Zimbabwe
Ministry of Primary and Secondary Education

NUMERACY

IGATE
Module 2
Early addition and subtraction

IGATE Improving Gender Attitudes, Transition and Education Outcomes
care
World Vision
The Open University
Girls' Education Challenge
UKaid from the British people

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www.wvi.org/education-and-life-skills/igate-improving-girls-access-through-transforming-education

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Revised Module 2 (MoPSE)

Foundational numeracy

Module 2: Early addition and subtraction

Module contents

About these modules

This is the second of six Teacher Professional Development (TPD) modules for all teachers working with learners whose attainment in numeracy is below their Grade or Form level. The modules are also appropriate for Initial Teacher Education (ITE) – particularly during school placements or practicum.

Module 1: What is a number?

Module 2: Early addition and subtraction

Module 3: Addition and subtraction of bigger numbers

Module 4: Multiplication and division – part 1

Module 5: Multiplication and division – part 2

Module 6: Fractions and decimals

The modules were collaboratively developed for the Ministry of Primary and Secondary Education (MoPSE) by the Open University, World Vision and CARE International. The modules have been tried and tested in hundreds of primary and secondary schools across Zimbabwe, strengthening the teaching of foundation skills and improving learning outcomes. Our thanks to everyone who contributed – especially teachers, school heads and schools' inspectors.

MoPSE's highest priority is to empower ALL learners through strong foundations in literacy and numeracy. Whatever their Grade or Form, all learners need strong foundations in literacy and numeracy to succeed in other learning areas.

Learners must **learn to read and use number** so they can **read and use number to learn**.

Using the modules

Teachers will benefit most by using the modules within reflective-practice cycles in their schools, as shown below.

Read an activity.
Plan how you will use the activity.
Do the activity with your learners.
Reflect on what learners learned from doing the activity.

- What worked well?
- What would you change next time?

Share your experiences with your colleagues.

- The modules can be used by:
  - individual teachers
  - pairs or groups of teachers
  - whole schools
  - cluster meetings or district workshops.
- The modules provide classroom activities and guidance for effective use.

Learner attainment

We describe learners who can do an activity confidently and successfully as ‘higher attaining’ and learners who cannot do an activity well as ‘lower attaining’.

No one knows what a learner will be able to do given the chance. Every learner has the potential for growth. Teachers have often been surprised when they found a learner who was ‘higher attaining’ for one activity was then ‘lower attaining’ for another – and vice versa. So, we don’t label learners with words like ‘fast’ or ‘slow’.

A learner may have different levels of attainment in different learning areas, or in different aspects of one learning area. That’s why assessment is a big part of the activities. It is important to find out, as often as possible, what learners know and can or can’t do. Then they can be given activities at a level that will help them progress.

Working in groups

Learning takes place as a result of **doing** an activity, **thinking** about it, and **understanding** the ideas it contains.

In order to make sure that all learners are **doing**, activities are designed so learners work together in pairs or small groups for most of the lesson. Pairs, or groups of four to six learners, work best because everyone can take part. Sometimes the teacher will need to demonstrate the activity first.

There are several ways in which learners can be put into groups. Teachers should choose the one that works best for their learners.

- **Learners choose their own groups:** Sometimes this can result in friends working (or not working!) together, while other learners are left out.
- **Learners at a similar level of attainment work together:** This can work well, as learners are working at their preferred pace, but learners who need help have to find it from outside the group.
- **Learners at mixed levels of attainment work together:** This type of grouping has the advantage that higher-attaining learners can help lower-attaining ones. This gives lower-attaining learners personal and prompt support, and higher-attaining learners a chance to talk about what they have learned, which helps to deepen their understanding.

Collecting and storing resources

Many of the activities in these modules rely on learners using physical resources. Some activities need large quantities of resources so that learners can work through the activities in small groups. How can you make sure you have enough?

Here are some suggestions from teachers who trialled these materials.

Sticks

- Ask each learner to bring in at least ten small sticks the length of their middle finger.
- Ask learners to hunt for small sticks in the school grounds during break times. (This worked well in rural communities, but not so well in city schools.)
- As an alternative to sticks, ask learners to bring in at least ten toothpicks or drinking straws.
- Bundle up the sticks into ‘tens’ using elastic bands (if you have them, as these work better for subtraction) or short lengths of wool or string.

Counters

- Ask each learner to bring in a collection of counters. In rural communities, learners usually brought in small stones, beans or seeds. In city schools, learners often brought in plastic beads or bottle tops.

Place value counters

- Ask local bottle stores, lodges and hotels to collect tops from drink bottles. Write the value of the counter on the top with a permanent marker.
- Cut up manila or paper into 2-cm squares and write the value of the counter on each piece.

Making resources together

- Some teachers organised a weekly after-school resource-making meeting to make sure all classes had the equipment they needed. This was especially helpful when making number cards or arrow cards!

Sharing resources

- Share a set of resources between two classes. So, for example, while one class has a literacy lesson, the other class does numeracy, and vice versa. The teachers quickly swop resources between lessons!
- Split the class into two halves. Work with one half of the class on activities that use resources, while the other half work in their books.

Storing resources

- Teachers often reused the same resources many times, for lots of different activities. They realised that they would save a lot of time by storing resources carefully, either in their classroom or a common store-cupboard.

Unit 1: Developing fluency, connecting addition and subtraction

Contents

Introduction

Learners are only ready to start thinking about addition and subtraction when they have a good understanding of numbers up to ten, and how those numbers work together. If they have not yet achieved this, offer some of the activities in **Module 1.**

The activities in this module will help you to provide your learners with a good understanding of how numbers work together when adding and subtracting. This provides a firm foundation for using column methods in addition and subtraction in **Module 3**. Only introduce the column method when learners have demonstrated their understanding of addition and subtraction using practical materials and diagrams.

Key words and phrases

- **bar model** – a diagram that shows a relationship between numbers (see **Resource C Bar model**, page 32)
- **efficiency** – using the quickest, accurate, calculating strategy
- **fluency** – the ability to calculate accurately and efficiently
- **number bonds to 10** – pairs of numbers that add up to 10

1.1: Ten nice things

Aim

It is important to engage learners in a variety of activities involving addition and subtraction so they can continue to develop their fluency. This activity is a game that learners can play at any stage in their learning. It helps them to become familiar with number facts to ten.

What the learners will do

Learners try to win the game by collecting all ten ‘nice things’. They turn a card over and take the number of objects shown on the card. As they take the objects, they work out how many more they need to make ten and win the game. They may like to use ten frames and counters to help them.

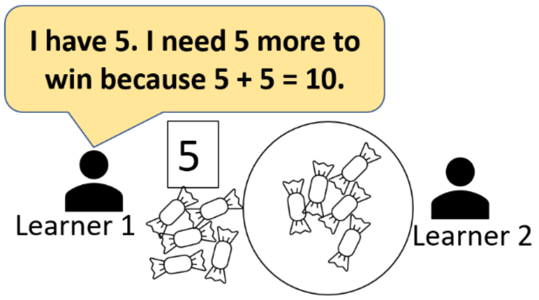
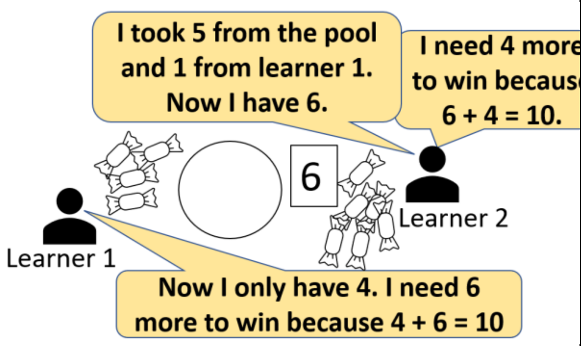
Resources

