

# Using a number line and the expression 'Imagine if ...': positive and negative numbers



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through School-based  
Support in India  
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*TESS-India OERs have been collaboratively written by Indian and international authors to address Indian curriculum and contexts and are available for online and print use (<http://www.tess-india.edu.in/>). The OERs are available in several versions, appropriate for each participating Indian state and users are invited to adapt and localise the OERs further to meet local needs and contexts.*

*TESS-India is led by The Open University UK and funded by UK aid from the UK government.*

### **Video resources**

*Some of the activities in this unit are accompanied by the following icon: . This indicates that you will find it helpful to view the TESS-India video resources for the specified pedagogic theme.*

*The TESS-India video resources illustrate key pedagogic techniques in a range of classroom contexts in India. We hope they will inspire you to experiment with similar practices. They are intended to complement and enhance your experience of working through the text-based units, but are not integral to them should you be unable to access them.*

*TESS-India video resources may be viewed online or downloaded from the TESS-India website, <http://www.tess-india.edu.in/>. Alternatively, you may have access to these videos on a CD or memory card.*

Version 2.0 EM03v1  
All India - English

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## What this unit is about

In this unit you will explore ways to encourage your students to think about the meaning of numbers and why the concept of 'negative numbers' was developed.

Students first meet the minus sign when it is used to indicate the mathematical operation of subtraction; therefore, careful introduction to its use in negative numbers will be needed. Explaining that the sign is used differently and exploring why it is used for negative numbers will help your students to recognise and understand the similarities and differences in using this sign.

Through the activities in this unit you will also think about developing the use of a number line to help your students visualise the movements indicated by positive and negative numbers. Actually making those movements themselves will further help the students understand what is meant by 'positive' and 'negative'. The value of saying 'Imagine if ...' to trigger imagination when teaching mathematics is also explored.

## What you can learn in this unit

- Some ideas to help your students understand the difference between positive and negative numbers.
- The role of saying 'Imagine if ...' to trigger the imagination of your students when learning mathematics.
- How to use a number line to understand positive and negative numbers.

This unit links to the teaching requirements of the NCF (2005) and NCFTE (2009) outlined in Resource 1.

## 1 The meaning of numbers

Numbers were probably invented for the purpose of counting animals or other possessions. Numbering systems were originally thought to only have words for 'one', 'two' and 'many', as that was all that was needed. Further developments allowed herds to be counted, and as trade between people increased, the number system we use today was developed – including zero and negative numbers. Number names are almost always structured using a logical procedure so that they can express numbers that are to all intents and purposes, infinite.

Numbers are used to represent:

- quantity, for answering questions like 'How many?' or 'How far?'
- relationships between numbers, for answering questions like 'How many more?' or 'How many less?'
- transformations in terms of quantities, for answering questions like 'Imagine if Zuree owed Mary three rupees. She gave Mary one rupee. How much does she owe now?', or 'Manu won three marbles in the first match and lost five marbles in the second. How many marbles did he lose altogether?'



### Pause for thought

Reflect on how and where your students may have encountered negative numbers. They may, for example, have come across the idea that the temperature in an ice cream freezer is below zero. Where else might they have encountered these ideas?

## Zero is a number

Zero plays a key role in understanding numbers. Mathematically, there are several uses or meanings attributed to zero that students have to work with. With one meaning the quantity 'nothing' is represented. This may mean 'none' as in 'a team scored no goals in a football match', or it may mean 'no tens and no units' as in the number 600.

Zero is also used as a coordinate of an arbitrary point of reference or origin, for example (0, 0). From this point at least two mutually opposite directions can be considered. Understanding that zero has all these different meanings is important when teaching negative numbers.

## Negative numbers

When a negative or minus sign is attached to a number as a prefix, it indicates the polarity of that number with respect to zero. Natural numbers are considered as positive numbers.

Both positive and negative numbers have both magnitude and direction. Negative numbers can cause confusion between magnitude and order. For example,  $-4$  is conventionally less than  $-1$ , despite  $-4$  appearing to have a magnitude of more than  $-1$ .



### Pause for thought

Think back to when you were learning negative numbers. Did it all seem straightforward?

