

REIES – Lesson 4a: Heat Delivery

Answer Key

You have worked through the exercises — well done! Use this document to check your answers and deepen your understanding. Pay particular attention to the COP vs. efficiency distinction and the comparison between different heat delivery technologies: these are core concepts throughout the course.

Section A – Knowledge Questions

Answer – Exercise 1

Knowledge

Technology	Primary energy source	Delivers heat only / also electricity?
Geothermal system	Earth's internal heat (geothermal)	Both: heat directly, or electricity via steam turbine + generator
Combustion boiler	Chemical energy of fuel (gas, oil, biomass, coal)	Mainly heat; combined with steam turbine it can also produce electricity (CHP)
Solar thermal collector	Solar radiation (sunlight)	Both: low-temperature collectors supply heat only; concentrating CSP systems can also produce electricity
Heat pump	Ambient heat (air, ground, water) + electrical energy for the compressor	Heat only (or cooling if reversed); it does not generate electricity

On $COP > 1$: A combustion boiler converts chemical energy into heat with efficiency ≤ 1 . No combustion process can 'create' energy, so $\eta \leq 1$. A heat pump, however, does not generate heat from its electrical input — it moves ambient heat from one place to another. Because it harvests free environmental heat, the delivered heat can exceed the electrical input, giving $COP > 1$. Geothermal systems, similarly, pump existing underground heat using a small amount of electrical energy, enabling very high COPs (up to 20). Solar thermal collectors have an 'efficiency' (useful heat out / solar radiation in) that is always ≤ 1 .

Answer – Exercise 2

Knowledge

Formulas:

η (efficiency) = useful energy output / energy input [dimensionless, $0 \leq \eta \leq 1$]
 $COP = Q_{\text{delivered}} / E_{\text{electrical}}$ [dimensionless, can be > 1]

Key difference: Efficiency compares useful output to total input for a conversion process and cannot exceed 1 (First Law of Thermodynamics). COP is used specifically for heat transfer systems (heat pumps, geothermal) where heat is moved rather than generated; the system can deliver more heat energy than the electrical energy it consumes, because it harvests additional free ambient heat.

Combustion boiler: its COP equals its efficiency — always ≤ 1 , typically around 0.85–0.95. A boiler can only convert its fuel, never harvest ambient energy.

*Heat pump COP = $Q_{\text{delivered}} / E_{\text{electrical}} = 5 \text{ kWh} / 1 \text{ kWh} = 5$
The 'extra' 4 kWh came from the ambient environment (air, ground, or water).
The compressor did 1 kWh of work to 'lift' that ambient heat to a higher temperature.*

Geothermal COP up to 20: the underground reservoir contains enormous amounts of stored thermal energy. Only a small amount of electricity is needed to run the pumps that circulate the water. The ratio of heat delivered to pump electricity used is very large.

Answer – Exercise 3

Knowledge

Conduction: transfer of heat through direct molecular contact (vibration), primarily in solids. Example: a metal spoon getting hot when left in a saucepan.

Convection: transfer of heat through the bulk movement of a fluid (liquid or gas). Example: hot air rising from a radiator, creating air currents that heat a room.

Radiation: transfer of heat via electromagnetic waves (infrared radiation) without need for a physical medium. Example: feeling warmth from the sun or from a fire across a room.

Geothermal installation:

- (a) Moving thermal energy underground — convection: the hot water flows through the borehole and pipes, carrying thermal energy by bulk fluid movement.
- (b) In the heat exchanger — conduction through the metal walls of the exchanger, then convection on both fluid sides.

Solar thermal flat-plate collector:

- (a) Sun delivering energy to the absorber plate — radiation (electromagnetic waves travelling through the vacuum of space and the atmosphere).
- (b) Absorber plate heating the water — conduction (through the plate metal) and then convection (the water flows through the tubes and carries the heat away).

Answer – Exercise 4

Knowledge

Location constraints:

- Geothermal: suitable underground reservoirs with sufficient temperature and permeability must exist. In the Netherlands, depths of 2+ km are needed for water above 80 °C. Not every location has the right geology.
- Solar thermal: output depends on solar irradiance, which varies strongly by latitude, season, and local climate. Northern European locations receive much less sunshine than southern ones, especially in winter.

