Elementary Mathematics

TESS India

Using real-life contexts: the formal division algorithm

Teacher Education through School-based Support in India
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TESS-India OERs have been collaboratively written by Indian and international authors to address Indian curriculum and contexts and are available for online and print use (http://www.tess-india.edu.in/). The OERs are available in several versions, appropriate for each participating Indian state and users are invited to adapt and localise the OERs further to meet local needs and contexts.

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Video resources

Some of the activities in this unit are accompanied by the following icon: . This indicates that you will find it helpful to view the TESS-India video resources for the specified pedagogic theme.

The TESS-India video resources illustrate key pedagogic techniques in a range of classroom contexts in India. We hope they will inspire you to experiment with similar practices. They are intended to complement and enhance your experience of working through the text-based units, but are not integral to them should you be unable to access them.

TESS-India video resources may be viewed online or downloaded from the TESS-India website, http://www.tess-india.edu.in/). Alternatively, you may have access to these videos on a CD or memory card.

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All India - English

What this unit is about

In this unit you will learn about introducing your students to the division algorithm through contexts that they are familiar with. You will also consider how students can express the division algorithm for themselves, giving it real meaning rather than just being an abstract idea.

Through activities you will think about developing your students' ability to work together to understand quite complex ideas, sharing out the work so that more ideas can be explored and more connections understood. You will also think about how helping the students visualise what is going on can help them to be able to use mathematical ideas with greater control.

What you can learn in this unit

- How to help your students understand the mathematical ideas behind the division algorithm.
- Some suggestions for using group work in the maths classroom.
- Some ideas to help your students see the connections between mathematical ideas and the real world.

This unit links to the teaching requirements of the NCF (2005) and NCFTE (2009) outlined in Resource 1.

1 The formal division algorithm

The formal division algorithm below describes very explicitly and formally what division is:

> Given any strictly positive integer d (divisor) and any integer a, there exist unique integers q (quotient) and r (remainder) such that
>
> $a = qd + r$
>
> and
>
> $0 < r < d$

A strictly positive integer is an integer that is greater than 0.

The description of division given by the two conditions $a = qd + r$ and $0 < r < d$ is operational. This means that the two conditions give a very explicit way of testing whether or not q is the quotient and r the remainder when the first number (a) is divided by the second (d).

However, these two conditions are not procedural, which means that they do not provide a method for actually finding the quotient and remainder (Lady, 2000). Knowing that 45 is equal to some number multiplied by six, plus some other number, does not in itself give you the procedures to find out what those numbers are.

The formal division algorithm leans towards finding the number that you must multiply the quotient by in order to find a number that is very close to the number a. This is important to note, because if a student's only secure conception of division is that of sharing (e.g. if I share 45 sweets equally between 6 children how many do they get each? How many are left over?), or grouping (e.g. how many groups of 6 can I make out of 45? How many are left over?), then they will have problems understanding the formal algorithm.

Encouraging your students to think deeply about division is important, and they are much more likely to understand the division algorithm if they have thought about the ideas behind it in a context meaningful to them which they can then express for themselves.

Pause for thought

Think about how you have taught division in the past. How do you naturally talk about division?

For example, if one of your students is having trouble working out 24 ÷ 6, what first comes to your mind? Is it the inverse of multiplication ('Think about your six times table – how many sixes make 24?'), grouping ('How many groups of six could you make out of 24?') or something else?

Have you talked to your students about what is the same and what is different about these various ways of talking about division? How could these different ways of talking about division confuse your students?

2 Being able to see what is happening in division

The concept of division can be difficult for students to grasp, partly because there are many ways of talking about it. When you have a problem such as '42 divided by 6', you can read it as:

- 'How many times does 6 go into 42?'
- 'How many groups of 6 can be made out of 42?'
- 'How many would be in each of the six groups?'
- 'What is one-sixth of 42?'

Although the answer is always 7, the ways you can get to that answer can be very different, and can result in confusion for some students.

Helping students to be aware that there is more than one way to think about division and to be alert to the possibility of ambiguity in everyday language will support their mathematical development. It is important to teach the students to remember to think very carefully about the meaning of every problem they encounter that asks them to divide, such as 'divide 42 into 6'.