Try to express why negative numbers seemed straightforward to you (if they did). Maybe it was because the negative numbers fitted in with the ideas you already had about natural numbers and extended those ideas in a satisfactory way? Try to remember how you came to understand how to perform mathematical operations on negative numbers – did you learn rules by rote first?

Think about some students in your classroom and the difficulties they have with natural numbers. Think about students you have taught and how they can get mixed up about when to apply the rule 'two negatives make a positive'. How can your students be helped to understand negative numbers and not rely solely on remembering rules?

## 2 The need for negative numbers

The activities in this unit will help you to develop your students' understanding of why negative numbers are used and how useful they can be. They will also offer ideas about how to help your students understand how to work with negative numbers rather than just remembering rules. The first activity is designed to help students appreciate the need for negative numbers as part of the numbering system.

Before attempting to use the activities in this unit with your students, it would be a good idea to complete all, or at least part, of the activities yourself. It would be even better if you could try them out with a colleague, as that will help you when you reflect on the experience. Trying them for yourself will mean you get insights into a learner's experiences, which can, in turn, influence your teaching and your experiences as a teacher.

## Activity 1: Understanding the need for negative numbers

### Preparation

This activity suggests three different ideas to help students appreciate the need for negative numbers as part of the number system. Using all the ideas, not necessarily in the same lesson, will give the students a wider exposure to thinking about negative numbers.

To illustrate 'positive' and 'negative', try to find pictures of mountains and deep seas so that the ideas of 'above' and 'below' can be discussed, along with a zero that is at sea level. Can you think of other situations where positive and negative would be obvious or intuitive to the students?

### The activity

#### Idea 1: Above and below sea level

Draw a large picture, either on a large piece of paper on the wall or on the blackboard. Your picture should show the sea, mountains above the sea, and space below sea level. Use the pictures that you have collected from magazines or drawn. Suitable items would be a plane, an octopus, a whale, a boat, a house, a car, a fish, etc.

Ask the students where they would place the items on your picture. Encourage them to say 'above the sea level' or 'under the sea level'. When all of the items are stuck on, discuss how high the plane might be and how far under the sea the octopus might be, and so on. Introduce the minus sign to indicate 'under the sea level'.

#### Idea 2: Robot steps

Make a space down the centre of the classroom, ensuring that all the students can see this pathway. Mark its centre with a chalk cross and ask a student to stand on the cross. Tell the class to imagine that the student is a robot who only moves forwards and backwards in a straight line. Use pieces of paper or chalk marks to number paces forward from the cross.

Ask the robot to move to 2, then ask them to move two spaces back. Ask the students to say what number should be put on the cross – hopefully they will say zero.

Ask others to give the robot instructions to move to a certain number and then back to a certain number. Now ask the robot to move to 3 and then move four spaces backwards. They have gone beyond zero! What number can be used to represent one step back from zero? Introduce other numbers beyond zero and ask the students to practise saying negative numbers by telling the robot where to move to.

#### Idea 3: A game with benches

Place as many benches as you can across the front of the room and divide the benches into individual seats with chalk lines. Write with chalk a zero on one of the seats (not at the end) and then number the other seats on the benches to the right of zero as 1, 2, 3 and so on. Ask the students how the seats to the left could be numbered. Suggest the negative sign if they do not think of it.

Then play games that involve negative and positive numbers. For example:

- Stand a student behind a seat. The class call out the seat they want the student to move to, for example '5' or '-2' and so on.

- Sit a student on the seat and ask the class to say which seat the student should sit on. Encourage them to use just '3' or '5', and so on for numbers to the right, and 'negative 2', 'negative 4' and so on for numbers to the left of zero.

Next, make the task more difficult. Sit a student on the seat labelled 5 and ask the class what 'move' has to be made to go to seat 2. This is more difficult because 'negative 3' can indicate a position relative to zero and can indicate the action of moving three to the left. Make sure you discuss these two meanings.

Now ask the student to make a move and then ask what move would 'undo' that move.

Use games like this as often as you can to help build confidence. You could stick the numbers to the wall rather than use chairs. In this way the students will use addition and subtraction of negative numbers naturally for the game.