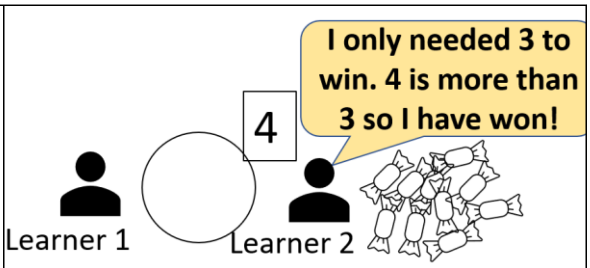
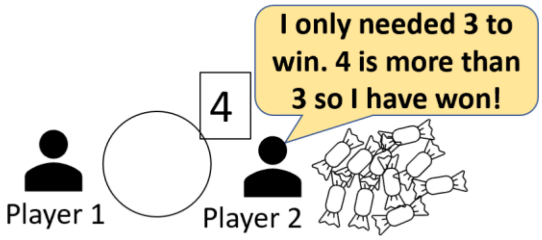
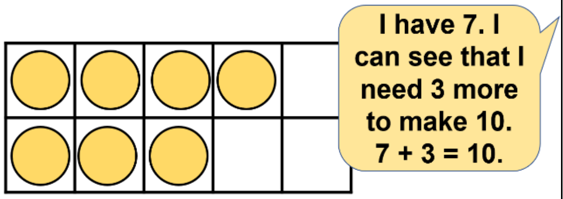
Each pair of players will need:

- four sets of 1–6 number cards (**Resource A Number cards**)
- ten ‘nice things’ – these can be any small objects (beads, seeds, pencils, toys etc.), which learners can collect themselves
- ten frames (**Resource B Ten frame**) and counters (see ‘Collecting and storing resources, page 3) – both ten frames and counters are optional.

Activity

<table>
<tr>
<td>1. The ten objects are placed between the learners. This collection is called the ‘pool’.

Learner 1 turns over a card and takes that many ‘nice things’ from the pool. They say how many more they need to win and how they know this.</td>
<td>
</td>
</tr>
<tr>
<td>2. Learner 2 turns over a card and tries to take that many nice things from the pool. If there are not enough, some have to be taken from Learner 1. They say how many more they need to win and how they know this.</td>
<td>
</td>
</tr>
</table>

3. Learners continue to take turns, taking nice things from each other and saying how many more they need to win.	
4. Repeat until one learner has all the items. The learner with all the items is the winner.	
5. Some learners may use ten frames to help them find how many more they need to win.	

Assessment

How do learners find the answer to 'How many more do I need to win?'

Do they:

- use a ten frame and count how many spaces are left
- count on their fingers
- 'just know'?

In practice

Mr Matenda used this game with his class. It was very popular! They used all sorts of 'nice things': some learners just broke up twigs so that they could play the game.

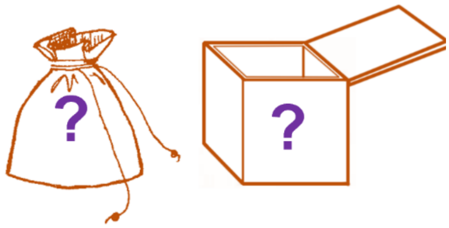
Mr Matenda was very impressed by the amount of adding and subtracting that went on. Because he wanted his learners to reason, he insisted they use the sentence: *'I need … to win,'* and that they stated how many objects they would have altogether, once they had collected them.

Some of the higher-attaining learners in the class quickly learned to play the game with ten nice things, so he extended their learning by asking, *'What if you try using 20 nice things?'* or *'What if you change one of the rules of the game?'* ***'What if …'*** is such a useful question.

1.2: What's in the bag?

Aim

The purpose of this activity is to help learners to 'see' addition and subtraction. It will help them to become better at adding and subtracting small numbers quickly and accurately.



What the learners will do

Learners work in groups. The 'group leader' puts in or removes counters from a box or bag. The rest of the learners keep track of the total number of counters in the bag or box. They will say, then write, the calculation that describes the action.

Resources

Each group of four to six learners will need:

- at least ten counters
- a bag (or box) in which to hide the counters.

Activity

Carry out a short (5-minute) demonstration of the activity. Learners will then work in groups of four to six, taking turns to be the group leader.

1. The group leader puts a small number of counters into the bag one at a time, so they cannot be seen in the bag. Everyone counts as they are put in: *'one, two, three …'* and so on.
2. The group leader adds more counters, making sure the learners see each counter as it goes into the bag, but without showing the counters inside. The group leader then asks: *'How many counters are in the bag now?'*
3. Learners give answers, then look in the bag and count to check.
4. Learners say the number sentence together, for example, '*Three plus one equals four,'* so they practise use of mathematical language.
5. All the learners in the group write the calculation in their books.
6. The bag and counters are passed to the next learner for their turn as group leader. Sometimes the group leader should remove some counters from the bag and ask, *'How many counters are in the bag now?'*

Note: Higher-attaining learners can work with a larger number of counters.

Assessment

Do the following as you watch learners doing the activity.

- Ask: *'How do you know there are xx counters in the bag?'* (This will tell you the strategy learners are using to calculate, and who needs extra support.)

- Suggest the same number of counters are taken out as were placed in the bag. Ask, *'How many are in the bag now? How do you know?'* (Learners who can tell you straight away have understood that addition and subtraction are 'opposite' operations.)

In practice

Mrs Sibanda had used the counting activities in **Module 1** with her class and was surprised by how much practice most of them needed before they were really fluent with counting. She realised that the learners who found counting challenging were also finding addition a real problem.

Mrs Sibanda found some clean hessian sacks that had been used to deliver vegetables and some big coloured plastic spoons. She used these to demonstrate the activity a few times to the class. She made sure that the class first counted the spoons that went into the bag, then said, for example, *'Three plus two equals five,'* and finally counted out the five spoons.

Mrs Sibanda liked the way that the accurate mathematical language of addition was introduced in a way that helped learners to understand it. For example, she was careful to say 3 + 2 = 5 as 'three plus two equals five', not 'three and two make five'. For 3 – 2 = 1, she said 'three subtract two equals one' rather than 'three take away two leaves one'.

1.3: Number sentences

Aim

Through this practical activity, learners begin to understand the relationship between addition and subtraction. They also begin to develop instant recall of important number facts.

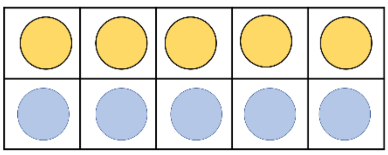
What the learners will do

Learners write four number sentences using the same three numbers. They can use ten frames, counters and a bar model to help them see the connections between the number sentences.

Resources

Each group of four learners will need:

- four sets of 1–5 number cards, mixed together (**Resource A Number cards**)
- a ten frame (**Resource B Ten frame)**
- five each of two different types of counters.



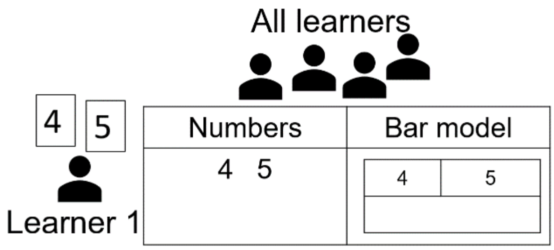
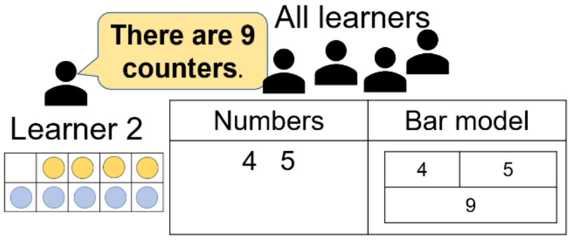
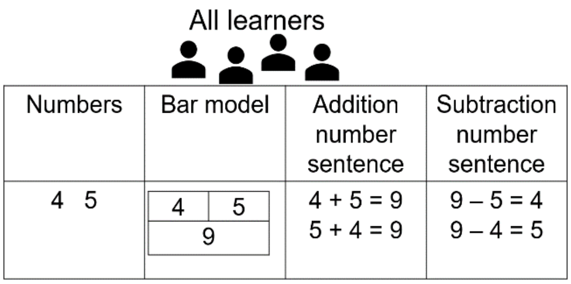
Each learner needs:

- to draw the following table in their books and complete the first line as you demonstrate the activity.

