

Zimbabwe
Ministry of Primary and Secondary Education


# IGATE Module 3 

Addition and subtraction of bigger numbers

For information about the IGATE project see:
www.wvi.org/education-and-life-skills/igate-improving-girls-access-through-transforming-education

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Revised Module 3 (MoPSE)

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## Foundational numeracy

## Module 3: Addition and subtraction of bigger numbers

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## About these modules

This is the third of six Teacher Professional Development (TPD) modules for all teachers working with learners whose attainment in numeracy is below their Grade or Form level. The modules are also appropriate for Initial Teacher Education (ITE) - particularly during school placements or practicum.

Module 1: What is a number?
Module 2: Early addition and subtraction
Module 3: Addition and subtraction of bigger numbers
Module 4: Multiplication and division - part 1
Module 5: Multiplication and division - part 2
Module 6: Fractions and decimals

The modules were collaboratively developed for the Ministry of Primary and Secondary Education (MoPSE) by the Open University, World Vision and CARE International. The modules have been tried and tested in hundreds of primary and secondary schools across Zimbabwe, strengthening the teaching of foundation skills and improving learning outcomes. Our thanks to everyone who contributed - especially teachers, school heads and schools' inspectors.

MoPSE's highest priority is to empower ALL learners through strong foundations in literacy and numeracy.
Whatever their Grade or Form, all learners need strong foundations in literacy and numeracy to succeed in other learning areas. Learners must learn to read and use number so they can read and use number to learn.

## Using the modules

Teachers will benefit most by using the modules within reflective-practice cycles in their schools, as shown below.

Read an activity.
Plan how you will use the activity.
Do the activity with your learners.
Reflect on what learners learned from doing the activity.


- What worked well?
- What would you change next reflect plan time? your experiences with your colleagues.
> The modules can be used by:
individual teachers
pairs or groups of teachers
whole schools
cluster meetings or district workshops.
The modules provide classroom activities and guidance for effective use.


## Learner attainment

We describe learners who can do an activity confidently and successfully as 'higher attaining' and learners who cannot do an activity well as 'lower attaining'.
No one knows what a learner will be able to do given the chance. Every learner has the potential for growth. Teachers have often been surprised when they found a learner who was 'higher attaining' for one activity was then 'lower attaining' for another - and vice versa. So, we don't label learners with words like 'fast' or 'slow'.
A learner may have different levels of attainment in different learning areas, or in different aspects of one learning area. That's why assessment is a big part of the activities. It is important to find out, as often as possible, what learners know and can or can't do. Then they can be given activities at a level that will help them progress.

## Working in groups

Learning takes place as a result of doing an activity, thinking about it, and understanding the ideas it contains.
In order to make sure that all learners are doing, activities are designed so learners work together in pairs or small groups for most of the lesson. Pairs, or groups of four to six learners, work best because everyone can take part. Sometimes the teacher will need to demonstrate the activity first.

There are several ways in which learners can be put into groups. Teachers should choose the one that works best for their learners.
$>$ Learners choose their own groups: Sometimes this can result in friends working (or not working!) together, while other learners are left out.
> Learners at a similar level of attainment work together: This can work well, as learners are working at their preferred pace, but learners who need help have to find it from outside the group.
> Learners at mixed levels of attainment work together: This type of grouping has the advantage that higher-attaining learners can help lower-attaining ones. This gives lower-attaining learners personal and prompt support, and higher-attaining learners a chance to talk about what they have learned, which helps to deepen their understanding.

## Collecting and storing resources

Many of the activities in these modules rely on learners using physical resources. Some activities need large quantities of resources so that learners can work through the activities in small groups. How can you make sure you have enough?
Here are some suggestions from teachers who trialled these materials.
Sticks
> Ask each learner to bring in at least ten small sticks the length of their middle finger.
> Ask learners to hunt for small sticks in the school grounds during break times. (This worked well in rural communities, but not so well in city schools.)
$>$ As an alternative to sticks, ask learners to bring in at least ten toothpicks or drinking straws.
$>$ Bundle up the sticks into 'tens' using elastic bands (if you have them, as these work better for subtraction) or short lengths of wool or string.

## Counters

> Ask each learner to bring in a collection of counters. In rural communities, learners usually brought in small stones, beans or seeds. In city schools, learners often brought in plastic beads or bottle tops.

## Place value counters

> Ask local bottle stores, lodges and hotels to collect tops from drink bottles. Write the value of the counter on the top with a permanent marker.
> Cut up manila or paper into $2-\mathrm{cm}$ squares and write the value of the counter on each piece.
Making resources together
$>$ Some teachers organised a weekly after-school resource-making meeting to make sure all classes had the equipment they needed. This was especially helpful when making number cards or arrow cards!