Being able to visualise what is happening is an important step to understanding the algorithm. In the following activity you will ask the students to use division intuitively and in context to find the quotient and the remainder. In later activities you will build on the thinking the students have developed in this activity to more deeply explore the division algorithm.

Before attempting to use the activities in this unit with your students, it would be a good idea to complete all, or at least part, of the activities yourself. It would be even better if you could try them out with a colleague, as that will help you when you reflect on the experience. Trying them for yourself will mean you get insights into learners' experiences that can in turn influence your teaching and your experiences as a teacher.

Activity 1: Dividing up lengths

Preparation

This activity sets out to challenge students' mathematical understanding of division. If your students are younger or have less experience with division, use easier numbers – it is the picture they build and the thinking they do that are important here.

This activity is best done with groups of four to six students working together. You may want to look at Resource 2, 'Using groupwork', to help you prepare for this activity.

The activity

Tell your students the following:

Rajni is to have new ceramic floor tiles in her bedroom (Figure 1). The length of the bedroom floor is 5,273 mm and its breadth is 4,023 mm.

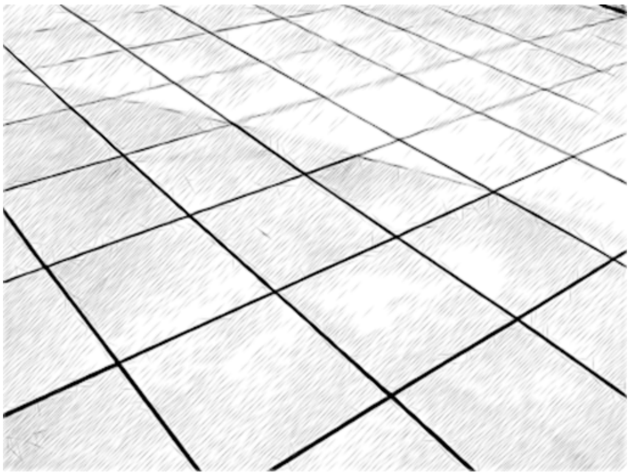

Figure 1 Tiles for Rajni's bedroom floor.

She has looked at a catalogue and shortlisted two designs of tiles:

- Pink Sparrow
- Rosewood Matte.

Pink Sparrow is a square tile of 600 mm in length and Rosewood Matte is a square tile of 450 mm in length.

- What is the shape of Rajni's bedroom floor?
- Draw Rajni's bedroom floor in your notebook. Before you start, think about this: you cannot draw a rectangle of length 5273 mm and breadth 4023mm in a notebook. How will you manage to draw a rectangle that represents Rajni's bedroom floor? Discuss in your group.
- (Note for the teacher: If your class is younger or you have not yet covered scales, skip to the next question). Now:
    - o Draw three different-sized diagrams, using different scales of Rajni's bedroom floor as accurately as you can. (Use just one scale if different scales makes the activity too difficult.) Remember to note the scale used next to the diagram. Describe the differences between the three diagrams.
    - o In each of the three drawings, cover the floor diagram with Pink Sparrow tiles and Rosewood Matte tiles.
    - o Is the number of tiles used to cover the diagrams of different sizes the same? Why?
- How many rows of tiles would Rajni use to cover the floor with each type of tile? Would these rows cover the entire floor? Why, or why not?
- How many columns of tiles would Rajni use to cover the floor with each type of tile? Would these columns cover the entire floor? Why, or why not?

Video: Using groupwork

http://tinyurl.com/video-usinggroupwork

Case Study 1: Mrs Agarwal reflects on using Activity 1

This is the account of a teacher who tried Activity 1 with her elementary students.

I tried this activity with a class that was having major problems with the division algorithm. It seemed as though they could not understand what it really meant when they were dividing two quantities.

I reduced the numbers for this activity to three-digit and two-digit numbers so that they could do the arithmetic more easily, as I wanted them to think about and visualise what they were doing. I thought that some of them would have a problem representing the room on paper, so I did some exercises with their atlases where we saw how scales were used to represent large distances. After that, when I actually did this activity, most of them had the idea and could draw the three bedroom floor figures.