### Video: Using local resources

<http://tinyurl.com/video-usinglocalresources>

## Case Study 1: Mrs Kapur reflects on using Activity 1

*This is the account of a teacher who tried Activity 1 with her elementary students.*

I remember that my classes developed a dislike for negative numbers because there seemed to be so much to remember and it was easy to get mixed up.

I decided to play some of the games in Activity 1 with them. They already knew about negative numbers so they were quick to say that the octopus would be at negative 8 metres. I drew the picture on paper on the wall with a scale marked positive and negative, and left it there after we had done this brief activity. In the morning many of the students arrived with pictures they had drawn so we put them in their correct places on the big picture and had another occasion to think about positive and negative numbers.

Later in the term we played the bench game. They enjoyed this and, although they sometimes found it hard working out a move that is across the zero – from say 5 to  $-2$  – they practised this a lot just because they wanted to keep playing. I definitely think making the actual moves themselves or instructing others to do so helped them to be able to visualise what was happening when we started to do the exercises on negative numbers in the textbook.

To help make the step to using the textbook even easier, I think I will repeat some of these ideas and then also have discussions with the students about how we could record what we are doing in mathematical notation and write this on the blackboard. Hopefully they will then see how the actions relate to the mathematical notation and sums, and to what is asked in the textbooks.

## Reflecting on your teaching practice

When you do such an exercise with your class, reflect afterwards on what went well and what went less well. Consider the questions that led to the students being interested and being able to progress, and those you needed to clarify. Such reflection always helps with finding a 'script' that helps you engage the students to find mathematics interesting and enjoyable. When you reflect on how the ideas in Activity 1 went with your class, make a note, as Mrs Kapur did, of some quite small things that made a difference.



## Pause for thought

In the case study, Mrs Kapur said that she was thinking about repeating some of the activities and recording the outcomes on the blackboard using mathematical notation and sums. What do you feel might be the advantages of doing this after the students have a lot of experience of the activities and games?

Now think about the following questions:

- How did the activities and games go with your class?
- What responses from students were unexpected? What did these tell you about their understanding of positive and negative numbers?
- What questions did you use to probe your students' understanding?
- What points did you feel you had to reinforce?
- What will you do differently next time you use these activities in your teaching?

## 3 Using number lines to develop understanding of positive and negative numbers

A number line such as Figure 1 is a geometrical idea that can be imagined as a set of points arranged in particular order on a straight line. A mathematical line has infinite length and is infinite in mutually opposite directions, but it is always centred at the origin, or zero. A number line can help students make sense of negative numbers and begin to understand adding and subtracting them.



Figure 1 A number line.

A number line can be so useful that it is a good idea to construct and display a large line divided into equally spaced intervals in any classroom where mathematics is learned, as in Figure 2.



Figure 2 A blank number line

Making the line in such a way that the numbers it represents can be written or attached separately will mean that it can be used for thinking about any part of the number system. Each division can then represent:

- units, tens or hundreds, etc.
- fractions or decimals, including very small decimals
- standard form

– as well as many other mathematical ideas.

Once students are used to seeing a number line on a wall or on their desks, they will be able to imagine the line in order to check their reasoning.

The idea of a negative number only has existence in relation to positive numbers using zero as the origin. That is, a point is selected on a number line and zero is assigned to it so that one side of zero is positive and the other negative. In order to help students think in terms of opposites, it is conventional to use the right-hand side of a horizontal line to represent positive numbers and the left-hand side to represent negative numbers. However, it is also a good idea to also use a vertical line, where the numbers above zero are represented by positive numbers and the negative numbers are below zero.

Whether using a horizontal or a vertical line, moving the point that is assigned to zero can help students understand that this is just a part of an infinite line, and that this part is being considered in lessons on negative numbers because zero is where things change.

The following activity uses a number line drawn on a blackboard in a way that will help the students think about how to use negative numbers and how to add and subtract those numbers. The activity also uses the expression 'Imagine if ...'. This expression can help the students to use their imagination and not be limited by their belief that mathematics can only be 'right' or 'wrong'. Knowing this is especially important in mathematical modelling (such as in word problems) where a model represents a perceived situation, which is not necessarily valid in all cases or may not even reflect a true real-life situation (Bruner, 1986).