Why a northern European building owner might choose a combustion boiler over solar thermal:

- Intermittency and low winter output: solar thermal delivers little in winter (December–January), exactly when heating demand is highest. A backup system is always needed anyway, making the combined cost high.
- High upfront installation cost and roof area requirements, especially if the roof is shaded, north-facing, or already used for other purposes.

Heat pump multifunctionality: the same heat pump device can provide (1) space heating in winter (compressor runs, condenser heats indoor air or water) and (2) space cooling in summer (cycle reverses, the evaporator cools indoor air, condenser rejects heat outside). Some models also provide (3) domestic hot water heating year-round.

CHP efficiency advantage: in a conventional setup, a power plant burns gas at ~36–40 % efficiency to make electricity, and a boiler burns gas at ~92 % efficiency to make heat — each from separate gas inputs. A CHP unit uses one gas input and captures both the electricity and the waste heat from the engine or turbine exhaust. Because the waste heat is captured rather than discharged, the combined thermal + electrical efficiency can reach 80–90 %, compared to ~36 % for electricity alone. The lesson shows an example where CHP requires 43 % less primary energy than separate systems for the same services.

Section B – Calculation Questions

Answer – Exercise 5

Calculation

Thermal energy in a water tank

- $Q = m \cdot c_p \cdot \Delta T = 300 \text{ kg} \times 4180 \text{ J/(kg}\cdot\text{K)} \times (65 - 15) \text{ K}$
 $Q = 300 \times 4180 \times 50 = 62\,700\,000 \text{ J} = 62.7 \text{ MJ}$
Convert to kWh: $62\,700\,000 / 3\,600\,000 = 17.42 \text{ kWh}$
- Daily use = 100 litres = 100 kg
 $Q_{\text{daily}} = 100 \times 4180 \times 50 = 20\,900\,000 \text{ J} = 5.81 \text{ kWh}$
Fraction of tank: $100/300 \approx 0.33 \rightarrow$ about 1/3 of a tank per day
- Collector power = Area $\times$ Irradiance $\times$ Efficiency
 $= 4 \text{ m}^2 \times 500 \text{ W/m}^2 \times 0.65 = 1\,300 \text{ W} = 1.3 \text{ kW}$
Daily energy (6 h sunshine): $1.3 \text{ kW} \times 6 \text{ h} = 7.8 \text{ kWh}$
Required: 5.81 kWh/day $\rightarrow$ YES, the collector output (7.8 kWh) exceeds the daily demand.

Note: In practice, the collector operates less than 6 full-sun hours in winter in the Netherlands, so a backup heat source (boiler or heat pump) is still needed for cloudy or winter days. The 4 m² collector is adequate on an annual average but not reliable year-round.

Answer – Exercise 6

Calculation

Latent heat and phase change

- $Q_{\text{latent}} = m \cdot L_f = 500 \text{ kg} \times 334\,000 \text{ J/kg} = 167\,000\,000 \text{ J} = 167 \text{ MJ}$
Convert: $167\,000\,000 / 3\,600\,000 = 46.4 \text{ kWh}$
- $Q_{\text{sensible}} = m \cdot c_p \cdot \Delta T = 500 \times 4180 \times (20 - 0) = 41\,800\,000 \text{ J} = 41.8 \text{ MJ}$
 $= 11.6 \text{ kWh}$
- Total = $167 + 41.8 = 208.8 \text{ MJ}$
% from phase change = $167 / 208.8 \times 100 = 80 \%$

The phase change accounts for 80 % of all the heat absorbed, even though the temperature only changed by 20 °C during the sensible heating phase. This shows why latent heat storage has high energy density — a large amount of energy is stored at a constant temperature (0 °C), which is ideal for stable cooling applications.