Arun wanted to know what I meant by 'three sizes'. Mita answered his query by saying that maybe we could take different scale sizes. I gave them sheets of squared paper and they all got busy with their drawings. It was taking some time, so I gave them a time limit and said that they should delegate the different sizes to different students in their group.

When they had done their drawings of the bedroom, I asked them if they could make copies so that each student in the group had one copy to use to make the tile layout. I then asked them to cover the floor with either Pink Sparrow tiles or Rosewood Matte tiles.

I remembered to ask: 'Why is the number of tiles the same, even though the scales are different?' At first they were unsure of how to answer but eventually someone said 'Because it's the same room' and another said 'We had to use the scale for the tiles as well'. I was pleased with these answers because they weren't just mechanically doing what was asked, but were thinking about what the mathematics meant.

Then we had the discussion on the last two questions with contributions from the entire class. They were a lot quicker at coming up with answers to the 'Why?' questions this time and I was able to help them see that they were dividing up the space and doing divisions, which helped them a great deal. The remainder meant something here as well – 'the bit left over' that you would have to cut a tile to fill was a real concept for them.

Reflecting on your teaching practice

When you do such an exercise with your class, reflect afterwards on what went well and what went less well. Consider the questions that led to the students being interested and able to progress, and those you needed to clarify. Such reflection always helps with finding a 'script' that helps you engage the students to find mathematics interesting and enjoyable. If students do not understand and cannot do something, they are less likely to become involved. Use this reflective exercise every time you undertake the activities, noting as Mrs Agarwal did some quite small things that made a difference.

Pause for thought

Good questions to trigger reflection are:

- How did it go with your class?
- What responses from students were unexpected? Why?
- What questions did you use to probe your students' understanding?
- Did you feel you had to intervene at any point?
- What points did you feel you had to reinforce?
- How well did your students understand the mathematical concept?
- Did you modify the task in any way? If so, what was your reasoning for this?

3 Making connections

The first activity shows how division is important in everyday calculations and how the division algorithm can be made more understandable by using it in a context that the students may be familiar with. In mathematics, division is an important part of many topics. In fact, the big concept of proportionality, which incorporates fractions and ratio, relies on students' understanding of division.

For example, a proportional division of a cake is one that gives each person at least a $\frac{1}{n}$ portion, which can be called a 'proportional share'.

The next activity starts to extend the context of Activity 1 in order to help the students build their ideas of division.

Activity 2: The meaning of a remainder

Tell your students the following.

A box contains four tiles. Rajni has to calculate the number of boxes of tiles that she will need to buy to cover the floor.

- From the diagram that you drew in Activity 1, calculate the total area of the rectangle that represents the floor. How is that area related to the total area of the actual bedroom?
- What is the area that is covered by one tile of each kind?
- What is the total area that can be covered if Rajni buys one box of tiles of each kind?
- How many boxes will Rajni need in total?
- Will Rajni be able to completely use all the tiles she buys? How many tiles will remain unused? Is the remainder the same for both designs of tile, Pink Sparrow and Rosewood Matte?

Case Study 2: Mrs Nagaraju reflects on using Activity 2

This was done the next day after doing Activity 1. I had collected the diagrams made by the students when we did Activity 1 so that they would not have to draw them again. I distributed the drawings back to the groups that they had worked in yesterday. Then I asked them to now work on the questions for this activity.

I reminded the groups that they would all have to be prepared to answer the questions I put up on the blackboard, so all of them should understand what the group is doing and not leave it to one or two to do all the calculations, and more importantly to know why they had done the calculations that they had.

Of course, they all had their carefully (or not so carefully) drawn diagrams from yesterday showing the tiles all laid out. Many groups started a discussion on whether they should just count the tiles drawn on their diagrams. Some of them did the counting and then the division and so both verified their answers, which showed they were really starting to understand the connections. But not all tried it by the same method.

It was an interesting discussion about how the remainder in both the cases was different and why that was. I also liked how the students linked 'what remains' to the word 'remainder' in the whole-class discussions and I noticed that in some of the group discussions they also used the words 'what is left over' and 'leftover' to describe this.

