

REIES – Lesson 4b: Motion Delivery

Answer Key

You have worked through the exercises — well done! Use this document to check your answers and deepen your understanding. Pay particular attention to the wind power formula and its cubic dependence on wind speed: this is one of the most important relationships in renewable energy engineering.

Section A – Knowledge Questions

Answer – Exercise 1

Knowledge

Technology	Primary energy input	Output type	Motion final / intermediate?
Combustion engine	Chemical (fuel)	Shaft rotation (→ linear via drivetrain)	Final (propulsion)
Gas turbine (power plant)	Chemical (gas/biomass)	Shaft rotation	Intermediate → electricity via generator
Gas turbine (aircraft)	Chemical (kerosene)	Thrust (linear, via exhaust jet)	Final (propulsion)
Steam turbine	Thermal (steam)	Shaft rotation	Intermediate → electricity via generator
Water turbine	Kinetic/potential energy of water	Shaft rotation	Intermediate → electricity via generator
Wind turbine	Kinetic energy of wind	Shaft rotation	Intermediate → electricity via generator

Gas turbine compressor: the compressor must continuously supply compressed air to the combustion chamber for the reaction to sustain. Without compression, the fuel-air mixture would not combust efficiently. The turbine extracts enough shaft power to drive the compressor (the 'parasitic' load), and the remainder drives the generator. In a jet engine, the remainder accelerates the exhaust gases through a nozzle to produce thrust.

Answer – Exercise 2

Knowledge

Four strokes in order:

- Intake: piston moves down; intake valve opens; air-fuel mixture drawn into cylinder.

- Compression: both valves closed; piston moves up; mixture compressed, raising its temperature and volatility.
- Power (combustion): spark plug fires; compressed mixture ignites; rapid gas expansion forces piston down — this is the only stroke producing useful work.
- Exhaust: exhaust valve opens; piston moves up; burned gases pushed out.

Useful work is produced only during the power stroke (1 of 4 strokes).

Energy losses: (1) Heat rejected through the exhaust gases — the hot burned gases leaving the cylinder carry substantial thermal energy. (2) Heat rejected through the engine cooling system (radiator) — friction between moving parts and the combustion heat transferred to the cylinder walls is removed by the coolant. Together these losses account for ~70–75 % of the fuel energy.

Answer – Exercise 3

Knowledge

Betz limit: the theoretical maximum fraction of wind energy that any turbine can extract is $16/27 \approx 59.3\%$. Physically, if the turbine extracted 100 % of the wind's kinetic energy, the air would stop completely behind the turbine and no new air could flow through — which is impossible. Some air velocity must remain downstream to allow continuous flow. The Betz limit represents the optimal balance between energy extraction and maintaining flow. In practice, modern turbines achieve Cp of 35–45 %.

$P = C_p \cdot \frac{1}{2} \cdot \rho \cdot A \cdot v^3$: C_p is the power coefficient (fraction of wind energy actually converted to mechanical energy). $\frac{1}{2} \cdot \rho \cdot v^3$ is the kinetic energy flux per unit area of the wind (power per m^2 of flow). A is the rotor swept area, determining how much wind is intercepted. The cubic dependence on v arises because kinetic energy is $\frac{1}{2}mv^2$ (quadratic in v), and the mass flow rate through the rotor per second is $\dot{m} = \rho \cdot A \cdot v$ (linear in v): multiplying gives $\frac{1}{2} \cdot \rho \cdot A \cdot v^3$.

Shear exponent α : α describes how quickly wind speed increases with height above ground; it depends on the surface roughness. Highest α : urban areas with tall buildings ($\alpha = 0.40$); lowest α : smooth surfaces like lakes or ocean ($\alpha = 0.10$). Practical implication: in rough urban terrain, building tall towers provides a much larger wind speed gain than in open coastal terrain, making tower height especially valuable in cities. This is also why offshore turbines need lower hub heights to access good wind speeds.

Tidal vs. wind: water density $\rho_{\text{water}} \approx 1025 \text{ kg/m}^3$ vs. air $\rho_{\text{air}} \approx 1.225 \text{ kg/m}^3$ — a ratio of ~836. Since $P \propto \rho \cdot v^3$, the same power can be extracted from tidal currents at much lower velocity. For example, to generate the same power as a 10 m/s wind, a tidal current only needs $v = 10 \times (1.225/1025)^{1/3} \approx 2.3 \text{ m/s}$. This is why tidal turbines can be much smaller than wind turbines for equivalent output.

