Zimbabwe
Ministry of Primary and Secondary Education

NUMERACY

IGATE
Module 4
Multiplication and division – part 1

For information about the IGATE project see:
www.wvi.org/education-and-life-skills/igate-improving-girls-access-through-transforming-education

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Revised Module 4 (MoPSE)

Foundational numeracy

Module 4: Multiplication and division – part 1

Module contents

About these modules

This is the fourth of six Teacher Professional Development (TPD) modules for all teachers working with learners whose attainment in numeracy is below their Grade or Form level. The modules are also appropriate for Initial Teacher Education (ITE) – particularly during school placements or practicum.

Module 1: What is a number?

Module 2: Early addition and subtraction

Module 3: Addition and subtraction of bigger numbers

Module 4: Multiplication and division – part 1

Module 5: Multiplication and division – part 2

Module 6: Fractions and decimals

The modules were collaboratively developed for the Ministry of Primary and Secondary Education (MoPSE) by the Open University, World Vision and CARE International. The modules have been tried and tested in hundreds of primary and secondary schools across Zimbabwe, strengthening the teaching of foundation skills and improving learning outcomes. Our thanks to everyone who contributed – especially teachers, school heads and schools' inspectors.

MoPSE's highest priority is to empower ALL learners through strong foundations in literacy and numeracy.

Whatever their Grade or Form, all learners need strong foundations in literacy and numeracy to succeed in other learning areas. Learners must **learn to read and use number** so they can **read and use number to learn**.

Using the modules

Teachers will benefit most by using the modules within reflective-practice cycles in their schools, as shown below.

Read an activity.
Plan how you will use the activity.
Do the activity with your learners.
Reflect on what learners learned from doing the activity.

- What worked well?
- What would you change next time?

Share your experiences with your colleagues.

- The modules can be used by:
  - individual teachers
  - pairs or groups of teachers
  - whole schools
  - cluster meetings or district workshops.
- The modules provide classroom activities and guidance for effective use.

Learner attainment

We describe learners who can do an activity confidently and successfully as ‘higher attaining’ and learners who cannot do an activity well as ‘lower attaining’.

No one knows what a learner will be able to do given the chance. Every learner has the potential for growth. Teachers have often been surprised when they found a learner who was ‘higher attaining’ for one activity was then ‘lower attaining’ for another – and vice versa. So, we don’t label learners with words like ‘fast’ or ‘slow’.

A learner may have different levels of attainment in different learning areas, or in different aspects of one learning area. That’s why assessment is a big part of the activities. It is important to find out, as often as possible, what learners know and can or can’t do. Then they can be given activities at a level that will help them progress.

Working in groups

Learning takes place as a result of **doing** an activity, **thinking** about it, and **understanding** the ideas it contains.

In order to make sure that all learners are **doing**, activities are designed so learners work together in pairs or small groups for most of the lesson. Pairs, or groups of four to six learners, work best because everyone can take part. Sometimes the teacher will need to demonstrate the activity first.

There are several ways in which learners can be put into groups. Teachers should choose the one that works best for their learners.

- **Learners choose their own groups:** Sometimes this can result in friends working (or not working!) together, while other learners are left out.
- **Learners at a similar level of attainment work together:** This can work well, as learners are working at their preferred pace, but learners who need help have to find it from outside the group.
- **Learners at mixed levels of attainment work together:** This type of grouping has the advantage that higher-attaining learners can help lower-attaining ones. This gives lower-attaining learners personal and prompt support, and higher-attaining learners a chance to talk about what they have learned, which helps to deepen their understanding.

Collecting and storing resources

Many of the activities in these modules rely on learners using physical resources. Some activities need large quantities of resources so that learners can work through the activities in small groups. How can you make sure you have enough?

Here are some suggestions from teachers who trialled these materials.

Sticks

- Ask each learner to bring in at least ten small sticks the length of their middle finger.
- Ask learners to hunt for small sticks in the school grounds during break times. (This worked well in rural communities, but not so well in city schools.)
- As an alternative to sticks, ask learners to bring in at least ten toothpicks or drinking straws.
- Bundle up the sticks into 'tens' using elastic bands (if you have them, as these work better for subtraction) or short lengths of wool or string

Counters

- Ask each learner to bring in a collection of counters. In rural communities, learners usually brought in small stones, beans or seeds. In city schools, learners often brought in plastic beads or bottle tops.

Place value counters

- Ask local bottle stores, lodges and hotels to collect tops from drink bottles. Write the value of the counter on the top with a permanent marker.
- Cut up manila or paper into 2-cm squares and write the value of the counter on each piece.

Making resources together

- Some teachers organised a weekly after-school resource-making meeting to make sure all classes had the equipment they needed. This was especially helpful when making number cards or arrow cards!

Sharing resources

- Share a set of resources between two classes. So, for example, while one class has a literacy lesson, the other class does numeracy, and vice versa. The teachers quickly swop resources between lessons!
- Split the class into two halves. Work with one half of the class on activities that use resources, while the other half work in their books.

Storing resources

- Teachers often reused the same resources many times, for lots of different activities. They realised that they would save a lot of time by storing resources carefully, either in their classroom or a common store-cupboard.

Unit 1: Introduction to multiplication and division

Contents

Introduction

In this unit, learners begin to develop the concepts of multiplication and division. They learn that multiplication can be thought of as repeated addition. Arrays are introduced as a useful representation of multiplication and division that helps learners to 'see' that 2 x 3 = 3 x 2. Arrays also help learners 'see' that multiplication and division are inverses (opposites).

Key words and phrases

- **array** – an arrangement with the same number of columns in each row (for example, if the first row has five columns, every other row must also have five columns)
- **column** – a vertical arrangement of objects
- **commutative** – a word meaning the order of the numbers in a calculation does not change the answer (applies to addition and multiplication, not to subtraction and division)
- **row** – a horizontal arrangement of objects
- **times** – multiplied by, lots of

1.1: Counting in jumps

Aim

This activity helps learners to see multiplication as repeated addition.

What the learners will do

Ideally, this activity takes place outside the classroom., Learners are given consecutive numbers. They line up in groups of two or three as directed by the teacher. The learner at the front of each line says their number loudly, the other learners whisper their numbers.

Next, one learner jumps along a giant number line in twos, then threes as the rest of the learners count.

Finally, learners return to the classroom and work in pairs to find multiplication facts by counting in jumps on their own number lines.

Resources

You will need:

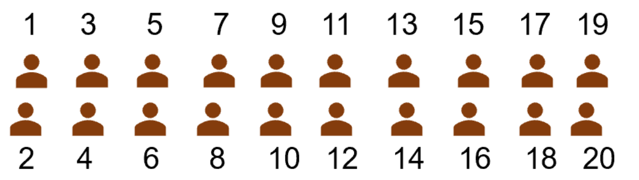
- to draw a 0–20 number line outside in the dust, on the classroom floor or on paper. If you prefer, you can put 0–20 number cards in a row with equal spaces between them and draw a line above them.

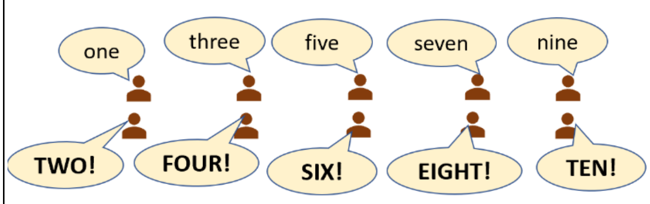
Each pair of learners will also need:

- a 0–30 number line (**Resource A Number line 0–30**, page 33). If learners have rulers, they can use these as number lines.