Pause for thought

- What responses from students were unexpected? What did these responses tell you about their understanding of the meaning of a remainder?
- What points did you feel you had to reinforce?
- Did you modify the task in any way? If so, what was your reasoning for this?

4 The division algorithm again

The next activity returns to considering the formal division algorithm. It is framed to ask the students to start to formulate the division algorithm for themselves.

Activities 1 and 2 have asked the students to visualise what is happening when they use division and also to start to talk about remainders. Activity 3 begins the process of formalising these ideas into the kind of mathematical language that can be found in mathematical textbooks. The activity also starts the process, (continued in Activity 4), of asking the students to write mathematics as mathematicians do.

Activity 3: The division algorithm

Tell your students the following:

Rajni finally decides to use the Pink Sparrow tiles for her floor.

- How many complete tiles will be used along the length of the floor? What is the total length covered by these tiles? How much is the length of the floor that remains uncovered after the maximum number of tiles have been put along the length?
- Let q = number of complete tiles used along the length of the room.
- Let r = the length of the room that remains uncovered after the maximum number of complete tiles have been put along the length.
- Write a relation between: length of room, length of one tile, q and r.
- Repeat the first step using the breadth of the floor.

Video: Assessing progress and performance

http://tinyurl.com/video-assessingprogress

Case Study 3: Mrs Agarwal reflects on using Activity 3

The students had done the two previous activities where they had seen that the tiles could completely cover the area of the floor lengthwise, except for one small bit, therefore they could tell how much of the length was not covered by the tiles.

I told them that this activity was going to ask them to be mathematicians and to use symbols to represent what they were doing when they used division. This seemed to spur them on and they worked hard in their groups for a short time, quickly writing the relationship between these various quantities in symbols. I asked several students to come to the blackboard and write down the relationship that their group had decided on. There were some small differences but the class quickly agreed on the way that we, as mathematicians, should write the division algorithm for this situation.

After they had done the work on the lengthwise relationship, I asked them to work in the same way on the breadth and they came up with that relationship very quickly as well.

It was good to see that even the students who normally would not take part in maths activities were contributing to the whole exercise. I think they could relate to this context, visualising what was going on, much better than if I had just told them the division algorithm. When I told them what the relationship they had found was called, the division algorithm, they all felt very important writing mathematics as mathematicians and coming up with such a formal sounding relationship.

After that I asked them to use the division algorithm to verify their answers to the previous activities and they could all do it and were able to, I think, understand what division actually means.

Pause for thought

Mrs Agarwal commented that even students who normally were reluctant to take part in maths activities made a contribution. This is great! What approaches might she have used to find out for certain that they had understood the division algorithm, or whether they still needed further opportunities to consolidate their understanding? After reflecting on this, you may find it useful to have a look at the key resource 'Assessing progress and performance' (http://tinyurl.com/kr-assessingprogress).

5 Working on a bigger scale

Asking the students to work on a real-life scale can be useful in helping them to see division in the world around them. The next activity continues to ask the students to explore the division algorithm and to use symbolic mathematical language, but this time they will be working on a bigger scale.

Thinking about a real situation and preparing an answer to a problem they have identified will help the students further develop their powers of visualisation and give visual meaning to symbolic mathematics.

Activity 4: Using the division algorithm in a real-life situation

Part 1: In the school grounds

Preparation

In this activity students will create a tiling plan for a particular part of their school, for example an area of the playground.

When taking students to work in the school grounds (Figure 2), you should always make sure your students are aware of safety hazards they might encounter, such as moving vehicles or building works, and prepare for changes in the weather.

Figure 2 Using the school ground for an activity.

The activity

Divide the class into groups of two or three students. Ask each group to do the following:

- Identify some part or portion of the school where they would like to lay tiles. The constraint is that the shape of the portion must be a rectangle. They can choose stairs or steps, hallways, rooms, open areas, floors or walls, etc.
- Measure the length L and breadth B of the floor or wall they would tile.
- Choose the length (l) of the square tile with which they want to tile the floor or wall.
- For the chosen tile, calculate the value of q and r (refer to previous activity) for both L and B.

Once they are done, every group comes back to the classroom to report.

