Zimbabwe
Ministry of Primary and Secondary Education

NUMERACY

IGATE
Module 6
Fractions and decimals

iGATE Improving Gender Attitudes, Transition and Education Outcomes
care
World Vision
The Open University
Girls' Education Challenge
UKaid from the British people

For information about the IGATE project see:
www.wvi.org/education-and-life-skills/igate-improving-girls-access-through-transforming-education

This material has been funded by UK aid from the UK government.

Revised Module 6 (MoPSE)

Foundational numeracy

Module 6: Fractions and decimals

Module contents

About these modules 2
Unit 1: What is a fraction? 5
Unit 2: Calculating with fractions 17
Unit 3: Decimal fractions 28
Acknowledgements..43

About these modules

This is the sixth of six Teacher Professional Development (TPD) modules for all teachers working with learners whose attainment in numeracy is below their Grade or Form level. The modules are also appropriate for Initial Teacher Education (ITE) – particularly during school placements or practicum.

Module 1: What is a number?

Module 2: Early addition and subtraction

Module 3: Addition and subtraction of bigger numbers

Module 4: Multiplication and division – part 1

Module 5: Multiplication and division – part 2

Module 6: Fractions and decimals

The modules were collaboratively developed for the Ministry of Primary and Secondary Education (MoPSE) by the Open University, World Vision and CARE International. The modules have been tried and tested in hundreds of primary and secondary schools across Zimbabwe, strengthening the teaching of foundation skills and improving learning outcomes. Our thanks to everyone who contributed – especially teachers, school heads and schools' inspectors.

MoPSE's highest priority is to empower ALL learners through strong foundations in literacy and numeracy.

Whatever their Grade or Form, all learners need strong foundations in literacy and numeracy to succeed in other learning areas. Learners must **learn to read and use number** so they can **read and use number to learn**.

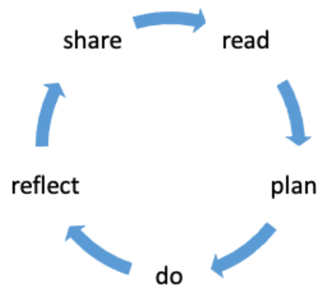
Using the modules

Teachers will benefit most by using the modules within reflective-practice cycles in their schools, as shown below.

Read an activity.
Plan how you will use the activity.
Do the activity with your learners.
Reflect on what learners learned from doing the activity.

- What worked well?
- What would you change next time?

Share your experiences with your colleagues.



- The modules can be used by:
  - individual teachers
  - pairs or groups of teachers
  - whole schools
  - cluster meetings or district workshops.
- The modules provide classroom activities and guidance for effective use.

Learner attainment

We describe learners who can do an activity confidently and successfully as ‘higher attaining’ and learners who cannot do an activity well as ‘lower attaining’.

No one knows what a learner will be able to do given the chance. Every learner has the potential for growth. Teachers have often been surprised when they found a learner who was ‘higher attaining’ for one activity was then ‘lower attaining’ for another – and vice versa. So, we don’t label learners with words like ‘fast’ or ‘slow’.

A learner may have different levels of attainment in different learning areas, or in different aspects of one learning area. That’s why assessment is a big part of the activities. It is important to find out, as often as possible, what learners know and can or can’t do. Then they can be given activities at a level that will help them progress.

Working in groups

Learning takes place as a result of **doing** an activity, **thinking** about it, and **understanding** the ideas it contains.

In order to make sure that all learners are **doing**, activities are designed so learners work together in pairs or small groups for most of the lesson. Pairs, or groups of four to six learners, work best because everyone can take part. Sometimes the teacher will need to demonstrate the activity first.

There are several ways in which learners can be put into groups. Teachers should choose the one that works best for their learners.

- **Learners choose their own groups:** Sometimes this can result in friends working (or not working!) together, while other learners are left out.
- **Learners at a similar level of attainment work together:** This can work well, as learners are working at their preferred pace, but learners who need help have to find it from outside the group.
- **Learners at mixed levels of attainment work together:** This type of grouping has the advantage that higher-attaining learners can help lower-attaining ones. This gives lower-attaining learners personal and prompt support, and higher-attaining learners a chance to talk about what they have learned, which helps to deepen their understanding.

Collecting and storing resources

Many of the activities in these modules rely on learners using physical resources. Some activities need large quantities of resources so that learners can work through the activities in small groups. How can you make sure you have enough?

Here are some suggestions from teachers who trialled these materials.

<u>Sticks</u>

- Ask each learner to bring in at least ten small sticks the length of their middle finger.
- Ask learners to hunt for small sticks in the school grounds during break times. (This worked well in rural communities, but not so well in city schools.)
- As an alternative to sticks, ask learners to bring in at least ten toothpicks or drinking straws.
- Bundle up the sticks into ‘tens’ using elastic bands (if you have them, as these work better for subtraction) or short lengths of wool or string

<u>Counters</u>

- Ask each learner to bring in a collection of counters. In rural communities, learners usually brought in small stones, beans or seeds. In city schools, learners often brought in plastic beads or bottle tops.

<u>Place value counters</u>

- Ask local bottle stores, lodges and hotels to collect tops from drink bottles. Write the value of the counter on the top with a permanent marker.
- Cut up manila or paper into 2-cm squares and write the value of the counter on each piece.

<u>Making resources together</u>

- Some teachers organised a weekly after-school resource-making meeting to make sure all classes had the equipment they needed. This was especially helpful when making number cards or arrow cards!

<u>Sharing resources</u>

- Share a set of resources between two classes. So, for example, while one class has a literacy lesson, the other class does numeracy and vice versa. The teachers quickly swop resources between lessons!
- Split the class into two halves. Work with one half of the class on activities that use resources, while the other half work in their books.

<u>Storing resources</u>

- Teachers often reused the same resources many times, for lots of different activities. They realised that they would save a lot of time by storing resources carefully, either in their classroom or a common store-cupboard.

Unit 1: What is a fraction?

Contents

Introduction 5
1.1: Fractions of shapes...6
1.2: Fractions as part of a set 9
1.3: Fractions as numbers 12

Introduction

In this unit, learners are introduced to the idea of fractions. In Unit 1, they work with shapes, are introduced to how fractions are written, and understand the meaning of ‘numerator’ and ‘denominator’. In Unit 2, they progress to finding fractions of a group of counters. Unit 3 introduces learners to the idea of fractions as numbers, and to the relative sizes of fractions, whole numbers and mixed numbers.

Key words and phrases

- **denominator** – the bottom number of a fraction, which shows how many equal parts the whole is divided into
- **equivalent** – the same size as
- **mixed number** – a number consisting of a whole number and a fraction
- **numerator** – the top number of a fraction, which shows how many equal parts of the whole are being considered

Unit 1: What is a fraction?

1.1: Fractions of shapes

Aim

This activity helps learners understand:

- the role of the top part (numerator) and bottom part (denominator) of a written fraction
- that fractions are parts of a whole
- that all the parts of the whole are equal
- the relative size of fractions
- that some fractions are equivalent to others.

What the learners will do

Each learner folds a sheet of paper first in half, then into quarters and finally eighths. With their talk partner, they use their sheet of paper to discover some equivalent fractions.

Resources

You will need:

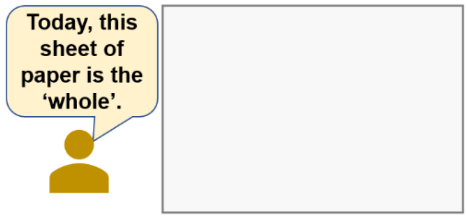
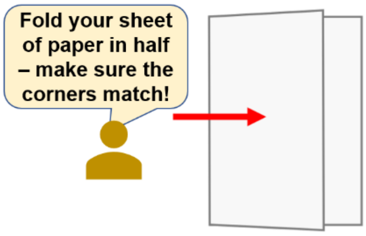
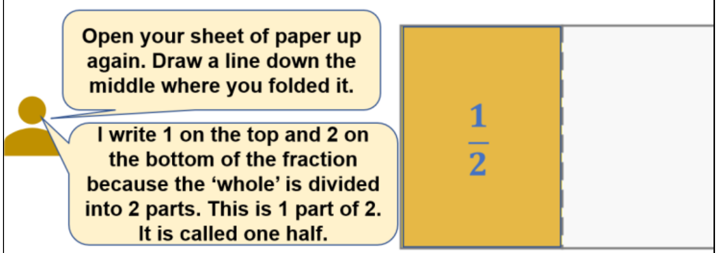
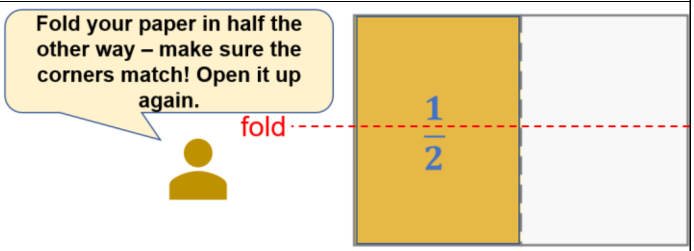
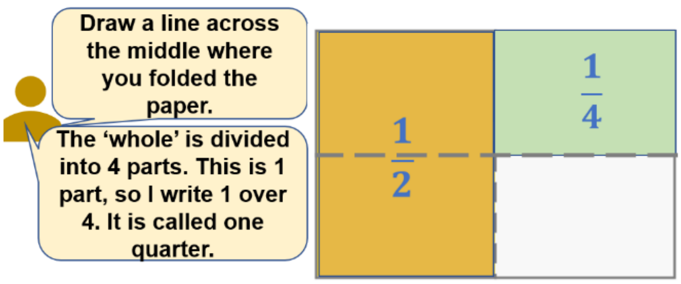
- a sheet of paper, A4 or larger
- a marker pen
- a ruler.