Activity

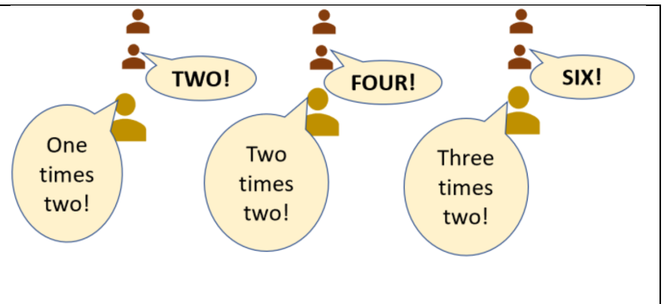
If possible, take learners outside for the start of this activity as it requires a lot of space. If this is not possible, move the desks and chairs so there is a large open space in the classroom.

<table>
<tr>
<td>1. Ask 20 learners to stand in two rows, facing the rest of the group. Give each learner a number as shown.</td>
<td>


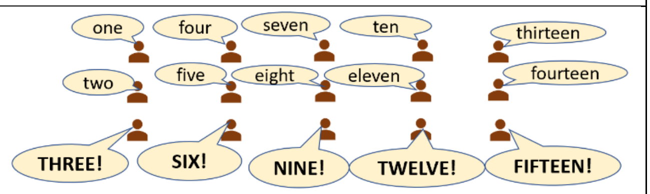
</td>
</tr>
<tr>
<td>2. Tell learners to say their numbers in order, but the learners at the back must whisper their number; the ones at the front must say their number loudly.</td>
<td>


</td>
</tr>
</table>

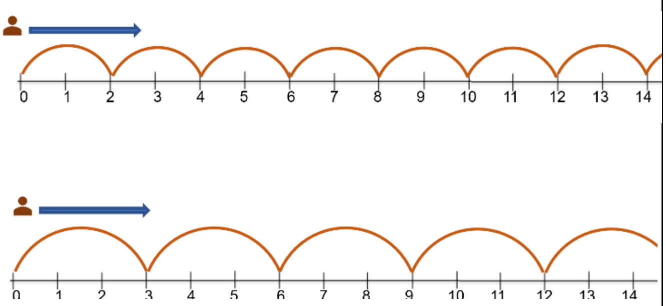
3. Next, call, *'One times two!'* and point at the first row. The learner at the front of the row says their number. Repeat with two times two, three times two and so on.



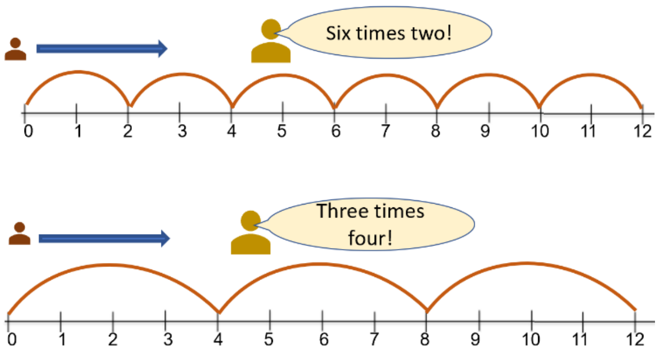
4. Repeat steps 1–3, this time asking 30 learners to line up in three rows. Call, *'One times three!'*, *'Two times three!'*, and so on.



5. Now ask a volunteer to jump in twos along your giant number line, starting at zero. Ask the whole class to count as the volunteer jumps. Repeat, counting in threes. Ask what is the same and what is different between the lining up activity and the number line activity. *(Both activities count in groups, first twos, then threes. One is counting objects, the other is counting jumps on the number line.)*



6. Ask learners to suggest how the volunteer could find the answer to six times two using the number line. *(Jump six lots of two.)* Choose another multiplication fact (with an answer less than 20) for the volunteer to find.



7. Ask learners to work in pairs to find the answer to multiplication calculations that you call out by making jumps on their own number lines.

Assessment

When working with the whole class, it can be difficult to notice which learners are progressing well and which ones still need a bit more help. One way of making sure you have noticed every learner is to take answers from learners whose names begin with certain letters, for example A, B, C, D, E or F for one question, then learners whose names begin with G, H, I, J, K or L to answer the next question and so on.

- Note down names of learners who are quick to give the answer. Ask them how they knew. They may already know the multiplication facts. Do not make them use the number line, as it will slow them down.
- Also note down the names of learners who add the two numbers instead of multiplying. They need more help with understanding that multiplication is repeated addition. One of the numbers tells them what number they are adding, the other one tells them how many times they need to add it.

In practice

When Mr Makoni began to teach his class about multiplication, he found that many of them found the idea very confusing. He realised that he had introduced the language of multiplication tables (for example ‘two times three equals six’) without properly explaining that multiplication was repeated addition.

His learners had enjoyed previous activities on addition and subtraction that involved jumping so he took them outside to do this activity. They enjoyed standing in a block (array) shouting and whispering numbers and they began to see patterns as they modelled different numbers. Jumping along a number line reinforced these patterns and he felt that they now had a good understanding of the meaning of multiplication.

1.2: What is an array?

Aim

This activity uses practical equipment to show learners that numbers can be reversed when multiplying because multiplication is commutative. Learners also become familiar with the mathematical symbol for multiplication (**x**).

What the learners will do

Learners search around the school for examples of arrays, draw them and write two multiplication sentences to show the different ways of 'seeing' them.

Resources

You will need:

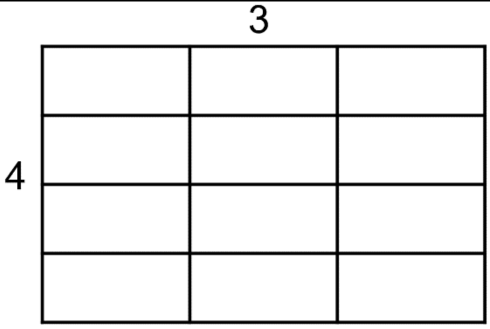
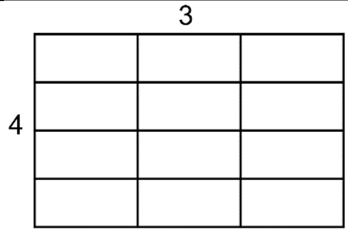
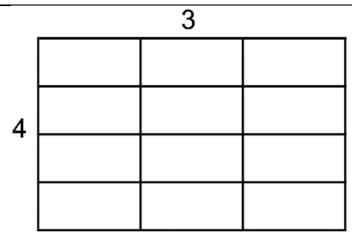
- to make sure you know where to find arrays around the classroom and the school site
- to make some arrays in the classroom from bottle tops, pieces of paper, books, pencils etc. for the learners to find.

Each learner will need:

- an exercise book and pen.

Activity

1. Choose an array that can easily be seen in your classroom. This may be the window, if it has suitable small panes, or the desks, or some chairs arranged as an array. This picture of a window is an array that shows 4 x 3. A 3 x 5 array could be made from three rows of five chairs, or three rows of five posters on the wall, for example.	