Part 2: Back in the classroom

Preparation

On the blackboard, make a table as shown in Table 1 with as many rows as there are groups.

Table 1 *Using the division algorithm in a real-life situation.*

	L	l	q	r	B	l	q	r
Group 1								
Group 2								
Group 3								
Group 4								
Group 5								
Group …								

The activity

Ask the groups to fill in their row in the table on the blackboard with their findings from Part 1 of this activity. Discuss the following with the class:

- What is the same and what is different between the observations of each group of students?
- Did some of you get the value of r as 0? Why do you think you got this value?
- If you wanted to make sure that r = 0, how would you change the values of l?
- If for both L and B the value of r = 0, what is the relation between L and B?
- How is the relation between (L, l, q and r) linked to the division algorithm?

Case Study 4: Mr Chadha reflects on using Activity 4

Now this was an extremely interesting activity for the students to take part in. I had to, of course, get tape measures for them to take the measurements. They all had a great time making a plan of the different areas they could measure and they enjoyed discussing where tiles would be a very good idea.

Some of the groups were talking about trying to measure some difficult-shaped areas, so I advised them to keep it simple. They went out quickly once their plans were made because I told them they had just ten minutes to get their measurements and be back in the classroom! I stood outside the classroom with my watch reminding them to be quick about what they had to do.

When they were all back in the classroom they had to decide whether to use big or little tiles; some of them thought that big tiles would be best but then did not like the division sums that they had set themselves so went for smaller ones. After about another five minutes they all had their answers ready to go into the table which I had drawn on the blackboard.

The discussion about whether any of them got r = 0 was interesting and there was a lot of talk about how and when they could get that. It also led to the talk about multiplicands and the divisibility of numbers and when we could say a number can be completely divided by another. We also had a talk about how remainders can be any number left when we remove a certain multiple of the divisor and why we try to remove the maximum numbers of times of the divisor. This included thinking about how the remainder could not be more than the divisor.

> I think by the end of this activity I can say that most of my students were able to understand the division algorithm and would know why they were doing what they were doing when using it.

Pause for thought

- What responses from students were unexpected? What did this reveal about their understanding of the division algorithm?
- What questions did you use to probe your students' understanding?
- Did you modify the task in any way? If so, what was your reasoning for this?

6 Summary

This unit has focused on the division algorithm, how to help students understand about division and how divisional algorithms work.

In reading this unit you have thought about how to enable your students to use contexts that they are likely to experience in everyday life in order to be able to visualise what they are doing. In this way they will develop an understanding of what the division algorithm means.

You can use these ideas for other mathematical concepts. Setting students a 'real-life' problem engages them and helps them to see how mathematics is useful outside of school. Working in groups enables all students to discuss and develop ideas collaboratively. Dialogue is important in the development of their mathematical understanding.

From this unit you will have considered how to help students understand each part of the division algorithm by coming up with the relationship for themselves rather than being presented with the algorithm unrelated to their own world. You will have noticed that asking the students to look at the concept of division in different ways helps them make important connections.

You have also seen how reflecting on your teaching is important in becoming better at supporting the learning of your students.

Pause for thought

Identify three techniques or strategies you have learned in this unit that you might use again in your classroom.

Resources

Resource 1: NCF/NCFTE teaching requirements

This unit links to the following teaching requirements of the NCF (2005) and NCFTE (2009) and will help you to meet those requirements:

- View learners as active participants in their own learning and not as mere recipients of knowledge; how to encourage their capacity to construct knowledge; how to shift learning away from rote methods.
- Let students see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.

Resource 2: Using groupwork

Groupwork is a systematic, active, pedagogical strategy that encourages small groups of students to work together for the achievement of a common goal. These small groups promote more active and more effective learning through structured activities.

The benefits of groupwork

Groupwork can be a very effective way of motivating your students to learn by encouraging them to think, communicate, exchange ideas and thoughts, and make decisions. Your students can both teach and learn from others: a powerful and active form of learning.

Groupwork is more than students sitting in groups; it involves working on and contributing to a shared learning task with a clear objective. You need to be clear about why you are using groupwork for learning and know why this is preferable to lecturing, pair work or to students working on their own. Thus groupwork has to be well-planned and purposeful.

