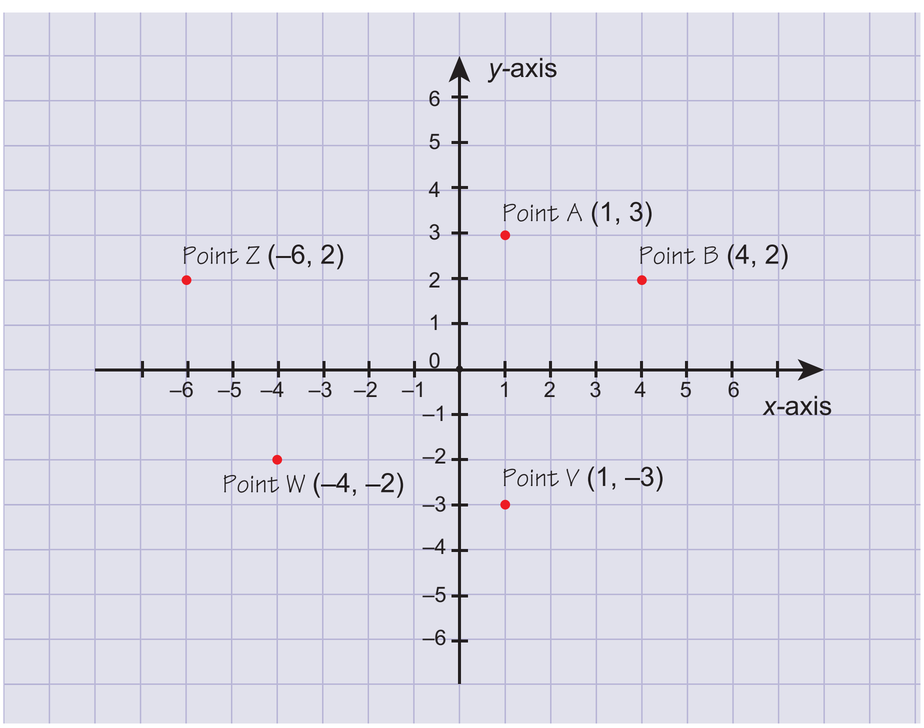
There are two parts to this section. In each part, there is something to read followed by some examples to follow – read the examples carefully several times. Then try to complete the tasks.

Axes and coordinates

The following diagram is an example of a typical set of axes, with some coordinates.

It has two axes – to avoid any problems please stick to the following nomenclature:

- the *x*-axis is the horizontal axis
- the *y*-axis is the vertical axis.



Each axis is like a number line: there are positive and negative sides. In theory the numbers go on into positive and negative infinity. In reality you would only make each axis as long as you need for plotting your coordinates.

You can see there are some points (points A, B, ...) on the diagram. Coordinates are used to describe a point's position on the diagram.

Coordinates are always written as two numbers, inside round brackets and separated by a comma. It is essential to stick to this way of writing coordinates. Points with the coordinates (1, 3), (4, 2), (1, –3), (–6, 2) and (–4, –2) are shown on the diagram.

In the brackets, the **first** number refers to the *x*-coordinate and the **second** number to the *y*-coordinate.

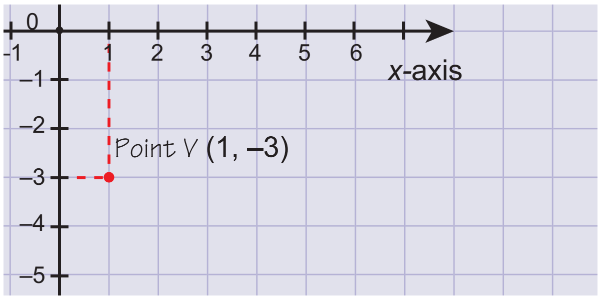
Coordinates are written alphabetically – you can remember this by thinking '*x* comes before *y*' in the alphabet, so the coordinates are written as (*x, y*).

A special point is the point where the axes cross and this point is called the **origin**. The coordinates of the origin are (0, 0).

In the next diagram, you can see point V with coordinates (1, –3).

The first number in the brackets is the *x*-coordinate; it is 1 and means that the point is at 1 on the *x*-axis.

The second number in the brackets is the *y*-coordinate; it is –3 and means that the point is at –3 on the *y*-axis.



Task 1: These are for you to try.

Complete the following table using the first diagram. Some answers are already filled in for you. You could add some more points to the diagram and write their coordinates in the table.