2. Draw a diagram to show the array you have chosen in your classroom. Label each side with the number it represents. In the above example, 4 is written on the vertical side to show four rows, and 3 on the horizontal side to show three columns.	
3. Write 4 x 3 = 12 on the board and explain that this is the multiplication sentence for the array as the array shows four rows of three.	 4 x 3 = 12
4. Ask if learners can think of any other multiplication that is shown by the array. *(It also shows three columns of four, written as 3 x 4 = 12.)*	 4 x 3 = 12 3 x 4 = 12
5. Send your learners in pairs on an 'array hunt'. Tell them to find an array, for example a part of a path, a different window or the bottle tops you have arranged in the classroom. When they find one, they must draw it and write the two multiplication sentences it represents.	   
Back in the classroom, ask learners, in groups of six, to share with each other the arrays they have found. Ask them to say if they agree with the multiplication sentences that have been written to match each array.	

Towards the end of the session, draw these diagrams on the board.

Ask learners to discuss which ones are and which ones are not arrays. Point to each one in turn. If learners think the one you are pointing to is an array, they put their thumbs up, if they think it is not, they put them down.

Assessment

As learners look for arrays around the school, notice and note who:

- is quick to spot arrays and to say both the multiplication sentences that match them – these learners have understood that, when multiplying, numbers can be in any order
- struggles to write two multiplication sentences – showing them a piece of paper with an array on might help as you can turn it round so they can 'see' the array in a different way.

During the activity at the end of the lesson, notice and note who:

- has not realised that an array must have an equal number in each row, and an equal number in each column – they may need more practice with making and labelling arrays.

In practice

Mr Mbiza realised that, when learners understand what they are doing, multiplication is easier. They can work things out for themselves instead of trying to remember a lot of unrelated facts. His learners found finding arrays around the school really helpful in visualising rows and columns and had no difficulty in writing multiplication sentences to describe them.

After working on a few arrays, most of the learners had written a pair of multiplication sentences for each array. At the end of the lesson, he chose a pair of sentences, such as 2 x 3 = 6 and 3 x 2 = 6. He asked the class, *'What is the same?'* and *'What is different?'* about these two sentences. He was pleased to find that his learners had understood that 2 x 3 is the same as 3 x 2. This will halve the number of multiplication facts that his learners need to remember!

1.3: Lines and jumps

Aim

This activity introduces learners to the concept of division and begins to show how division and multiplication are linked.

What the learners will do

Outside the classroom, groups of learners arrange themselves in lines of two, then three. They count the number of lines to find how many lines of two or three there are in different numbers.

Next, a volunteer jumps along a giant number line to find how many jumps of two or three they need to reach different numbers.

Finally, working in pairs, learners find division facts by counting in jumps on their own number lines.

Resources

You will need:

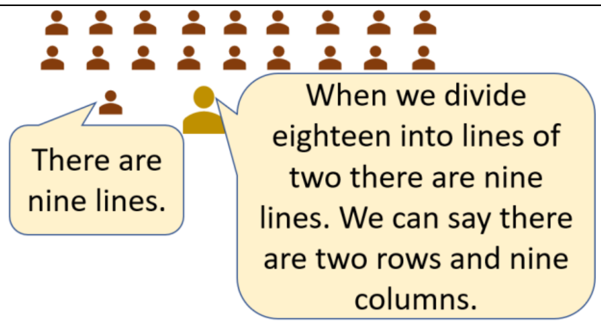
- to draw a 0–15 number line outside in the dust, on the classroom floor or on paper. If you prefer, you can put 0–15 number cards in a row with equal spaces between them and draw a line above them.

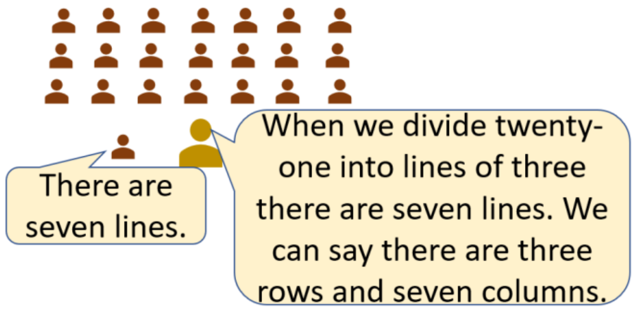
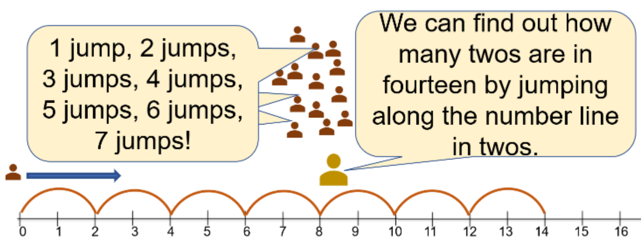
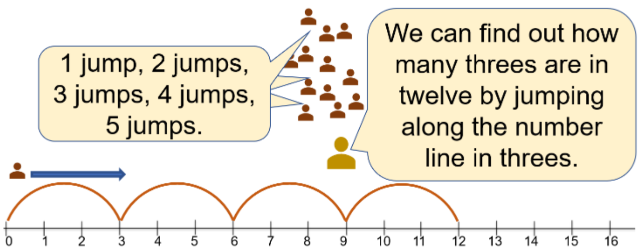
Each pair of learners will need:

- a 0–30 number line (**Resource A Number line 0–30**, page 33). If learners have rulers, they can use these as number lines.

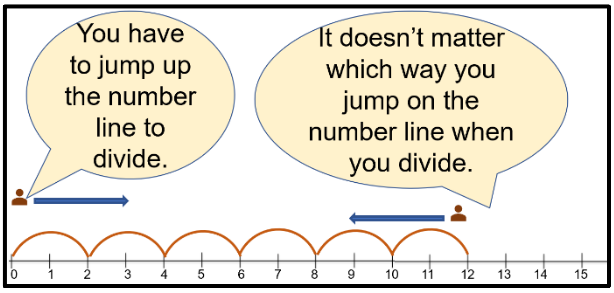
Activity

If possible, take learners outside for the start of this activity as it requires a lot of space. Otherwise move the desks and chairs so there is a large open space in the classroom.

1. Ask 18 learners to arrange themselves in lines of 2, so pairs of learners are standing behind the other, facing the rest of the group.	
2. Ask another learner to count the number of lines. Say, *'When we divide eighteen into lines of two, there are nine lines. We can say there are two rows and nine columns.''*	

<table>
<tr>
<td>3. Ask 21 different learners to arrange themselves in lines of three.
Ask another learner to count the number of lines.
Say, ‘When we divide twenty-one into lines of three, there are seven lines. We can say there are three rows and nine columns.’</td>
<td>
</td>
</tr>
<tr>
<td>4. Now ask a volunteer to jump in twos along your giant number line until they get to 14. Ask the rest of the learners to count the jumps.
Ask how many jumps it took. (7) Say, ‘We can find out how many twos are in fourteen by jumping along the number line in twos.’</td>
<td>
</td>
</tr>
<tr>
<td>5. Ask another volunteer to jump in threes along the number line until they get to 12, with the rest of the learners counting. Perhaps this time a learner can tell you what you have just found out.</td>
<td>
</td>
</tr>
<tr>
<td colspan="2">6. Back in the classroom, explain that what they have been doing outside is dividing. They divided 18 by 2 when they lined up in twos and found there were 9 lines. They divided 12 by 3 when they jumped along the number line in threes and found they made 4 jumps.</td>
</tr>
<tr>
<td colspan="2">7. Ask learners to work in pairs to find the answer to division calculations that you call out by making jumps on their own number lines.</td>
</tr>
</table>

Assessment



Draw the cartoon above on the board. Ask learners to think about which of the learners is correct. Learners who understand that the learner on the right is correct are developing a good sense of number in relation to multiplication and division. Learners who think the learner on the left is correct are simply following a procedure rather than understanding what they are doing. Focus on those learners in the next unit.