Numbers	Bar model	Addition number sentence	Subtraction number sentence

Activity

At the beginning of the session, ask the learners to draw the empty table in their books and show learners how to complete it for the numbers 4 and 5. Learners will then be able to do the activity in their groups for the rest of the session.

<table>
<tr><td>1. Learner 1 turns over two cards. All learners write the numbers in the first column of the table in their books and draw a bar model in the second column. They write the numbers on the cards in the top two boxes.</td><td>
</td></tr>
<tr><td>2. Learner 2 places counters on the ten frame to represent the numbers on the cards and adds the two numbers together. Everyone writes the total in the bottom box of the bar model.</td><td>
</td></tr>
<tr><td>3. All learners think of two addition number sentences and two subtraction number sentences. They write them in the correct columns in their books.</td><td>
</td></tr>
<tr><td colspan="2">4. Learners compare their number sentences to see if they are all the same. If they are not, they discuss the reasons for this and decide whose answers are correct.</td></tr>
<tr><td colspan="2">5. Learners repeat steps 1–4. This time Learner 3 turns over the cards and Learner 4 places the counters on the ten frame.

They repeat the activity as often as time allows.</td></tr>
</table>

Note: More confident learners can use sets of 0–9 cards and will not need to use the ten frame.

Assessment

What strategies do learners use to generate the number sentences?

Do they:

- write the first one, then automatically know the other three
- count the counters in ones for each number sentence
- do something else?

In practice

Miss Maphosa's learners hadn't grasped that addition and subtraction are related, so she decided to give **Activity 1.3 Number sentences** a try. She showed her learners the bar model on the board and that helped a lot.

At first, there was a lot of discussion in the groups about how to use the two numbers to fill in the gaps in the table. However, learners soon began to see a pattern and moved on quickly to new pairs of numbers.

One group did not seem to be getting on very well, so she sat down with them. She realised that they were unsure about adding, so she modified the game. Learners used stones to make a model of the calculation. They discussed how to make the number statements and, after a time, were able to complete the task without using stones.

Miss Maphosa thought that this activity really helped her learners to remember their number bonds and to understand the connections between addition and subtraction.

Unit 2: Teens and beyond – patterns and bonds

Contents

Introduction

Working with numbers beyond ten requires many skills. Learners may be able to recite number names or count a small number of objects, but this does not mean they fully understand the number system. This unit looks at some areas of potential difficulty and suggests strategies for overcoming them.

Investing time in counting will bring many benefits. Counting in tens from any number helps learners to calculate more accurately and efficiently. It also gives learners a strong foundation on which they can build their understanding of place value.

Key words and phrases

- **bar model** – a diagram that shows a relationship between numbers (see **Resource C Bar model**, page 32)
- **efficiency** – using the quickest, accurate, calculating strategy
- **fluency** – the ability to calculate accurately and efficiently
- **pattern** – something that repeats, for example a sequence of numbers
- **talk partners** – a pair of learners who discuss a question posed by the teacher

2.1: Switch

Aim

This activity enables learners to become familiar with the names and order of numbers beyond ten. It helps them to be able to count on and back from any number.

What the learners will do

Learners throw the ball to each other, counting on or back as they do so, using the range of numbers chosen by the teacher. Learners have to pay attention all the time as the ball can be thrown to anyone in the circle!

Resources

You will need:

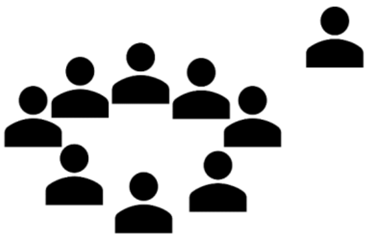
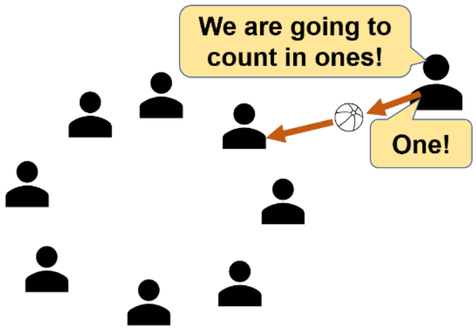
- a 0–20 number card display where everyone can see it (Resource F Number card display, page 34). (If you have completed **Module 1** with your learners, you will already have a 0–10 number card display).

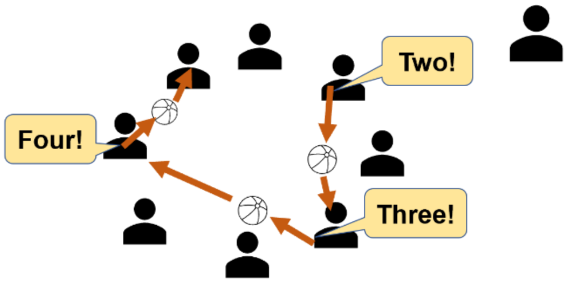
Each group of six to eight learners will need:

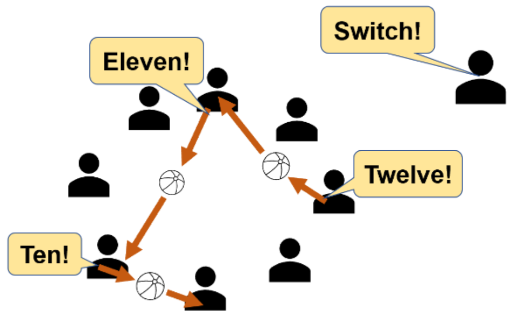
- a ball or something similar to throw. This could be something like a plastic bag stuffed with paper and wrapped in sticky tape.

Activity

Begin the activity as a whole class. When you are sure learners understand what to do, put them in groups of six to eight.

1. Learners take turns to be the group leader. All the other learners stand in a circle.	
2. The group leader tells their group whether they are counting in ones, twos or tens. They throw the ball to one of the learners, saying the first number.	

<table>
<tr><td>3. Learners throw the ball to each other, saying the next number as they throw the ball. (Remind them to make sure everyone gets a go!)</td><td>
</td></tr>
<tr><td>4. The group leader calls ‘SWITCH!’ When the next learner throws the ball, they have to begin counting backwards.

5. After a few more throws, the group leader calls ‘SWITCH’ for the second time. The next learner begins counting forwards again.</td><td>
</td></tr>
<tr><td colspan="2">Learners continue throwing the ball and counting forwards and backwards as the group leader calls ‘SWITCH’. The group leader should change after two or three calls of ‘SWITCH’.</td></tr>
</table>

Assessment

Watch learners as they do the activity. Look for learners who:

- count forwards easily, but struggle counting backwards
- find it difficult to remember the names of the ‘teen’ numbers (remind learners that **Resource F Number card display**, page 34, can help them)
- say thir**ty,** for**ty**, fif**ty** etc. instead of thir**teen**, four**teen***,* fif**teen** etc.
- find the counting easy. These learners can move on to counting higher numbers or counting in steps.

In practice

To generate starting numbers for **Activity 2.1: Switch**, Mr Mataka picked a number from a pile of number cards. He found it interesting to listen to different groups as they threw the ball and counted. Many learners found it very challenging to count backwards through the teen numbers.

What shocked him most was the number of children who had misconceptions about ‘ty’ and ‘teen’ numbers. This resulted in some very muddled counting, such as *‘ten, eleven, twelve, thirty, forty, fifty … ninety, twenty’*. He had not picked up on this confusion before. He stopped the counting and asked two children from different groups to write, on the ground, the number after 15. One wrote ‘60’, the other ‘16’. He was shocked to discover that the learners were divided in their opinions as to which was right.

He decided to go back to basics. The class used counters and ten frames to make 60 and 16, 80 and 18, etc. He then asked these questions *‘What’s the same about 60 and 16? What is different about 60 and 16?'* He now uses ‘What is the same and what is different?’ quite often to check understanding in his numeracy lessons.

2.2: Ten and some ones

Aim

Learners build on what they already know about pairs of numbers that make 10 to find pairs of numbers that make 20.