## Sharing resources

$>$ Share a set of resources between two classes. So, for example, while one class has a literacy lesson, the other class does numeracy, and vice versa. The teachers quickly swop resources between lessons!
$>$ Split the class into two halves. Work with one half of the class on activities that use resources, while the other half work in their books.

## Storing resources

Teachers often reused the same resources many times, for lots of different activities. They realised that they would save a lot of time by storing resources carefully, either in their classroom or a common storecupboard.

## Unit 1: Place value

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## Introduction

In this unit, learners are introduced more formally to the idea of place value, and the links between 'hundreds', 'tens' and 'ones'. These activities reuse the resources you made for previous modules, including sticks and ten frames. You will also use a new resource - arrow cards, which are used to show place value. For example, learners can take apart arrow cards for 253 to show that the 2 means 2 'hundreds' or 200, the 5 means 5 'tens' or 50 , and the 3 means 3 'ones'. Keep all the resources you make, as part of your 'professional toolkit' for numeracy teaching.
Throughout the modules, you will notice that place value charts refer to 'hundreds', 'tens' and 'ones'. Although older textbooks refer to 'hundreds', 'tens' and 'units', many teachers now prefer to use 'ones' instead of 'units'. 'One' is a familiar word and learners know what it means. 'Unit' is less familiar, most often used in measure (for example centimetres and metres are units of length). Using 'ones' makes calculations easier to understand.

## Key words and phrases

place value - the value of a digit, depending on its position in a number
$>$ ten frame - an arrangement of ten squares (in a $2 \times 5$ grid) that helps learners to 'see' numbers


## 1.1: Using mathematical language

## Aim

This activity introduces some key features of decimal place value:
> In a two-digit number, the first digit represents 'tens' and the second represents 'ones'.
$>$ A digit in any column is worth ten times the digit in the column to its right.
> No column can contain more than nine.

## What the learners will do

Learners work in two teams of three. Teams take turns to pick a card from the top of a pile of number cards and place that number of counters in their ten frame. They say and write the total number of counters. The first team to fill all three ten frames is the winner.

## Resources

Each team of three learners will need:

- three ten frames - these could be drawn on paper (Resource A Ten frame, page 37) or made from egg trays cut into $5 \times 2$ rectangles (see photo)

- three pieces of paper or cloth with 'one group of ten' written on each of them (Resource B One group of ten, page 37) - each must be big enough to cover a ten frame
- 30 counters (see ‘Collecting and storing resources, page 3)
- one place value chart (Resource C Place value chart, page 38), drawn in the dust, on the classroom floor or on paper
Each group of six will need:
- two sets of 1-6 number cards (Resource D Number cards, page 39).


## Activity

1. Each team takes a place value chart and puts one empty ten frame in the ones column of the place value chart.
2. They mix up the number cards and put them face down in the middle of the table.


| 3. One person from each team turns over a card, placing the number of counters shown on the card in the first ten frame. They return the card to the bottom of the pile. | Tens |  |
| :---: | :---: | :---: |
| 4. On their next turn, a new member of the team takes a card, adds that number of counters to the number of counters already on their frame, says how many counters they have altogether and writes the number. | Tens We had 3. I added another 4. Now we have 7. |  |
| 5. When one ten frame is full, it is moved to the 'tens' column. |  | Ones <br> This ten <br> frame has <br> ten ones in it <br> sol have <br> moved it to <br> the tens <br> column. |
| 6. After the ten frame has been moved to the 'tens' column, the team puts one of their pieces of paper or cloth saying 'one group of ten' on top of the full ten frame to cover it. |  |  Ones <br>   |
| 7. The team puts a new ten frame in the 'ones' column of the place value chart. They put any counters needed to finish their turn on the new ten frame. |  |  Ones <br>  $\square$ <br>  $\square$ <br>   <br>   <br>   <br>   |


| 8. After each turn, learners say the total number of counters on their place value chart and write down the number. |  |  |  |
| :---: | :---: | :---: | :---: |
| The first team to fill all three ten fra | the winner. |  |  |

## Assessment

This activity can become quite competitive. As you move around the room listening to and watching learners, make sure they are using mathematical language as well as enjoying the activity!
Check that learners are making connections between the position of the counters and the digits they use to make the written number. (For example, one full frame in the 'tens' column is represented by '1'.) You can do this by pointing to one digit in a written number and asking, 'How many counters is this digit worth?'

## In practice

Mrs Ncube had spent a lot of time helping her learners to understand the structure of numbers using ten frames. She found that some learners still had problems making the link between ten 'ones' and one 'ten'. For example, in the number 15, they did not realise that the digit 1 means one 'ten' and is the same as ten 'ones'.

She found that this activity really helped these learners because they were moving counters in groups of ten from the 'ones' column to the 'tens' column. This helped them to see that ten 'ones' are the same as one 'ten'. Explaining what they were doing to the rest of the group helped to reinforce this idea.