Planning groupwork

When and how you use groupwork will depend on what learning you want to achieve by the end of the lesson. You can include groupwork at the start, the end or midway through the lesson, but you will need to allow enough time. You will need to think about the task that you want your students to complete and the best way to organise the groups.

As a teacher, you can ensure that groupwork is successful if you plan in advance:

- the goals and expected outcomes of the group activity
- the time allocated to the activity, including any feedback or summary task
- how to split the groups (how many groups, how many students in each group, criteria for groups)
- how to organise the groups (role of different group members, time required, materials, recording and reporting)
- how any assessment will be undertaken and recorded (take care to distinguish individual assessments from group assessments)
- how you will monitor the groups' activities.

Groupwork tasks

The task that you ask your students to complete depends on what you what them to learn. By taking part in groupwork, they will learn skills such as listening to each other, explaining their ideas and working cooperatively. However, the main aim is for them to learn something about the subject that you are teaching. Some examples of tasks could include the following:

- **Presentations:** Students work in groups to prepare a presentation for the rest of the class. This works best if each group has a different aspect of the topic, so they are motivated to listen to each other rather than listening to the same topic several times. Be very strict about the time that each group has to present and decide on a set of criteria for a good presentation. Write these on the board before the lesson. Students can the use the criteria to plan their presentation and assess each other's work. The criteria could include:
  - Was the presentation clear?
  - Was the presentation well-structured?
  - Did I learn something from the presentation?
  - Did the presentation make me think?

- **Problem solving:** Students work in groups to solve a problem or a series of problems. This could include conducting an experiment in science, solving problems in mathematics, analysing a story or poem in English, or analysing evidence in history.
- **Creating an artefact or product:** Students work in groups to develop a story, a piece of drama, a piece of music, a model to explain a concept, a news report on an issue or a poster to summarise information or explain a concept. Giving groups five minutes at the start of a new topic to create a brainstorm or mind map will tell you a great deal about what they already know, and will help you pitch the lesson at an appropriate level.
- **Differentiated tasks:** Groupwork is an opportunity to allow students of different ages or attainment levels to work together on an appropriate task. Higher attainers can benefit from the opportunity to explain the work, whereas lower attainers may find it easier to ask questions in a group than in a class, and will learn from their classmates.
- **Discussion:** Students consider an issue and come to a conclusion. This may require quite a bit of preparation on your part in order to make sure that the students have enough knowledge to consider different options, but organising a discussion or debate can be very rewarding for both you and them.

Organising groups

Groups of four to eight are ideal but this will depend on the size of your class, the physical environment and furniture, and the attainment and age range of your class. Ideally everyone in a group needs to see each other, talk without shouting and contribute to the group's outcome.

- Decide how and why you will divide students into groups; for example, you may divide groups by friendship, interest or by similar or mixed attainment. Experiment with different ways and review what works best with each class.
- Plan any roles you will give to group members (for example, note taker, spokesperson, time keeper or collector of equipment), and how you will make this clear.

Managing groupwork

You can set up routines and rules to manage good groupwork. When you use groupwork regularly, students will know what you expect and find it enjoyable. Initially it is a good idea to work with your class to identify the benefits of working together in teams and groups. You should discuss what makes good groupwork behaviour and possibly generate a list of 'rules' that might be displayed; for example, 'Respect for each other', 'Listening', 'Helping each other', 'Trying more than one idea', etc.

It is important to give clear verbal instructions about the groupwork that can also be written on the blackboard for reference. You need to:

- direct your students to the groups they will work in according to your plan, perhaps designating areas in the classroom where they will work or giving instructions about moving any furniture or school bags
- be very clear about the task and write it on the board in short instructions or pictures. Allow your students to ask questions before you start.

During the lesson, move around to observe and check how the groups are doing. Offer advice where needed if they are deviating from the task or getting stuck.