In practice

After Miss Tshuma and her colleague Mrs Nyoni had used all three of the activities in this unit, they sat down together one day to talk about what they had noticed.

Their first reflection was about how much the learners had enjoyed the activities. Miss Tshuma was particularly pleased about this, as she sometimes found her class difficult to manage and had been reluctant to take her class outside.

Their second reflection was that they could see that each part of the activity had a clear purpose and the parts linked together. As the teachers watched learners jumping along the number line in their groups, and listened to what they were saying, the teachers could see that learning was taking place. If a mistake was made, other learners in the group helped to correct it. The teachers realised that they found out much more by working in this way than they did from looking at learners' books after the lesson.

Their final reflection was about the first and third activities in the unit. They noticed that they looked the same, but they weren't. This led to a long discussion, exploring these two activities in detail. In the end, Mrs Nyoni said it looked like division and multiplication were opposites, but she hadn't realised it until now.

They decided to do the activities in the next unit to see whether the activities supported their conclusion and to meet for another chat afterwards.

Unit 2: Noticing patterns

Contents

Introduction

This unit uses the many patterns and connections between numbers to help learners make sense of the idea of multiplication. It continues to link multiplication to division. It is important to read through and try out these activities with a colleague before using them with a class. Some activities will take one lesson; longer activities may take a week or two.

Key words and phrases

- **array** – an arrangement with the same number of columns in each row (for example, if the first row has five columns, every other row must also have five columns)
- **column** – a vertical arrangement of objects
- **row** – a horizontal arrangement of objects

2.1: Four facts in one!

Aim

This activity helps learners to see the connections between multiplication and division. It also introduces learners to the division symbol (÷).

What the learners will do

Learners work in groups of four to six. They take turns to make an array and write down the multiplication and division sentences that the array represents.

Resources

You will need:

- a piece of paper or card showing a 2 x 4 array (see step 1 below).

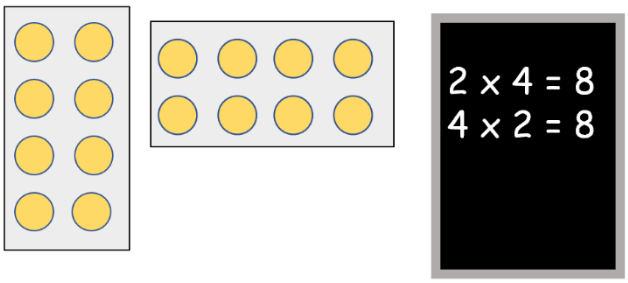
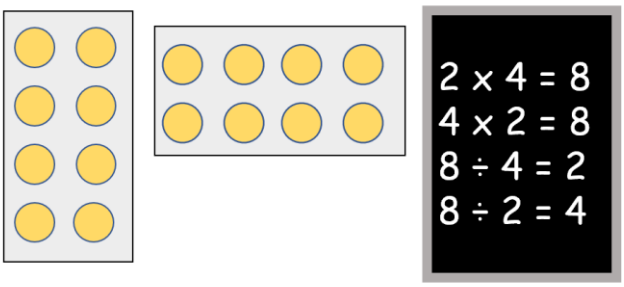
Each group of four to six learners will need:

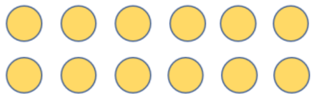
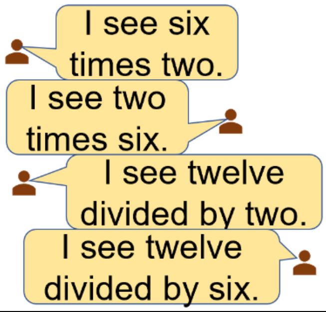
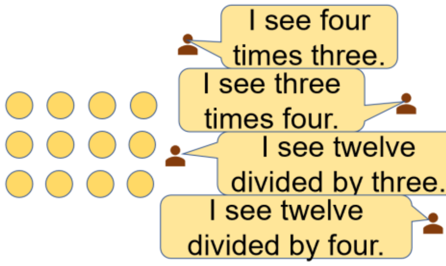
- at least 30 counters.

Each learner will also need:

- an exercise book and pen.

Activity

1. At the beginning of the session, show learners a simple 4 x 2 array on a large sheet of paper or card. Ask them to tell you any multiplication sentences that they see. If learners struggle, you can turn the paper or card round to help them 'see' another sentence. Write the multiplication sentences on the board.	
2. Now ask learners to tell you any division sentences they can 'see'. Write these on the board, reading them out as you write so they understand the meaning of the '÷' symbol.	

3. One learner in each group takes 12 counters and makes them into an array (they could choose 3 x 4, 2 x 6 or 1 x 12).	
4. Learners discuss the multiplication and division sentences that the array represents. They all write the four number sentences in their books.	 6 x 2 = 12 2 x 6 = 12 12 ÷ 2 = 6 12 ÷ 6 = 2
5. Another learner in each group makes a different array with 12 counters and learners repeat step 4.	 4 x 3 = 12 3 x 4 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3
6. When all learners in the group think they have made all the arrays they can using 12 counters, they should take 15 counters and repeat steps 3 to 5.	
Some groups will go further and make arrays using 20, 24 or 30 counters.	

Assessment

As you move around the classroom watching learners working together in groups, notice and note learners who:

- are able to identify the two multiplication sentences, but find it more difficult to see the division sentences shown by an array
- do not yet seem to understand the importance of equal sized groups in multiplication
- are accurate and efficient in finding all four number sentences represented by the array.

In practice

Mrs Sitole usually taught multiplication and division separately. Her learners were usually happy with the idea of multiplication, but found division very puzzling. She discovered that using arrays, and linking the ideas of multiplication and division together, made it much easier for learners to understand the meaning of division. Writing four number sentences for each array reinforced both the meaning of multiplication and division and the links between them. Mrs Sitole feels her learners are now much better prepared to learn about multiplication and division using more formal methods.

2.2: Multiplication facts

Aim

Learners become familiar with multiplication facts through a variety of different activities. These activities help them connect multiplication facts with objects arranged in arrays and pictorial representations such as a number line and multiplication square.

What the learners will do

Learners work in groups of six. They think about an individual 'times table' for two days, then think about a different 'times table' for the next two days.

On the first day of the two days, they spend time making the 'times table' with arrays and jumping along a number line. On the second day, they play 'Switch' and write multiplication facts in a multiplication square.

Resources

Day 1

You will need:

- to set up two activities. If you have a large class, you may need two tables for each activity. Activity 1 will need 100 counters. Activity 2 will need a large number line marked in jumps of the 'times table' of the day.

Day 2

You will need:

- to draw a blank multiplication square on the board (Resource B Blank multiplication square, page 33)
- to set up two activities. If you have a large class, you may need two tables for each activity. Activity 1 will need something to throw (a ball or paper scrunched up in a plastic bag, for example). Activity 2 will need a large multiplication square with the 'times table' of the day completed.

