Asking questions that challenge thinking: fractions

Teacher Education through School-based Support in India
www.TESS-India.edu.in

TESS-India (Teacher Education through School-based Support) aims to improve the classroom practices of elementary and secondary teachers in India through the provision of Open Educational Resources (OERs) to support teachers in developing student-centred, participatory approaches. The TESS-India OERs provide teachers with a companion to the school textbook. They offer activities for teachers to try out in their classrooms with their students, together with case studies showing how other teachers have taught the topic and linked resources to support teachers in developing their lesson plans and subject knowledge.

TESS-India OERs have been collaboratively written by Indian and international authors to address Indian curriculum and contexts and are available for online and print use (http://www.tess-india.edu.in/). The OERs are available in several versions, appropriate for each participating Indian state and users are invited to adapt and localise the OERs further to meet local needs and contexts.

TESS-India is led by The Open University UK and funded by UK aid from the UK government.

Video resources

Some of the activities in this unit are accompanied by the following icon: . This indicates that you will find it helpful to view the TESS-India video resources for the specified pedagogic theme.

The TESS-India video resources illustrate key pedagogic techniques in a range of classroom contexts in India. We hope they will inspire you to experiment with similar practices. They are intended to complement and enhance your experience of working through the text-based units, but are not integral to them should you be unable to access them.

TESS-India video resources may be viewed online or downloaded from the TESS-India website, http://www.tess-india.edu.in/). Alternatively, you may have access to these videos on a CD or memory card.

What this unit is about

In this unit you will think about how to introduce fractions to your students.

Some students can see fractions as a very difficult topic to understand. There are many reasons for this, but making sure that your students have rich and varied experiences of working with fractions will help them to develop their understanding.

In this unit you will explore the fact that a fraction only has meaning when looked at in relation to a whole, and consider how to help your students to get to know about different ways to read the symbolic representations of fractions.

Through activities you will also think about the value of asking your students interesting and challenging questions, of getting your students to ask questions themselves and talking about fractions.

What you can learn in this unit

- How to ask effective questions that are interesting and challenging.
- Some ideas to help your students construct their own understanding of fractions.
- Some ideas to help your students talk about fractions.

This unit links to the teaching requirements of the NCF (2005) and NCFTE (2009) outlined in Resource 1.

1 What's so difficult about fractions?

One of the reasons fractions can seem so difficult is that there is a lot to understand. For example, half of something can be smaller than a quarter of something else. An example of this is 'half of six is three' and 'a quarter of sixteen is four'. So learning about fractions by folding pieces of paper or by dividing circles may mislead students, especially if the paper is always the same size. Students must be taught to ask 'A fraction of what?'

Developing an understanding of fractions is not so different from learning to understand other mathematical concepts. For example, very young children are offered many different experiences as they learn to generalise the concept of 'three'.

Despite being older when they learn about fractions, elementary students similarly will need a great many rich and varied experiences if they are to begin to develop a good understanding of fractions.

Many students will have had experiences that help them to develop some understanding of fractions. In her research, Nunes (2006) found that primary school students already have insights into fractions when solving division problems:

> They understand the relative nature of fractions: if one student gets half of a big cake and the other gets half of a small one, they do not receive the same amount. They also realise, for example, that you can share something by cutting it in different ways: this makes it 'different fractions but not different amounts'. Finally, they understand the inverse relation between the denominator and the quantity: the more people there are sharing something, the less each one will get.

Talking fractions: using the language

Encouraging the students to talk about fractions and use the vocabulary will help them understand some of the difficult vocabulary associated with fractions. The questions you use should show the students how important the correct vocabulary is, so that everyone knows what is being referred to.

First, model some ways of talking about fractions and drawing attention to how words are used. Then focus on getting your students talking. The more the students use the words themselves, the more they will build their understanding of fractions. Asking the students to make up questions to ask one another is a good way to get them talking. Another way is to ask the students to explain the reasoning they used to arrive at their answers.

The first activity is for you to think about issues of learning fractions in your classroom.

Activity 1: Thinking about your students learning fractions

Think about what your students need to know about fractions, and make some notes on the different ideas. Use your textbooks. If you have a multigrade class, you will need to think about what different students need to know about fractions:

- how to find out a fraction of a quantity
- what fraction one quantity is of another
- how to add fractions together.