## Activity 2: Learning from misconceptions and mistakes

### Part 1: How positive was that?

Draw a number line on the blackboard going from  $-10$  to  $10$ . Ask the students to imagine positive things that can happen and ask them to imagine where they would put that on the number line. For example, 'Someone gives me Rs. 10' is a little positive; 'Someone gives me Rs. 100' is more positive.

Then ask them to suggest negative things, for example, 'My new outfit was splashed with mud when a rickshaw went past and I didn't notice', or 'My cricket team lost a match'. Each time ask them to imagine where to put the idea on the number line, asking them to imagine 'How positive do you feel?' or 'How negative was that?'

### Part 2: The 'happiness' model

The ideas in Part 1 can then be extended to adding and subtracting negative numbers.

Say to the class:

I feel OK today; imagine I score 2 (pointing to the number line) on this happiness scale.

Imagine if someone gave me nine sweets (a positive!), how would I feel then? Yes, I could move up 4 to 6.

Now imagine if someone told me I had to stay after school (negative) how would I feel then? Yes, down 1, to 5.

Imagine if you took away seven of my chocolates? How would I feel? Sadder? Yes, I need to go down 7, to  $-2$ .

What if you told me I could go home early?

...

Adding something positive or taking away something negative improves the situation (go up the number line).

Adding something negative or taking away something positive makes the situation worse (go down the number line).

(Source: Part 2 was adapted from NRICH, undated.)



### Video: Talk for learning

<http://tinyurl.com/video-talkforlearning>

## Case Study 2: Mrs Agarwal reflects on using Activity 2

I used the ideas in Activity 2 to explain positive and negative numbers to my class. Before I started I said, 'I believe that adding and subtracting with negative numbers makes sense'.

I wrote a big number line along the top of my blackboard. With the students, I brainstormed on 'Things that are POSITIVE' and 'Things that are NEGATIVE'. We talked for quite a while about how you feel if someone gives you a positive thing, or if someone takes one away. We also talked about how you feel if someone gives you a negative thing, or if someone takes one away.

Then we went on to use the happiness model. I went through getting sweets and losing my sweets, pointing to where I was on the happiness scale and then writing down the mathematical expression of what I was saying. I asked several of the students to tell their own story using the scale and to start with I wrote the sums as they were telling the story.

I then asked the students to work in groups of three or four. They drew a number line in chalk on their desks then one told a story while another pointed to where they were on the number line and another wrote the addition and subtractions that they were doing. I have never seen so many smiles!



### Pause for thought

- How well did Activity 2 go with your class?
- What responses from students were unexpected? Why?
- Did you modify the task in any way? If so, what was your reasoning for this?
- What did you learn about your students' understanding of positive and negative numbers?

## 4 Meanings of addition and subtraction processes

Addition and subtraction are mutually inverse mathematical operations. For example:

$$5 + 1 = 6$$

$$6 - 1 = 5$$

$$6 - 5 = 1$$

According to some researchers (Linchevski and Williams, 1999; Bruno and Martinon, 1999) subtraction skills assist students to learn the concept of 'negative numbers'. The processes of addition and subtraction become interchangeable in the case of integers. For example:

$$3 + 5 = 8 = 3 - (-5)$$

The next activity aims to help your students focus on the thinking processes involved in calculating positive and negative numbers.

### Activity 3: Adding positive and negative numbers

#### Part 1: Using the 'counters' model to understand addition and subtraction with negative numbers

For this activity you will need a quantity of counters or pieces of card in two different colours. One colour should feature positive signs and the other colour should feature negative signs. An important part of this activity is getting students to talk about addition and subtraction with negative numbers and to explain their thinking. When you are planning your lesson you may want to have a look at Resource 2, 'Talk for learning'.

Tell your students that all of the following arrangements of counters add up to four.