Each learner will also need:

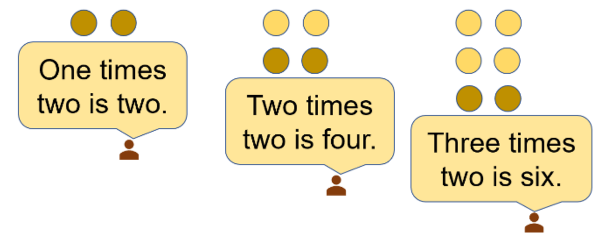
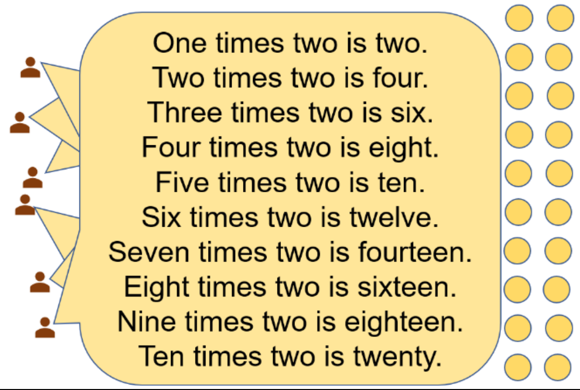
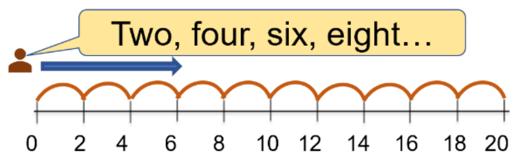
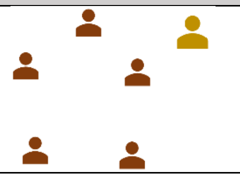
- an exercise book and pen.

Activity

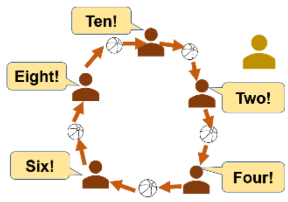
When you follow these activities with your learners for the first time it will take some time to explain what learners will be doing. Demonstrate each activity using the two times table, then ask learners to work on that set of multiplication facts in their groups. In subsequent sessions, focus on tens, then fives. This will help learners to revise known facts while they are getting used to the activities. After this, focus on threes and fours, followed by sevens, eights and nines.

Learners should spend about ten minutes on each activity, then move to the next one. In a 30-minute lesson, this will give five minutes for introducing the task and five minutes for the end-of-session discussion.

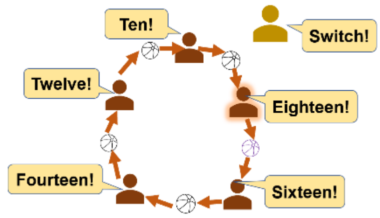
At the beginning of the first session, ask learners to copy the blank multiplication square into their books so they will be ready for the second activity on Day 2.

Day 1 Activity 1	
1. Learners take turns to add rows to an array for the 'times table' they are working with. As they add their row, they say the multiplication fact that the array represents.	
2. When there are ten rows, learners say the multiplication facts altogether for the 'table' with which they are working, pointing to each row as they say each multiplication fact.	
Day 1 Activity 2	
1. Learners take turns to jump along the number line starting from zero saying the numbers as they jump. When all learners have jumped forwards ten jumps from zero, they should take turns to jump forwards ten jumps then back again to zero, saying the numbers as they jump.	
Day 2 Activity 1	
1. One learner is the leader. The rest of the learners stand in a circle.	

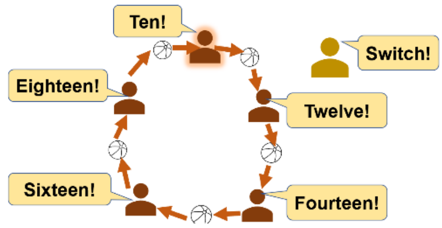
2. Learners throw the ball round the circle to each other, counting in the steps of the times table with which they are working.



3. After the ball has been thrown a few times, the leader calls, *'Switch!'* The next learner to catch the ball begins to count back towards zero.



4. When the leader calls, *'Switch!'* again, the next learner to catch the ball begins to count forwards again. (The leader must make sure they call 'Switch' before the count gets to zero!)



The leader should change after they have called 'Switch' two or three times.

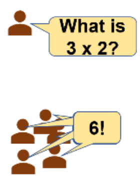
Day 2 Activity 2

1. Learners complete the appropriate row and column of the multiplication square in their books, copying from your large copy if necessary.

x	1	2	3	4	5	6	7	8	9	10
1		2								
2	2	4	6	8	10	12	14	16	18	20
3		6								
4		8								
5		10								
6		12								
7		14								
8		16								
9		18								
10		20								

2. Learners take turns to ask other learners in their group a multiplication fact, for example, '3 x 2'. All the other learners find it on their multiplication square and agree on the answer.

x	1	2	3	4	5	6	7	8	9	10
1		2								
2	2	4	6	8	10	12	14	16	18	20
3		6								
4		8								
5		10								
6		12								
7		14								
8		16								
9		18								
10		20								



Towards the end of each session, gather the class back together. Ask different groups to show what they have been doing and to explain what they have learned.

Assessment

As learners do the activities, notice and note names of learners who are finding the activity difficult or are very quick and accurate. If you find that all the learners in one group are struggling, reorganise the groups so higher-attaining learners can help those who are finding the task more difficult.

You may also consider moving some higher-attaining learners on to more challenging multiplication facts, such as the seven or eight times tables, while allowing other learners more time to practice the less challenging facts (fives, threes and fours, for example).

Occasionally, ask an individual learner questions such as:

- What number would be next if you carried on jumping after the end of the number line? How do you know?
- What number will you say if the leader calls 'Switch' just as the ball gets to you next time?
- What number will you write in this box? (Point to a box further down the column or across the row.)

In practice

Mr Mhlanga noticed that this was a longer activity than usual, so he made sure that he practised it with a colleague before using it with his class. As they worked through the activity together, they could see why the activity was split into two parts. The first session gave learners practice in counting in twos and building an array in twos, so that they were confident in thinking in twos before the second session. The 'Switch' activity provided another opportunity to practice counting in twos. After completing these activities, learners had a good understanding of how numbers work together in 'times tables' and did not find it difficult to complete the multiplication square.

Mr Mhlanga already had a multiplication square on his classroom wall, but hadn't realise how confusing some learners found it!

2.3: Finding patterns

Aim

Learners will spot patterns within and between individual 'times tables'. This will help them check they have remembered a multiplication fact correctly.

What the learners will do

Working in pairs, learners look for patterns of doubles in their multiplication squares. They then go on to look for patterns within columns.

Resources

You will need:

- a completed multiplication square displayed on the wall or written on the board (**Resource C Completed multiplication square**, page 34).

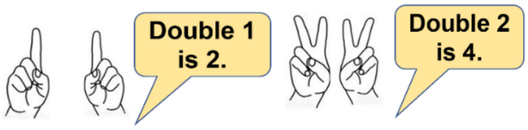
Individual learners will need:

- a completed multiplication square in their books.

Activity

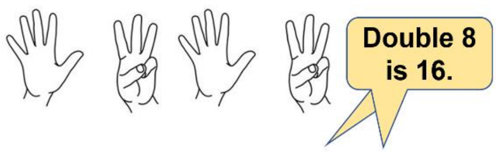
1. Show learners one hand with one finger up. Say, *'One.'* Show a finger on the other hand. Say, *'Double one is two.'*

   Repeat with two fingers on each hand.



2. Ask learners to work in pairs to find double three, double four and double five, then to use both their pairs of hands to find double six, seven, eight, nine and ten.

   Make sure they understand that a number is doubled when it is added to itself.