For each of the ideas associated with fractions, write down how the vocabulary associated with those ideas and the way it is used to express ideas. For example 'half of ten' means 'divide 10 by 2', but it can also mean 'multiply 10 by $\frac{1}{2}$'. The students might also see $\frac{10}{2}$, which has the same outcome and is thus equivalent in meaning but which may also be expressed as '10 divided by 2' or '10 shared between 2 people'.

Think about some specific students in your class. What activities might help them to understand the different ways that fractions can be expressed and the different meanings given to those interpretations?

---

2 Developing an understanding of fractions

The second activity focuses on students physically representing the concepts of fractions. This is also called embodiment. You will ask them to use their bodies to represent mathematical ideas. If the students move themselves to make fractions of a whole, they will begin to develop their concept of what a fraction is and how they can work with fractions.

Before attempting to use the activities in this unit with your students, it would be a good idea to complete all, or at least part, of the activities yourself. It would be even better if you could try them out with a colleague as that will help you when you reflect on the experience. Trying them for yourself will mean you get insights into a learner's experiences, which can, in turn, influence your teaching and your experiences as a teacher.

Activity 2: Physically representing fractions

Preparation

First create a space, and ask eight students to come to the front of the class or somewhere where the rest of the class can see them.

The activity

- Ask your students to arrange themselves into a rectangle.
- Ask someone else to divide the group into half.
- Reform the rectangle, then ask another student to divide the group in half in a different way.
- Ask the students what is the same and what is different about the new half of the group.
- Now ask another student to divide the eight students into quarters (fourths). Again ask whether there is a different way to do this division, and what is the same and what is different about the new way of dividing into quarters.
- Now change the number of students and go through the process above again. It may be that dividing into quarters is difficult but depending on the chosen number, continue asking for $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$ and so on, until a fraction that cannot be done is reached. Ask the students why you cannot find that fraction of these students. Dividing one student into bits is not allowed!
- Ask the students to work in groups of 12. You could appoint a leader in each group to note down ideas if the class does not split evenly into groups of 12. Ask them to work out all the fractions they can divide 12 students into.

Video: Using questioning to promote thinking
http://tinyurl.com/video-usingquestioning

Case Study 1: Mrs Rawool reflects on using Activity 1

This is the account of a teacher who tried Activity 1 with his elementary students.

First, I invited eight students to come to the front of the class and to form themselves into a rectangular shape where the rest of the class could see them. I then asked student Anoushka to come and divide these eight students in half, which was easy to do.

I then asked the class if the group of eight students could be divided in half in another way. This proved to be a little challenging, as the students were used to mathematics questions having just one answer, so they wondered at first if Anoushka was wrong. They needed clarification about what 'different' meant here. Of course, whichever way they divided the students in half, there were always four students in each half. Since this was the answer I was looking for, I gave them time to talk about these ideas.

Next, I asked student Nita to come to the front and divide the group into quarters. This time the students were able to suggest different ways to achieve this, and they were happy there would always be two students in each part.

I then asked another group of students to come to the front, this time with six students. This time I asked them to divide themselves into half in two ways. I asked 'Do you always get the same answer?' 'Yes sir!' they said. Then I asked 'What other fraction can you divide yourselves into?' They tried to divide themselves into quarters but they could not, but what they did find was that they could divide themselves into three parts and discussed what this fraction was called.

I then put the class into groups of 12 and asked them what fractions they could embody in their groups. One group came up with twelfths, but most worked happily on halves, quarters, thirds and sixths.

Reflecting on your teaching practice

When you do such an exercise with your class, reflect afterwards on what went well and what went less well. Consider the questions that led to the students being interested and being able to progress, and those you needed to clarify. Such reflection always helps with finding a 'script' that helps you engage the students to find mathematics interesting and enjoyable. If they do not understand and cannot do something, they are less likely to become involved. Use this reflective exercise every time you undertake the activities, noting as Mrs Rawool did some quite small things that made a difference.

Pause for thought

Good questions to trigger such reflection are:

- How did it go with your class?
- What responses from students were unexpected? Why?
- What questions did you use to probe your students' understanding?
- Did you feel you had to intervene at any point?
- What points did you feel you had to reinforce?
- Did you modify the task in any way? If so, what was your reasoning for this?

3 Asking questions effectively

Teachers ask a lot of questions in their work – some research suggests that teachers ask up to 400 questions every day when they are teaching! The better the questions that teachers ask, the better their teaching will be.