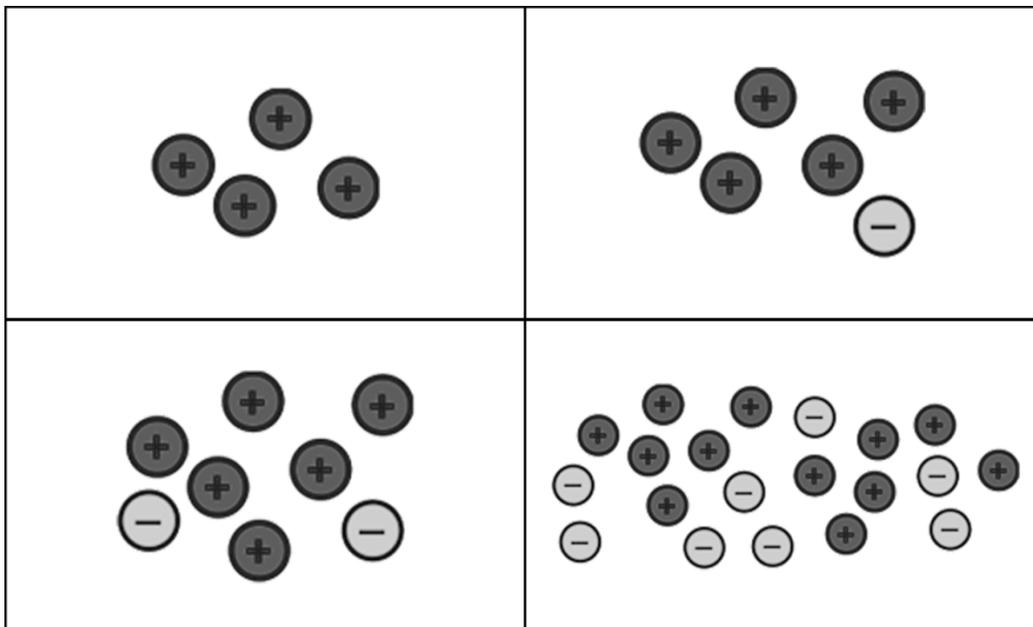


Figure 3 Various arrangements of counters that add up to four.

- Ask the students to suggest some other possibilities.
- Then ask them to explain how all these representations represent 4.
- Now use positive and negative counters to represent  $-2$  in lots of different ways, starting with just two 'negative' counters. Be sure to ask the students to make suggestions themselves.
- Can they explain why all the different representations represent  $-2$ ?

#### Part 2: Using the 'counters' model for larger numbers, in small groups

Put your students into small groups and tell them:

- Using counters or pieces of coloured paper, choose a number under ten and make at least four representations of that number using positive and negative counters.
- Record your representations as addition sums.
- Choose a negative number and do the same.
- If you have time, choose another number that you think will challenge you.
- Write out the sums for each of your representations.

- Swap with another group to mark them.

If you can, go to the NRIC website to find several more ways to develop ideas for using positive and negative counters.

(Source: Part 1 adapted from NRIC, undated.)



### Video: Using local resources

<http://tinyurl.com/video-usinglocalresources>

## Case Study 3: Mrs Nagaraju reflects on using Activity 3

I used the positive and negative counters with my class as they were having difficulty understanding how to work with negative numbers. I started off with some paper plates with positive and negative signs on them and got the students to hold the plates at the front of the class.

They came up with some good ideas of combinations that made 4. It took longer for them to start making suggestions for negative 2 but they soon did. I wrote the total number of positives and the total number of negatives on the blackboard and then asked the students for the sign that meant put them together. They quickly suggested the 'add' sign.

I put the students into groups of six for the next activity because there were nearly 60 in my class and they worked well together. Each group had ten pieces of paper in two colours and they wrote the positive and negative signs on for themselves. They made three different representations for each number they chose and I also made sure that they chose different numbers to the neighbouring groups. They wrote their addition sums on pieces of paper that we stuck on the wall so that everyone could see.

I wanted to see if I could use the same ideas to help them understand what happens when you take away a negative, and of course you can! I got out the paper plates again and made 5 from 8 positives and 3 negatives. I asked what number we got when we took away 2 negatives and they could all tell me the answer was now 7.

I wrote on the blackboard:

$$5 - (-2) = 7$$

For homework I asked them to look in their textbook for three similar examples like  $5 - (-2) = 7$  and to make a counter drawing of what these sums could look like.



### Pause for thought

In the case study, Mrs Nagaraju asked a question to the whole class about which sign was needed to put the combinations together. The correct answer was given, but do you think she could be sure that this point was understood fully by all of the students? What other strategies might she have used to ensure that all of the students were involved in thinking about and discussing the answer?