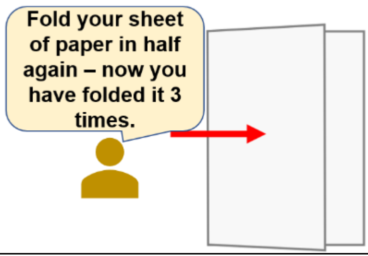
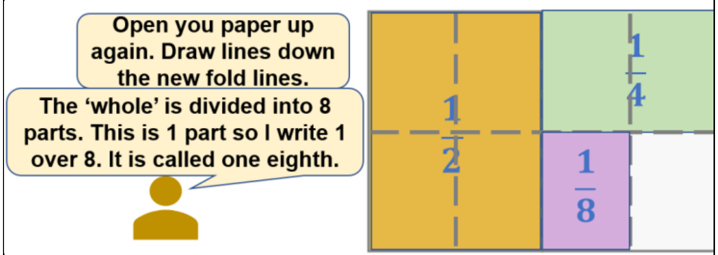
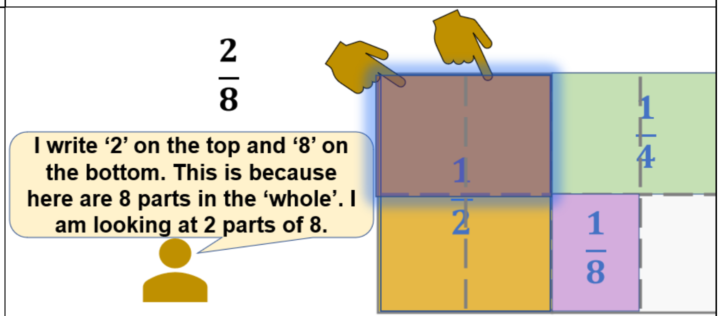
Individual learners will need:

- a sheet of A4 paper
- a pen or pencil
- a ruler (this can be shared between learners if necessary).

Activity

The first eight steps of the activity are teacher led, but learners are actively involved in folding and writing on their paper. You will need to read the left-hand side as well as the right-hand side of the instructions, as questions addressed to learners are not shown in the diagrams. In step 9, learners work with talk partners.

1. Explain that fractions are parts of a 'whole'. Over the next few sessions, they will see the 'whole' as different things, but today the piece of paper they have in front of them is the 'whole'.	
2. **Ask learners to copy you as you demonstrate each of the next steps**. First, fold your paper in half and ask learners to do the same.	
3. Now open up your sheet. Draw a line down the fold line. Ask, *'How many parts is my 'whole' divided into?'* (2) Point to one section. Ask, *'How many parts are here?'* (1) Say, *'I write '1' on the top and '2' on the bottom of the fraction because the 'whole' is divided into 2 parts. This is 1 part of 2 parts. It is called 'one half'.'*	
4. Fold your paper in half the other way and ask learners to do the same. Open up your paper.	
5. Draw a line across the new fold line. Ask, *'How many parts is my 'whole' divided into?'* (4) Point to one section. Ask, *'How many parts are here?'* (1) Say, *'I write '1' on the top and '4' on the bottom of the fraction because the 'whole' is divided into 4 parts. This*	

is 1 part of 4 parts. It is called ‘one quarter’.’	
6. Fold your paper along the fold lines you have already made, then fold it again. It will now have been folded three times.	
7. Open up your sheet. Draw a line across the new fold lines. Ask, *‘How many parts is my ‘whole’ divided into?’* (8) Point to one section. Ask, *‘How many parts are here?’* (1) Say, *‘I write ‘1’ on the top and ‘8’ on the bottom of the fraction because the ‘whole’ is divided into 8 parts. This is 1 part of 8 parts. It is called ‘one eighth’.’*	
8. Point to two ‘eighth’ sections. Ask, *‘How many parts am I pointing to?’* (2). Ask, *‘How many equal parts are in the ‘whole’?’* (8). As you write $\frac{2}{8}$ on the board, say, *‘I write ‘2’ on the top and 8 on the bottom. This is because there are 8 parts in the ‘whole’. I am looking at 2 parts of 8 parts.’*	

9. Ask learners to work with their talk partner, each using their folded paper. Ask each question below. After each question, leave at least a minute for learners to discuss their answers, then choose two or three pairs to share with the whole class.
   - *What fraction is smaller than* $\frac{1}{4}$? (Answer $\frac{1}{8}$)
   - *What fraction is bigger than* $\frac{1}{4}$? (Answer $\frac{1}{2}$)
   - *What fraction is equivalent to* $\frac{1}{4}$? (Answer $\frac{2}{8}$)
   - *Can you write two fractions that are equivalent to* $\frac{1}{2}$? (Answer $\frac{2}{4}$, $\frac{4}{8}$)

- *Find three fractions that are bigger than* $\frac{1}{8}$. (e.g. $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$)
  How do you know?
- *Find three fractions that are smaller than* $\frac{1}{2}$. (e.g. $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$)
- *What is the biggest fraction you have written on the folded paper?* (Answer $\frac{1}{2}$)
- *Which is the smallest?* (Answer $\frac{1}{8}$)
- *How do you know which is the biggest and which is the smallest?*

Assessment

Walk around the room as learners fold their paper to make sure they are folding the paper and writing the fractions correctly.

During step 9, notice and note learners' answers to the questions. In particular, pay careful attention to the *'How do you know?'* parts of the questions. These will tell you whether learners understand the role of the numerator and the denominator, and whether they understand that, as the fraction gets smaller, the denominator gets bigger.

In practice

This was the first time that Mrs Matsika had introduced fractions using a paper-folding activity. As she has a large class and not much paper, she used scrap paper, which worked well. Because each learner was folding their own piece of paper, she found that they really understood the meaning of the numbers above and below the line. This understanding will help when fraction addition and subtraction are introduced.

1.2: Fractions as part of a set

Aim

This activity helps learners to make connections between:

- fractions of shapes and fractions of quantities
- arrays and fractions
- division and fractions.

What the learners will do

Learners work first in groups of six under the direction of the teacher, sharing counters equally onto paper divided into halves and quarters. Next, they work together, making arrays to help them to answer questions from the board.

Resources

You will need:

- to write the two lists in *Figure 1* on the board.

Each group of six learners will need:

- 25 counters
- chalk (if drawing on the classroom floor) or paper and pens.

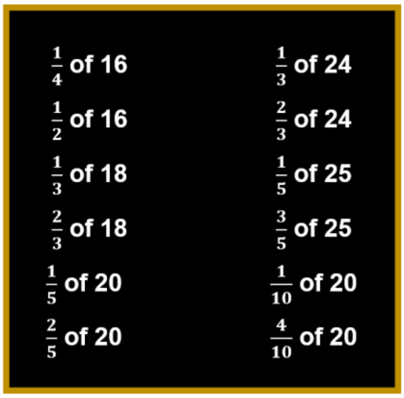
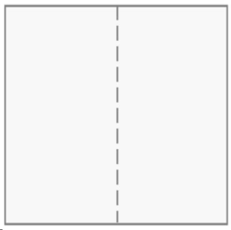
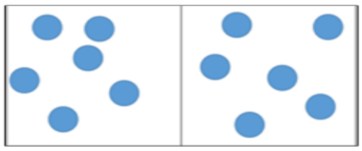


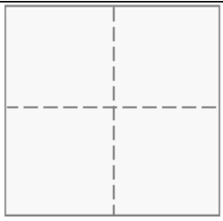
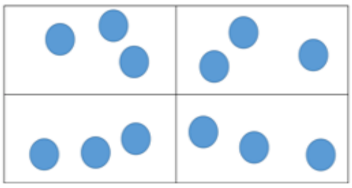
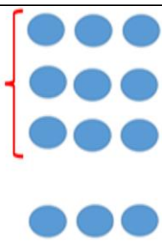
Figure 1

Activity

Although the first part of this session is teacher-led, each learner in every group has a role. This ensures that all learners are taking part in the activity.

Explain that in the previous activity their piece of paper was the 'whole' and the sections were the parts. In this activity, they are going to look at the 'whole' in two different ways. As a square that they draw, and as a quantity of counters.

1. After grouping your class into sixes, ask them to number themselves from 1–6. If a group has fewer than six, some learners can have two numbers.	
2. To begin, ask Learner 1 to draw a large square on the floor (or on paper). Ask them to draw a line to divide the square into halves. (Remind them that each part must be equal in size.)	
3. Ask Learner 2 to count out 12 counters. Say this is their 'whole'. Now ask learners, in their group, to discuss how they can use their square and counters to find $\frac{1}{2}$ of 12. Remind them that to find half they need to divide the 'whole' (12) into two equal parts, then say how many are in one part.	
4. Once the group has agreed, Learner 3 shares the counters equally between the two halves of the square they have drawn.	
5. Ask learners to discuss how to write a number sentence that says what their arrangement of counters on the square shows. Once the group has agreed, Learner 4 writes the number sentence.	$\frac{1}{2}$ of 12 = 6

6. Ask Learner 1 to draw another line on the square to divide it into quarters. Ask learners to discuss how they can use the square and their counters to find $\frac{1}{4}$ of 12. Remind them that this means they will divide the 'whole' (12) into four equal parts, then say how many counters are in one part.	
7. After the group has discussed the problem, Learner 5 moves the counters so they are divided equally between the four sections of the square. Learner 6 writes the number sentence.	 $\frac{1}{4}$ of 12 = 3
8. Ask learners to take the counters off the square and arrange them in an array of four rows. Help them to see that they can find $\frac{1}{4}$ of 12 by sharing 12 equally between 4 rows.	$\frac{1}{4}$ of 12 = 3 
9. Ask learners how they would use the array to find $\frac{3}{4}$ of 12. Remind them that there are four parts in the 'whole' and they want to find out how many counters are in three parts.	$\frac{3}{4}$ of 12 = 9 

10. Ask each group of learners to choose a list of fraction problems from the board. Say that the second list is more challenging than the first. They should choose the list of problems that they think they will be able to solve.

Note: Learners take different roles in the group:

- Learner 1 takes the correct number of counters, says how many parts the whole is divided into, names the fraction and makes an array.
- Learner 2 says how many counters are in each part.
- Learner 3 says how many parts are needed and moves that many 'parts' away from the rest of the array.
- Learner 4 gives the answer.
- Learners 5 and 6 say if they agree or disagree and why.

Learners change roles for each new problem.

Towards the end of the session, give learners some 'true' or 'false' fractions statements, such as $\frac{1}{10}$ of 30 = 5 *(false)*, and $\frac{2}{3}$ of 18 = 12 *(true)*. Ask learners to decide in their groups whether the statement is true or false. Choose different groups to answer and to explain their reasons.

Assessment

Listen carefully to learners as they discuss the fraction problems together. Make sure you visit groups that include any learners who, during the previous activity, seemed unsure of what a fraction was.

During the 'true' or 'false' activity, make a note of learners who give good explanations, and also those who do not seem to understand.