Much research has been undertaken about good questions, for example by Wragg and Brown (2001) and Hattie (2008).The research concludes that effective questions:

- are firmly linked to the learning of the lesson
- build on students' previous knowledge
- involve, interest and motivate the students
- are sequenced to encourage higher-order thinking (but not too soon!)
- enable the students to build their own knowledge
- reveal misconceptions and misdirections
- prompt and challenge thinking and reasoning

Pause for thought

Reflect on the questions you asked in the last lesson you taught.

- Did they challenge the students to think?
- Could a small change have uncovered more about the students' current learning?
- Could your questions have encouraged the students to build their own learning more?

Activity 3 asks you to first prepare for asking effective questions, and then to try out these questions when teaching your students.

Activity 3: Asking effective questions about fractions

Part 1: Preparing to ask effective questions

If you can do this part of the activity with another teacher, you may find that it is easier.

Think about the next lesson in which you will teach on fractions. What is it you want the students to know? Write some notes about that now.

What previous knowledge do you think they will need in order to understand the ideas you want them to learn? Write a question which will enable you to know whether or not they have that prior knowledge. For example, you could ask your students: 'Can you give me an example of …? And another? And another? And another? And another?' Asking for more examples could help you to find out the extent of their knowledge and some of the students' misconceptions.

Think about some of the ways that fractions are used in the real world. Write a question that might interest or engage the students because it is based on something they know about and use.

Now write an easy question for the particular topic you have to teach and then write a hard question. Write a sequence of questions that will challenge your students – but not too much!

Think about all the ways that misconceptions can happen in fractions. Write two or three questions that will help you check whether or not your students have these misconceptions. You can find some examples of such questions in Case Study 2. It is also important to think ahead about how you might respond to your students' answers in the best way to reinforce learning and extend their thinking. You can use Resource 2 to help you think about some ideas for how to receive your students' responses.

Now write a question that will encourage your students to reason their way to a solution. For example, 'Your big sister never believes what you say. How will you convince her that your method works?'

Part 2: Using your effective questions in the classroom

Now you have written these questions, use them with a class.

Did you think that the class learned more because they used these questions?

Don't forget to use real objects to allow your students to work with ideas on fractions and to approach challenging questions through a process of reasoning.

Video: Planning lessons

http://tinyurl.com/video-planninglessons

Case Study 2: Mrs Mohanty questions the students to check their understanding of fractions

When thinking about Part 1 of Activity 3, I decided I would use my normal introduction to fractions by demonstrating fractions on the blackboard as usual, but to be very precise and repetitive in the questions and instructions I was going to use. I wrote them down on a piece of paper, and put them on my desk so I would not forget them.

These are the questions and prompts I prepared:

- Show me how you divide this circle into halves/quarters/eighths.
  - How do you know this is correct?
  - Please describe your method clearly.
  - Would anyone do it another way?
- Show me on this circle one half/one quarter/one eighth.
  - How do you know this is correct?
  - Please describe your method clearly.
  - Would anyone do it another way?
- Show me on this circle one third/one sixth/one twelfth.
  - How do you know this is correct?
  - Please describe your method clearly.
  - Would anyone do it another way?
- Show me on this circle one third/one fifth/one seventh.
  - How do you know this is correct?
  - Please describe your method clearly.
  - Would anyone do it another way?
- Show me on this circle three quarters/six eights.
  - How do you know this is correct?
  - Please describe your method clearly.
  - Would anyone do it another way?

I drew the circle using chalk. I then invited students to come to the blackboard, and asked them the questions. Having the questions written down really helped me to focus and helped to avoid diversions from what I had intended to do. I also noticed that as a result there was less 'teacher talk' and more student talk and student work.

Pause for thought

- What questions did you use to probe your students' understanding?
- Did you feel you had to change your planned questions at any time? Why?
- How effective did you feel your responses to the students' answers were in reinforcing learning and helping you to understand the way your students think?

4 Effective questioners give students time to think

Mary Budd Rowe (1986) researched the 'wait time' that teachers allowed after asking a question. 'Wait time' is the length of quiet time that teachers allow after asking a question before they expect a student to answer, or before they rephrase the question or even answer the question themselves. Her team analysed 300 tape recordings of teachers asking questions over six years. They found the mean wait time was 0.9 seconds.

If you ask a question that requires the students to think, are you really giving them enough time to think, or are you only giving them time to instantly react?