Answer – Exercise 4

Knowledge

Pumped hydro charging: Surplus electricity (e.g. from wind) drives electric motors connected to pumps, which lift water from the lower to the upper reservoir. Conversions: Electrical → Mechanical (motor) → Kinetic (pump) → Gravitational potential energy (water elevated).

Discharging: Water flows down from upper to lower reservoir through penstocks. Conversions: Gravitational potential → Kinetic (water in penstocks) → Rotational mechanical (water turbine) → Electrical (generator).

CHP efficiency: A conventional gas turbine exhausts hot gases at several hundred degrees Celsius after generating electricity. This thermal energy (~60 % of the original fuel input) is normally vented to the atmosphere as waste. In a CHP system, a heat recovery unit captures the exhaust heat and uses it for space heating, domestic hot water, or industrial processes. Because the same fuel now provides both electrical and useful thermal output, the combined efficiency rises from ~38–40 % (electricity only) to 70–80 % (electricity + heat). The efficiency 'jump' is not due to the turbine becoming more efficient — it is due to using energy that was previously wasted.

Staged expansion reasons: (1) Efficiency: each stage is designed with blade geometry optimised for a specific pressure and temperature range. Extracting energy in stages allows each stage to operate near its optimal design point, improving overall efficiency. (2) Moisture and blade damage: if steam expands too much in a single stage, its temperature drops below the saturation point, causing condensation (liquid droplets). These droplets erode the blade surface at high velocity. By reheating steam between stages, condensation is prevented and blade life is extended.

Section B – Calculation Questions

Answer – Exercise 5

Calculation

Kinetic energy and velocity conversions

$$1. \text{ Convert: } 120 \text{ km/h} \times (1000 \text{ m/km}) / (3600 \text{ s/h}) = 33.33 \text{ m/s}$$

$$2. \text{ KE} = \frac{1}{2} \cdot m \cdot v^2 = 0.5 \times 1400 \text{ kg} \times (33.33 \text{ m/s})^2 = 0.5 \times 1400 \times 1111 = 777\,700 \text{ J} \approx 778 \text{ kJ}$$

$$3. \text{ KE in kWh} = 777\,700 / 3\,600\,000 = 0.216 \text{ kWh}$$

(This is the energy that must be braked away to stop – equivalent to ~20 min of a 60W lamp)

The 0.216 kWh needed to bring a car from 120 km/h to rest is recovered by a regenerative braking system in an electric vehicle. A conventional car converts this entirely to heat in the brake pads — a direct loss.

Answer – Exercise 6

Calculation

Wind turbine power output

1. Swept area: $A = (\pi/4) \cdot D^2 = (\pi/4) \times 126^2 = 0.7854 \times 15876 = 12\,469 \text{ m}^2$
2. $P = C_p \cdot \frac{1}{2} \cdot \rho \cdot A \cdot v^3$
 $= 0.42 \times 0.5 \times 1.225 \times 12\,469 \times 10^3$
 $= 0.42 \times 0.5 \times 1.225 \times 12\,469 \times 1000$
 $= 0.42 \times 0.6125 \times 12\,469\,000$
 $= 0.42 \times 7\,637\,225 = 3\,207\,634 \text{ W} \approx 3\,208 \text{ kW} \approx 3.21 \text{ MW}$
3. $\text{Power} \propto v^3 \rightarrow \text{factor} = (12/10)^3 = 1.2^3 = 1.728$
 $P_{\text{new}} = 3.21 \text{ MW} \times 1.728 = 5.55 \text{ MW}$
A 20% increase in wind speed produces a 72.8% increase in power.
Implication: small differences in average wind speed between sites translate into very large differences in energy yield – siting is critical.
4. $\text{Annual energy} = 3.21 \text{ MW} \times 2\,500 \text{ h} = 8\,025 \text{ MWh} \approx 8.0 \text{ GWh}$

This 3.21 MW turbine produces about 8 GWh/year, sufficient to supply approximately 2 300 average Dutch households (each using ~3 500 kWh/year). Modern offshore turbines (15–20 MW) produce 5–6× more energy per turbine.