In practice

Mr Dhliwayo found paper-folding a really useful way to teach learners about fractions. When he read through this activity, he thought that it might be useful to check that learners could fold their papers into the fractions in *Figure 1*.

He asked his learners to fold a piece of paper into halves and then quarters. Most learners repeated what they had done in **Activity 1.1**, but some learners folded their paper along the diagonals. This resulted in an interesting class discussion on whether both sheets of paper had been folded into quarters. Eventually the class decided that any way of folding was acceptable, as long as all the parts of the whole were the same size.

1.3: Fractions as numbers

Aim

This activity helps learners to see fractions as:

- a way to describe numbers between whole numbers
- numbers 'in their own right'.

It also develops learners' understanding of equivalent fractions.

What the learners will do

Learners begin by counting in steps of a half, then a quarter, connecting this with a number line. Next, they work in groups of four to construct their own group fraction chart, using it to compare fractions.

Resources

You will need:

- to draw a long 0–10 number line, either on the classroom floor, outside in the dust or on a long sheet of paper, marked with a blank graduation halfway between each number (**Resource A Fraction number line**, page 40)
- to cut five pieces of paper into equal-sized circles, then cut these in half, so there are ten halves or semicircles (see *Figure 2*)
- scissors to cut halves into quarters.

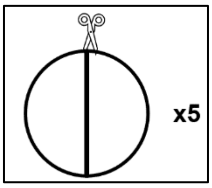


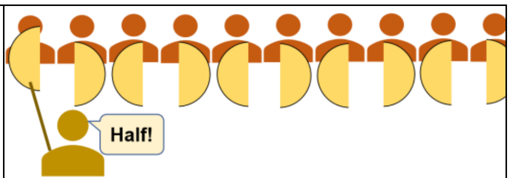
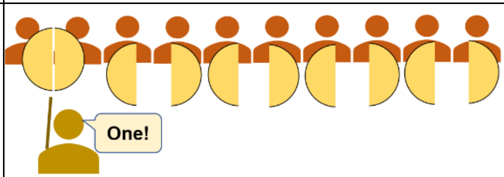
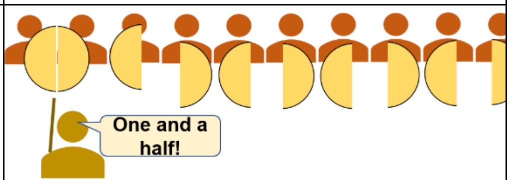
Figure 2

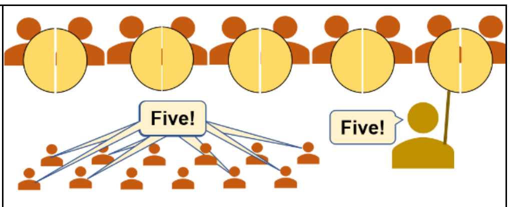
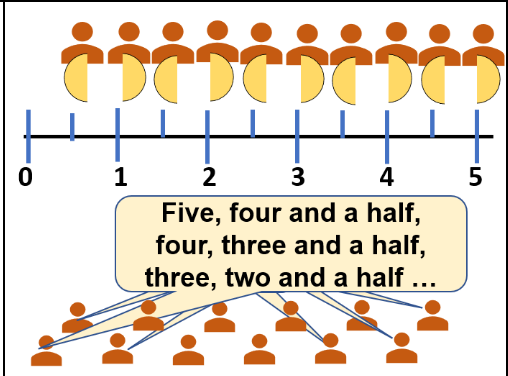
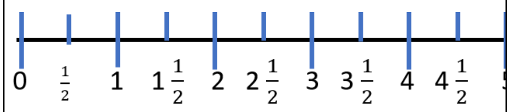
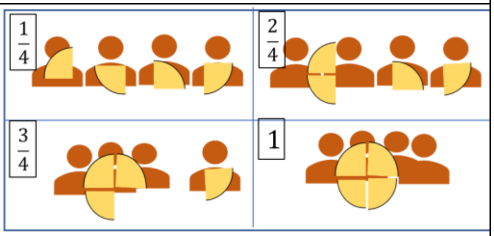
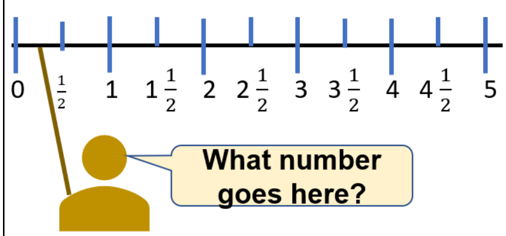
Each group of four learners will need:

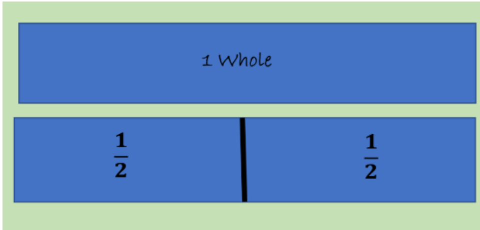
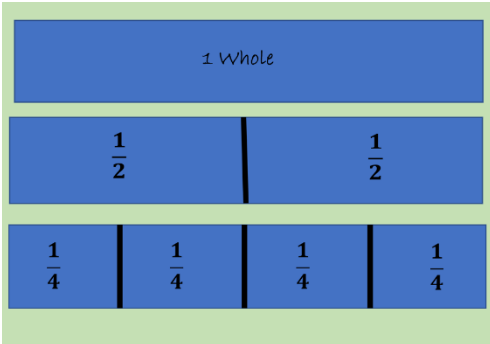
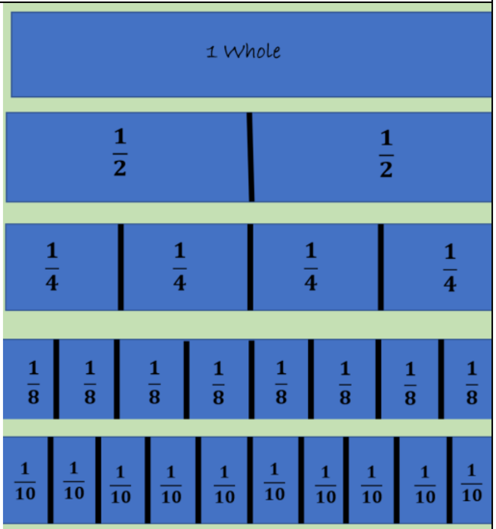
- five strips of paper (20 cm x approximately 2 cm)
- rulers
- pencils or pens
- glue
- a sheet of paper or card approximately A4 size.

Activity

The first half of this session is teacher-led. Take care to go through it at the pace of your learners. You may find you spend a long time on one part or another because you notice that many of your learners are finding the concept difficult. In this case, you may find it best to do the counting in one session and the fractions chart in a follow-up session.

1. Give the half circles that you have prepared to ten learners. Point to the first learner as they hold up their half and say: *'Half!'*	
2. When the next learner holds up their half, get the two to join their halves together to make a whole circle. Say: *'One!'*	
3. When the third learner holds up their half circle, say: *'One and a half!'*	

4. Ask the class to join with you in counting in halves to five. Each time a 'whole' is made, the two learners put their halves together.	
5. Ask the ten learners to stand where they think their number belongs on the number line. Ask the class to count back in halves from 5 to 0. Ask learners to give the shapes back and sit down.	
6. Write the 'half' numbers on the blank graduations of your number line to show that these are numbers that are between whole numbers.	
7. Cut two halves into quarters and ask four learners to hold them. Count in quarters with the learners. Ask them to join their quarters together as their number is counted. Ask what they notice about $\frac{2}{4}$. *(It is the same as $\frac{1}{2}$.)*	
8. Point halfway between 0 and $\frac{1}{2}$ on the number line. Ask learners to discuss, in pairs, what number they think belongs there. Then ask what number they think goes between $\frac{1}{2}$ and 1.	
Organise learners into mixed-attainment groups of five. Give each learner a number (Learner 5 will have the greatest challenge). Say that all learners work on their task at the same time.	
9. On the first strip of paper, Learner 1 writes '1 whole'. They stick it in the middle at the top of their large sheet of paper or card.	

10. Learner 2 folds their strip of paper carefully in two and draws a line on the fold. They write '$\frac{1}{2}$' in each section and stick it below the 'one whole' strip, lining it up carefully.	
11. Learner 3 folds their strip of paper carefully in half, then in half again, drawing lines along each of the folds with their ruler. They write '$\frac{1}{4}$' in each section and stick it on the chart, lining it up carefully.	
12. Learner 4 folds their strip into eighths, draws lines and writes '$\frac{1}{8}$' in each section. They stick it on the chart, lining it up carefully.	
13. Learner 5 measures 2 cm sections carefully along the top and bottom of the long side of their strip. They draw lines from top to bottom to join each pair of marks and write '$\frac{1}{10}$' in each section. They stick their strip below the 'eighths' strip.	
Ask learners to use their fraction chart to find as many equivalent fractions as they can.	

Assessment

Listen out for learners who use what they have learned in the first two activities in this unit to help them to answer questions. Asking learners, *'How do you know?'* will often tell you whether they are making connections with previous learning.

In practice

Mr Nyanthi spread this activity over two lessons, as his learners wanted to count forwards and backwards in fractions other than halves. They tried quarters and tenths, and finished the lesson with a game of 'Fraction Switch'. He was quite happy to do this, as it gave his learners lots of practice in saying fraction names and looking for patterns on the fraction number lines.

After all this practice, his class found making and using a fraction chart very straightforward and had no difficulty in spotting equivalent fractions and talking about them. He was very pleased that the extra time he had given to the activity had resulted in so much learning!

Unit 2: Calculating with fractions

Contents

Introduction 17
2.1: Bigger or smaller? 18
2.2: Adding fractions 21
2.3: Subtracting fractions 24

Introduction

In this unit, learners begin by consolidating their understanding of fractions of a set of objects. They then progress to learning how to add and subtract fractions.

As always, it is important to do the activities at the pace of the learners. Some learners find the idea of a fraction difficult to understand. They need time to get to know the nature of fractions before beginning to calculate with them.

Key words and phrases

- **exchange** – substitute two or more fractions of an equivalent value for a single fraction, or the opposite
- **non-unit fraction** – a fraction with more than one part, for example $\frac{2}{3}$

Unit 2: Calculating with fractions

2.1: Bigger or smaller?