You might want to change the groups during the task. Here are two techniques to try when you are feeling confident about groupwork – they are particularly helpful when managing a large class:

- **'Expert groups':** Give each group a different task, such as researching one way of generating electricity or developing a character for a drama. After a suitable time, re-organise the groups so that each new group is made up of one 'expert' from all the original groups. Then give them a task that involves collating knowledge from all the experts, such as deciding on what sort of power station to build or preparing a piece of drama.
- **'Envoys':** If the task involves creating something or solving a problem, after a while, ask each group to send an envoy to another group. They could compare ideas or solutions to the problem and then report back to their own group. In this way, groups can learn from each other.

At the end of the task, summarise what has been learnt and correct any misunderstandings that you have seen. You may want to hear feedback from each group, or ask just one or two groups who you think have some good ideas. Keep students' reporting brief and encourage them to offer feedback on work from other groups by identifying what has been done well, what was interesting and what might be developed further.

Even if you want to adopt groupwork in your classroom, you may at times find it difficult to organise because some students:

- are resistant to active learning and do not engage
- are dominant
- do not participate due to poor interpersonal skills or lack of confidence.

To become effective at managing groupwork it is important to reflect on all the above points, in addition to considering how far the learning outcomes were met and how well your students responded (did they all benefit?). Consider and carefully plan any adjustments you might make to the group task, resources, timings or composition of the groups.

Research suggests that learning in groups need not be used all the time to have positive effects on student achievement, so you should not feel obliged to use it in every lesson. You might want to consider using groupwork as a supplemental technique, for example as a break between a topic change or a jump-start for class discussion. It can also be used as an ice-breaker or to introduce experiential learning activities and problem solving exercises into the classroom, or to review topics.

Additional resources

- 'The division algorithm' by E.L. Lady: http://www.math.hawaii.edu/~lee/courses/Division.pdf
- A newly developed maths portal by the Karnataka government: http://karnatakaeducation.org.in/KOER/en/index.php/Portal:Mathematics
- National Centre for Excellence in the Teaching of Mathematics: https://www.ncetm.org.uk/
- National STEM Centre: http://www.nationalstemcentre.org.uk/
- National Numeracy: http://www.nationalnumeracy.org.uk/home/index.html
- BBC Bitesize: http://www.bbc.co.uk/bitesize/
- Khan Academy's math section: https://www.khanacademy.org/math
- NRICH: http://nrich.maths.org/frontpage
- Art of Problem Solving's resources page: http://www.artofproblemsolving.com/Resources/index.php
- Teachnology: http://www.teach-nology.com/worksheets/math/
- Math Playground's logic games: http://www.mathplayground.com/logicgames.html
- Maths is Fun: http://www.mathsisfun.com/
- Coolmath4kids.com: http://www.coolmath4kids.com/

- National Council of Educational Research and Training's textbooks for teaching mathematics and for teacher training of mathematics: http://www.ncert.nic.in/ncerts/textbook/textbook.htm
- AMT-01 *Aspects of Teaching Primary School Mathematics*, Block 1 ('Aspects of Teaching Mathematics'), Block 2 ('Numbers (I)'), Block 3 ('Numbers (II)'): http://www.ignou4ublog.com/2013/06/ignou-amt-01-study-materialbooks.html
- LMT-01 *Learning Mathematics*, Block 1 ('Approaches to Learning') Block 2 ('Encouraging Learning in the Classroom'), Block 4 ('On Spatial Learning'), Block 6 ('Thinking Mathematically'): http://www.ignou4ublog.com/2013/06/ignou-lmt-01-study-materialbooks.html
- *Manual of Mathematics Teaching Aids for Primary Schools*, published by NCERT: http://www.arvindguptatoys.com/arvindgupta/pks-primarymanual.pdf
- *Learning Curve* and *At Right Angles*, periodicals about mathematics and its teaching: http://azimpremjifoundation.org/Foundation_Publications
- Textbooks developed by the Eklavya Foundation with activity-based teaching mathematics at the primary level: http://www.eklavya.in/pdfs/Catalouge/Eklavya_Catalogue_2012.pdf
- Central Board of Secondary Education's books and support material (also including *List of Hands-on Activities in Mathematics for Classes III to VIII*) – select 'CBSE publications', then 'Books and support material': http://cbse.nic.in/welcome.htm

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Figure 2: Clare Lee

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