## 1.2: Arrow cards

## Aim

This activity helps learners to understand the value of each digit in a two-digit number. For example, in the number 25 , ' 2 ' represents two 'tens' (20), and ' 5 ' represents five 'ones' (5).

## What the learners will do

Learners make two-digit numbers from arrow cards, and using bundles of ten sticks and single sticks. They will tell the rest of their group how many 'tens' and how many 'ones' are in their number.

## Resources

You will need:

- a set of larger arrow cards for single-digit numbers 0-9 and for 'tens' numbers 10-90 (Resource E Arrow cards, page 40) to demonstrate the activity to the whole class.
Each group of four learners will need:
- one set of arrow cards for single-digit numbers 0-9 and for 'tens' numbers 10-90 (Resource E Arrow cards, page 40)
- bundles of ten sticks and single sticks (see 'Collecting and storing resources, page 3).


## Activity

1. Using the large set of arrow cards, start by drawing the learners' attention to the arrows on the right side of the cards. These arrows must always be on top of each other when making a number. Demonstrate by making two or three numbers, showing how the number is made up of 'tens' and 'ones'.
2. Call out a number between 11 and 59 , for example 25 . One learner from each group makes the number with the arrow cards (using the ' 20 ' arrow card with the ' 5 ' arrow card). A second learner makes the number with bundles of sticks and single sticks (using two bundles of ten sticks and five single sticks).

3. Repeat until all members of each group have had a turn making a number with both arrow cards and with sticks.

| 4. Next, ask every learner to pick a 'tens' arrow card and a 'ones' arrow card to make their own two-digit number. |  |
| :---: | :---: |
| 5. Learners tell each other about their number. For example: 'My number is ten and eight added together. It's eighteen. It is made up from one ten and eight ones.' | My number is ten and eight added together. It is made up from one ten and eight ones. It's eighteen. |
| 6. Finally, ask questions of the w <br> Who has a number with tw <br> Who has a number with thirty <br> Who has a number with mor <br> Find someone with the sam | le class, such as: tens? <br> and some more? <br> than three ones? <br> number of tens as you. |

## Assessment

Although this is a teacher-led activity, you do not need to be at the front of the class after step 1.

Move around the room throughout the rest of the lesson, checking that learners are doing the activity in the way that you demonstrated.
Look out for learners who:
> make two-digit numbers by using two 'ones' cards - they have not understood the value of the digit in the 'tens' column
> make the number using a large quantity of single sticks/straws instead of using bundles of ten and single sticks
$>$ find it difficult to talk about their number in terms of 'tens' and 'ones'.
These learners need more practice working with, and talking about, two-digit numbers.

Also look out for learners who are very confident. Consider giving these learners a leadership role in groups of learners who are not so confident.

## In practice

Miss Zhou had never seen arrow cards before. She made a small set from some scrap paper and tried out the activity herself. She saw immediately how useful arrow cards would be in helping learners to understand place value, but she was worried that she would not be able to make them for the whole class. Where would she get enough paper or card? How would she find time to make so many?
Miss Zhou believes that there is always a way to meet challenges, so she asked her learners to bring in old cartons or paper with a blank side that could be
written on. She realised that the sets of arrow cards needed to be exactly the right size, so she made several sets in thick cardboard, and then asked learners to copy their own sets from these. Some learners made cards at an after-school club and some took cards home so that their families could help to make more. Mrs Zhou found she soon had lots of sets of arrow cards. As a bonus, she found that all her helpers had a very good understanding of place value!

## 1.3: Ordering and comparing bigger numbers

## Aim

This activity will help learners to understand of the size of bigger numbers and the value of each digit within a two-digit and a three-digit number, including understanding the role of zero.

## What the learners will do

In groups of four, learners use arrow cards to order two-digit and three-digit numbers. They then, in groups of eight, use the symbols $<,>$ and $=$ to compare numbers. Finally, they discuss the meaning of zero in bigger numbers.

## Resources

You will need:

- the large set of arrow cards used for Activity 1.2
- a bundle of 100 sticks, made from ten bundles of ten sticks tied together
- a few bundles of ten sticks
- a few single sticks
- a place value chart showing 'hundreds', 'tens' and 'ones' (Resource C Place value chart, page 38). This could be drawn on the classroom floor.
Each group of four learners will need:
- the set of arrow cards they used for Activity 1.2
- a set of eight additional arrow cards showing 'tens' numbers from 60 to 90 and 'hundreds' numbers from 100-400 (Resource E Arrow cards, page 40).
Each group of eight learners (steps 4 and 5) will need:
- a set of $<,>$ and $=$ cards (Resource F Comparing cards, page 42). (You may already have these from Module 1 Activity 2.2.)