Aim

This activity prepares learners for adding and subtracting fractions by:

- reinforcing the connections between fractions of shapes and fractions of number
- making the link between fractions and division
- providing more experience of comparing fractions.

It also reminds learners of the use of the symbols **<** and **>**.

What the learners will do

Learners work in groups of six. They draw a 3 x 2 grid and place different quantities of counters in each section to help them find fractions of amounts. They then predict which of two fractions of numbers is bigger, before using a 3 x 2 or 2 x 2 grid to find the answer. Each group works as a team and keeps a score of their correct answers.

Resources

Each team of six learners will need:

- somewhere to draw a 3 x 2, then a 2 x 2 grid (this could be on the classroom floor with chalk, outside in the dust or on a large sheet of paper)
- 24 counters.

Each learner will also need:

- an exercise book and pen.

Activity

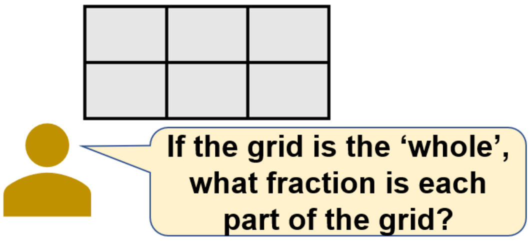
Before you begin this activity, remind learners of the meaning of the **<** and **>** symbols.

This activity is teacher led. After you have used one example to show the class how to do the activity, with learners following your instructions, learners work together in groups of six to answer the questions you give them – trying to gain the most points for their team.

1. Each team of six learners draws a 3 x 2 grid.

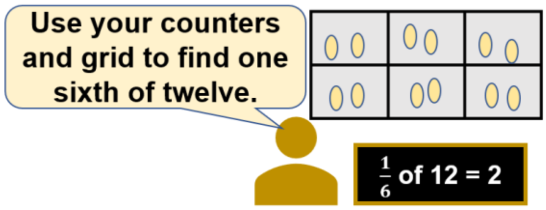
   Ask, 'If the grid is the 'whole', what fraction of the grid is each part?'

   ($\frac{1}{6}$)

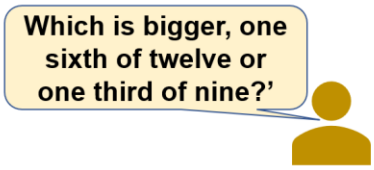


2. Say, *'Use your counters to find one sixth of twelve.'* Learners count out 12 counters and use the grid to find one sixth of twelve.

   Write $\frac{1}{6}$ **of 12 = 2** on the board.



3. Ask**, '***Which is bigger, one sixth of twelve or one third of nine?'* Learners agree on an answer in their teams.



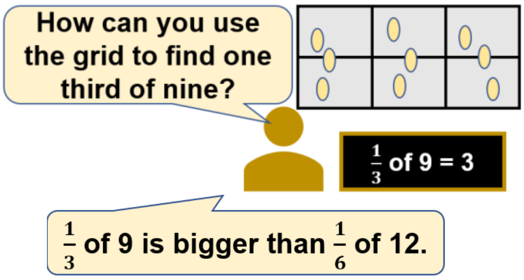
4. Ask how learners can use their grid to find the answer. *(Each column is one third of the whole.)*

   Learners share out nine counters between the three columns.

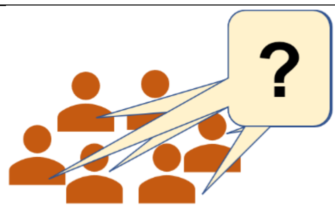
   Write $\frac{1}{3}$ of 9 = 3 on the board.

   Say, *'$\frac{1}{3}$ of 9 is bigger than $\frac{1}{6}$ of 12.'*

   Any team with the correct answer scores a point.

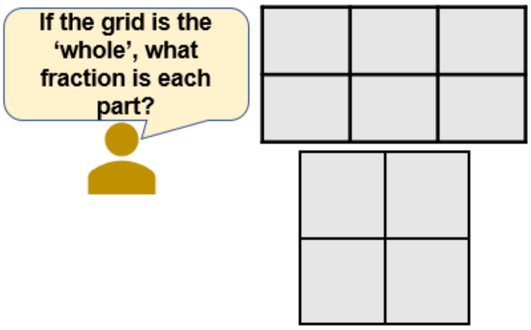


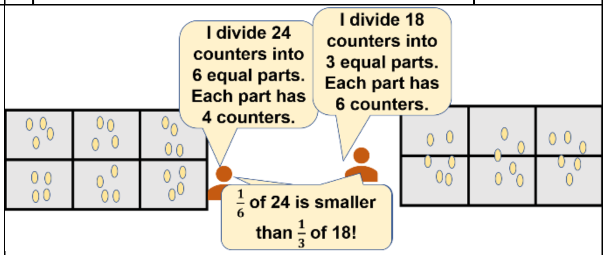
5. Learners discuss and agree whether to use < or > between the two number statements. All learners write '$\frac{1}{3}$ of 9 **>** $\frac{1}{6}$ of 12 ' in their books.



6. Ask each team to draw a 2 x 2 grid next to or under the 3 x 2 grid. Ask, *'If the grid is the 'whole, what fraction is each part?'* ($\frac{1}{4}$)

   Tell the teams that can use either grid to find the answers to the questions you are about to ask them.



7. Read out the pairs of fractions of numbers (in the box opposite) one at a time, saying *'Which is bigger, __ of __ OR __ of __?'* Learners write both questions in their books with a space between them. Together, they discuss whether they think < or > is going to go between them. All learners write the symbol that they think is correct in the space.	$\frac{1}{6}$ **of 24** or $\frac{1}{3}$ **of 18** $\frac{1}{4}$ **of 16** or $\frac{1}{2}$ **of 10** $\frac{1}{3}$ **of 18** or $\frac{1}{2}$ **of 14** $\frac{1}{2}$ **of 22** or $\frac{3}{4}$ **of 16** $\frac{2}{3}$ **of 18** or $\frac{2}{6}$ **of 24** $\frac{3}{4}$ **of 20** or $\frac{2}{3}$ **of 24**
8. Make sure that learners in each team **take turns** to use their grids to find out if their prediction is correct after the team has decided on each answer. If the team predicted correctly, they get a point. If they did not predict correctly, they write the correct answer in their books below their original answer.	
The team with the most points is the winner.	

Assessment

This activity provides rich opportunities for assessing your learners.

While learners are discussing questions together, listening to their discussions will help you see how each of them are thinking, and will help you plan future lessons.

Notice and note the strategies learners use to predict the answer.

- Do they make a connection with division, saying something like, *'You divide 24 by six.'*? If so, ask them to explain their thinking to the rest of the class, as this is an efficient strategy.
- Do they imagine putting counters on a grid, or putting counters into groups? This is a key strategy and shows the learner is making mathematical connections.

Watch as learners find the answers.

- Do they copy other learners in the team or wait for others to tell them what to do? These learners will benefit from extra small group work with you.
- Do they share counters equally between the sections on the shape? They are beginning to make a link with division, which should be encouraged.
- Do they quickly sort the counters as though they 'just know' the answer? These learners may well be ready to find the answer without counters.

In practice

When Miss Zhou read this activity through, she thought that there was nothing new in it. Then when she read the aim of the activity again, she realised that the activity made new connections between ideas that learners had previously encountered. She thought this was useful, as learners did not always make these connections for themselves.

She particularly liked step 7, as she found learners were sometimes puzzled by facts such as one quarter of 24 is bigger than one half of 8. They know that one half is a greater part of a whole than one quarter, but forget that 'wholes' are different sizes!

This part of the activity gave the class a lot to talk about and cleared up some of their misconceptions.

2.2: Adding fractions

Aim

This activity introduces learners to addition of:

- fractions with the same denominator
- non-unit fractions
- mixed numbers
- fractions with denominators that are multiples of the same number.

What the learners will do

Learners work in groups of six. They choose fraction cards and put them together to find the answer to questions involving addition of fractions. When necessary, they exchange smaller fraction cards for an equivalent larger fraction card or vice versa.

Resources

You will need:

- a set of fraction cards as follows (use the folded paper you used in **Unit 1 or Resource B Paper folding**, page 40, to make sure the cards are the correct size)
    - 1 x 1 (whole)
    - 2 x $\frac{1}{2}$
    - 4 x $\frac{1}{4}$
    - 8 x $\frac{1}{8}$
- something with which to stick fraction cards on the board

- a collection of extra fraction cards for learners to use as needed.

Each group of six learners will need:

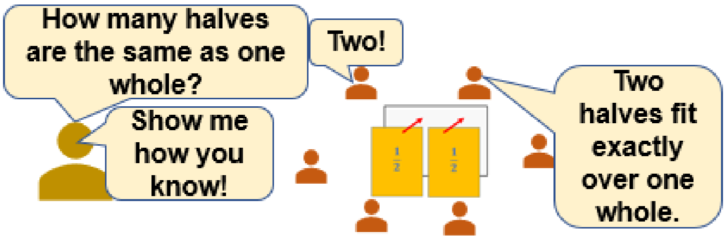
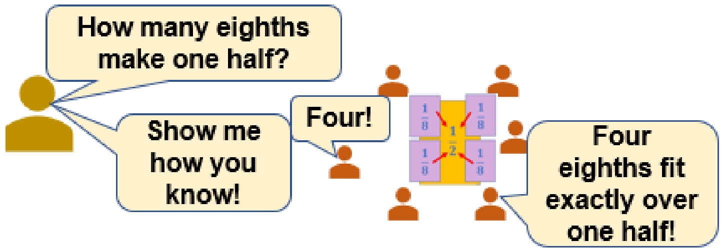
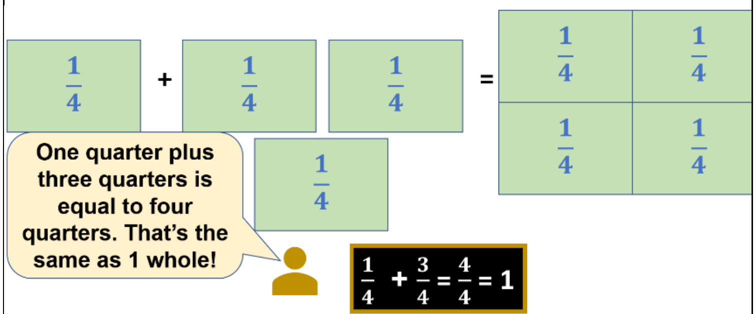
- a set of fraction cards, as above.