Now think about the following questions:

- When you carried out Activity 3, what questions did you use to probe your students' understanding?

- Did you feel you had to intervene at any point?
- Were there any points that you feel some or all of your students need to revisit to secure their understanding?

## 5 Summary

This unit has focused on negative numbers, understanding why they are needed and how they can be worked with. In reading this unit you have thought about how to enable your students to develop ways to visualise what negative numbers actually mean and ways to see addition and subtraction with negative numbers. You also used the expression 'Imagine if...' to trigger your students' imagination.

Students need rich and varied experiences of working with negative numbers if they are to see them as a normal part of using mathematics to represent the world. The activities in this unit have looked at many different ways to allow students to experience and understand negative numbers.

You have also seen how reflecting on learning and how learning happens is important in becoming better at teaching.

## Resources

### Resource 1: NCF/NCFTE teaching requirements

This unit links to the following teaching requirements of the NCF (2005) and NCFTE (2009) and will help you to meet those requirements:

- View students as active participants in their own learning and not as mere recipients of knowledge; how to encourage their capacity to construct knowledge; how to shift learning away from rote methods.
- Let students see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.

### Resource 2: Talk for learning

#### Why talk for learning is important

Talk is a part of human development that helps us to think, learn and make sense of the world. People use language as a tool for developing reasoning, knowledge and understanding. Therefore, encouraging students to talk as part of their learning experiences will mean that their educational progress is enhanced. Talking about the ideas being learnt means that:

- those ideas are explored
- reasoning is developed and organised
- as such, students learn more.

In a classroom there are different ways to use student talk, ranging from rote repetition to higher-order discussions.

Traditionally, teacher talk was dominant and was more valued than students' talk or knowledge. However, using talk for learning involves planning lessons so that students can talk more and learn more in a way that makes connections with their prior experience. It is much more than a question and answer session between the teacher and their students, in that the students' own language, ideas, reasoning and interests are given

more time. Most of us want to talk to someone about a difficult issue or in order to find out something, and teachers can build on this instinct with well-planned activities.

## Planning talk for learning activities in the classroom

Planning talking activities is not just for literacy and vocabulary lessons; it is also part of planning mathematics and science work and other topics. It can be planned into whole class, pair or groupwork, outdoor activities, role play-based activities, writing, reading, practical investigations, and creative work.

Even young students with limited literacy and numeracy skills can demonstrate higher-order thinking skills if the task is designed to build on their prior experience and is enjoyable. For example, students can make predictions about a story, an animal or a shape from photos, drawings or real objects. Students can list suggestions and possible solutions about problems to a puppet or character in a role play.

Plan the lesson around what you want the students to learn and think about, as well as what type of talk you want students to develop. Some types of talk are exploratory, for example: 'What could happen next?', 'Have we seen this before?', 'What could this be?' or 'Why do you think that is?' Other types of talk are more analytical, for example weighing up ideas, evidence or suggestions.

Try to make it interesting, enjoyable and possible for all students to participate in dialogue. Students need to be comfortable and feel safe in expressing views and exploring ideas without fear of ridicule or being made to feel they are getting it wrong.

## Building on students' talk

Talk for learning gives teachers opportunities to:

- listen to what students say
- appreciate and build on students' ideas
- encourage the students to take it further.

Not all responses have to be written or formally assessed, because developing ideas through talk is a valuable part of learning. You should use their experiences and ideas as much as possible to make their learning feel relevant. The best student talk is exploratory, which means that the students explore and challenge one another's ideas so that they can become confident about their responses. Groups talking together should be encouraged not to just accept an answer, whoever gives it. You can model challenging thinking in a whole class setting through your use of probing questions like 'Why?', 'How did you decide that?' or 'Can you see any problems with that solution?' You can walk around the classroom listening to groups of students and extending their thinking by asking such questions.

Your students will be encouraged if their talk, ideas and experiences are valued and appreciated. Praise your students for their behaviour when talking, listening carefully, questioning one another, and learning not to interrupt. Be aware of members of the class who are marginalised and think about how you can ensure that they are included. It may take some time to establish ways of working that allow all students to participate fully.