3. Next, in their pairs, learners compare the twos column and the fours column on their multiplication square. What do they notice? *(That the numbers in the fours column are always double those on the same row in the twos column.)*
   Can they spot any other columns where the numbers are doubles of those on the same row in other columns? *(Fours and eights, threes and sixes, fives and tens)*

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

4. Ask learners to tell their partner what they notice about the numbers in the fives column. *(They all end in 5 or 0)*
5. Ask them to look at other columns to see if they notice any patterns. For example:
   Tens: The numbers always end in 0. The digits in the 'tens' column go up in order (1, 2, 3, 4 etc.).
   Nines: The digits in the 'ones' column count down (9, 8, 7, 6 etc.). The digits in the 'tens' column count up (1, 2, 3, 4 etc. starting with 2 x 9).
   Twos: Numbers end in 2, 4, 6, 8, 0, then repeat the sequence.
   Fours: Numbers end in 4, 8, 2, 6, 0, then repeat the sequence.

Sixes: numbers end in 6, 2, 8, 4, 0, then repeat the sequence.
Some learners may be familiar with odd and even numbers. If so, they may notice that numbers in the two, four, six, eight and ten columns all end in even numbers. They may also notice that numbers in the threes column alternate between odd and even numbers.
Note: The idea is to have fun noticing patterns – learners may not notice any of the patterns above, or they may notice different ones. There is no 'right' or 'wrong' answer, so long as they have noticed a genuine pattern. The main benefit is that they are engaging with the multiplication square.

Towards the end of the session, ask pairs of learners to join two other pairs (six learners altogether) to tell each other about patterns they have spotted.

Finally, ask learners to work individually to play a game of 'thumbs up, thumbs down'. Say, and write on the board, different multiplication facts. Give learners a chance to check them on their multiplication square. If the 'fact' is correct, they put their thumbs up, if it is incorrect, they put their thumbs down.

Assessment

As learners work with their partners, listen carefully to their conversations. Learners who can spot patterns quickly have developed a good sense of number. During the 'thumbs up, thumbs down' game, notice and note learners who 'just know' multiplication facts without checking, and learners who do not yet understand how to find the answer on their multiplication square.

In practice

Miss Simango's class really enjoyed looking for patterns in the multiplication square. She set them the task of finding as many patterns as they could. This gave them the opportunity to practise their mathematical language as they explained the patterns to each other. One group noticed that nearly every product (answer to a multiplication calculation) occurred twice in the table. She used this observation to explain to learners that, since numbers to be multiplied can be written in any order, they have fewer multiplication facts to remember, for example 2 x 3 = 3 x 2 = 6.

Farai noticed that the numbers on the diagonal only occurred once on the table, so Miss Simango took the opportunity to draw these numbers as arrays on the board and to explain that these numbers were called square numbers. It wasn't planned as part of the lesson for that day, but she really enjoyed building a part of the lesson on the observation of a learner. Her class enjoyed it too! She knew this because she spotted them trying to count in square numbers at playtime!

Unit 3: Using mathematical language

Contents

Introduction

This unit focusses on the mathematical language of multiplication and addition. Some learners are able to complete calculations accurately, but are unable to explain what they have done. It is important that all learners can use mathematical language so that they can talk to each other about what they are thinking and doing. This is especially important when a mistake has been made, so that it can be discussed and corrected when learners are working in small groups.

Key words and phrases

- **remainder** – the number of objects that are left if it is not possible for the objects to be divided into equal groups

3.1: Fish ponds

Aim

Through this activity, learners 'see' division as sharing equally, and learn mathematical language to describe what they do. They begin to develop an understanding of remainders.

What the learners will do

One by one, learners turn over a card from each pile to find how many 'ponds' and how many 'fish' they need. They share the 'fish' equally between the 'ponds'. If there are any left over, they identify these as the 'remainder'. They use mathematical language to describe what they have done and write a number sentence.

Resources

Each group of six learners will need:

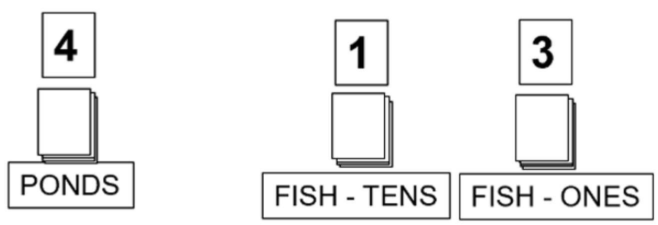
- two sets of 1–9 number cards (**Resource D Number cards**, page 35)
- four sets of 1–3 number cards (**Resource D Number cards**, page 35)
- two sets of 0–9 number cards (**Resource D Number cards**, page 35)
- 'Ponds' and 'Fish' labels (see step 1)
- 40 counters
- six 'ponds' (these can be pieces of scrap paper or card, drawn on the classroom floor with chalk or in the dust outside with a stick).

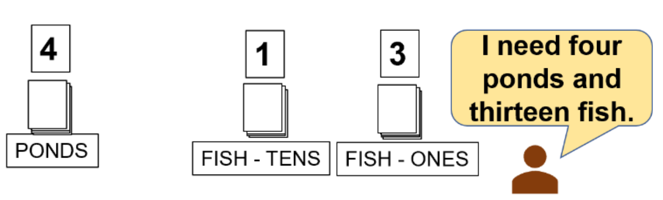
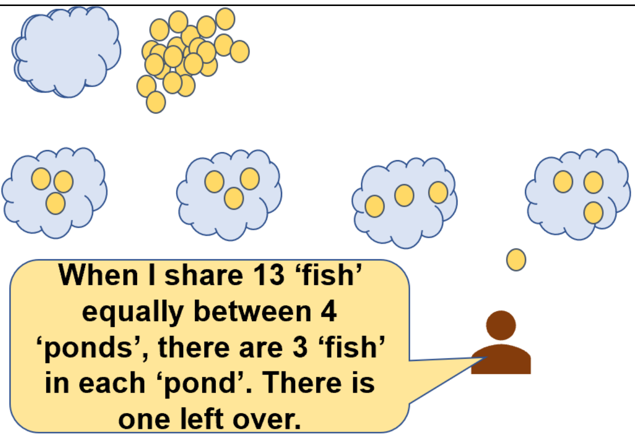
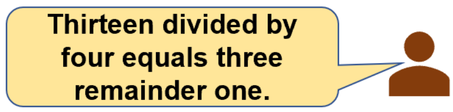
Each learner will also need:

- an exercise book and pen.

Activity

At the beginning of the session, put a set of resources on a table and make sure everyone can see what you are doing. Demonstrate the activity **once** so learners understand what to do. Ask learners to repeat the mathematical phrases as you say them (written in **bold** in the instructions below).

1. Each group places their three piles of number cards between them, putting ponds and fish labels under the correct pile.	

2. When it is their turn, learners turn over one number card from each pile. They say, '**I need __ ponds and __ fish**.'	
3. The learner places the correct number of 'ponds' on the table. They count out the correct number of fish and share them equally between the ponds.	
4. They say, '**When I share __ fish equally between __ ponds, there are __ fish in each pond.**' If there are any 'fish' left over, they say, '**There are __ fish left over.**'	
5. They write the number sentence that shows what they have done and say, '**__ divided by __ equals __**.' If there are any left over, they also say, '**remainder __**'.	$13 \div 4 = 3 \text{ r } 1$ 
6. Learners take turns to share 'fish' between 'ponds', taking care to write the correct division sentence and use correct mathematical language, until everyone has had a go.	

Assessment

Listen carefully to learners as they write and say the division sentences. If you notice that many learners are struggling to use mathematical language, stop the activity and remind the whole class of this language.

In practice

When Miss Ndou read the title of this activity, she wondered what fish ponds had to do with numeracy teaching! She knew what ‘fish’ were, but had to look up ‘ponds’ in her dictionary!