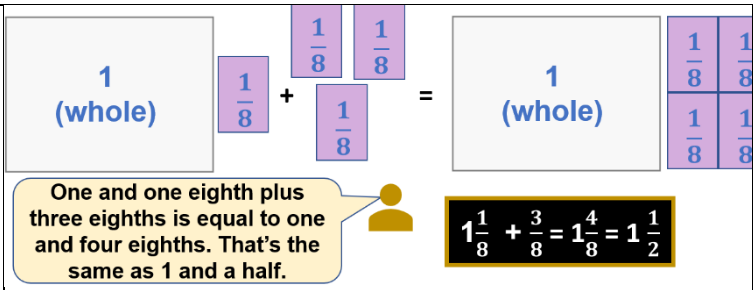
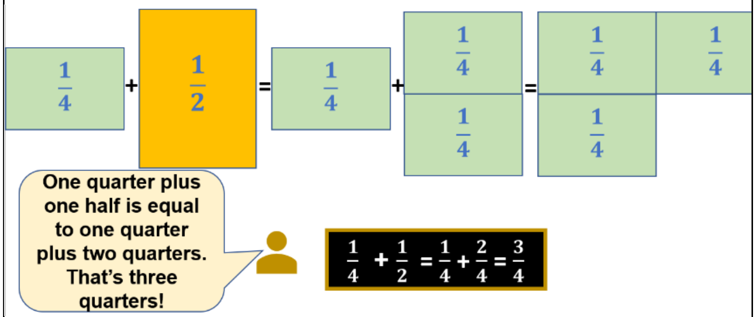
Each learner will also need:

- an exercise book and pen.

Activity

The first part of the session is teacher led. Make sure all learners are involved by telling the groups of six that they must all have a turn using the fraction cards on their table.

1. Ask, *'How many halves are the same as one whole?'* and choose a group to answer. Ask them to show how they know (and the other groups to check the answer) by putting two 'half' cards on top of the 'whole' card. Demonstrate using your set of cards.	
2. Ask other questions such as, *'How many quarters are the same as one whole?'* and, *'How many eighths are the same as one half?'* until all learners in the group have used the fraction cards.	
3. Write $\frac{1}{4} + \frac{3}{4}$ on the board. Ask learners to think, in their groups, about how they can use their cards to help them find the answer. Choose a group that is using the cards correctly to demonstrate. Complete the fraction number sentence on the board.	

<table>
<tr>
<td>4. Now ask groups to use their cards to find the answer to $1\frac{1}{8} + \frac{3}{8}$. Choose a group of learners who have solved the problem correctly to demonstrate. Complete the fraction number sentence on the board.</td>
<td>
</td>
</tr>
<tr>
<td>5. Finally, ask learners to see if they can use what they know along with their fraction cards to find the answer to $\frac{1}{4} + \frac{1}{2}$. Choose a group of learners who have solved the problem correctly to demonstrate. Complete the fraction number sentence on the board.</td>
<td>
</td>
</tr>
<tr>
<td colspan="2">6. Within their groups of six, learners now work in pairs.
○ One pair makes up a fraction addition question that uses halves, quarters, eighths and/or whole numbers.
○ The second pair uses the fraction cards to find the answer.
○ The third pair checks the answer.
○ All learners write the fraction number sentence in their books.
○ Learners change roles after each question.
Tell learners that if they need more fraction cards, they can help themselves from the pile on your desk.</td>
</tr>
</table>

Towards the end of the session, write on the board:

$$\frac{1}{2} + \frac{1}{2} = \frac{2}{4} = \frac{1}{2} \qquad \text{and} \qquad \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$$

Say that two learners in another school wrote these number sentences. One is right, one is wrong. Ask learners, in pairs, to decide which is the right answer and to think about what mistake the other learner has made.

Assessment

The activity at the end of the session shows a common misconception. The first learner has not understood the role of the denominator and the numerator and has just added the top numbers and added the bottom numbers. Make a note of learners who make the same mistake. Ask them to work with fraction cards using fractions with the same denominator a little longer, then gradually introduce them to mixed numbers. Only introduce fractions with denominators that are multiples of the same number when you are sure they understand fractions with the same denominator.

In practice

Mr Tambo noticed that, in this module, fractions were represented as parts of circles, squares or rectangles. He thought it was a good idea to show learners that many shapes can be divided into fractions. In **Activity 1.3**, where circles were used, it was easy for learners to see how many of each fraction made a whole because learners could see that they made a complete circle. Circles are less useful for fractions with larger denominators, as it is difficult to cut them up accurately.

Mr Tambo's learners enjoyed learning to add fractions using the rectangular cards. Even if they found the language of fractions difficult, they could choose the cards they needed and move them around on the table until they had answered the question.

When learners were confident that they had answered the question correctly, Mr Tambo encouraged them to talk about what they had done. He knew it was important that learners took every opportunity to practise their use of mathematical language.

2.3: Subtracting fractions

Aim

This activity introduces learners to subtraction of:

- fractions with the same denominator
- non-unit fractions
- mixed numbers
- fractions with denominators that are multiples of the same number.

What the learners will do

Learners work in groups of six. They choose fraction cards and put them together to find the answer to questions involving subtraction of fractions. When necessary, they exchange smaller fraction cards for an equivalent larger fraction card or vice versa.

Resources

You will need:

- a set of fraction cards as follows (use the folded paper you used in Unit 1 or Resource B Paper folding, page 40, to make sure they are the correct size)

  - 1 x 1 (whole)
  - 2 x $\frac{1}{2}$
  - 4 x $\frac{1}{4}$
  - 8 x $\frac{1}{8}$
- something with which to stick fraction cards on the board
- to write the questions in *Figure 1* on the board
- a collection of extra fraction cards for learners to use as needed.

Each group of six learners will need:

- a set of fraction cards, as above.

Each learner will also need:

- an exercise book and pen.

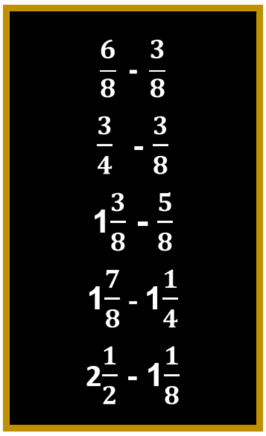
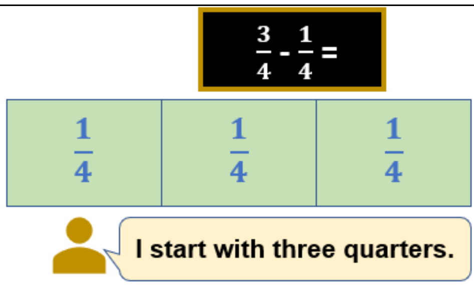
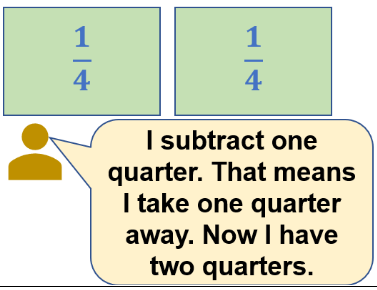
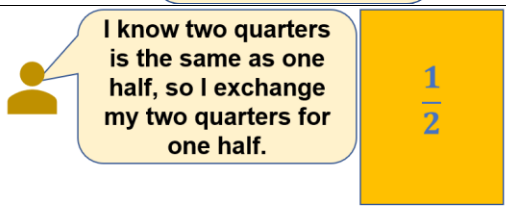


Figure 3

Activity

The first part of the session is teacher led. Make sure all learners are involved by telling the groups of six that they must all have a turn using the fraction cards on their table.

1. Write $\frac{3}{4} - \frac{1}{4} =$ on the board. Use your fraction cards to demonstrate finding the answer as follows. Ask learners to copy each step. Show three quarter cards and say, *'I start with three quarters.'*	
2. Remove one quarter and say, *'I subtract one quarter. That means I take one quarter away. Now I have two quarters.'*	
3. Say, *'I know two quarters is the same as one half, so I exchange my two quarters for one half.'*	

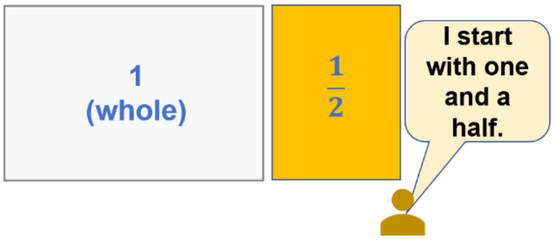
4. Complete the fraction sentence on the board to show what you have done.

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

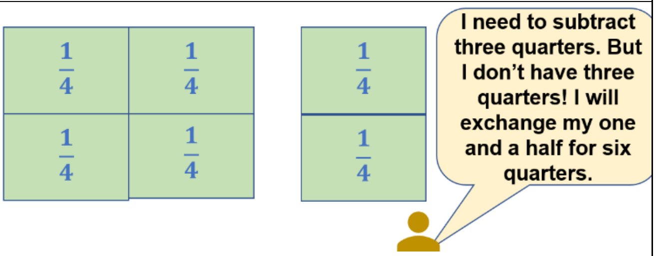
5. Write $1\frac{1}{2} - \frac{3}{4} =$ on the board.

   Use fraction cards to find the answer as follows.

   Say, *'I start with one and a half.'*



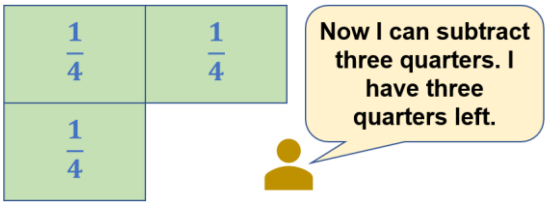
6. Say, *'I need to subtract three quarters. But I don't have three quarters! I will exchange my one and a half for six quarters.'*



7. Remove three quarters. Say, *'Now I can subtract three quarters. I have three quarters left.'*

   Complete the fraction sentence on the board to show what you have done.

   Explain that some of the questions on the board are easier than others. As they work in their groups, they can choose which questions they answer.



$$1\frac{1}{2} - \frac{3}{4} = \frac{6}{4} - \frac{3}{4} = \frac{3}{4}$$

8. Within their groups of six, learners now work in pairs.
   - One pair chooses a question from the board.
   - The second pair uses the fraction cards to find the answer.
   - The third pair checks the answer.
   - All learners write the fraction number sentence, including exchange if needed, in their books.
   - After each question, learners change roles.