Coordinates	Point
	V
(–6, 2)	Z
(1, 3)	
	B

Task 2: Plot the following points on the diagram below.

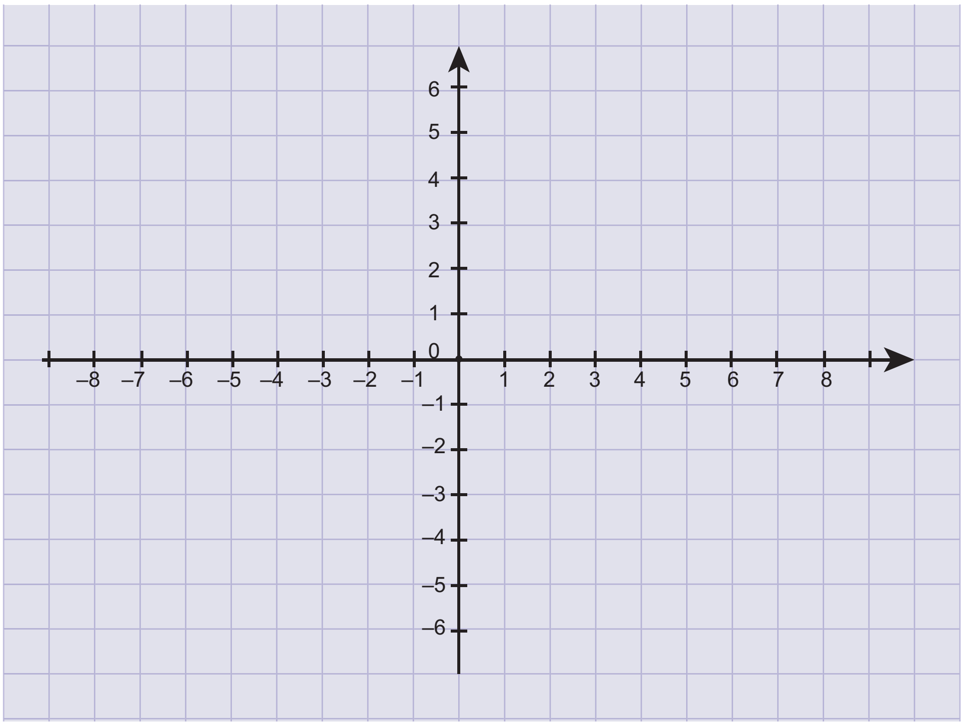
A (8, 5) B (5, –1) C (3, –5)

D (–2, –5) E (–5, –5) F (–4, –3)

G (–2, 1) H (0, 5) I (4, 5)

Now join up the points alphabetically from A to B to C, and so on, and finally join I to A.

What shape is it?



Make up another task like this and take it to your next tutorial to share with other Learning Assistants.

Drawing a straight-line graph using tables

When you are asked to draw a straight-line graph (that is, a graph consisting of a straight line, with no curves or bumps), you will be given an equation, such as $y = 3x + 1$.

To draw a graph for this equation you first need to find some coordinates to plot. A good method is using a table of values, as shown below.

Table of values for the equation y = 3x + 1:

$y = 3x + 1$

x	–1	0	1	2
y				
coordinates				

If x = –1, then y will be ...

If x = 0, then y will be ...

If x = 1, then y will be ...

If x = 2, then y will be ...

To work out what to put in this table, you substitute the values for x into the equation, just as you did in the previous section:

The equation $y = 3x + 1$ is the same as $y = 3 \times x + 1$.