Answer – Exercise 7

Calculation

Wind speed profile and hub-height correction

Open terrain ($\alpha = 0.16$):
 $v_{100} = 6 \times (100/10)^{0.16} = 6 \times 10^{0.16} = 6 \times 1.445 = 8.67 \text{ m/s}$

Power at 8.67 m/s ($D=126\text{m}$, $C_p=0.42$, $\rho=1.225$):
 $P = 0.42 \times 0.5 \times 1.225 \times 12\,469 \times 8.67^3$
 $8.67^3 = 651.5$
 $P = 0.42 \times 0.6125 \times 12\,469 \times 651.5 = 0.42 \times 4\,975\,000 = 2\,090\,000 \text{ W} \approx 2.09 \text{ MW}$

Suburban terrain ($\alpha = 0.30$):

$$v_{100} = 6 \times (100/10)^{0.30} = 6 \times 10^{0.30} = 6 \times 1.995 = 11.97 \text{ m/s}$$

Power at 11.97 m/s:

$$11.97^3 = 1715$$

$$P = 0.42 \times 0.6125 \times 12\,469 \times 1715 = 0.42 \times 13\,088\,000 = 5\,497\,000 \text{ W} \approx 5.50 \text{ MW}$$

Comparison: suburban terrain gives 5.50 vs 2.09 MW for open terrain – 2.6× more power.

This is counterintuitive: rougher terrain (higher α) slows wind near the ground more, but the wind also increases more steeply with height, so at sufficient hub height the suburban site actually outperforms the open site in this example. The lesson is clear: wind resource assessment must always use the actual wind speed at hub height, never ground-level measurements directly.

Answer – Exercise 8

Calculation

Water turbine power and comparison

$$1. P_{\text{hydraulic}} = \phi_V \cdot \rho \cdot g \cdot H = 80 \times 1000 \times 9.81 \times 25 = 19\,620\,000 \text{ W} = 19.62 \text{ MW}$$

$$2. P_{\text{turbine}} = \eta \times P_{\text{hydraulic}} = 0.88 \times 19.62 = 17.27 \text{ MW}$$

$$3. \text{Drought: } \phi_V \text{ drops to } 40 \text{ m}^3/\text{s}$$

$$P_{\text{hydraulic_drought}} = 40 \times 1000 \times 9.81 \times 25 = 9.81 \text{ MW}$$

$$P_{\text{turbine_drought}} = 0.88 \times 9.81 = 8.63 \text{ MW}$$

$$\text{Reduction: } (17.27 - 8.63) / 17.27 \times 100 = 50 \% \text{ drop}$$

(Power is proportional to flow rate when head is constant.)

$$4. \text{Wind turbines needed (3.21 MW each from Ex6):}$$

$$N = 17.27 \text{ MW} / 3.21 \text{ MW} \approx 5.4 \rightarrow 6 \text{ turbines}$$

But 6 turbines only produce ~8.0 GWh each/year at 29% capacity factor, while the hydro plant runs at higher capacity factor with predictable output.

The hydro plant produces steady, controllable power; the wind farm is variable and needs storage or grid balancing. The hydro plant occupies a much smaller land footprint (intake, penstock, powerhouse) than 6 wind turbines spread across a wind farm. However, hydro requires a suitable river head and large dams carry ecological and social costs.

Answer – Exercise 9

Calculation

Gas turbine efficiency and CHP

1. $P_{\text{electrical}} = 500 \text{ MW} \times 0.38 = 190 \text{ MW}$
2. $P_{\text{waste}} = 500 - 190 = 310 \text{ MW}$ (62% of fuel becomes waste heat)
- 3a. $\text{Heat recovered} = 310 \times 0.60 = 186 \text{ MW}$ (useful thermal)
- 3b. $\text{Overall CHP efficiency} = (190 + 186) / 500 = 376 / 500 = 0.752 = 75.2 \%$
4. 75.2 % is within the lesson's stated range of 70–80%. ✓
Energy still wasted = $500 - 376 = 124 \text{ MW} = 24.8\%$ of fuel energy
(This remaining heat is in the lowest-temperature exhaust, too low to use economically.)

This exercise shows that CHP does not make the turbine more thermodynamically efficient — the turbine's electrical efficiency stays at 38 %. What changes is that previously wasted heat is now productively used. The key is having a nearby heat demand (buildings, industrial process) that can absorb the recovered heat, which is why CHP plants are typically located near district heating networks or industrial sites.