Answer – Exercise 7

Calculation

Geothermal heat exchanger

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1.  $P = \dot{m} \cdot c_p \cdot \Delta T = 20 \text{ kg/s} \times 4\,180 \text{ J/(kg}\cdot\text{K)} \times (85 - 35) \text{ K}$ 
 $P = 20 \times 4\,180 \times 50 = 4\,180\,000 \text{ W} = 4\,180 \text{ kW} = 4.18 \text{ MW}$ 

2.  $\text{COP} = Q_{\text{delivered}} / E_{\text{electrical}} = 4\,180 \text{ kW} / 50 \text{ kW} = 83.6$ 
(This is a theoretical maximum; real installations account for additional losses.)

3. Annual energy:  $4\,180 \text{ kW} \times 8\,000 \text{ h} = 33\,440\,000 \text{ kWh} = 33\,440 \text{ MWh}$ 
Homes supplied:  $33\,440\,000 \text{ kWh} / 17\,000 \text{ kWh} = \text{approx. } 1\,967 \text{ homes} \approx \sim 2\,000 \text{ homes}$ 

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This illustrates why geothermal heat is so attractive for urban district heating: a single installation can supply thousands of homes with renewable, constant heat. The Leyweg project in The Hague (visible in the lesson slides) follows exactly this model.

Answer – Exercise 8

Calculation

Heat pump vs. gas boiler — energy and cost comparison

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1.  $E_{\text{electricity}} = Q_{\text{heat}} / \text{COP} = 14\,000 / 4.8 = 2\,917 \text{ kWh/year}$ 

2.  $E_{\text{gas}} = Q_{\text{heat}} / \eta_{\text{boiler}} = 14\,000 / 0.92 = 15\,217 \text{ kWh/year}$ 

3. Heat pump cost =  $2\,917 \times \text{€}0.30 = \text{€}875/\text{year}$ 
   Gas boiler cost =  $15\,217 \times \text{€}0.12 = \text{€}1\,826/\text{year}$ 
   → Heat pump is cheaper by €951/year at these prices.

4. Break-even electricity price:
    $2\,917 \times p_{\text{elec}} = 15\,217 \times 0.12$ 
    $2\,917 \times p_{\text{elec}} = 1\,826$ 
    $p_{\text{elec}} = 1\,826 / 2\,917 = \text{€}0.626/\text{kWh}$ 
   → The heat pump is cheaper as long as electricity costs below €0.626/kWh.

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This calculation shows that at typical 2024 Netherlands energy prices (~€0.25–0.35/kWh electricity, ~€0.10–0.12/kWh gas), a ground-source heat pump is already cost-competitive for running costs. The higher upfront installation cost is the main financial barrier. The break-even electricity price of €0.63/kWh is well above current retail prices, confirming the running cost advantage.

Answer – Exercise 9

Calculation

Temperature conversions and granite storage

Conversion formulas: $K = ^\circ C + 273.15$ | $^\circ F = (^\circ C \times 1.8) + 32$ | $^\circ C = (^\circ F - 32) \times 5/9$

$20\ ^\circ C \rightarrow K = 293.15\ K \rightarrow ^\circ F = 68\ ^\circ F$

$212\ ^\circ F \rightarrow ^\circ C = (212-32) \times 5/9 = 100\ ^\circ C \rightarrow K = 373.15\ K$

$300\ K \rightarrow ^\circ C = 26.85\ ^\circ C \rightarrow ^\circ F = 80.3\ ^\circ F$

$-10\ ^\circ C \rightarrow K = 263.15\ K \rightarrow ^\circ F = 14\ ^\circ F$

Kelvin [K]	Celsius [°C]	Fahrenheit [°F]
293.15	20	68
373.15	100	212
300	26.85	80.3
263.15	-10	14

Granite storage (2 000 kg, 20 °C to 200 °C):

$Q = 2\ 000 \times 790 \times (200 - 20) = 2\ 000 \times 790 \times 180 = 284\ 400\ 000\ J = 284.4\ MJ$
 $= 79.0\ kWh$

Water storage (300 kg, 20 °C to 65 °C):

$Q = 300 \times 4\ 180 \times (65 - 20) = 300 \times 4\ 180 \times 45 = 56\ 430\ 000\ J = 56.4\ MJ = 15.7\ kWh$

Per kg – granite: $284\ 400\ 000 / 2\ 000 = 142\ 200\ J/kg$

Per kg – water: $56\ 430\ 000 / 300 = 188\ 100\ J/kg$

→ Water stores more energy per kg in this scenario (lower c_p is offset by a ΔT 4× larger for granite).