## Activity

This is a teacher-led activity, but all the learners are actively engaged throughout. Take as much time on each step as your learners need. It is important for them to understand each step before you move on. Learners begin working in groups of four, then two groups of four join to make a group of eight.

| 1. First, each learner makes a twodigit number using the arrow cards on their table. In their groups of four, they put their numbers in order from the smallest to the biggest number. |  |
| :---: | :---: |
| 2. They repeat step 1 until you are sure they are confident with two-digit numbers, then begin making and ordering three-digit numbers. | $\begin{aligned} & \begin{array}{\|l\|l\|l\|l\|l\|} \hline 1 & 3 & 2 \\ \hline & 1 & 4 \\ \hline \end{array} \\ & \begin{array}{l\|l\|l\|l\|l\|l\|} \hline 4 & 2 & 2 & 5 \\ \hline \end{array} \end{aligned}$ |
| 3. Ask learners to keep the last three-digit number they made and to join with another group. Each group will now have eight learners, so there will be eight three-digit numbers to work with. |  |

4. Remind learners of the meaning of > (more than), < (less than) and = (equal to).
5. Learners take turns to compare their number with another number in the group, using <, > or = symbols.
If it is not possible to use the $=$ symbol, ask the group to see if they can make two numbers equal by changing an arrow
 card in one of the numbers.

Ask learners to go back to their small groups of four and face you.

```
6. Ask one volunteer to make the number one hundred and two using your large arrow cards.
At the same time, ask another volunteer to make the same number with bundles of sticks on your place value chart. What do learners notice? (That they do not use any 'tens' arrow cards, and that there are no sticks in the 'tens' column.) Explain that 0 means that a column has nothing in it. For example, 100 is made up from one 'hundred', no 'tens' and no 'ones'; 102 is made up from one 'hundred', no 'tens' and two 'ones'.
```

If time allows, ask learners, in their groups, to make other 3-digit numbers with arrow cards, some with, some without tens.

## Assessment

Move around the room as learners work.
During steps 1 and 2, you could ask learners which part of the number they look at first when they are deciding on the order. This will tell you if they are developing understanding of the value of each digit.
During step 5, you could ask some of the following questions.
> Could another symbol be used if the order were to be changed? For example, $37<41$ could become $41>37$.
$>$ Would the symbol change if you added 10 to one of the numbers?
$>$ For example, adding 10 to 37 would change from $37<41$ to $47>41$.
$>$ What if one of the numbers was 11 bigger, or 9 smaller?
$>$ Can you think of a number that could go between the two numbers?
> Can you think of a number that could not go between the two numbers?

## In practice

Mr Kumalo found that some learners made mistakes when writing numbers. They were quite happy to say that 32 was thirty-two, or 102 was one hundred and two, but they wrote the numbers down as 302 (thirty followed by two) or 1002 (one hundred followed by 2). These learners clearly understand that 3 in 32 means 30, and 1 in 102 means 100, but they did not know how to write this down as a number.

Using a place value chart helped learners to understand how to write larger numbers correctly. They really enjoyed making numbers using the arrow cards and taking the numbers apart if they were unsure of the meaning of each digit. Mr Kumalo has stored his arrow cards carefully so that he can use them again the next time his learners need help in understanding place value.

## Foundational numeracy <br> Unit 2: Using columns to add and subtract

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## Introduction

In this unit, learners begin to add and subtract using columns. Notice that the method is introduced using single sticks and bundles of sticks. Before learners begin to add and subtract using numbers, they develop an understanding of the concepts using images. This is especially important when the process of exchange is being taught.

Learners' knowledge of place value is extended to 'hundreds', and links between 'hundreds', 'tens' and 'ones'. Resources from the previous two modules are used again.

## Key words and phrases

$>$ exchange - convert a number from one place value unit to another (for example exchanging one 'ten' for ten 'ones', or one 'hundred' for ten 'tens'

## 2.1: Using columns and sticks to add

## Aim

This activity helps learners to understand column addition of two-digit numbers. It shows them which column to add first and where to write the answer.

## What the learners will do

Learners make two-digit numbers using bundles of ten sticks and single sticks. They use these to help them understand column addition. They write two-digit column addition calculations in their books, making sure the digits are in the correct positions.

## Resources

You will need:

- to write the numbers in the box to the right on the board before the session - these numbers are chosen so learners do not need to exchange
- a large place value chart showing 'tens' and 'ones' (Resource C Place value chart, page 38)

- nine bundles of ten sticks
- nine single sticks.

Each group of six learners will need:

- nine bundles of ten sticks
- nine single sticks
- a place value chart showing 'tens' and 'ones' (Resource C Place value chart, page 38).

Each learner will need:

- an exercise book and pen.


## Activity

Before learners begin working on their own in groups, talk them through two calculations.

For the first calculation, ask one volunteer to arrange your sticks on your large place value chart and another volunteer to write the numbers in a place value chart drawn on the board.

For the second calculation, ask learners (in their groups) to arrange their sticks on their own place value chart and to write the numbers in columns in their books.



Learners repeat steps 1-6 until everyone in the group has had a turn arranging the sticks.