Answer – Exercise 10

Calculation

Wind → pumped hydro → electricity round-trip

1. $\dot{m} = P_{\text{pump}} \times \eta_{\text{pump}} / (g \times H)$
 $= 200 \times 10^6 \text{ W} \times 0.85 / (9.81 \times 300)$
 $= 170\,000\,000 / 2943 = 57\,800 \text{ kg/s}$
2. Mass stored in 6 h: $m = 57\,800 \times 6 \times 3600 = 57\,800 \times 21600 = 1.248 \times 10^9 \text{ kg}$
Gravitational PE = $m \cdot g \cdot H = 1.248 \times 10^9 \times 9.81 \times 300 = 3.673 \times 10^{12} \text{ J} = 3\,673 \text{ GJ}$
Convert: $3\,673 \text{ GJ} / 3600 = 1\,020 \text{ MWh}$
3. Electrical energy recovered: $E_{\text{out}} = 1\,020 \text{ MWh} \times 0.90 = 918 \text{ MWh}$
4. Energy input from wind farm: $E_{\text{in}} = 200 \text{ MW} \times 6 \text{ h} = 1\,200 \text{ MWh}$
Round-trip efficiency = $E_{\text{out}} / E_{\text{in}} = 918 / 1\,200 = 0.765 = 76.5 \%$
Losses: pump (15%) + turbine (10%) + minor hydraulic losses

A round-trip efficiency of ~75–80 % is typical for pumped hydro and is the highest of any large-scale energy storage technology. For comparison, lithium-ion batteries achieve ~85–90 % round-trip but are far more expensive per MWh of storage capacity. Pumped hydro dominates global storage capacity (>90 % of installed GWh) precisely because it is cheap, durable, and has long storage duration.

Section C – Internet Exploration

Answer – Exercise 11

Internet Exploration

Reference: <https://www.geminiwindpark.nl> | <https://gwec.net>

Gemini Wind Park (verify current figures on site): 600 MW total installed capacity, 150 turbines, 4 MW each (Siemens SWT-4.0-130), supplying approximately 785 000 Dutch households. It is located 85 km off the coast of Groningen in the North Sea.

Global wind (GWEC, check for current year): total global capacity exceeded 1 000 GW in 2023. China has by far the most installed capacity (~400+ GW). Annual additions have been 80–100 GW in recent years. The Netherlands has ~6–7 GW onshore and a rapidly growing offshore fleet.

Why offshore is better for the Netherlands: the Netherlands is a low-lying, densely populated country — onshore wind is constrained by proximity to homes, nature areas, and aviation corridors, all creating planning conflicts. Offshore, the North Sea provides vastly larger available area with much stronger, more consistent winds. Using the wind profile formula, offshore wind speeds at hub height are higher due to low roughness ($\alpha \approx 0.10$ for open sea), and there are no buildings or hills causing turbulence. Higher and more consistent wind speeds directly translate into higher capacity factors (40–50 % offshore vs ~25–30 % onshore), producing more energy per installed MW.

Go deeper: <https://www.rvo.nl/onderwerpen/wind-op-zee> | <https://gwec.net/global-wind-report/>

Answer – Exercise 12

Internet Exploration

Reference: <http://www.goronadelviento.es/en/> | <https://orbitalmarine.com> | <https://www.iea.org/energy-system/renewables/hydropower>

El Hierro / Gorona del Viento (verify current figures): ~11.5 MW wind capacity, ~11.3 MW hydro turbine capacity, ~380 000 m³ upper reservoir. In recent years the island has achieved 50–100 % renewable electricity on many days, with annual renewable share regularly exceeding 50–60 %. The system demonstrates the wind-pumped-hydro concept from the lesson at island scale.

Orbital Marine O2: rated power 2 MW, rotor diameter approximately 20 m (two rotors of ~20 m each), deployed at the European Marine Energy Centre (EMEC) in Pentland Firth, Scotland — one of the strongest tidal currents in the world. Comparison with Ex 6 wind turbine (D=126 m,

3.21 MW at 10 m/s wind): the O2 has a swept area of $\sim 314 \text{ m}^2$ per rotor vs $12\,469 \text{ m}^2$ for the wind turbine. Power per m^2 : O2 $\approx 2\,000/628 = 3.2 \text{ W/m}^2$; wind turbine $\approx 3\,210\,000/12\,469 = 257 \text{ W/m}^2$. The tidal turbine extracts significantly more power per m^2 because water is ~ 836 times denser than air, allowing high power at much lower current speeds ($\sim 2\text{--}3 \text{ m/s}$).

IEA hydropower: hydropower provides approximately 15–17 % of global electricity and about 60 % of all renewable electricity generation. Main challenges to expansion: (1) Most of the best dam sites in developed countries are already used; (2) Large dams displace communities and flood ecosystems; (3) Climate change causes drought variability, reducing reliability; (4) High capital costs and long construction times. Run-of-river and pumped storage are growing, but large new dams face strong environmental opposition.