→ Granite stores more total energy in absolute terms (79.0 vs. 15.7 kWh) due to the much larger ΔT .

The key insight is that granite can be heated to much higher temperatures than liquid water, compensating for its lower specific heat. High-temperature thermal storage (using rock, molten salt, or ceramics) is used in industrial-scale energy storage systems precisely for this reason.

Answer – Exercise 10

Calculation

Nuclear power plant heat balance and Carnot efficiency

1. $P_{\text{electrical}} = P_{\text{thermal}} \times \eta = 3\ 000\ MW \times 0.33 = 990\ MW$

2. $P_{\text{waste}} = P_{\text{thermal}} - P_{\text{electrical}} = 3\ 000 - 990 = 2\ 010\ MW$

3. $Homes = 2\ 010\ 000\ kW / 10\ kW = 201\ 000\ homes$

This illustrates two important points. First, even the most efficient thermal power stations waste more heat than they convert to electricity — a fundamental thermodynamic constraint (Second Law). Second, if this waste heat were used for district heating, the overall useful energy fraction would rise from 33 % to over 80 %, which is why nuclear-combined-heat-and-power is discussed in some European contexts.

Section C – Internet Exploration

Answer – Exercise 11

Internet Exploration

Reference: <https://www.ehpa.org/market-data> | <https://www.iea.org/energy-system/buildings/heat-pumps>

European heat pump market: approximately 3 million units were sold in Europe in 2023 (verify current year on EHPA site). Norway consistently has the highest sales per capita — a combination of cheap hydropower electricity, cold winters, and long-standing subsidy programmes. Sales dipped in 2023–2024 after the post-energy-crisis peak of 2021–2022.

IEA global picture: heat pumps currently supply around 10 % of global space and water heating in buildings. The IEA Net Zero Emissions scenario requires this to roughly double by 2030 and reach over 50 % by 2050. Annual sales need to grow from ~15 million units today to ~200 million by 2030 — a 13-fold increase in 7 years.

Cold-climate performance: modern air-source heat pumps (ASHPs) can operate down to -20°C or lower, but COP drops significantly. At $+7^{\circ}\text{C}$ outdoor air a good ASHP might achieve COP 3.5–4.5. At -10°C , COP typically falls to 2.0–2.5. At very low temperatures some units switch to auxiliary electric resistance heating (COP 1), which is why well-insulated buildings are important for heat pump performance in cold climates. Ground-source heat pumps avoid this problem because ground temperature is stable year-round.

Go deeper: <https://www.iea.org/reports/the-future-of-heat-pumps> | <https://www.ehpa.org>

Section D – Home Experiments

Answer – Exercise 12

Home Experiment – Measuring Specific Heat with a Kettle

Expected results for a 1 500 W kettle, 500 ml water, 60 s heating:

*Energy input: $E = P \times t = 1\,500\text{ W} \times 60\text{ s} = 90\,000\text{ J}$
Expected ΔT : $\Delta T = E / (m \times c_p) = 90\,000 / (0.5 \times 4\,180) = 43.1\text{ }^\circ\text{C}$
Actual ΔT will be smaller – perhaps 30–38 $^\circ\text{C}$ – because the kettle body, heating element, and evaporation also absorb some energy.*

*Measured c_p (if $\Delta T = 35\text{ }^\circ\text{C}$ is observed):
 $c_p = 90\,000 / (0.5 \times 35) = 5\,143\text{ J/(kg}\cdot\text{K)}$ → higher than 4 180 J/(kg·K)
% error = $(5\,143 - 4\,180) / 4\,180 \times 100 \approx 23\%$ overestimate*

Why the measured value is higher than 4 180 J/(kg·K): some of the 90 000 J goes into heating the metal kettle body, the heating element, steam evaporation, and heat losses to the surrounding air. Since you assumed all energy went into the water, the denominator ($m \times \Delta T$) is too small, making the calculated c_p too large.