The equation $y = 3x + 1$ is the same as $y = 3 \times x + 1$

If $x = -1$, then $y = 3 \times x + 1$

$\downarrow$ substitute

$y = 3 \times (-1) + 1$

$y = -3 + 1$

$y = -2$ and the coordinates are $(-1, -2)$

If $x = 0$, then $y = 3 \times x + 1$

$\downarrow$ substitute

$y = 3 \times (0) + 1$

$y = 0 + 1$

$y = 1$ and the coordinates are $(0, 1)$

If $x = 1$, then $y = 3 \times x + 1$

$\downarrow$ substitute

$y = 3 \times (1) + 1$

$y = 3 + 1$

$y = 4$ and the coordinates are $(1, 4)$

If $x = 2$, then $y = 3 \times (2) + 1$

$y = 6 + 1$

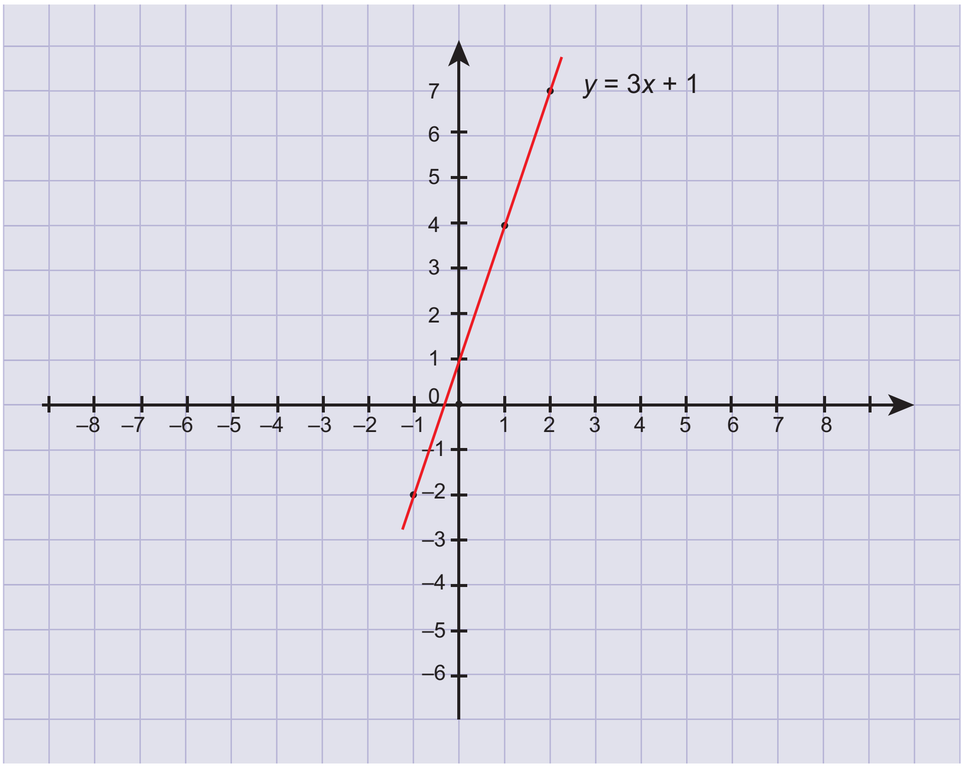
$y = 7$ and the coordinates are $(2, 7)$

You can then complete the table: $y = 3x + 1$

x	–1	0	1	2
y	–2	1	4	7
Coordinates	(–1, –2)	(0, 1)	(1, 4)	(2, 7)

Once you have the values for x and y, you simply plot these coordinates on a diagram, and join up the points. You will get a straight line. Next to that line write the equation y = 3x + 1 to make clear what graph this line represents.

When constructing your table of values, you can pick any values for x. Because it is a straight line, you actually only need to find two points. However, by working out a third or fourth point, and plotting these, you can see immediately whether or not you have made a mistake. If the line is not straight, then you have miscalculated somewhere!