What the learners will do

Learners work with ‘talk partners’ to find different ways of making 20. Then they practise the skills they have just learned, still working with their talk partners.

Resources

All learners will need:

- books and pens.

Some learners will need:

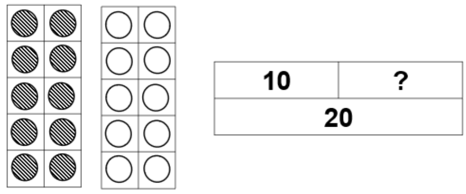
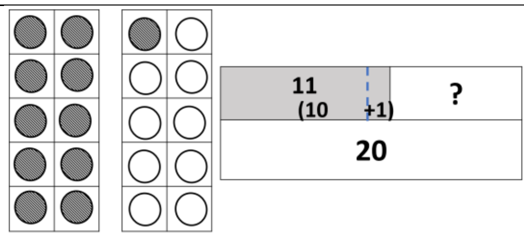
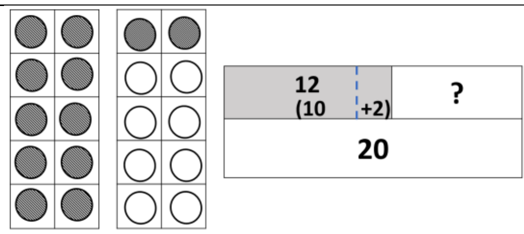
- two ten frames (Resource B Ten frame)
- double-sided counters (for example bottle tops, some showing the top and others turned upside down).

Activity

Suggestion: In the first, shorter, part of the activity, the teacher guides learners and their talk partners through the activity. For most of the activity, they work together using numbers they have made up.

<table>
<tr>
<td>1. Draw a bar model on the board with ‘10’ in the bottom and two question marks in the top two boxes.

Ask learners, with their talk partners, to think of as many pairs of numbers as they can that might go in the top two boxes. Say the two boxes might not be the same size. Ask pairs of learners to give you one answer each until all pairs</td>
<td>
<table>
<tr><td>?</td><td>?</td></tr>
<tr><td colspan="2">10</td></tr>
</table>
</td>
</tr>
</table>

of numbers that equal ten have been given.	
2. Tell learners that if they know all pairs of numbers that equal 10, they can also know all the pairs of numbers that equal 20. Draw the images shown in the diagram to the right on the board. Ask learners to decide, with their talk partner, what number goes with 10 to equal 20. *(10)*	
3. Shade another counter on the board. Ask learners to tell their talk partners what they see. *(10 and 1, that's 11)* Change the bar model to show 11 as 10 + 1. Ask learners, after discussion with their talk partners, to show on their fingers the number they think should go in the other box. Write 11 + 9 = 20 on the board.	
4. Repeat step 3 to show 12 + 8 = 20, then 13 + 7 = 20.	
5. Write 14 + ? = 20 on the board. Ask learners, after discussion with their talk partners, to show on their fingers what number they think is represented by the question mark. Ask a volunteer to show on the ten frames and the bar model how they know this is true.	**14 + ? = 20**
6. Write 8 + ? = 20 on the board. Ask pairs of learners to show on their fingers, together, what they think is represented by the question mark. (They will need more than ten fingers!) Ask a learner to show, using the ten frames and the bar model, how they know this is true.	**8 + ? = 20**

7. Learners carry on working in pairs. One learner says a number between 1 and 19. The second learner says the other number that goes with it to equal 20. They both draw a bar model and ten frames in their books alongside the number sentence to show the number fact they have just worked on.

Assessment

- During step 1 of this activity, make sure all learners can find pairs of numbers that equal ten. Let learners who find the activity challenging to continue to work on Unit 1 activities for a bit longer.
- During the rest of the activity, keep an eye out for learners who wait for their partner to give the answer. If you see this happening, rearrange the class so less-confident learners work together in pairs.
- Notice learners who are able to give the missing number quickly without counting on or using ten frames. They can be given extra ‘missing number’ challenges such as 34 + ? = 40.

In practice

When Mrs Mpofu first read the activity, she thought it seemed very long and complicated for such a simple idea. She talked it over with her colleague, Miss Ndlovu, who pointed out that each part of the activity was a small step in the development of learners' understanding of the teen numbers. They agreed to try out the activity with their learners and to talk about how it went after the lesson.

Mrs Mpofu found that, by the end of the activity, most of her learners understood the idea of the teen numbers being ‘ten and some ones’, although they did find the English words for the numbers challenging compared to the words in their home language.

Miss Ndlovu found that she had to go more slowly with her learners. All the learners took part in steps 1–6 of the activity in her lesson, but she didn't have much time for the independent pair work (step 7). The teachers agreed it was important that the lesson proceeded at the pace of the learners, and that Miss Ndlovu’s class needed two lessons to complete the activity.

They thought that, another time, it would be interesting to try splitting the class. Lower-attaining learners might continue to practise **Activity 2.1** while the rest of the class were introduced to **Activity 2.2**. Higher-attaining learners could then go on to work together in pairs while the teacher introduced **Activity 2.2** to lower-attaining learners.

2.3: Counting in tens from any number

Aim

The aim of this activity is to help learners to see some of the patterns in the decimal number system and begin to connect the way a number is written with the quantity it represents.

What the learners will do

Learners work in groups of six, deciding on, then collecting, the correct number of bundles and single sticks to put on the giant number grid. They discuss how the number is written. Groups then take turns to count in tens from any number with the support of the number grid.

Resources

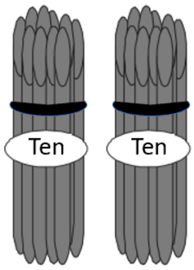
You will need:

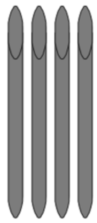
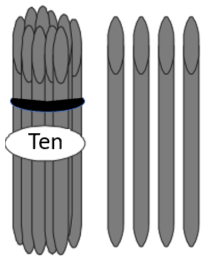
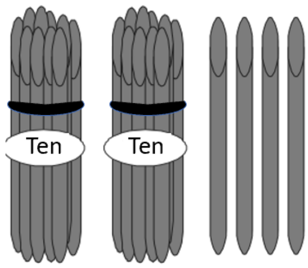
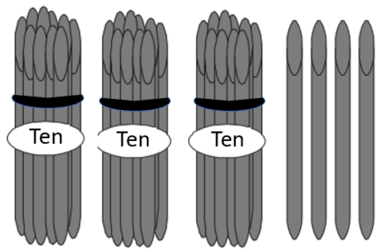
- to draw a large 10 x 10 grid on the ground, on the classroom floor or on a sheet of plain paper or fabric
- to write numbers in the squares along the top and down the right-hand side as illustrated
- to make at least 20 bundles, each containing 10 sticks, tied together with wool, string or elastic bands You will also need about 40 single sticks.

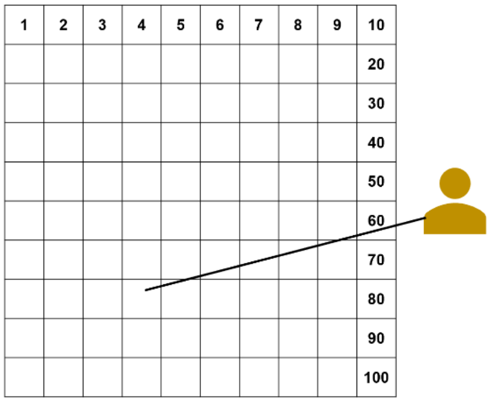
1	2	3	4	5	6	7	8	9	10
									20
									30
									40
									50
									60
									70
									80
									90
									100

Activity

This is a teacher-led activity, with each group of learners taking turns to place sticks on the grid and write numbers.