Doubling the water: yes, ΔT should approximately halve (for 1 000 ml, expect $\Delta T \approx 15\text{--}20\text{ }^\circ\text{C}$ rather than 30–38 $^\circ\text{C}$). This confirms the inverse relationship between mass and temperature rise: $Q = m \cdot c_p \cdot \Delta T \rightarrow$ for fixed Q and c_p , $\Delta T \propto 1/m$.

Vegetable oil comparison ($c_p \approx 1\,670\text{ J/(kg}\cdot\text{K)}$, density $\approx 0.92\text{ kg/litre}$):

*Mass of 500 ml oil = $0.5 \times 0.92 = 0.46\text{ kg}$
 $\Delta T = E / (m \times c_p) = 90\,000 / (0.46 \times 1\,670) = 117\text{ }^\circ\text{C}$
Compare to water: $\sim 43\text{ }^\circ\text{C} \rightarrow$ oil heats up $\sim 2.7\times$ faster
This is why oil-based industrial processes can store heat at much higher temperatures.*

References for further reading:

- https://www.engineeringtoolbox.com/specific-heat-capacity-d_391.html — specific heat values for many materials
- <https://www.khanacademy.org/science/ap-physics-2/ap-thermodynamics> — thermodynamics fundamentals

Answer – Exercise 13

Home Experiment – Building a Simple Solar Thermal Collector

Expected results: on a sunny day with direct sunlight, the collector bottle should be 10–25 $^\circ\text{C}$ warmer than the control after 30 minutes. On a cloudy day, the difference may be only 2–5 $^\circ\text{C}$. The actual temperature rise depends on solar irradiance, outdoor temperature, and how well the box is insulated.

Sample calculation ($\Delta T = 15\text{ }^\circ\text{C}$ after 30 minutes):

$$Q = m \cdot c_p \cdot \Delta T = 0.2 \text{ kg} \times 4180 \text{ J/(kg}\cdot\text{K)} \times 15 \text{ K} = 12\,540 \text{ J} = 0.0035 \text{ kWh} = 3.5 \text{ Wh}$$

$$\text{Power absorbed: } 12\,540 \text{ J} / (30 \times 60 \text{ s}) = 6.97 \text{ W}$$

*If the bottle has an absorbing area of $\sim 0.02 \text{ m}^2$ (bottle side facing sun):
Irradiance received $\approx 6.97 \text{ W} / 0.02 \text{ m}^2 \approx 350 \text{ W/m}^2$ (reasonable for partial cloud)*

Why the black surface improves performance: black surfaces have high absorptivity for solar (shortwave) radiation. According to Kirchhoff's Law, good absorbers are also good emitters — but solar radiation is shortwave (from the high-temperature sun), while the bottle re-emits longwave infrared. The black paint maximises absorption of the incoming shortwave radiation.

Role of the cling film (glazing): it transmits incoming shortwave solar radiation (allowing sunlight in) but blocks the outgoing longwave infrared radiation emitted by the warm bottle. This is the greenhouse effect: energy enters easily but cannot escape as easily. It also reduces convective heat losses from the bottle surface. Real flat-plate collectors use tempered glass for the same reason.

Limitations compared to a real flat-plate collector:

- No selective absorber coating: a real collector uses a spectrally selective surface that absorbs shortwave solar radiation very well but emits very little longwave radiation, minimising heat loss. A bottle painted black has much higher thermal emission.
- No insulation: a real collector is well-insulated on the back and sides to prevent heat loss to the ambient air. The cardboard box provides minimal insulation.
- No tracking or optimal tilt optimisation; no pump to continuously remove heat (so water stagnates and equilibrium is reached quickly); and water temperature cannot be maintained at a set level.

References for further reading:

- <https://www.energy.gov/energysaver/solar-water-heaters> — overview of solar water heating systems
- <https://newheat.com/en/how-does-solar-thermal-energy-work/> — how solar thermal collectors work (source used in the lesson)
- <https://www.sciencedirect.com/topics/engineering/flat-plate-collector> — academic overview of flat-plate collector design