## Assessment

Watching and listening as learners talk and work together will tell you a lot about who understands the concept of place value and who needs more help. Make sure all learners are joining in so you can check on the progress of the quieter ones.

Towards the end of the session, gather learners together.
Write the two calculations in Figure 1 on the board.
Say that two learners in another school (you may like to give them names) were asked to add 34 and 5 , using column addition. They wrote these addition calculations in their books. One of them got the right answer, the other one didn't.

Ask learners, in pairs, to decide which answer is right, and to say how they

would help the learner with the incorrect answer to understand where they went wrong. This will help you to see which learners understand the importance of lining up the numbers in the correct columns.

## In practice

Mrs Ndlovu did not usually write anything on the board unless it was correct because she was afraid that her learners would remember what she had written, think it was correct and repeat it in their books. She had often seen the error in Figure 1 (above) in learners' books and corrected it there.

She found that getting learners to talk about the error in pairs worked better than correcting errors in learners' books, as learners didn't always look at what she had written. It was noisy, but after a minute or two, every learner in the class knew which calculation was correct and why. She found that, when she next looked at learners' books, there were fewer errors of this type than before.

## 2.2: Using columns and sticks to subtract

## Aim

This activity helps learners to understand column subtraction of two-digit numbers. It shows them which column to subtract first and where to write the answer.

## What the learners will do

Learners make two-digit numbers using bundles of ten sticks and single sticks. They use these to help them understand column subtraction. They write twodigit column subtraction calculations in their books, making sure the digits are in the correct positions.

## Resources

You will need:

- to write the two sets of numbers to the right (Board 1 and Board 2) on the board before the session
- a large place value chart showing 'tens' and 'ones' (Resource C Place value


Board 1 chart, page 38)

- nine bundles of ten sticks
- nine single sticks.

Each group of six learners will need:

- nine bundles of ten sticks
- nine single sticks
- a place value chart showing 'tens' and 'ones' (Resource C Place value chart, page 38).
Each learner will need:
- an exercise book and pen.


## Activity

Before learners begin working on their own in groups, talk them through two calculations.

For the first calculation, ask a volunteer to arrange your sticks on your large place value chart and another volunteer to write the numbers in a place value chart drawn on the board.

For the second calculation, ask learners, in their groups, to arrange their sticks on their own place value chart and to write the numbers in columns in their books.


| 3. Starting with the ones, Learner 1 says how many need to be taken away. They remove that number of sticks. They place the sticks that are left between the two horizontal lines at the bottom of the 'ones' column. All learners count the number of sticks that are left. | Itake <br> away <br> two <br> owes. <br> $\vdots$There <br> are <br> ater <br> fones <br> onef. <br> left |  |  |  | Ones |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4. All learners write the number of 'ones' remaining between the thicker lines in the 'ones' column. |  | Tens | Ones <br> 6 <br> 2 <br> 4 |  |  |
| 5. Learner 1 says how many 'tens' need to be taken away. They remove that number of bundles of sticks and place the bundles that are left between the two horizontal lines at the bottom of the 'tens' column. All learners count the number of bundles that are left. |  |  |  |  |  |
| 6. All learners write the number of tens remaining between the thicker lines in the 'tens' column. |  |  | Tens <br> 5 <br> 4 <br> 1 | Ones <br> 6 <br> 2 <br> 2 <br> 4 |  |

Learners repeat steps 1-6 until everyone in the group has had a turn arranging the sticks.

## Assessment

Move around the room while learners work in groups. Notice and note learners who:
$>$ need help with setting the numbers out correctly in columns - these learners may need a bit more help with understanding place value
> say how many will be left in each column immediately without counting - these learners are developing fluency in addition and subtraction facts.

## In practice

Miss Hove used Activities 2.1 and 2.2 with her class, in separate lessons. She felt that the activities gave learners a good understanding of both column addition and column subtraction, but she felt that learners needed more practice.
About a week after learners had completed the activities, she put some mixed addition and subtraction questions on the board. She found that some learners ignored or misread the + and - signs. Others had forgotten the column method.

She realised that learners needed to revisit previous topics frequently, to make sure they don't forget what they have learned. Now, once a week, she gives her class five quick questions on previous work. Learners are very pleased when they show that they remember what they have been taught.

## 2.3: More about column addition

## Aim

This activity takes column addition a step further by introducing the concept of exchange. It helps learners to understand what to do when they have more than nine in any column.

## What the learners will do

Learners make two-digit numbers using bundles of ten sticks and single sticks. When adding the two numbers together, learners sometimes exchange ten 'ones' for one bundle of ten sticks. They may also sometimes exchange ten bundles of ten for one bundle of one hundred.

## Resources

You will need:

- a large place value chart showing 'hundreds', 'tens' and 'ones' (Resource C Place value chart, page 38)
- 18 bundles of ten sticks
- 18 single sticks.