   Tell learners that if they need more fraction cards, they can help themselves from the pile on your desk.

Assessment

Move around the room as learners work on the questions on the board. Look out for learners who still confuse the top and bottom numbers, and for learners who cannot yet tell you simple equivalents. Make a small group with these learners and work with them, then ask them to do some simple addition and subtraction of fractions using the cards.

In practice

Mrs Banda decided that her fraction cards were likely to be used frequently, so she decided to make them from coloured manila, using different colours for each fraction.

She liked the way the activity introduced fraction subtraction without mentioning 'multiplying numerator and denominator by the same number' or 'cancelling a fraction to its simplest form'. She felt that it was important that learners first understood what they were doing using the cards. When that understanding was secure, they drew pictures in their books to help them with fraction exchange.

Unit 3: Decimal fractions

Contents

Introduction 28
3.1: Introducing decimal fractions.. 29
3.2: Addition of numbers containing decimal fractions 31
3.3: Subtraction of numbers containing decimal fractions 35

Introduction

In this unit, decimal fractions are introduced. Formal methods of addition and subtraction of whole numbers are extended to numbers containing decimal fractions, supported by a new place value counter representing ‘one tenth’.

Key words and phrases

- **decimal fraction** – a fraction where the denominator is a power of 10 (for example 10, 100, 1000) and where the numerator is written to the right of the decimal point
- **proper fraction** – a fraction where the numerator is smaller than the denominator

3.1: Introducing decimal fractions

Aim

This activity helps learners understand the meaning of 'one tenth' and to understand different ways of writing it.

What the learners will do

Learners work in pairs, using strips of paper to make, name and write numbers that include tenths as proper fractions and decimal fractions.

Resources

You will need:

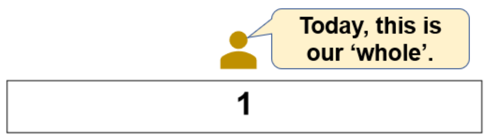
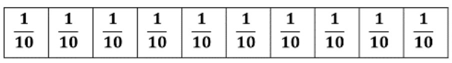
- a demonstration-sized strip of paper 'A' and **two** of strip 'B' (**Resource C Decimal fraction strips**, page 40)
- to draw a 0–2 number line marked in tenths on the board (**Resource D Number line in tenths**, page 41)
- something to stick tenths and decimals to the board.

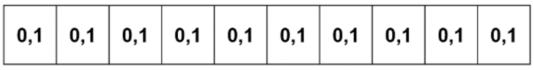
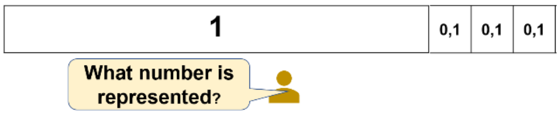
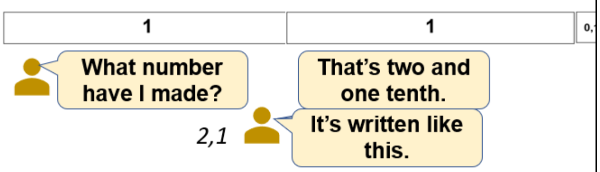
Each pair of learners will need:

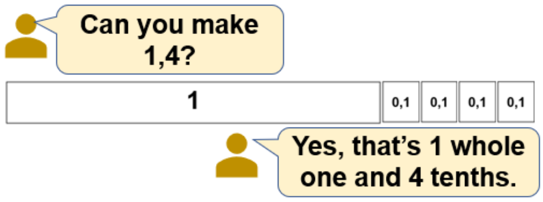
- three strips of paper – **two** of strip 'A' and **one** of strip 'B' (**Resource C Decimal fraction strips**, page 40)
- a pen and paper.

Activity

Steps 1–8 are teacher led. Learners talk and work with strips of paper in pairs. Most of the lesson should be spent on Steps 9 and10, where learners make up questions for each other.

1. Show learners strip 'A'. Ask pairs of learners to find one the same. Say, *'Today, this is our 'whole'.'*	
2. Show learners strip 'B'. Ask pairs of learners to find one on their table that is the same. Ask learners to discuss the following questions with their partners: '*How many parts is our 'whole' divided into?*' (10) '*What is each part called?*' (One tenth) '*How can we write that?*' ($\frac{1}{10}$)	

Write $\frac{1}{10}$ in each section of your strip.	
3. Pick up your second strip 'B'. Write 0,1 in the first section. Explain that this is another way to represent a tenth and is used as part of the decimal number system. Draw attention to the comma, which separates whole numbers and decimal numbers. Write 0,1 in each of the sections and ask learners to do the same with their strip 'B'.	
4. Cut the strip you made in Step 3 into tenths. Ask learners to do the same.	
5. Stick your strip 'A' and three '0,1' sections on the board. Ask learners to talk to their partners about what number is represented. (1,3 or one and three tenths.) Ask them to use their finger to write in the air how it is written.	
6. Write 2,5 on the board. Ask pairs of learners to use their strips to make the number.	
7. Tell learners that they are going to work in pairs, taking turns to ask and answer questions.	
8. Learner 1 uses some of their strips and '0,1' sections of paper to make a number. Learner 2 says what the number is and writes it as a decimal number. They each have two turns to ask and answer questions.	

<table>
<tr>
<td>9. Learner 1 says a decimal number between 0 and 2. Learner 2 makes the number with strip and sections.</td>
<td>
</td>
</tr>
<tr>
<td colspan="2">Towards the end of the session, gather the class together. Ask them to count with you along the 0–2 decimal number line that you drew on the board. Ask, 'What number would come next?' (2,1) Ask other questions such as, 'What number is one whole number bigger than 2,1?' (3,1)</td>
</tr>
</table>

Assessment

During steps 7 and 8, notice and note learners who either seem to be working very quickly and efficiently or seem to be finding it difficult to identify or make decimal numbers. Take care to spend extra time in the next two activities with those who are finding the concept difficult. Ask higher-achieving learners where they think knowing about decimal fractions might be helpful in real life.

In practice

Mr Dube thought that this was such an effective way to introduce decimal fractions to learners that he wondered why he hadn't thought of it himself! Each learner had a role, so no one was left out. Mr Dube was free to walk round the classroom and watch what learners were doing. He was pleased to find that they all seemed to understand what they were doing and to be placing the comma correctly between the 'ones' and the 'tenths' columns.

3.2: Addition of numbers containing decimal fractions

Aim

This activity helps learners understand that they can use the same strategies to add decimal fractions as they use for whole numbers.

What the learners will do

Learners work in groups of six. They use number cards to make two-digit numbers with one decimal place. They use place value counters to help them add the two numbers together.

Resources

You will need:

- a place value chart (**Resource E Place value chart**, page 41)
- place value counters representing ‘tens’, ‘ones’ and ‘tenths’ (**Resource F Place value counters**, page 41)
- two sets of number cards (**Resource G Number cards**, page 42
- two decimal comma cards (**Resource H Comma card**, page 42).
- A small collection of counters representing ‘hundreds’ for learners to use if needed (see step 6).

Each group of six learners will need:

- a place value chart (**Resource E Place value chart**, page 41)
- place value counters representing ‘tens’, ‘ones’ and ‘tenths’ (**Resource F Place value counters**, page 41)
- two sets of 1–9 number cards (**Resource G Number cards**, page 42)
- two decimal comma cards (**Resource H Comma card**, page 42).

Each learner will also need:

- an exercise book and pen.

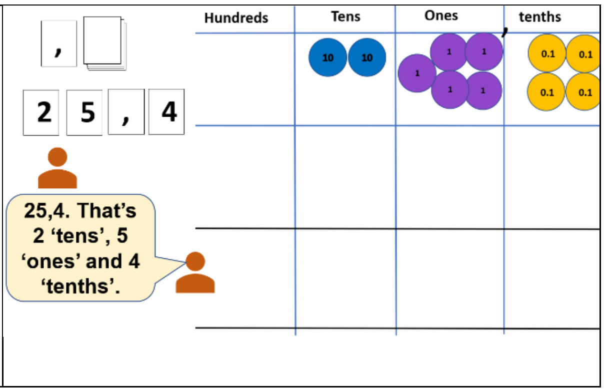
Activity

Before learners begin working on their own in groups, talk them through two calculations.

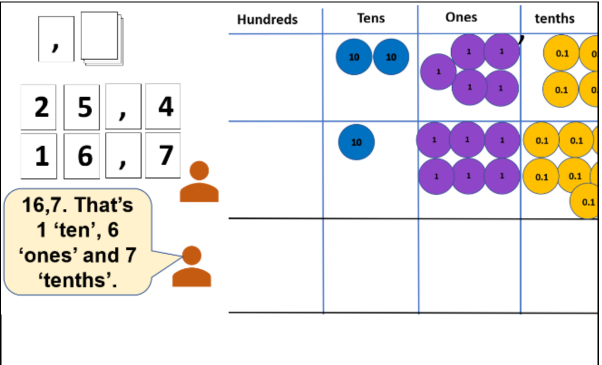
Ask a volunteer to arrange your place value counters on your large place value chart and another volunteer to write the numbers in a place value chart drawn on the board.

For the second calculation, ask learners, in their groups, to arrange their place value counters on their own place value chart and to write the numbers in columns in their books.

1. Learner 1 turns over three cards. They place the decimal comma between the second and third card to make a two-digit number with one decimal place. Learner 2 makes the number on the group’s place value chart using place value counters.



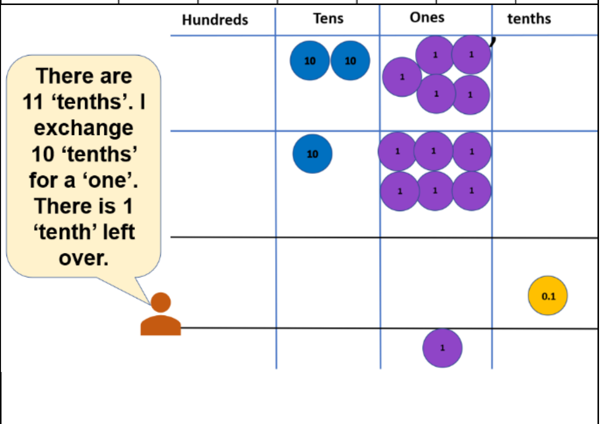
2. Learner 3 turns over another three cards and places these under the first three numbers, placing the decimal comma between the second and third card. Learner 4 makes the second number below the first number on the group's place value chart.