1. Ask a learner to jump down the 'tens' column from 10 to 100 as the whole class counts in tens. Explain that each bundle contains ten sticks. Place one bundle of ten on the '10' of the number square.	**10** 
2. Tell learners you are going to add another ten. Ask them how many sticks you will have now and where the two bundles should go on the number grid. Place two bundles of ten on the '20'. Ask groups to decide where the bundles should go if another ten is added. Choose a group to put the bundles on the grid and to say the number. Ask if anyone has noticed anything yet. (The first part of the number	**20** 

tells you how many bundles of ten there are. Each time another ten is added, the number is just below the one before.) Ask how many bundles would be placed on 70. *(7)*	
3. Place four single sticks on the '4'. Ask learners to discuss, in their groups, what would happen if they added a bundle of ten to the four sticks. Help them to see that if you add a bundle of ten to four sticks you get 14. Ask a representative from one group to place one bundle of ten sticks and four ones in the correct square for 14 and tell you how to write the number. *(They should put the sticks in the square below 4.)*	4  14 
4. Ask a different group of learners to decide what would happen if they added another bundle of ten. They should put two bundles of ten and four ones on the square below 14 and tell you what number to write. Ask other groups if they agree.	24 
5. Choose another group to show what happens if they add another ten. Ask what is the same when they added tens starting from 10 and adding tens starting from 4. (*The first part of the number tells them how many bundles of ten there are, the second part of the number is the same each time. The second part of the number tells them how many ones there are.*) Ask what is different. (*The first part of the*	34 

number changes, the second part stays the same.)	
6. Point to the square that will represent 84. Ask a group of learners to predict the number of bundles and single sticks that would be needed for that square, and the number that would be written there.	
7. Help learners to realise that when they have understood the pattern, they can count in tens from any number. The first part of the number says how many tens there are, so it goes up by one each time, the second part of the number stays the same.	
8. Ask each group in turn to stand by the number grid. They choose a single-digit number to count from. All the group counts in tens from that number as one member of the group jumps down the chosen column on the number grid.	

Assessment

When learners are working in big groups, it is difficult to make sure each learner understands. Watch for learners who do not seem to be joining in.

Occasionally, ask an individual learner one of these questions:

- *Do you agree with the rest of your group? Why?*
- *How do you know that is the right number?*
- *What number do you think will go in 'this' square?* (Point to a blank square close to one that has a number in.)

In practice

When doing the first question, all Miss Kativa's learners correctly predicted that there would be one bundle of ten and four single sticks, and she thought they would find it easy to work out the answer to the next question (24). When she listened to their conversations, she realised that quite a number of them had counted in ones to get to 14. She had to ask a number of leading questions to get them to notice that the ones stayed the same and the tens increased by one each time.

She decided that next time she used this activity, she would include a few minutes of **Activity 2.1: Switch** to give learners practice in counting forwards and backwards in tens from any number, for example from 6 or 28.

Her biggest learning experience was when she asked what number would be below 94 if there was another line on the grid. As well as 104, she received the answers 114, 204 and 1004. Asking learners to explain their answers helped her to uncover the misconceptions behind them. She realised that all the number grids or number lines she was using used stopped at 100, which encouraged some learners to think numbers stop at 100! Now she's working with learners to make a number grid up to 200 **(Resource D 1–200 number grid)** to go on the classroom wall.

When I used this activity, the learners worked in groups. I asked them to choose their own groups for this activity, as I find that they are more willing to take risks if they are with friends. This seems to be especially true for girls. Sometimes I had to rearrange the groups so that learners got more practice with the numbers they found difficult.

Unit 3: Addition and subtraction on a number line

Contents

Introduction

In the first two activities of this unit, learners return to working with small numbers to help them understand the concept of adding and subtracting on a number line. **Activity 3.3** reintroduces larger numbers.

Number lines and number 'jumps' are visual tools that can help with both understanding and calculation. A number line can be easily drawn in books, on a poster, on a wall or traced in the dust on the ground. It can be horizontal, vertical or diagonal. It is important to vary its presentation, so that learners realise that the orientation of the line does not matter.

Number lines model numbers using distances rather than objects, in preparation for the study of decimal and negative numbers in the future. However, as with everything in numeracy, making connections – for example, between counting objects and numbers on a number line – is important.

Before learners begin **Activity 3.3**, they must be able to count in tens from any number and have a good understanding of **Activities 3.1** and **3.2**. Some learners may need to spend more time on the earlier activities. Remember that we all learn at different speeds. It would be easier for teachers if we all learned at the same rate, but we don't!

Key words and phrases

- **blank number line** – a number line without numbers, used to record the progress of a calculation
- **efficiency** – using the quickest, accurate, calculating strategy
- **graduated number line** – a number line where numbers are evenly spaced along its length

3.1: Jumping along a number line

Aim

This activity will help learners to find the solution to number stories by jumping along a number line.

What the learners will do

After a short whole-class demonstration, learners work in pairs, taking it in turns to make up their own number stories and to find the solution using a number line.

Resources

You will need:

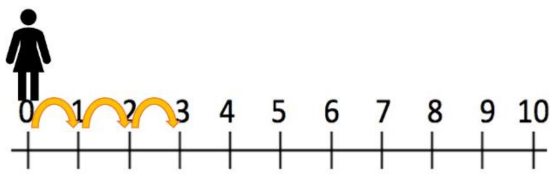
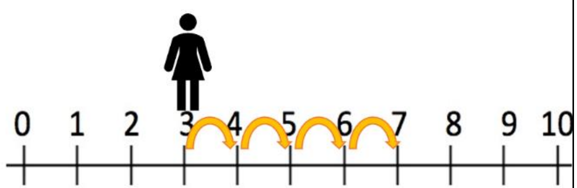
- to draw a giant 0–10 number line (**Resource E Graduated number line**, page 33) on the classroom floor, in the dust outside or on paper. Make sure the marks on the number line are equally spaced. You will also need somewhere to write calculations.

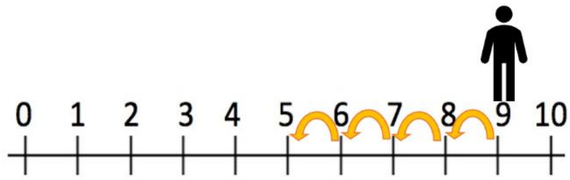
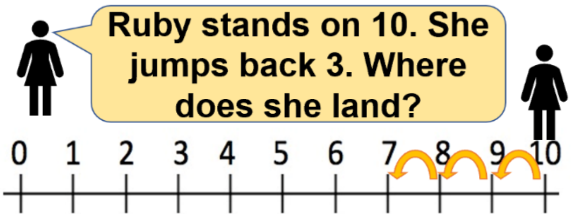
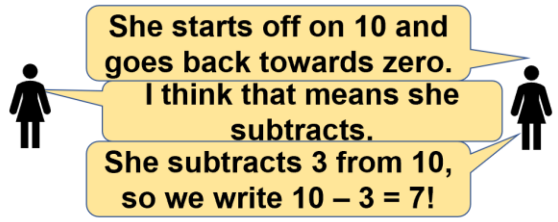
Learners will need:

- to draw 0–10 number lines (**Resource E Graduated number line**, page 33). These can either be large ones similar to the one you use to demonstrate, or they can be drawn in their books.

Activity

Begin by gathering the class around your giant number line, making sure everyone can see. Tell the following simple number stories, asking for volunteers to be Tanaka and Bongani. Make sure that learners spend most of the activity making up number stories in pairs (steps 6 and 7).

1. Say: *'Tanaka jumps along a number line. She starts by standing on zero. First, Tanaka makes three jumps.'* The volunteer jumps to 3 from 0 while the rest of the class counts.	
2. Say: *'Now Tanaka jumps another four. Where does she land?'* The volunteer jumps on another four while the rest of the class counts.	

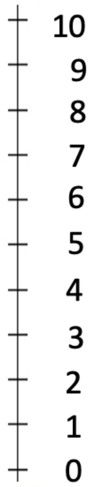
3. Ask learners how they could write down what Tanaka did. Help them to realise that counting on 3 then counting on another 4 is adding 3 and 4.	**3 + 4 = 7**
4. Say: *'Bongani stands on 9. He makes four jumps back towards zero. Where does he land?'* The new volunteer stands on 9 then jumps back 4 while the whole class counts.	
5. Ask learners how they could write down what Bongani did. Help them to realise that by starting on 9 then counting back 4, they are subtracting 4 from 9.	**9 – 4 = 5**
6. Ask learners to work in pairs, taking turns to make up number stories, jump on the number line or draw jumps in their books.	
7. For each story, the pair of learners should decide together how to write down what they have done.	