## Encourage students to ask questions themselves

Develop a climate in your classroom where good challenging questions are asked and where students' ideas are respected and praised. Students will not ask questions if they are afraid of how they will be received or if they think their ideas are not valued. Inviting students to ask the questions encourages them to show curiosity, asks them to think in a different way about their learning and helps you to understand their point of view.

Using a number line and the expression 'Imagine if ...': positive and negative numbers

You could plan some regular group or pair work, or perhaps a 'student question time' so that students can raise queries or ask for clarification. You could:

- entitle a section of your lesson 'Hands up if you have a question'
- put a student in the hot-seat and encourage the other students to question that student as if they were a character, e.g. Pythagoras or Mirabai
- play a 'Tell Me More' game in pairs or small groups
- give students a question grid with who/what/where/when/why questions to practise basic enquiry
- give the students some data (such as the data available from the World Data Bank, e.g. the percentage of children in full-time education or exclusive breastfeeding rates for different countries), and ask them to think of questions you could ask about this data
- design a question wall listing the students' questions of the week.

You may be pleasantly surprised at the level of interest and thinking that you see when students are freer to ask and answer questions that come from them. As students learn how to communicate more clearly and accurately, they not only increase their oral and written vocabulary, but they also develop new knowledge and skills.

## Additional resources

- A newly developed maths portal by the Karnataka government: <http://karnatakaeducation.org.in/KOER/en/index.php/Portal:Mathematics>
- National Centre for Excellence in the Teaching of Mathematics: <https://www.ncetm.org.uk/>
- National STEM Centre: <http://www.nationalstemcentre.org.uk/>
- National Numeracy: <http://www.nationalnumeracy.org.uk/home/index.html>
- BBC Bitesize: <http://www.bbc.co.uk/bitesize/>
- Khan Academy's math section: <https://www.khanacademy.org/math>
- NRICH: <http://nrich.maths.org/frontpage>
- Art of Problem Solving's resources page: <http://www.artofproblemsolving.com/Resources/index.php>
- Teachnology: <http://www.teach-nology.com/worksheets/math/>
- Math Playground's logic games: <http://www.mathplayground.com/logicgames.html>
- Maths is Fun: <http://www.mathsisfun.com/>
- Coolmath4kids.com: <http://www.coolmath4kids.com/>
- National Council of Educational Research and Training's textbooks for teaching mathematics and for teacher training of mathematics: <http://www.ncert.nic.in/ncerts/textbook/textbook.htm>
- AMT-01 *Aspects of Teaching Primary School Mathematics*, Block 1 ('Aspects of Teaching Mathematics'), Block 2 ('Numbers (I)'), Block 3 ('Numbers (II)'): <http://www.ignou4ublog.com/2013/06/ignou-amt-01-study-materialbooks.html>
- LMT-01 *Learning Mathematics*, Block 1 ('Approaches to Learning') Block 2 ('Encouraging Learning in the Classroom'), Block 4 ('On Spatial Learning'), Block 6 ('Thinking Mathematically'): <http://www.ignou4ublog.com/2013/06/ignou-lmt-01-study-materialbooks.html>
- *Manual of Mathematics Teaching Aids for Primary Schools*, published by NCERT: <http://www.arvindguptatoys.com/arvindgupta/pks-primarymanual.pdf>
- *Learning Curve* and *At Right Angles*, periodicals about mathematics and its teaching: [http://azimpremjifoundation.org/Foundation\\_Publications](http://azimpremjifoundation.org/Foundation_Publications)

- Textbooks developed by the Eklavya Foundation with activity-based teaching mathematics at the primary level: [http://www.eklavya.in/pdfs/Catalouge/Eklavya\\_Catalogue\\_2012.pdf](http://www.eklavya.in/pdfs/Catalouge/Eklavya_Catalogue_2012.pdf)
- Central Board of Secondary Education's books and support material (also including *List of Hands-on Activities in Mathematics for Classes III to VIII*) – select 'CBSE publications', then 'Books and support material': <http://cbse.nic.in/welcome.htm>

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Using a number line and the expression 'Imagine if ...': positive and negative numbers

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Video (including video stills): thanks are extended to the teacher educators, headteachers, teachers and students across India who worked with The Open University in the productions.