3. All members of the group write the calculation in their books.

	H'reds	Tens	Ones	tenths
		2	5 ,	4
+		1	6 ,	7

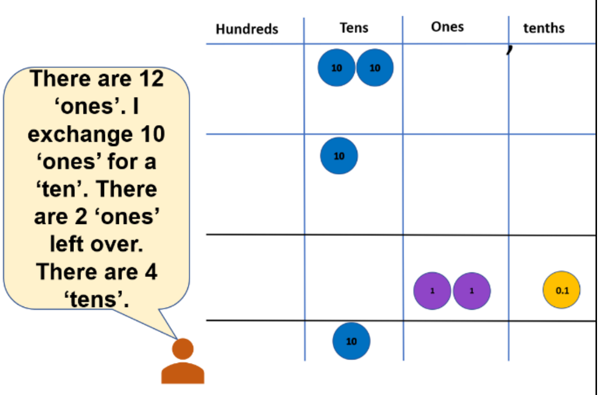
4. Learner 5 collects the 'tenths' together and counts them. If there are more than nine 'tenths', they exchange ten 'tenths' for a 'one' and place it under the line in the 'ones' column. Any 'tenths' that are left stay in the 'tenths' column.



5. All learners write the total of the 'tenths' in their books, writing '1' below the line in the 'ones' column if ten 'tenths' have been exchanged for a 'one'.

	H'reds	Tens	Ones	tenths
		2	5 ,	4
+		1	6 ,	7
				1
			1	

6. Learner 6 collects the 'ones' together and counts them. They must remember to include the 'one' that has been exchanged (if there is one). If there are more than nine 'ones', they exchange ten 'ones' for a 'ten'. Any 'ones' that are left over stay in the 'ones' column. Learner 6 also collects the 'tens' together and counts them. If there are more than nine 'tens', they exchange ten 'tens' for a 'hundred'.



7. All learners complete the calculation in their books.

	H'reds	Tens	Ones	tenths
		2	5,	4
+		1	6,	7
		4	2,	1
		1	1	

Learners repeat steps 1–7 as often as time allows. Learners change jobs with each new calculation.

Assessment

Make sure you make a note of which learners are working together in groups.

When you look at their books, check the books of all learners from the same group at the same time. It is likely they will have the same answers because they have been working together.

To find out what individual learners understand, you will need to listen to them talking together as they work on the calculations.

In practice

Mrs Taibu really liked the clear link that this activity made with the addition of whole numbers. When her learners had understood that there were ten 'tenths' in one, just as there were ten 'ones' in ten, they were able to make up questions for each other and to answer them accurately. They did find the language tricky, especially the difference between 'tens' and 'tenths', but they were able to talk about it in their groups until they all understood what it meant.

While the learners were doing group work, Simba asked Mrs Taibu a really good question. He said, *'I notice a pattern. 100 is 10 'tens', 10 is 10 'ones', 1 is 10 'tenths'. As you move right across the place value chart, the value of the place value counters gets smaller and smaller. What comes next?'*

Mrs Taibu shared Simba's question with the rest of the class and the class decided it would be ‘hundredths’. They were so pleased to have worked this out for themselves that some of the learners began to set questions for each other using ‘hundredths’.

Mrs Taibu was delighted to find her learners teaching themselves.

The bottle tops learners brought in to use as place value counters were an assortment of colours. I wrote the value on the top of each with a permanent marker. I realised that it is the **value** that is important, not the **colour**.

3.3: Subtraction of numbers containing decimal fractions

Aim

This activity helps learners understand that they can use the same strategies to subtract decimal fractions as they use for whole numbers.

What the learners will do

Learners work in groups of six. They use number cards to make two 2-digit numbers with one decimal place. They subtract the smaller number from the bigger number.

Resources

You will need:

- a place value chart (**Resource E Place value chart**, page 41)
- place value counters representing ‘tens’, ‘ones’ and ‘tenths’ (**Resource F Place value counters**, page 41)
- two sets of number cards (**Resource G Number cards**, page 42)
- two decimal comma cards (**Resource H Comma card**, page 42).

Each group of six learners will need:

- a place value chart (**Resource E Place value chart**, page 41)
- place value counters representing ‘tens’, ‘ones’ and ‘tenths’ (**Resource F Place value counters**, page 41)
- two sets of 1–9 number cards (**Resource G Number cards**, page 42)
- two decimal comma cards (**Resource H Comma card**, page 42).

Each learner will also need:

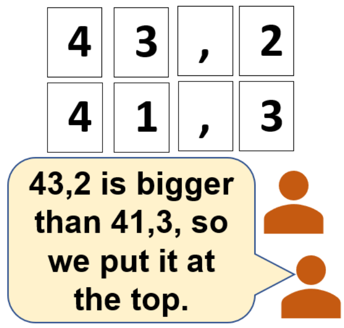
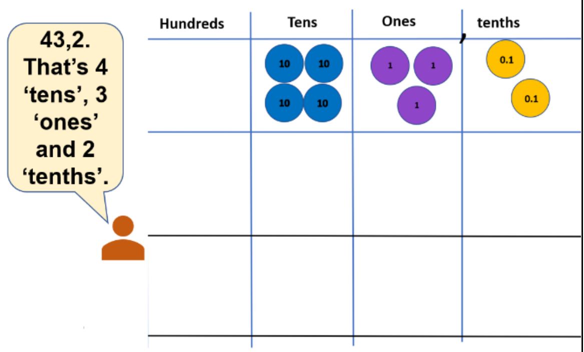
- an exercise book and pen.

Activity

Before learners begin working on their own in groups, talk them through two calculations.

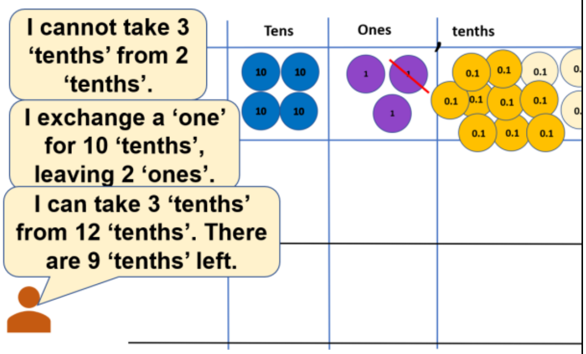
Ask a volunteer to arrange your place value counters on your large place value chart and another volunteer to write the numbers in a place value chart drawn on the board.

For the second calculation, ask learners, in their groups, to arrange their place value counters on their own place value chart and to write the numbers in columns in their books.

<table>
<tr><td>1. Learner 1 turns over three cards. They place the decimal comma between the second and third card to make a two-digit number with one decimal place. Learner 2 turns over another three cards and places these under the first three numbers, placing the decimal comma between the second and third card. They arrange the cards so the bigger number is above the smaller number.</td><td>
</td></tr>
<tr><td>2. All members of the group write the calculation in their books.</td><td><table>
<tr><td></td><td>H'reds</td><td>Tens</td><td>Ones ,</td><td>tenths</td></tr>
<tr><td></td><td></td><td>4</td><td>3 ,</td><td>2</td></tr>
<tr><td>-</td><td></td><td>4</td><td>1 ,</td><td>3</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
</table></td></tr>
<tr><td>3. Learner 3 makes the bigger number on the place value chart with place value counters.</td><td>
</td></tr>
</table>

4. Learner 4 subtracts the 'tenths'. If there are not enough 'tenths' to subtract the required amount, they must exchange a 'one' for ten 'tenths'. They place the ten 'tenths' in the 'tenths' column.

   They subtract the required amount and tell the rest of their group how many are left.



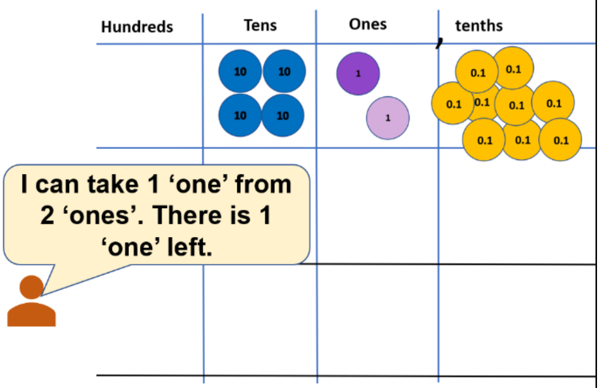
5. All learners write what Learner 4 has done in their books. If a 'one' has been exchanged, they change the number in the 'ones' column to show how many are left, and put a '1' beside the digit in the 'tenths' column to show that ten 'tenths have been exchanged for the 'one'.

   They write how many 'tenths' are left between the thick lines in the 'tenths' column.

	H'reds	Tens	Ones	tenths
		4	~~3~~ 2	12
-		4	1	3
				9

6. Learner 5 subtracts the 'ones'. If there are not enough 'ones' to subtract the required amount, they must exchange a 'ten' for ten 'ones'. They place the ten 'ones' in the 'ones' column.

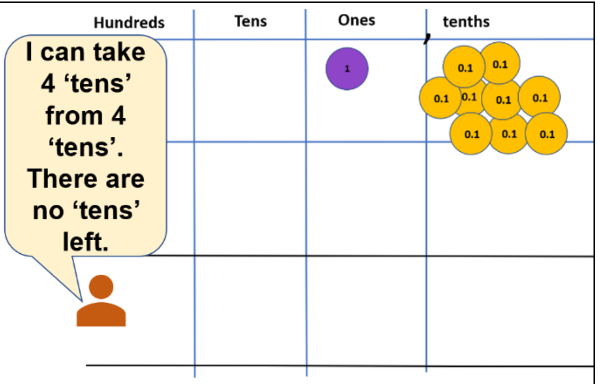
   They subtract the required amount and tell the rest of their group how many are left.



7. All learners write what Learner 5 has done in their books. If a 'ten' has been exchanged, they change the number in the 'tens' column to show how many are left, and put a '1' beside the digit in the 'ones' column to show that a 'ten' has been exchanged for ten 'ones'.

   They write the number of 'ones' that are left between the thick lines in the 'ones' column.