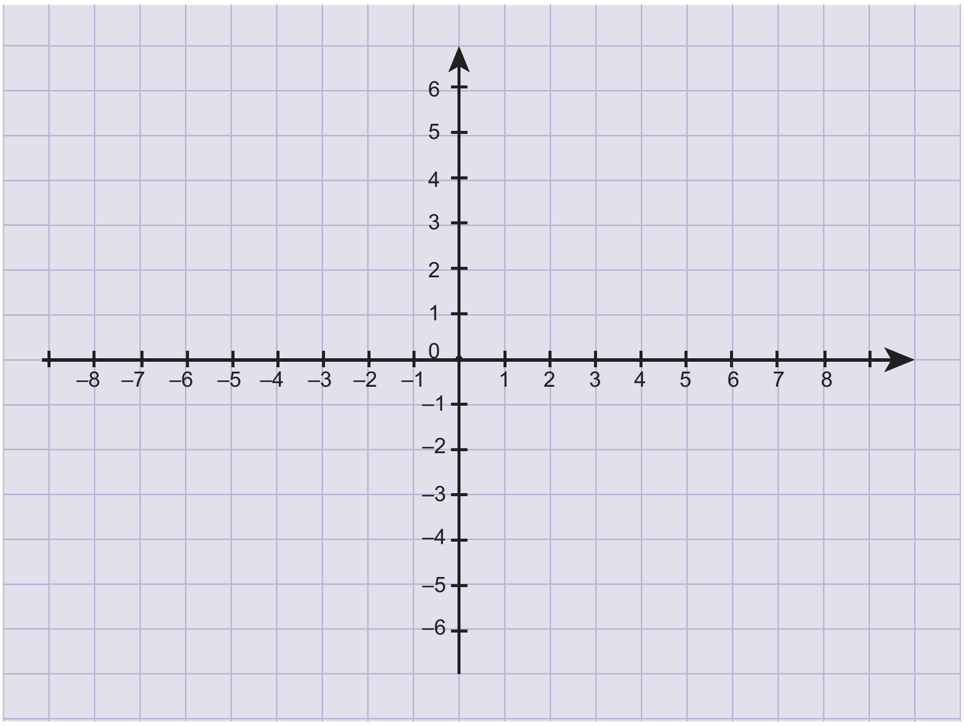


Task 3: This is for you to try.

Complete the table of values for the equation $y = 2x - 1$, and then plot the points on the diagram below.

$y = 2x - 1$

x	–1	0	1	2
y				
Coordinates				



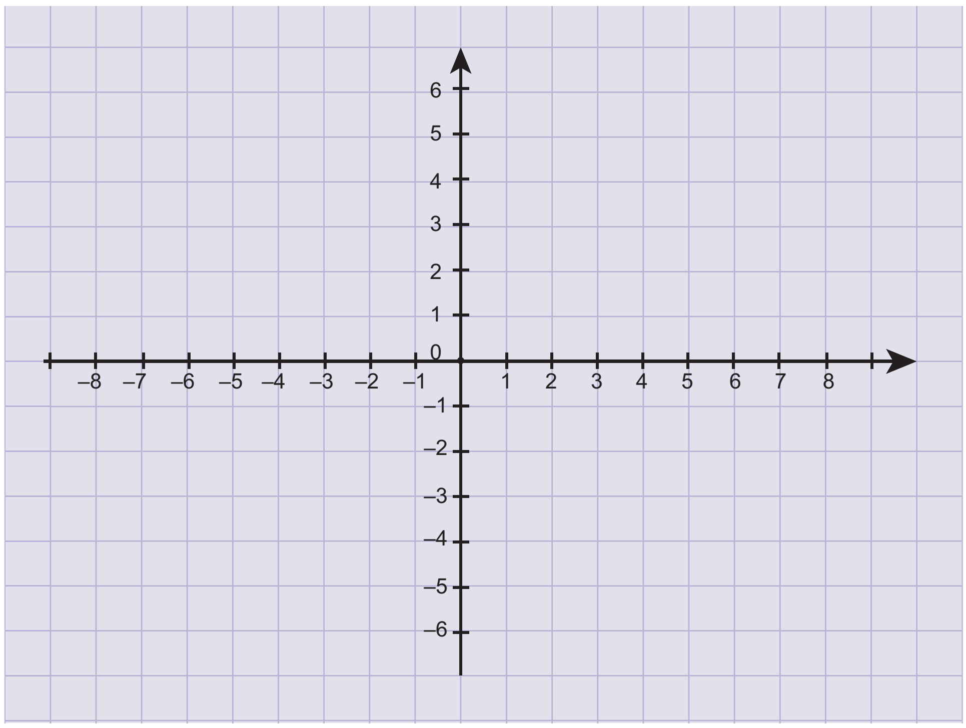
You can also make up an equation, work out a table of values and plot the graph for it.

Well done for completing these tasks! Try to compare your answers with another Learning Assistant – did you get the same answers? Are there any questions that you find difficult? Mark these questions and remember to ask your tutor about them at the next Maths tutorial.

Exam question

Here is an exam question for you to try. Take your answers to the next Maths tutorial.

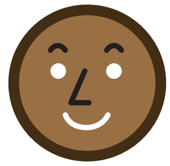
Draw the graph of $y = x + 1$.



Hint: Use the following table to work out the coordinates:

x				
y				
Coordinates				

How am I doing?

	 **Easy** (Tick this box if you feel confident that you **understand this section well**)	 **Fine** (Tick this box if you still **need a little work** on this section)	 **Difficult** (Tick this box if you still **need a lot of work** on this section)
I can plot axes of a graph			
I can plot coordinates on a graph			
I can read coordinates on a graph			
I can find coordinates of a graph by making a table of values			
I can draw a straight-line graph			

Notes on what to do next:

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