The teachers in Budd Rowe's research were trained so that they became able to increase their wait time to between three and five seconds. The increased wait time resulted in:

- an increased length of student response
- an increased number of unsolicited, but appropriate, replies
- a decreased failure to respond
- an increased confidence of response
- an increase in the incidence of students comparing their answers to those from another student
- the number of alternative explanations offered multiplying.

In other words, the students had more time to think and that increased the level (and quality) of discussion that went on in the classroom, which in turn meant the teachers learnt more about their students thinking and were able to act on any misconceptions. Increasing wait time is not easy to do and can feel odd when you start, but if your students are to think, they must be given sufficient time.

Activity 4 asks you to experiment with increasing the wait time in your classroom in a similar way.

Activity 4: increasing the wait time

Like the teachers in Budd Rowe's research, in your next lesson, increase your wait time for students to respond to five seconds. After the lesson, reflect on whether you observed:

- an increased length of student response
- an increased number of unsolicited, but appropriate, replies
- a decreased failure to respond
- an increased confidence of response
- an increase in the incidence of students comparing their answers to those from another student
- the number of alternative explanations offered multiplying.

The next activity links together many of the ideas that have been discussed so far. It suggests that you:

- ask the students to work with concrete objects to answer some challenging questions
- ask the students to work together so that they can support one another
- give them more time to think.

Activity 5: Learning about fractions

Preparation

This activity is an example of the kinds of rich activity that students need to build their understanding of fractions. For this task you will need a quantity of paper plates, or card cut into rectangles of the same size.

Arrange the students to work in groups of three or four, and give them a pile of paper plates or cards. You may want to look at the key resource 'Using groupwork' (http://tinyurl.com/kr-usinggroupwork) to help you prepare for this.

The activity

- Ask your students first to show you half of a plate, then a quarter of a plate. It is important not to tell them to take one plate here; let them think this out for themselves.
- Then ask each group to show you 'half of six' using the plates.

Make sure everyone is able to do this before carrying on.

- Ask the students to suggest several fraction problems that they can solve using the plates. Each time ask the class to show you the solution using the plates. If they don't suggest questions using just one plate then prompt them.

The idea is to give everyone time to play a little with fractions and to think about what fractions are.

- Ask the students what is the same and what is different about the two types of questions they have worked on so far. This will help the students recognise some of the different ways that fractions are used in mathematics.

Figure 1 A group of students using plates to learn about fractions.

Now move on to problems that mix the two ideas.

- Ask the class to find $\frac{1}{4}$ of 12, then ask them to find $\frac{1}{4}$ of 13.
- When the class have had time to solve this, ask one group to explain the process they went through in order to solve the second problem.
- Now ask the students to suggest how they could record the problem and the answer using mathematical notation. Spend time on this, as it is important that the students recognise the relationship between the way the problems are written and what they are doing with the plates.
- Now ask the class to solve other 'hard fraction problems' using the plates, for example $\frac{1}{5}$ of 12, or $\frac{1}{4}$ of 10, which require two plates to be shared out.
- Again, ask for suggestions and discuss how these ideas might be written down.
- Ask each group to make up an easy problem and a hard problem with fractions to give to another group to do. Ask each group to record their answers.

Case Study 3: Mr Bhatia reflects on using Activity 5

I gave each group 12 paper plates. The plates were to help to support the students' thinking that finding fractions is about sharing out equally.

First, I set them the task of dividing the plates into quarters. I asked several of the groups to talk about the process of dividing into quarters. Then I asked them to divide their 12 plates into thirds. When they had done this, I once again asked the students to explain how they did it. I made sure that everyone was

comfortable sharing out the objects that they were working with, in this case the plates. The students enjoyed working and collaborating together in groups and completing the task.

I then decided that the class was ready for a more challenging question. I gave each group one more plate, so that they then had 13 plates, and again asked them to divide the plates into quarters and then thirds. This time the students discovered that they needed to subdivide the extra plate in order to share out the plates equally into quarters and thirds.

This time I spent more time on the feedback session in order to make sure that everyone understood the reason why one of the plates had to be subdivided. I then asked the class to divide the plates into thirds, and this time I offered them scissors as well. Several students gave some good reasons why they needed to divide up the extra plate, but working in groups helped them all to try out their ideas first before telling the whole class.

Pause for thought

- What questions did you use to probe your students' understanding?
- Did you feel you had to intervene at any point?
- What points did you feel you had to reinforce?
- Did you modify the task in any way like Mr Bhatia did? If so, what was your reasoning for this?