Each group of six learners will need:

- 18 bundles of ten sticks
- 18 single sticks
- two sets of 1-9 number cards (Resource D Number cards, page 39)
- a place value chart showing 'hundreds', 'tens' and 'ones' (Resource C Place value chart, page 38).
Each learner will need:
- an exercise book and pen.


## Activity

Before learners begin working on their own in groups, talk them through two calculations.
For the first calculation, choose numbers that will involve exchanging ten 'ones' for one 'ten'. Ask a volunteer to arrange your sticks on your large place value chart and another volunteer to write the numbers in a place value chart drawn on the board.
For the second calculation, choose numbers that will involve exchanging ten 'tens' for one 'hundred'. Ask learners, in their groups, to arrange their sticks on their own place value chart and to write the numbers in columns in their books.


| 3. Learner 5 collects the ones together and counts them. If there are more than nine ones, they make a bundle of ten and place it under the line in the 'tens' column. Any ones that are left stay in the 'ones' column. |  |  |  | 5 ones plus 6 ones. That's 11 ones. I make 1 bundle of ten and put it under the line in the 'tens' column. 1 is left in the 'ones' column. |
| :---: | :---: | :---: | :---: | :---: |
| 4. All learners write the total of the ones in their books, writing ' 1 ' below the line in the 'tens' column if a bundle of ten has been carried over. | + | H'reds Tens <br>  4 <br>  1 <br>   <br>  1 | Ones <br> 5 <br> 6 <br> 1 |  |
| 5. Learner 6 collects the bundles of ten together and counts them. They must remember to include the bundle of ten that has been carried (if there is one). If there are more than nine bundles of ten, they tie ten bundles of ten together to make one bundle of a hundred in the 'hundreds' column. Any bundles of ten that are left, stay in the 'tens' column. |  | Tens | 9 mom |  Ones <br>   <br>   <br>   |
| 6. All learners complete the calculation in their books. |  |  Hreds $^{\prime}$ Ten <br> +   <br> +   <br>    <br>    | ens Ones <br> 4 5 <br> 1 6 <br> 6 1 <br> 1  |  |

Learners repeat steps 1-6 as often as time allows. Learners change jobs with each new calculation.

## Assessment

On the board, write the calculation and the speech bubbles in the illustration to the right.

Ask learners to discuss, with a partner, which answer they think is right and what the other learners may have misunderstood.


## Note:

Prudence wrote ' 12 ' in the answer box instead of carrying the ten and putting it under the line.

Courage forgot to add the extra 'ten' that he had carried after totalling the ones.
Charity has the correct answer.

## In practice

Mr Masuku had never heard of using sticks in the classroom. He decided to try this activity because he had noticed that many of his learners often got confused when using column methods, especially if exchange was needed. He asked his learners to bring in some small sticks and together they tied some of them into bundles of ten.

He found that it was important to do every step of the activity, and to check that learners understood each step. He thought it was better to go slowly and take two lessons if necessary. Once learners understood the idea of exchange, they were usually able to practise it in their groups without his help and quickly progressed to working without sticks because they understood what they were doing.

## Unit 3: More subtraction, and place value counters

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## Introduction

This unit introduces learners, in small steps, to more complicated subtraction calculations using bigger numbers and exchange. Learners progress from using bundles of sticks to place value counters, linking both to column addition methods. With practice, they are able to use column methods accurately and with understanding.

## Key words and phrases

> exchange - converting a number from one place value unit to another (for example exchanging one 'ten' for ten 'ones', or one 'hundred' for ten 'tens'
> place value counters - counters representing 'ones', 'tens' and 'hundreds'
$>$ representation - a group of objects or a picture that helps learners to understand a mathematical concept or idea

## 3.1: Race to zero!

## Aim

This activity takes column subtraction a step further by introducing exchange. It helps learners understand what they need to do when the bottom digit in a column is more than the top digit.

## What the learners will do

Each group starts with five bundles of ten sticks. When it is their turn, learners take a card and subtract that number of sticks. Learners sometimes exchange one bundle of ten sticks for ten single sticks. The first group to have no sticks left is the winner.

## Resources

You will need:

- the large place value chart that you used for Activity 2.3 (Resource C Place value chart, page 38)
- nine bundles of ten sticks
- nine single sticks.

Each group of six learners will need:

- five bundles of ten sticks, tied up in such a way (for example with elastic bands or wool) that they can be untied and made into ten single sticks
- nine single sticks
- two sets of 1-9 number cards (Resource D Number cards, page 39)
- the place value chart that they used for Activity 2.3 (Resource C Place value chart, page 38).
Each learner will also need:
- an exercise book and pen.


## Activity

Before learners begin working on their own in groups, talk them through two calculations, asking volunteers to move the sticks and write the numbers. Choose numbers that will involve exchange.