Go deeper: <https://www.iea.org/reports/hydropower-special-market-report> | <https://orbitalmarine.com/o2/>

Section D – Home Experiments

Answer – Exercise 13



Home Experiment – Kinetic Energy on a Bicycle (or by Walking)

Sample results for a person (+ bike) of total mass 80 kg:

$$\begin{aligned} 5 \text{ km/h} &= 1.39 \text{ m/s} \rightarrow KE = 0.5 \times 80 \times 1.39^2 = 0.5 \times 80 \times 1.93 = 77 \text{ J} \\ 15 \text{ km/h} &= 4.17 \text{ m/s} \rightarrow KE = 0.5 \times 80 \times 4.17^2 = 0.5 \times 80 \times 17.4 = 695 \text{ J} \\ 30 \text{ km/h} &= 8.33 \text{ m/s} \rightarrow KE = 0.5 \times 80 \times 8.33^2 = 0.5 \times 80 \times 69.4 = 2\,776 \text{ J} \end{aligned}$$

Doubling speed from 15 to 30 km/h: KE ratio = $2\,776/695 = 4.0 \rightarrow KE$ quadruples

This is consistent with $KE \propto v^2$ – doubling v squares the kinetic energy.

When you doubled speed, KE quadrupled — not doubled. This follows directly from $KE = \frac{1}{2}mv^2$: squaring the velocity means squaring the energy. This is why stopping a car from 100 km/h requires four times the braking distance of stopping from 50 km/h.

Regenerative braking: when the driver brakes in an EV, the wheels drive the electric motor in reverse (generator mode). The kinetic energy of the vehicle is converted into electrical energy and stored back in the battery, instead of being wasted as heat. The same machine — the electric motor/generator — handles both acceleration and braking, connecting to the lesson's point about reversible converters.

Check: $60 \text{ kJ fuel} \times 0.28 \text{ efficiency} = 16\,800 \text{ J}$ reaches the wheels. $KE = \frac{1}{2} \times 1\,200 \times 30^2 = 0.5 \times 1\,200 \times 900 = 540\,000 \text{ J}$. So 60 kJ is far less than needed to reach 30 m/s — the car would have needed roughly $540\,000 / 0.28 \approx 1\,929 \text{ kJ}$ of fuel. The question illustrates that 60 kJ only produces 16.8 kJ of KE (useful work), demonstrating the low efficiency visually.

References for further reading:

- <https://www.khanacademy.org/science/physics/work-energy-and-power/kinetic-energy/a/what-is-kinetic-energy> — kinetic energy explained with examples
- <https://www.fueleconomy.gov/feg/atv.shtml> — US Dept of Energy: advanced technology vehicles and regenerative braking

Answer – Exercise 14



Home Experiment – Build a Simple Wind Turbine Anemometer

Expected observations: with any noticeable wind outdoors, the anemometer should spin faster at 2 m than at 1 m. The difference may be small (~5–15 %) because the height ratio is only 2:1 and typical shear exponents are 0.1–0.3. Repeated trials are essential because wind is naturally gusty and variable.

Why the cups spin: the open face of a cup has high drag (the cup 'catches' the wind, creating a large force). The curved back presents a lower drag profile. The net torque on the arm is always in the direction of the open-face cup side, so the wheel rotates. This is a drag-based device.

Lift vs. drag (for further reading): real wind turbine blades are aerofoils (cross-sections similar to aircraft wings). When wind flows over an aerofoil, it accelerates over the upper curved surface and slows over the lower surface, creating a pressure difference (Bernoulli effect) that produces a lift force perpendicular to the flow. Lift-based turbines can extract energy much more efficiently than drag-based devices because the blade tip can travel faster than the wind itself (tip-speed ratio > 1), intercepting a larger volume of air per second. Drag-based cups always move slower than the wind.

Wind power ratio calculation:

*Power $\propto v^3$ (with same C_p , ρ , A)
 $P_{2m} / P_{1m} = (v_{2m})^3 / (v_{1m})^3 = (2.3)^3 / (2.0)^3 = 12.17 / 8.0 = 1.52$
So the wind at 2 m height carries about 52% more power than at 1 m – a significant gain
from just doubling the measurement height, illustrating why tower height matters greatly.*

References for further reading:

- <https://www.windpowerengineering.com/how-does-a-wind-turbine-blade-generate-lift/> — blade aerodynamics explained
- <https://www.energy.gov/eere/wind/how-do-wind-turbines-work> — US Department of Energy overview of wind turbine operation
- https://energyeducation.ca/encyclopedia/Wind_profile_power_law — wind profile power law with examples