Assessment

Watch learners as they work together.

- Check that learners count the **jumps**, not the numbered marks.
- Listen out for learners who are finding it difficult to count forwards and back as they jump. They may need more practice with counting activities.
- Look for learners who are eager to make bigger jumps. Some may be ready to make one jump of 3 back from 10, for example, because they 'just know' that they will land on 7. These learners should move on to **Activity 3.2: Secret jumps**.

In practice

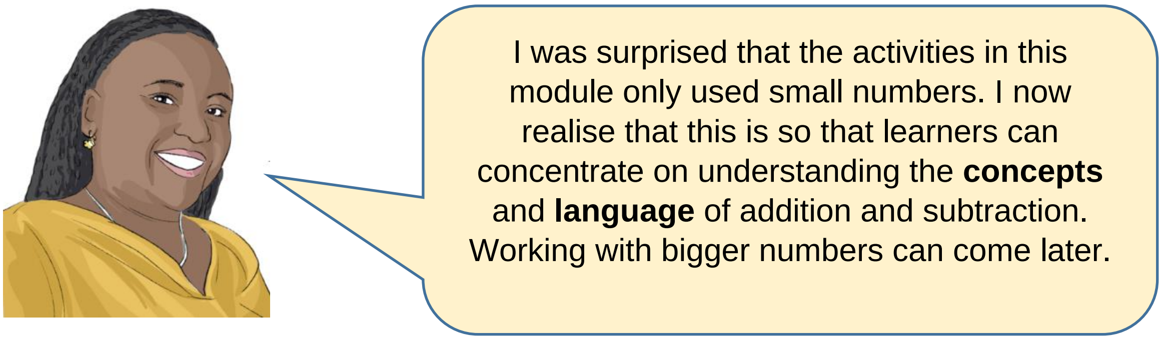


Mrs Taibu had never used number lines for addition and subtraction, but she thought that they would help learners to see addition in another way. She decided to paint the lines on a brick wall just outside the classroom so that she could check progress and easily spot learners who were looking hesitant or needed help.

After doing the activity in the classroom she painted 0–10 number lines on one wall – vertical ones on the high bit and horizontal ones where it was a bit lower.

The big number lines let her see what the learners were thinking. Mostly learners were telling stories and puzzling out their partners' stories. They were doing so much practice and reasoning without doing pages of sums – she was delighted. Several pairs moved on to the other wall where they could number their own lines up to 20. One pair decided they were going up to 50!

Once the jumps were recorded, Mrs Taibu could see exactly how much learners were doing in the ten minutes' practice they had each lesson. She was surprised to find that most of them did more calculations than if they had been working from the board.



3.2: Secret jumps

Aim

This activity helps learners to use a number line to solve simple 'missing number' equations (for example 5 + ☐ = 8).

What the learners will do

After a short demonstration, learners take turns to make up 'missing number' stories for each other and to use a number line to solve them. They discuss together ways to write a calculation that shows what they have done.

Resources

As with **Activity 3.1**, you will need:

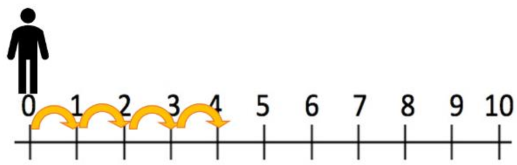
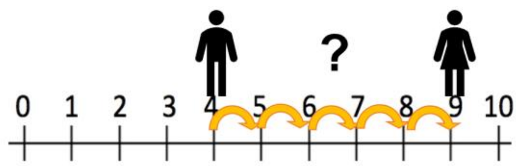
- to draw a giant 0–10 number line (**Resource E Graduated number line**) on the classroom floor, in the dust outside or on paper – make sure the marks on the number line are equally spaced
- somewhere to write calculations.

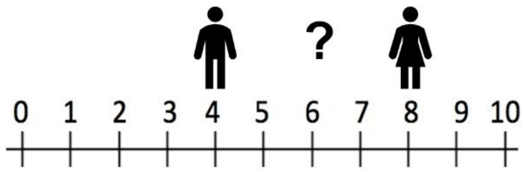
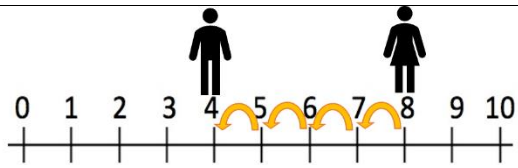
Learners will need:

- to draw 0–10 number lines (**Resource E Graduated number line**). These can either be large ones similar to the one you use to demonstrate, or they can be drawn in their books.

Activity

Begin the activity by demonstrating **one** addition and **one** subtraction secret jump. As in **Activity 3.1**, you will need a volunteer to jump along the number line as you tell the number story. Make sure that learners spend most of the session making up their own 'missing number' stories (step 7).

1. Say: '*Gift starts at 0. He jumps forwards four along the number line.*' The whole class counts as the volunteer jumps from 0 to 4.	
2. Say: '*Gift jumps forwards some more along the number line and lands on 9. How many does he jump?*' Ask a second volunteer to stand on 9. The whole class counts as the first volunteer jumps to meet the second volunteer (5 jumps).	
3. Write these calculations where learners can see them. Ask learners to discuss which one is best to show what they have just done. Why? *(The second one, as the unknown number is 5.)*	$4 + 5 = \square$ $\quad$ $4 + \square = 9$

4. Say: '*Peace starts on 8. She jumps back and lands on 4.*' Ask one volunteer to stand on 8 and a second volunteer to stand on 4. Say that Peace thinks she has to jump back 5, but Gift thinks she has to jump back 4. Ask learners, in pairs, to decide who they think is right and to say why they think that. **Note:** This helps learners to realise that they count the *jumps*, not the *marks*. Peace counted the marks, so included the number she was standing on in the count.	
5. The whole class counts as the first volunteer jumps back to the second volunteer to see who is right.	
6. How do learners think they might write what they have done?	$8 - \square = 4$
7. Ask learners to work in pairs, taking turns to make up 'missing number' stories and jump on the number line or draw jumps in their books, both writing the missing number calculation each time. 8. In a follow-up activity, learners can be asked to draw number lines to help them solve written missing number equations.	

Assessment

While learners are talking together about the question in step 3, listen to the discussion.

- Do all learners understand what the box represents in a missing number equation?
- Do some learners suggest different equations, such as 9 – 4 = 5 (showing they understand the relationship between addition and subtraction) or 4 + 9 = ☐ (showing they have not yet developed a good sense of number)?

While learners are deciding whether Peace or Gift are right (step 4), listen to their reasons.

- If learners think Peace is right, give them opportunities to practise using number lines alongside ten frames and counters so they have a way of self-correcting.

- Learners may have a variety of different explanations. Listening as they talk to their partners will tell you a lot about what they are thinking.

In practice

Miss Nkala and her colleague Mrs Gumbo decided to swap their numeracy and literacy lessons for two weeks so that Mrs Gumbo could use the number lines while Miss Nkala taught literacy and vice versa.

Miss Nkala told learners about Tanaka's 'secret jump' and showed them how to use the number line to puzzle out the secret. When she asked one of the learners to come to the wall and think of a secret jump story for themselves, Constance came up and showed her story on the number line. Constance was a bit hesitant at first, but when she saw that it worked, she was full of confidence and started to help others when learners were practising by themselves.

After about a week, Miss Nkala knew that the class were confident thinking in this way about adding and subtracting. She could see that several learners knew what the secret jump was, without having to count. She was happy that the activity encouraged learners to move away from adding or subtracting by counting in ones.

3.3: Bigger jumps

Aim

To move learners on from using a graduated number line for finding the answer, to using a blank number line as a tool for keeping track of their thinking during a mental calculation.

What the learners will do

Learners work in pairs, discussing different jumps that they could make to find the solution to two-digit addition and subtraction calculations. They decide which jumps they think are the most efficient and explain their thinking.