After she had done the activity, she realised that it would help her learners to explore division and remainders in a practical way. She thought that her learners may not understand the meaning of some of the words in English, so she used a combination of English, the learners’ home language and actions to make sure they understood what they had to do. After she had used the activity, if learners couldn’t remember what a ‘remainder’ was, she just had to say, *‘Fish ponds!’* and they had a mental picture of the fish who didn’t get put in the pond!

3.2: Remainder!

Aim

Through this activity, learners ‘see’ division as dividing into equal groups and continue to develop mathematical language to describe what they do. They also further develop their understanding of remainders.

What the learners will do

In their groups, learners organise a handful of counters into groups according to the number on a card that has been turned over. Each learner writes the division number sentence that shows what they have done and reads it to the rest of the group. The group keeps score of the remainders (the number of counters each learner has left after the counters have been divided into equal groups). The learner with the lowest score is the winner.

Resources

Each group of six learners will need:

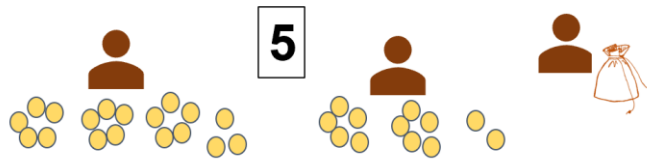
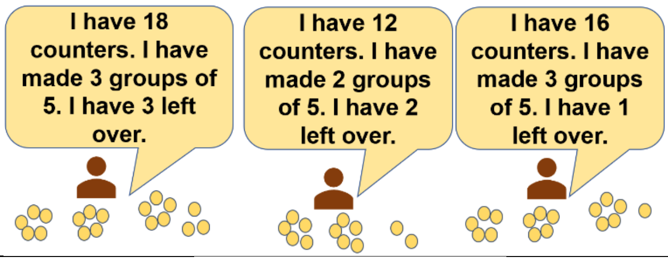
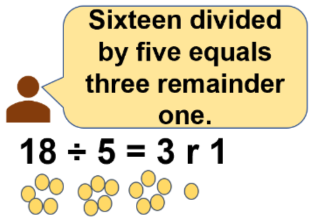
- four sets of 1–5 number cards (**Resource D Number cards**, page 35)
- a bag full of small counters.

Each learner will also need:

- an exercise book and pen.

Activity

At the beginning of the session, put a set of resources on a table and make sure everyone can see what you are doing. Ask two learners to help you demonstrate the activity so learners understand what to do. Ask all learners to repeat the mathematical phrases as they are spoken (written in **bold** in the instructions below).

1. One person in each group of learners turns over a card. Learners will make groups of this size with their counters.	
2. Each learner in the group takes a handful of counters out of the bag quickly without looking – they must not count them before they take them out. Each learner arranges their counters into groups the size of the number on the card.	
3. They say, **‘I have __ counters. I have made __ groups of __.’** If there are any counters left over, they say, **‘I have __ left over.’**	
4. They write the number sentence that shows what they have done and say, **‘__ divided by __ equals __.’** If there are any left over, they also say, **‘remainder __’.**	
5. The player who has the smallest remainder gets a point.	
6. The card is put back into the pile, the counters are returned to the bag and learners repeat steps 1–5 as time allows.	

Towards the end of the session, gather the learners back together. Ask them to work with talk partners to think about what is **the same** about the ‘Fish ponds’ and ‘Remainder!’ activities. *(Both activities are about division and both sometimes have remainders.)*

Ask them to think about what is **different** about the two activities. *(In ‘Fish ponds’, they share out the counters, one for you, one for you, one for you. They count how many are in each pond and how many are left over to find the answer. In ‘Remainder!’ they put counters into groups. They count the number of groups they make and how many are left over to find the answer.)*

Help learners to understand that sometimes a division problem will ask them to share things equally between a number of groups. In this case, the answer will be the **number in each group**. Sometimes it will ask them to put things groups of equal size. In this case, the answer will be the **number of groups**.

Assessment

Notice and note learners who seem to find it hard to connect making groups with division. In the next activity, make sure you spend some time working with these learners.

In practice

Mrs Mlambo enjoyed standing back and 'noticing and noting' what her learners were doing. She found it really interesting to listen to their thinking when they were discussing numeracy. Sometimes they came up with ideas she hadn't thought of, which pleased her a lot. She also realised that some learners had not fully grasped the ideas in the activity.

She thought for a while about how she could give extra help to learners who needed it. Sometimes she asked a group of learners to come in for an extra lesson at lunchtime. They were happy to do this, as they thought of the activities as games. Sometimes their friends asked if they could join in as well. She also made time to help small groups of lower-attaining learners in class, by giving the rest of the class work they could do on their own. She found that this worked well and that her learners appreciated the extra help.

3.3: What's wrong?

Aim

Learners develop reasoning skills as they explain why they think a division sentence does or does not match the array of counters.

What the learners will do

In their groups, learners write a multiplication or division sentence (for example 5 x 3 = 15) and make an array with their counters that matches what they have written. Next, they write another multiplication or division sentence. This time they make an array that is almost – but not quite – correct.

In pairs, learners move from table to table, deciding which of the arrangements of counters that other groups have made match the number sentences and which do not. They explain their thinking and say how they could change the wrong one to make it right.

Resources

You will need:

- to write the following questions on the board:
    - Which is correct?

- How do you know?
- How do you know the other one is not right?
- What can you change to make it right?

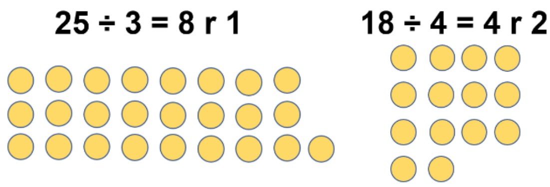
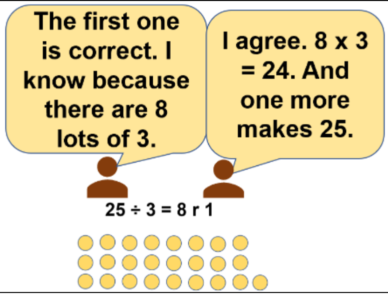
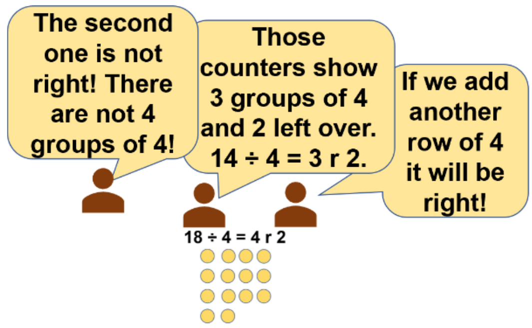
Each group of six learners will need:

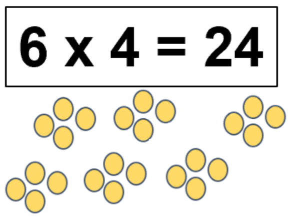
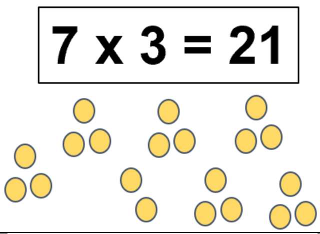
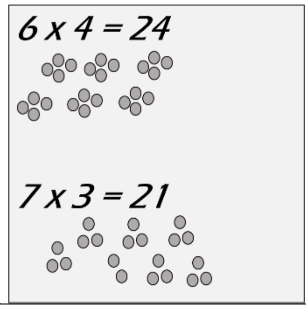
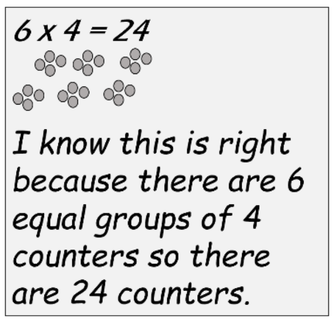
- up to 60 counters
- two sheets of paper.