5 Summary

This unit has focused on teaching fractions, but you have also looked at how to ask questions that require students to think and the importance of giving students sufficient time to think.

In studying this unit you have thought about how to enable your students to develop their ideas about fractions and about the necessity of providing rich and varied activities if students are to learn, understand and use ideas about fractions.

You have also seen how reflecting on learning, and how learning happens, is important in becoming better at teaching.

Pause for thought

Identify three techniques or strategies you have learnt in this unit that you might use in your classroom, and two ideas that you want to explore further.

Resources

Resource 1: NCF/NCFTE teaching requirements

This unit links to the following teaching requirements of the NCF (2005) and NCFTE (2009) and will help you to meet those requirements:

- View students as active participants in their own learning and not as mere recipients of knowledge; how to encourage their capacity to construct knowledge; how to shift learning away from rote methods.

- Let students see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.
- Let students learn important mathematics and see mathematics is more than formulas and mechanical procedures.

Resource 2: Receiving answers from students

Your response matters

The more positively you receive all answers that are given, the more students will continue to think and try. There are many ways to ensure that wrong answers and misconceptions are corrected, and if one student has the wrong idea, you can be sure that many more have as well. You could try the following:

- Pick out the parts of the answers that are correct and ask the student in a supportive way to think a bit more about their answer. This encourages more active participation and helps your students to learn from their mistakes. The following comment shows how you might respond to an incorrect answer in a supportive way: 'You were right about evaporation forming clouds, but I think we need to explore a bit more about what you said about rain. Can anyone else offer some ideas?'
- Write on the blackboard all the answers that the students give, and then ask the students to think about them all. What answers do they think are right? What might have led to another answer being given? This gives you an opportunity to understand the way that your students are thinking and also gives your students an unthreatening way to correct any misconceptions that they may have.

Value all responses by listening carefully and asking the student to explain further. If you ask for further explanation for all answers, right or wrong, students will often correct any mistakes for themselves, you will develop a thinking classroom and you will really know what learning your students have done and how to proceed. If wrong answers result in humiliation or punishment, then your students will stop trying for fear of further embarrassment or ridicule.

Improving the quality of responses

It is important that you try to adopt a sequence of questioning that doesn't end with the right answer. Right answers should be rewarded with follow-up questions that extend the knowledge and provide students with an opportunity to engage with the teacher. You can do this by asking for:

- a *how* or a *why*
- another way to answer
- a better word
- evidence to substantiate an answer
- integration of a related skill
- application of the same skill or logic in a new setting.

Helping students to think more deeply about (and therefore improve the quality of) their answer is a crucial part of your role. The following skills will help students achieve more:

- **Prompting** requires appropriate hints to be given – ones that help students develop and improve their answers. You might first choose to say what is right in the answer and then offer information, further questions and other clues. ('So what would happen if you added a weight to the end of your paper aeroplane?')
- **Probing** is about trying to find out more, helping students to clarify what they are trying to say to improve a disorganised answer or one that is partly right. ('So what more can you tell me about how this fits together?')

- **Refocusing** is about building on correct answers to link students' knowledge to the knowledge that they have previously learnt. This broadens their understanding. ('What you have said is correct, but how does it link with what we were looking at last week in our local environment topic?')
- **Sequencing** questions means asking questions in an order designed to extend thinking. Questions should lead students to summarise, compare, explain or analyse. Prepare questions that stretch students, but do not challenge them so far that they lose the meaning of the questions. ('Explain how you overcame your earlier problem. What difference did that make? What do you think you need to tackle next?')
- **Listening** enables you to not just look for the answer you are expecting, but to alert you to unusual or innovative answers that you may not have expected. It also shows that you value the students' thinking and therefore they are more likely to give thoughtful responses. Such answers could highlight misconceptions that need correcting, or they may show a new approach that you had not considered. ('I hadn't thought of that. Tell me more about why you think that way.')

As a teacher, you need to ask questions that inspire and challenge if you are to generate interesting and inventive answers from your students. You need to give them time to think and you will be amazed how much your students know and how well you can help them progress their learning.

Remember, questioning is not about what the teacher knows, but about what the students know. It is important to remember that you should never answer your own questions! After all, if the students know you will give them the answers after a few seconds of silence, what is their incentive to answer?