	H'reds	Tens	Ones	tenths
		4	~~3~~ 2	12
-		4	1	3
			1	9

<table>
<tr>
<td>8. Learner 6 repeats the process with the ‘tens’ column.</td>
<td>
</td>
</tr>
<tr>
<td>9. All learners complete the calculation in their books.</td>
<td>
<table>
<tr><td></td><td>H’reds</td><td>Tens</td><td>Ones ,</td><td>tenths</td></tr>
<tr><td></td><td></td><td>4</td><td>~~3~~ 2 ,</td><td>12</td></tr>
<tr><td>-</td><td></td><td>4</td><td>1 ,</td><td>3</td></tr>
<tr><td></td><td></td><td></td><td>1 ,</td><td>9</td></tr>
</table>
</td>
</tr>
<tr>
<td colspan="2">Learners repeat steps 1–9 as often as time allows. Learners change jobs with each new calculation.</td>
</tr>
</table>

Assessment

Watch learners carefully as they use the place value counters and write the calculations.

If they are not getting the correct answer, is it because:

- they do not understand how to exchange in subtraction
- they do not understand decimal numbers
- they do not understand how to write the calculation correctly in their books?

In practice

After completing this activity, Miss Moyo was pleased to find that her learners were able to add and subtract decimal numbers.

She planned to link this knowledge to the topic of measurement in her next lesson. She first checked that learners could measure in centimetres using their 30-cm rulers, then she asked them to find two objects and to measure them. Finally, she asked learners to find the total length of their objects and the difference in lengths. This really made them think about where they should put digits in their place value charts and which number they should write first when subtracting.

Reflection

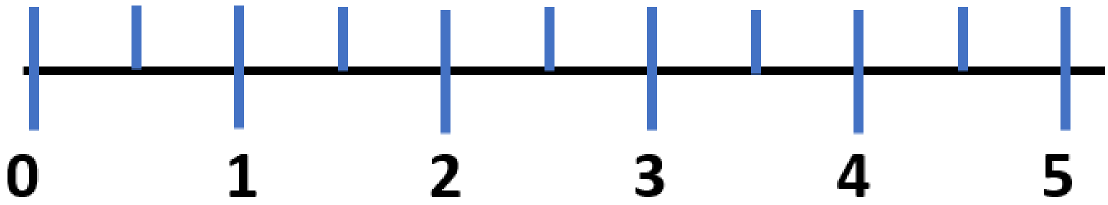
You have now completed the final module in this set of six modules, which offer activities about the learner journey from understanding the nature and language of numbers, to adding, subtracting, multiplying and dividing whole numbers, fractions and decimal fractions.

Reflect on what you have learned. You can do this by yourself, but, if you have the opportunity, it is better to do so with other teachers in your school or cluster. Perhaps you can meet after school or set up a WhatsApp group to work with teachers some distance away.

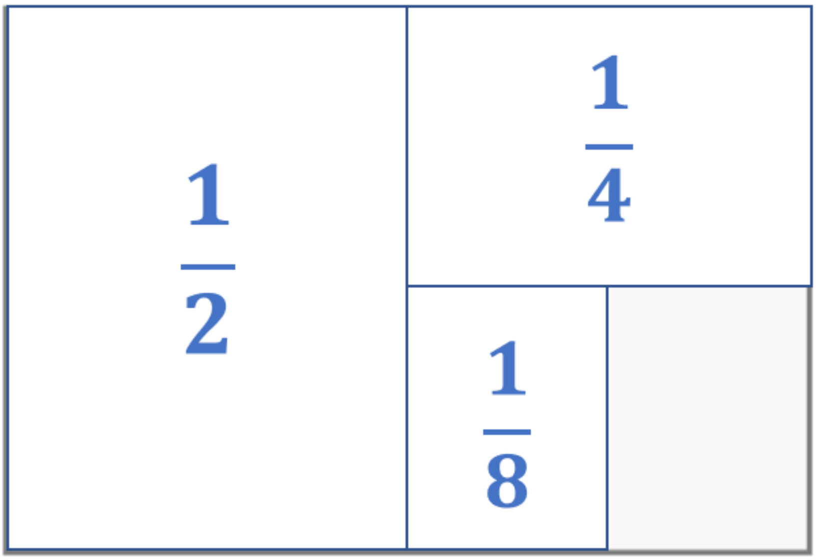
- Which activities did you find particularly successful? Why do you think they worked well?
- Which activity was least successful? Why was this? What did you do differently the next time you used the activity?
- Based on what you have learned from the modules, what advice would you give to someone who was new to numeracy teaching?

Resources

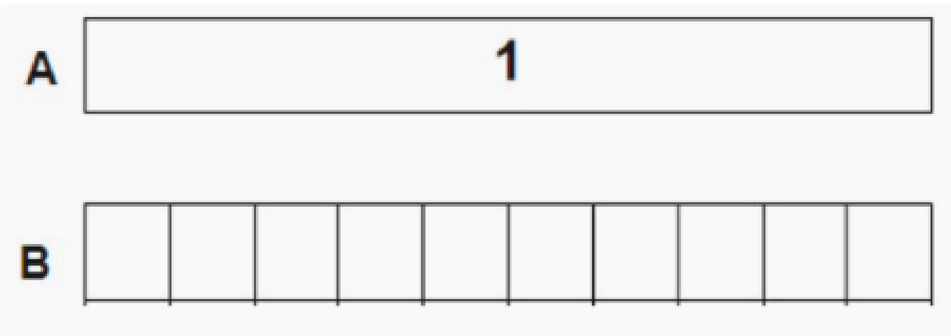
Resource A Fraction number line



Resource B Paper folding



Resource C Decimal fraction strips

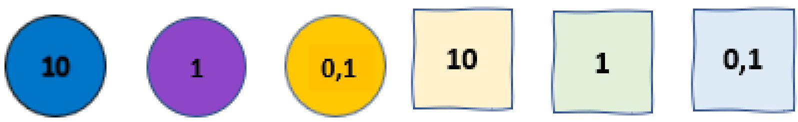


Resource D Number line in tenths

Resource E Place value chart

Hundreds	Tens	Ones	, tenths

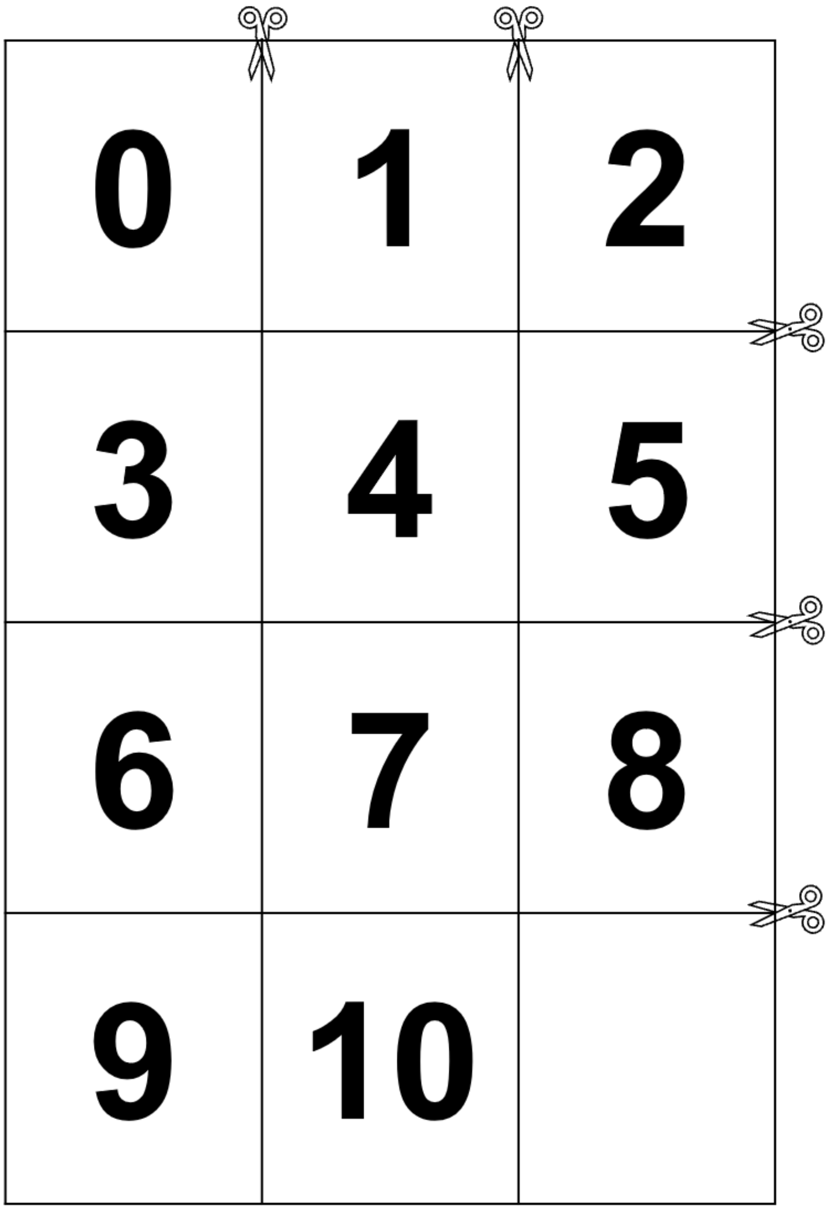
Resource F Place value counters



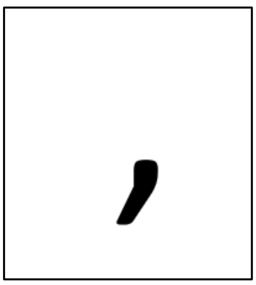
Place value counters can be made from bottle tops or pieces of torn paper of any colour. The value can be written on bottle tops with a permanent marker.

Resource G Number cards

You can make these cards by cutting a sheet of paper as shown and writing the numbers on the pieces, or you can cut up old cardboard boxes and write the numbers on those. Each card should be about 6 cm x 4 cm.



Resource H Comma card



Acknowledgements

Grateful acknowledgement is made to the following sources:

Cover photos © World Vision Zimbabwe and not included in the Creative Commons license.

Partner logos are not included in the Creative Commons license.

Every effort has been made to contact copyright holders. If any have been inadvertently overlooked, the publishers will be pleased to make the necessary arrangements at the first opportunity.

iGATE
Improving Gender Attitudes, Transition and Education Outcomes
care
World Vision
The Open University
Girls' Education Challenge
UKaid from the British people