Each learner will also need:

- an exercise book and pen.

Activity

1. Write 25 ÷ 3 = 8 r 1 on the board. Under it, draw counters to show the calculation. You could use arrays or groups. Now write 18 ÷ 4 = 4 r 2. Under it, draw an arrangement of counters that is almost correct, but not quite (there is one row missing!).	
2. Ask learners to decide, in their groups: ○ which is correct. ○ how they know.	
3. Ask learners to decide, in their groups: ○ how they know the other one is not right ○ what they can change to make it right.	
4. Before learners move on to the next part of the activity, write these 'sentence starters' on the board.	I know this is right because … I know this is not right because … I can make it right by …

5. Learners in each group think of a multiplication or division sentence. They write it on one of their sheets of paper. They arrange their counters **correctly** below it.	
6. Next, the learners write a different multiplication or division sentence on the other sheet of paper. Under it they arrange the counters in a way that is **almost, but not quite correct**.	
7. When all groups have finished steps 5 and 6, learners move to another table. They copy down the number sentences and arrangements of counters on that table into their books.	
8. In pairs, learners talk about the questions on the board. ○ Which is correct? ○ How do you know? ○ How do you know the other one is not right? ○ What can you change to make it right? Learners write their answers in their books, using the ‘sentence starters’ on the board. Then they move on to the next table (if time allows).	

Towards the end of the session, ask learners to return to their own tables. Ask each table in turn to tell the rest of the class which of their displays is right, and what they can do to make the other one right. The other learners check in their books to see if they agree.

Assessment

This is a very valuable assessment activity. It shows you which learners really understand the concepts of multiplication and division. As learners are moving around the tables, listen carefully to their conversations. Notice and note reasons learners give for knowing whether what they see is correct.

For example, a learner may say:

- *'I know that's not right because there are not the right number of counters.'* This shows a low level of reasoning. The learner may be relying on counting in ones.
- *'I know that is not right because the counters are not all in equal groups.'* This shows the learner understands that equal groups are important in multiplication and division.
- *'I know that's not right because 4 x 4 = 16, so they need four groups of four and two left over when they divide 18 by 4.'* This shows that the learner has a good understanding of the relationship between multiplication and division.

You may not have time to notice and note what all your learners are doing during the session. As you look at their books, make a note of the kind of reasoning skills you see in the written answers.

In practice

Mr Sithole's learners were surprised to be asked whether a calculation was right or wrong, and even more surprised to be asked to write an incorrect calculation!

The activity gave both the question-setters and the question-answerers practice in working with division. Learners really enjoyed working on questions that had been written by other learners, instead of by the teacher. They found explaining why something was right or wrong quite challenging, but the sentence starters helped a lot. It took a while to finish the first set of questions, but with practice they worked more quickly.

Mr Sithole had never thought of using an activity where learners set questions for each other and moved around the room. His learners asked him when they can do another lesson like this, so he thinks he will make up another activity like this to use soon.

Reflection

When you have completed this module, and tried out the activities in class, reflect on what you have learned from it. You can do this by yourself, but, if you have the opportunity, it is better to do so with other teachers in your school or cluster. Perhaps you can meet after school or set up a WhatsApp group to work with teachers some distance away.

In this module, learners progress in small steps from not knowing the meaning of multiplication and division to understanding both concepts and constructing their own multiplication square.

Read through the activities again, and make a note of what is being learned in each activity.

- Were any of the ways of working in this module new to you? Why do you think they were included?

- As a result of reading this module, will you make any changes to the way in which you introduce multiplication and division? If so, what will you do differently?

Resources

Resource A Number line 0–30

Resource B Blank multiplication square

X	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

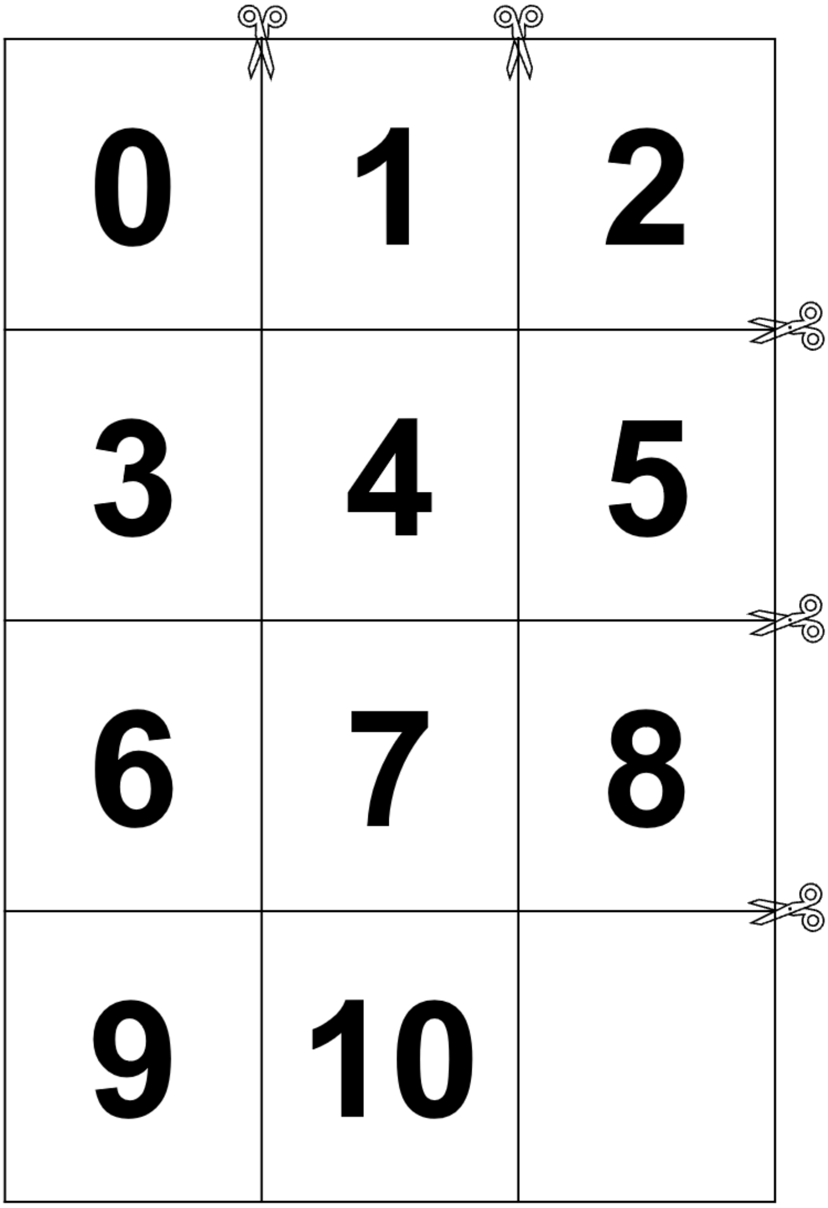
Resource C Completed multiplication square

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

(Shading not necessary, but may help learners follow the rows and columns.)

Resource D Number cards

You can make these cards by cutting a sheet of paper as shown and writing the numbers on the pieces, or you can cut up old cardboard boxes and write the numbers on those. Each card should be about 6 cm x 4 cm.



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