Additional resources

- A newly developed maths portal by the Karnataka government: http://karnatakaeducation.org.in/KOER/en/index.php/Portal:Mathematics
- National Centre for Excellence in the Teaching of Mathematics: https://www.ncetm.org.uk/
- National STEM Centre: http://www.nationalstemcentre.org.uk/
- National Numeracy: http://www.nationalnumeracy.org.uk/home/index.html
- BBC Bitesize: http://www.bbc.co.uk/bitesize/
- Khan Academy's math section: https://www.khanacademy.org/math
- NRICH: http://nrich.maths.org/frontpage
- Art of Problem Solving's resources page: http://www.artofproblemsolving.com/Resources/index.php
- Teachnology: http://www.teach-nology.com/worksheets/math/
- Math Playground's logic games: http://www.mathplayground.com/logicgames.html
- Maths is Fun: http://www.mathsisfun.com/
- Coolmath4kids.com: http://www.coolmath4kids.com/
- National Council of Educational Research and Training's textbooks for teaching mathematics and for teacher training of mathematics: http://www.ncert.nic.in/ncerts/textbook/textbook.htm
- AMT-01 *Aspects of Teaching Primary School Mathematics*, Block 1 ('Aspects of Teaching Mathematics'), Block 2 ('Numbers (I)'), Block 3 ('Numbers (II)'), Block 4 ('Fractions'): http://www.ignou4ublog.com/2013/06/ignou-amt-01-study-materialbooks.html
- LMT-01 *Learning Mathematics*, Block 1 ('Approaches to Learning') Block 2 ('Encouraging Learning in the Classroom'), Block 4 ('On Spatial Learning'), Block 5 ('Exploring Numbers'), Block 6 ('Thinking Mathematically'): http://www.ignou4ublog.com/2013/06/ignou-lmt-01-study-materialbooks.html

- *Manual of Mathematics Teaching Aids for Primary Schools*, published by NCERT: http://www.arvindguptatoys.com/arvindgupta/pks-primarymanual.pdf
- *Learning Curve* and *At Right Angles*, periodicals about mathematics and its teaching: http://azimpremjifoundation.org/Foundation_Publications
- Textbooks developed by the Eklavya Foundation with activity-based teaching mathematics at the primary level: http://www.eklavya.in/pdfs/Catalouge/Eklavya_Catalogue_2012.pdf
- Central Board of Secondary Education's books and support material (also including *List of Hands-on Activities in Mathematics for Classes III to VIII*) – select 'CBSE publications', then 'Books and support material': http://cbse.nic.in/welcome.htm

References/bibliography

Bell, A. (1987) 'Diagnostic teaching 3: provoking discussion', *Mathematics Teaching*, vol. 118, pp. 21–3.

Budd Rowe, M. (1986) 'Wait time: slowing down may be a way of speeding up!', *Journal of Teacher Education*, vol. 43, pp. 44–50. Abstract available from: http://jte.sagepub.com/cgi/content/abstract/37/1/43 (accessed 3 February 2014).

Hastings, S. (2003) 'Questioning', *TES Newspaper*, 4 July. Available from: http://www.tes.co.uk/article.aspx?storycode=381755 (accessed 22 September 2014).

Hattie, J. (2008) *Visible Learning: A Synthesis of Over 800 Meta-analyses Relating to Achievement*. New York, NY: Routledge.

Hattie, J. (2012) *Visible Learning for Teachers: Maximising the Impact on Learning*. Abingdon: Routledge.

National Council for Teacher Education (2009) *National Curriculum Framework for Teacher Education* (online). New Delhi: NCTE. Available from: http://www.ncte-india.org/publicnotice/NCFTE_2010.pdf (accessed 3 February 2014).

National Council of Educational Research and Training (2005) *National Curriculum Framework* (NCF). New Delhi: NCERT.

Nunes, T. (2006) *Fractions: Difficult but Crucial in Mathematics Learning*, Teaching and Learning Research Brief, Economic and Social Research Council, UK. Available from: http://www.tlrp.org/pub/documents/no13_nunes.pdf (accessed 3 February 2014).

Watson, A., Jones, K. and Pratt, D. (2013) *Key Ideas in Teaching Mathematics*. Oxford: Oxford University Press.

Wragg, E. and Brown, G. (2001) *Questioning in the Secondary School*. London: RoutledgeFalmer.

Zack, V. and Graves, B. (2001) 'Making mathematical meaning through dialogue: "Once you think of it, the Z minus three seems pretty weird"', *Educational Studies in Mathematics,* vol. 46, pp. 229–71.

Acknowledgements