Resources

Learners will need:

- two sets of 1–6 number cards (**Resource A Number cards**) per pair of learners
- individual exercise books and pens.

Activity

Don't forget to go through the activity with a colleague or on your own before you work with the class, so that you are really confident using a blank number line.

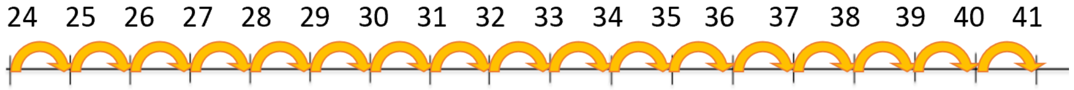
1. Write **24 + 17** on the board.

   Ask learners to discuss in pairs how they think they could use a number line to find the solution. Ask them to think about how they can use what they already know about pairs of numbers that make ten and about counting in tens from any number to help them.

2. Prompt learners with some of the following questions:

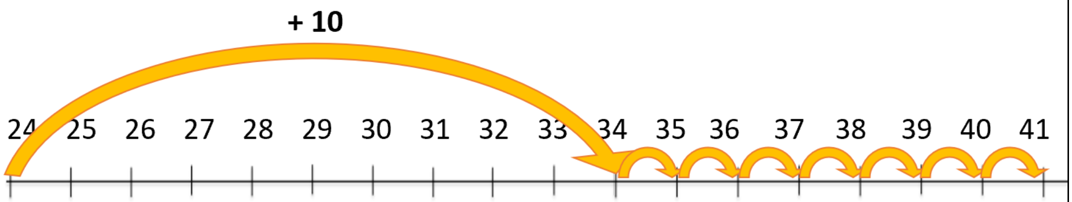
'Does a number line always have to start at 0?'

(No, for example it can start at 24. Learners just need to jump on 17.)



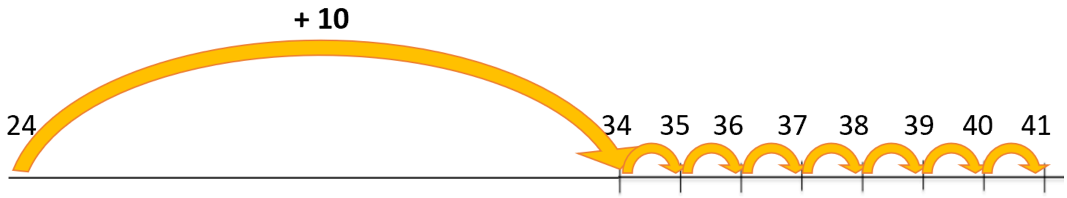
'Do you always have to jump in ones on a number line?'

(No, for example if you know how to count in tens from any number, you can make a jump of ten to begin with.)



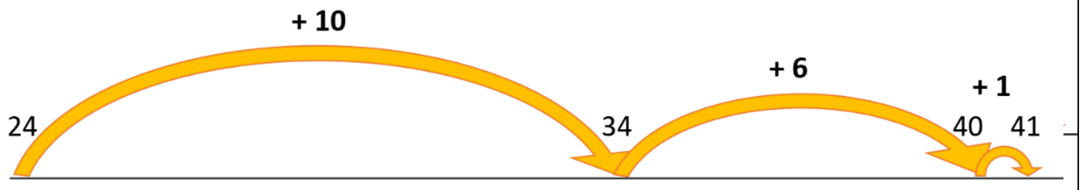
'Do you need to write all the numbers on the number line, or can you write the numbers as you make the jump?'

(You can write the numbers on a blank number line as you make the jumps, to remind you of the number you land on each time.)



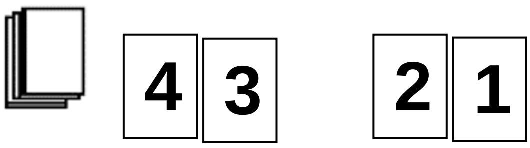
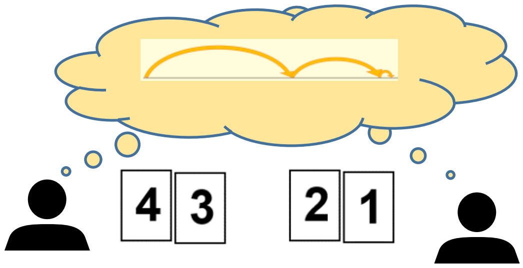
'What other ways could you jump?' (You do not always have to jump in ones or tens.)

(If you know that 4 + 6 = 10, you know that 34 + 6 = 40. You can jump 6 to the next ten [40] and add a jump of 1 to make 7 altogether.)



(You can count eight lots of two then add another one.)

Note: Learners may come up with different ways to jump. Any that result in adding 17 to 24 should be accepted. Learners can discuss which strategies they think they think are best and give their reasons (easiest to understand, most efficient etc.).

3. Ask learners to work in pairs. They turn over four number cards to make two two-digit numbers.	
4. Learners talk together about different ways that they could jump along a number line to add the two numbers together. They choose up to three different ways of jumping on a number line to record in their books. When they have finished one calculation, they should return the cards to the bottom of the pack and pick another four number cards. **Note:** Remind learners that they can add numbers in any order; they will not have to make so many jumps if they put the biggest number first!	

Assessment

Move around the class as pairs work together. Check the following:

- Are both learners in a pair equally involved? (Consider rearranging pairs if one learner seems to dominate.)
- Are learners choosing to jump in ones rather than steps *other* than ones? (They might just need a little more support but if you feel the activity is too hard for them at the moment, you might want to ask them to work with two single-digit numbers or try adding a single digit to a two-digit number.)
- Are higher-attaining learners using what they know about the way that numbers work together to make the most efficient jumps? (Spending a few minutes working with a small group of higher-attaining learners may help.)

Some learners may grasp the idea very quickly. Ask them to try use the same method for two-digit subtraction. Remind them that they need to put the bigger number first.

In practice

Mr Mazakadza found that talking their way through the calculations really helped his learners to be able to see the jumps in the calculation 56 + 32.

Using two-digit numbers really made the class think. At first, when he asked why he started with 56, learners just said, *'Well, it's in the sum,'* so he said, *'Would it* *have been as good to start at 32 then?'* Eventually Natasha said, *'No, because then you would have had to jump 56 along and you might get mixed up doing all that jumping.'*

Mr Mazakadza went straight on to using number lines to subtract. He had been one of those teachers who said, *'One idea at a time!'* but now he sees that looking at adding and subtracting together helps learners to see how they are related. Learners realised that you could start from either number when adding. They wanted to do the same thing when subtracting, so they tried it out. They found that, in this case, the calculations weren't the same at all.

Mr Mazakadza thinks that learners are much more likely to remember this because they have experienced it, rather than just being told that swopping the order of the numbers doesn't work for subtraction.

Reflection

When you have completed this module, and tried out the activities in class, reflect on what you have learned from it. You can do this by yourself, but, if you have the opportunity, it is better to do so with other teachers in your school or cluster. Perhaps you can meet after school or set up a WhatsApp group to work with teachers some distance away.

- Which activity worked best with your learners? Why do you think it worked well?
- What will you do differently as a result of reading this module?
- In this module, learners use sticks, ten frames and number lines. What does each of these teach them about numbers?

Module 1 introduced the 'Connections diagram'[1] shown on the right, which summarises the connections between different ways of 'seeing' numbers.

- Choose one activity from this module. Think about how it relates to each part of the Connections diagram.