For the first calculation, ask learners to repeat the phrases that you use as they move the sticks and write the numbers.

For the second calculation, ask learners to tell you what they think the volunteers need to do and say at each stage.

6. Learners take turns to turn over cards, subtract the correct number of sticks from the previous answer and write the calculation in their books.

7. The group that finishes their sticks first is the winner. (They do not need to get the exact number. So, for example, if they turn over a card that says 8 and they only have 3 sticks left, they have finished all their sticks, so they have won.)

## Assessment

Make sure learners are using mathematical language to explain what they are doing.

Notice and note those learners who:
$>$ seem to copy others or need someone else to tell them what to do these learners will need extra support
> do not understand when there are enough 'ones' to take away without exchange - they are just following a process and do not yet understand the purpose of exchange.

## In practice

After using this activity, Miss Ndou felt that her learners had become more confident and accurate subtracting using column methods. By using sticks and numbers side by side, they were able to understand the process of exchange and could describe in words what they were doing.

She felt that the next step was to help learners to use their knowledge of column addition and subtraction to solve word problems. Learners already knew how to talk about addition and subtraction using precise mathematical language, but she realised that many found the language in problems difficult to understand because there are so many words that can be used to ask a question involving addition or subtraction. She made a list of some of these words and asked her colleagues to add to her list.

| add | sum | more |
| :--- | :--- | :--- |
| altogether | total | morethan |
| lessthan | takeaway | greater than |
| difference | subtract | minus |

Next, she made some cards, each containing a question in words. Some were simple, such as: 'What is the difference between 16 and 32?' Some were reallife problems, such as: 'Farai has 23 cattle. Simba has 56 cattle. How many do they have altogether? How many more cattle does Simba have than Farai?'

When there were a few minutes to spare in a lesson, she gave a card to groups of learners and asked them to work together to answer the question. She made sure that there was a strong reader in each group so that lower-attaining readers could still work on the problem. She found doing this often, for just a few minutes, helped her learners to understand the written language of mathematical problems.

## 3.2: Using place value counters

## Aim

This activity introduces learners to place value counters (see 'Collecting and storing resources, page 3) to help them understand addition of numbers beyond 100.


## What the learners will do

Learners make three-digit numbers and find the total. They use place value counters to help them with the column addition. The calculations will sometimes involve exchange.

## Resources

You will need:

- the large place value chart that you used for Activity 2.3 (Resource C Place value chart, page 38). You may need to add a 'thousands' column.
- 16 place value counters with ' 1 ' written on
- 12 place value counters with ' 10 ' written on
- ten place value counters with ' 100 ' written on
- a small collection of counters with '1000' written on for learners to use if needed (see Step 7).

Each group of six learners will need:

- the place value chart that they used for Activity 2.3 (Resource C Place value chart, page 38) - they may need to add a 'thousands' column
- 16 place value counters with ' 1 ' written on
- 12 place value counters with ' 10 ' written on
- ten place value counters with ' 100 ' written on
- two sets of 0-9 number cards and two sets of 1-4 number cards.

Each learner will also need:

- an exercise book and pen.


## Activity

Explain that as numbers get bigger, the number of sticks get too big to work with, so from now on they are going to use place value counters to represent the numbers.

Before learners begin working on their own in groups, demonstrate using the place value counters for addition of three-digit numbers. Ask volunteers to move the place value counters and write the numbers.


8. All learners write the total of the 'hundreds' between the thicker lines in the 'hundreds' column.

|  | Hreds | Tens | Ones |
| :---: | :---: | :---: | :---: |
|  | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{8}$ |
| $\mathbf{+}$ | $\mathbf{2}$ | $\mathbf{4}$ | $\mathbf{3}$ |
|  | $\mathbf{7}$ | $\mathbf{0}$ | $\mathbf{1}$ |
|  | $\mathbf{1}$ | $\mathbf{1}$ |  |

9. The number cards are returned to the bottom of the pile and the counters are removed from the place value chart. Learners repeat the process as often as time allows.
Towards the end of the session, gather the class together. Write the column addition on the right on the board.

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$+672$

Ask learners to discuss in pairs, how they would complete this calculation. Help them to understand that every time there are more than nine in any column, ten of them are exchanged for one in the column to the left, in this case, one 'thousand'.

## Assessment

$>$ As learners work in groups, notice and note those learners who reverse the digits when exchanging, for example, when there are 12 'ones', the learner puts ' 2 ' under the line in the 'tens' column, and ' 1 ' between the thicker lines in the 'ones' column. These learners need more practice with bundles of sticks and small numbers.
$>$ During the whole class activity at the end of the session, notice and note those who are able to explain what to do when there are more than nine 'hundreds'. They clearly understand the process of exchange and may not need the support of place value counters in the future.