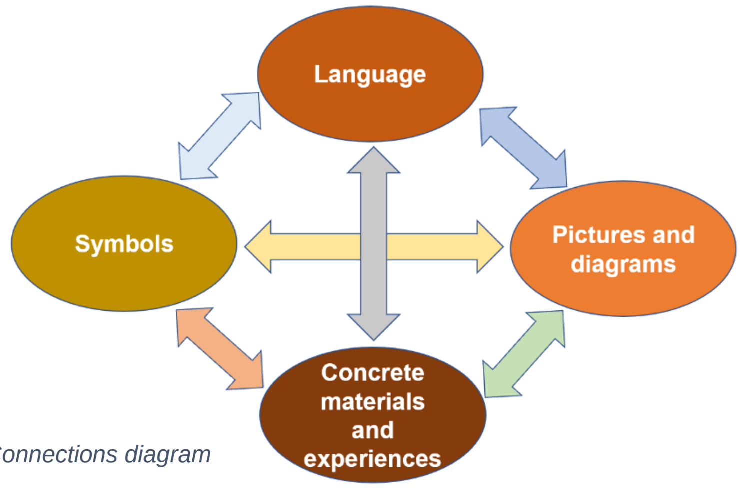


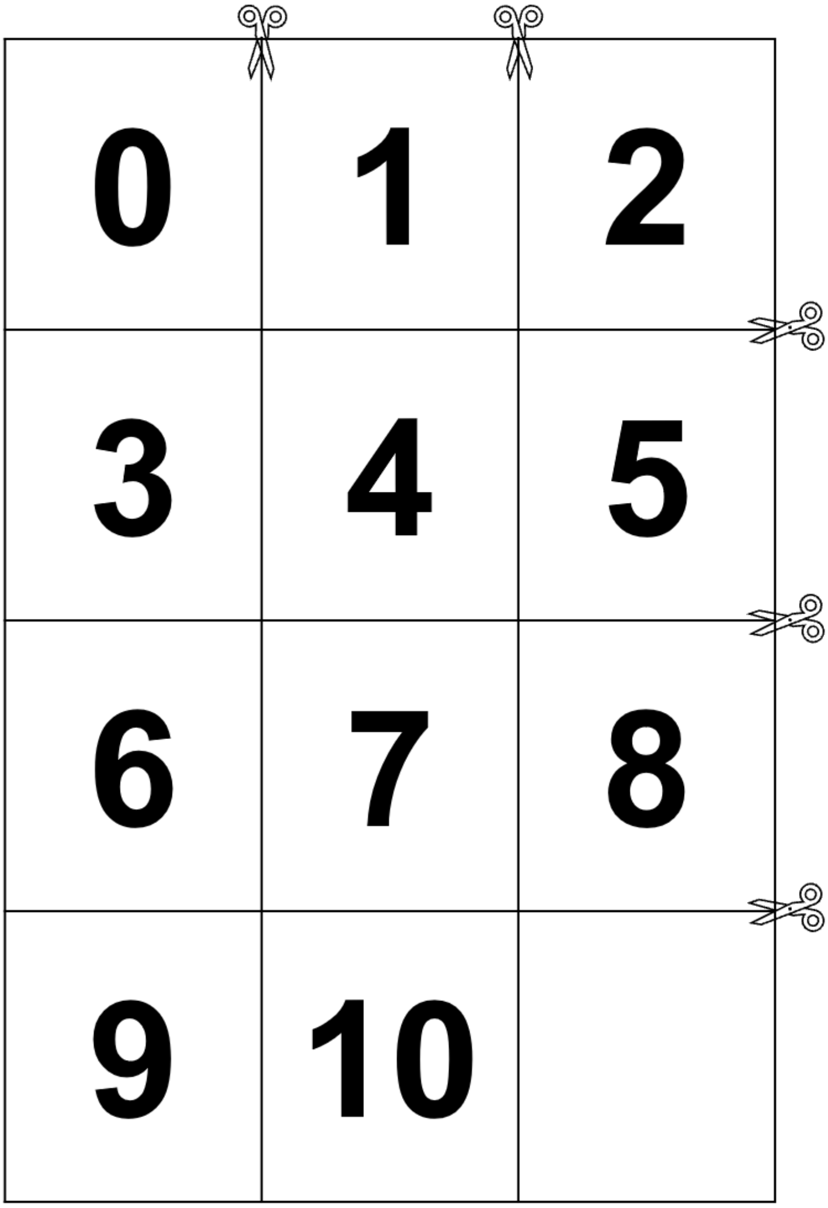
Figure 1 Connections diagram

[1] Haylock, D. and Cockburn, A. (2017:13) *Understanding Mathematics for Young Children* (5th edition). London: Sage

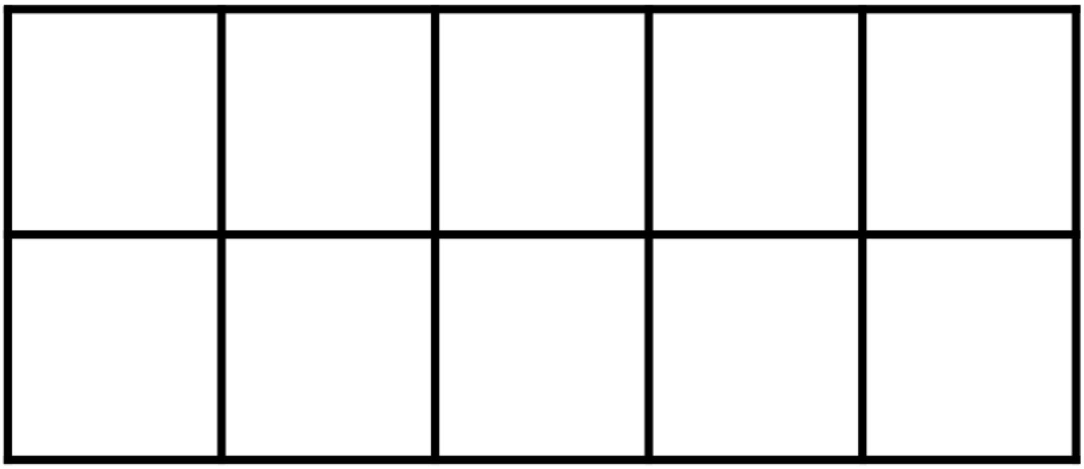
Resources

Resource A Number cards

You can make these cards by cutting a sheet of paper as shown and writing the numbers in the pieces, or you can cut up old cardboard boxes and write the numbers on those. Each card should be about 6 cm x 4 cm.

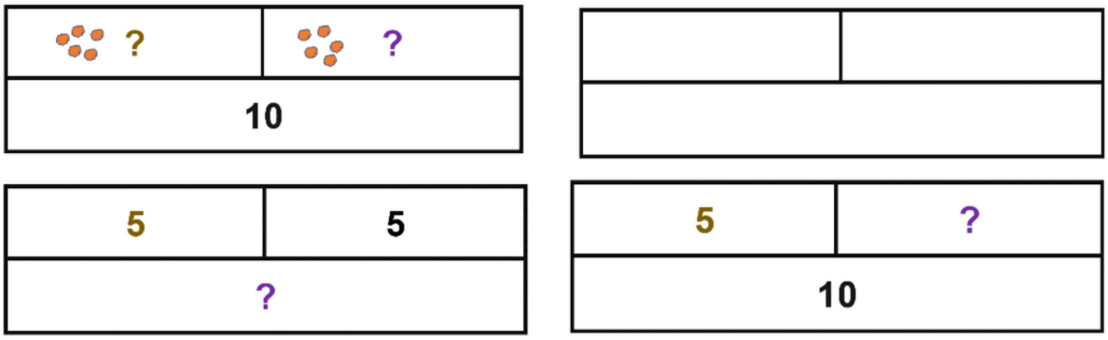


Resource B Ten frame



Resource C Bar model

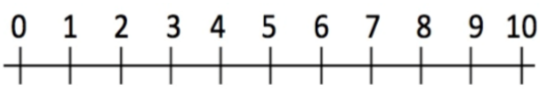
This is a diagram that can be used in many ways to help learners 'see' how numbers work together. Below are some ways that it can used to develop fluency in number bonds to 10.



Resource D 1–200 number grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

Resource E Graduated number line



Resource F Number card display

0 zero	1 one	2 two
3 three	4 four	5 five
6 six	7 seven	8 eight
9 nine	10 ten	11 eleven
12 twelve	13 thirteen	14 fourteen
15 fifteen	16 sixteen	17 seventeen
18 eighteen	19 nineteen	20 twenty

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