## In practice

Mrs Maposa found that using bundles of sticks really helped her learners to 'see' and understand place value and column addition involving exchange, but using them for larger numbers needed a lot of sticks and a lot of space!
She was pleased that this activity introduced learners to place value counters. They were smaller than bundles of sticks, and encouraged learners to talk about quantities of tens and hundreds, rather than thinking in ones all the time.

To begin with, she didn't have enough bottle tops for the class to use, so she asked friends to collect them for her, from their homes or local places selling food. In the meantime, she made place value counters by cutting up pieces of scrap paper. Her counters were square, as it took too long to cut out circles!

## 3.3: Count down from 301

## Aim

This activity helps learners understand what to do when exchange is necessary in order to subtract the ones when there is a zero in the tens column, for example, 203-135.

## What the learners will do

In groups of six, pairs of learners subtract two-digit numbers from 301. They continue to subtract two-digit numbers from their previous answer until they reach zero. The first pair of learners in each group to reach zero is the winner.

## Resources

You will need:

- the large place value chart that you used for Activity 2.3
- 18 place value counters with ' 1 ' written on
- 18 place value counters with ' 10 ' written on
- three place value counters with '100' written on.

Each pair of learners will need:

- two sets of 0-9 number cards
- a place value chart like the one they used for Activity 2.3
- 18 place value counters with ' 1 ' written on
- 18 place value counters with ' 10 ' written on
- three place value counters with '100' written on.

Each learner will also need:

- an exercise book and pen.


## Activity

Before learners begin working in their groups, explain the activity and demonstrate subtracting a two-digit number from 301 at least twice, once with learners watching and once with learners copying what you do in their groups.


| 5. They show what they have done in their books by changing the ' 10 ' in the 'tens' column to ' 9 ' and putting a ' 1 ' beside the ' 1 ' in the 'ones' column to say there are 11 'ones'. |  | - | H'eds <br> 2 S | Tens 109 4 | Ones <br> 11 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6. The learners can now complete the calculation. <br> They subtract 3 'ones' from 11 'ones', leaving 8 'ones'. | Hundreds (100 100 |  | $\sqrt{x}$ |  | Ones Q $11-1 X$ $1.1 . X$ |
| Next, they subtract 4 'tens' from 9 'tens', leaving 5 'tens'. <br> They do not need to subtract any 'hundreds', so they write ' 2 ' between the thicker lines in the 'hundreds' column. |  | - | Hreds <br> 2 S | Tens 10 9 4 4 5 | Ones <br> 11 <br> 3 <br> 8 |

7. Pairs of learners continue to subtract two-digit numbers from their answer until they reach zero. The pair in the group that reaches zero first wins.

## Assessment

Move around the room watching learners as they complete this activity. Notice and note learners who:
$>$ take the digit on the top from the digit on the bottom to avoid having to exchange - these learners will benefit from spending more time using bundles of sticks and talking through simpler subtraction calculations with you
$>$ forget to change the number in the column from which a counter was removed - spending a little time asking them to explain what they are doing will probably help them see where they have gone wrong
$>$ seem confused by the double exchange - they may need more practice working with single exchange subtraction so they fully understand what they are doing
$>$ are confident and accurate as they work through the subtraction calculations - they may be ready to work without place value counters.

## In practice

Mr Bhebhe did not have any place value counters, so he demonstrated the activity on the board by drawing counters in a place value chart. His learners did not understand what he was doing. When he went to the back of the room
he could see why. What he had written was hard to read, because the board need a new coat of paint and only white chalk was available.
On reflection, he realised that it was important that learners used the place value counters themselves, rather than just listening to him talk about them. He made some place value counters from paper and tried the activity again, following the instructions exactly. He was pleased to find that, this time, his learners understood the need to exchange twice, and were soon practising this type of calculation while enjoying playing the game.

## Reflection

When you have completed this module, and tried out the activities in class, reflect on what you have learned from it. You can do this by yourself, but, if you have the opportunity, it is better to do so with other teachers in your school or cluster. Perhaps you can meet after school or set up a WhatsApp group to work with teachers some distance away.
$>$ Make list of all the different ways in which numbers are represented in this module.
$>$ Think about what each representation teaches a learner about numbers.
$>$ Are some representations better used together? If so, which ones and why?
> Some learners fail to progress in numeracy because they don't know the meaning of some spoken or written English words. How can you identify these words and ensure that all learners know what they mean?
$>$ As a result of reading this module, will you make any changes to the way in which you teach column addition and subtraction? If so, what will you do differently?

## Resources

Resource A Ten frame


Resource B One group of ten


Resource C Place value chart

| Hundreds | Tens | Ones |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |

## Resource D Number cards

You can make these cards by cutting a sheet of paper as shown and writing the numbers on the pieces, or you can cut up old cardboard boxes and write the numbers on those. Each card should be about $6 \mathrm{~cm} \times 4 \mathrm{~cm}$.


Resource E Arrow cards



Resource F Comparing cards


## Acknowledgements

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