## Comparing and contrasting tasks: volume and capacity



TESS-India (Teacher Education through School-based Support) aims to improve the classroom practices of elementary and secondary teachers in India through the provision of Open Educational Resources (OERs) to support teachers in developing student-centred, participatory approaches. The TESS-India OERs provide teachers with a companion to the school textbook. They offer activities for teachers to try out in their classrooms with their students, together with case studies showing how other teachers have taught the topic and linked resources to support teachers in developing their lesson plans and subject knowledge.

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## Video resources

Some of the activities in this unit are accompanied by the following icon:
This indicates that you will find it helpful to view the TESS-India video resources for the specified pedagogic theme.

The TESS-India video resources illustrate key pedagogic techniques in a range of classroom contexts in India. We hope they will inspire you to experiment with similar practices. They are intended to complement and enhance your experience of working through the text-based units, but are not integral to them should you be unable to access them.

TESS-India video resources may be viewed online or downloaded from the TESS-India website, http://www.tess-india.edu.in/). Alternatively, you may have access to these videos on a CD or memory card.

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## What this unit is about

'Compare and contrast' is an activity to make students aware of mathematical properties and their applications. It is effective for learning about subtle samenesses and differences. When you compare, you identify what is the same; when you contrast, you identify what is different.

Measuring is a skill that is used frequently in everyday life. For example: measuring the quantity of water to be added for cooking, the quantity of fuel to fill up your car, the length of cloth for a new dress, etc. Estimation is often used in many such daily measurements, for example: add abouttwo cups of water, the car will need about half a tank of fuel, etc. In school mathematics exact measures and correct units are usually needed.

Capacity and volume are measurements related to three-dimensional objects which are often confused by students. In this unit you will think about helping your students to understand the similarities and differences between capacity and volume by using the teaching technique of 'compare and contrast'.

## What you can learn in this unit

- How to use the 'compare and contrast' technique to help students notice mathematical properties.
- Some effective ways to teach the difference between volume and capacity.
- Some teaching ideas to promote understanding measurement of three-dimensional objects.

This unit links to the teaching requirements of the NCF (2005) and NCFTE (2009) as in Resource 1 and will help you to meet those requirements.

## 1 'Compare and contrast' tasks to learn about mathematical properties

'Compare and contrast' is a technique to make students aware of mathematical properties and applications. It is effective for learning about subtle samenesses and differences. When you compare you identify what is the same, when you contrast you identify what is different.

The actions of comparing and contrasting force us to think about the properties of mathematical objects and to notice what is the same and what is different. While doing so, students may make connections they might not normally consider. They are prompted into mathematical thinking processes such as generalising, conjecturing about what stays the same and what can change (called 'variance' and 'invariance'), and then verifying these conjectures. This is an example of the national curriculum requirements of helping students use abstractions to perceive relationships, to 'see' structures, to reason things out for themselves, and to argue the truth or falsity of statements.

Volume and capacity are properties of three-dimensional objects. Volume is the space that a threedimensional object occupies or contains; capacity, on the other hand, is the property of a container and describes how much a container can hold. Students often get confused by these two concepts (Watson et al., 2013). Activity 1 will help your students to become aware of the characteristics and measurements of three-dimensional shapes. The activity also requires the students to start thinking intuitively about the difference between volume and capacity.

Before attempting to use the activities in this unit with your students, it would be a good idea to complete all, or at least part, of the activities yourself. It would be even better if you could try them out with a
colleague as that will help you when you reflect on the experience. Trying them for yourself will mean you get insights into learners' experiences which can, in turn, influence your teaching and your own experiences as a teacher.

When you are ready, use the activities with your students and, once again, reflect and make notes on how well the activity went and the learning that happened. This will help you to develop a more student-focused teaching environment.

## Activity 1: Exploring three-dimensional objects

- Ask your students to name any object that they have used during the previous day. As they name objects, write them on the black board. You will end up with a list of objects, e.g. glass, tube of toothpaste, plate, book, pen, pencil, coins, ruler, paper, bowl, knife, spoon, bottle, eraser, chalk, telephone, television, bucket, mug, towel, ball, etc.
- Once your students are done, circle some of these objects and ask them if they can find something that is common to all the circled objects. Choosing objects for which the students could easily estimate the three dimensions will save time.

Now arrange the students into small groups or pairs. Ask the students the following questions:

- For each circled object, estimate the following:

Table 1 Estimating template.

| Object | Length | Width/breadth | Height |
| :--- | :--- | :--- | :--- |
| Glass |  |  |  |
| Tube of toothpaste |  |  |  |
| Book |  |  |  |
| Pencil |  |  |  |
| Coin |  |  |  |
| Bottle |  |  |  |
| Television |  |  |  |

- If all these objects were made of gold, which would be the most expensive (or least expensive)? Then, arrange these objects in the increasing order of their worth.
- In comparing the worth of each object above, which measurement was most useful? Why?

Ask the students to present their findings to the whole class. Not all students have to agree. As long as their reasoning is based on mathematical properties and on logic, then all arguments are acceptable.

## Case Study 1: Mrs Meganathan reflects on using Activity 1

This is the account of a teacher who tried Activity 1 with her elementary students.
To get ideas for a list of objects the students had used the day before, I started with writing 'glass' and 'book' on the blackboard and said I had used these yesterday. Because asking them what objects they had used yesterday would be rather unusual for me to do, I thought it would help focus their minds.

They came up with lots of examples, which I wrote on the blackboard. To be honest, some of them would be really awkward and complicated to calculate the volume of, for example a bicycle! I could have left such examples on the blackboard for them to work with later in the activity but I was not sure how I would handle that as a teacher. So I said that I would now pick six of these objects, and picked the ones for which the three dimensions were easier to estimate. I think next time I actually will feel more confident in leaving the 'awkward' examples on the blackboard as well.

I put the students in groups of four or five - this I can do easily by asking every other row of students to turn around, so putting them into groups does not take too much time or hassle.

I drew the table on the blackboard as it says in the activity and wrote all the questions at once on the blackboard. I had been thinking whether to do that one step at a time but thought that having them all on the blackboard at once would:

- give the students an idea of how the activity would develop
- give more learning time to the students because they would not have to wait for others to finish each question.

This worked well, apart from at one stage, where I felt I was really running from one group to another to answer their questions, like 'How do we do this?' or 'What do we do next?' So I did stop the class after some time and said that if they had a question, please first check with the neighbouring group whether they knew. It became much more manageable for me after that!

The question about the worth of the object if it was made of gold did make them think about ideas to do with volume and capacity, without actually using those terms. The presentations and the discussions that followed developed these ideas further and proved very good scaffolding to lead to the thinking required in Activity 2.

## Pause for thought

What do you think about Mrs Meganathan's solution to finding herself running between groups? What additional strategies might she have used to make this part of the lesson more manageable? You may already have some good ideas for this, but if you are fairly new to working in this way have a look at Resource 2, 'Managing groupwork'.

## Reflecting on your teaching practice

When you do such an exercise with your class, reflect afterwards on what went well and what went less well. Consider the questions that led to the students being interested and being able to progress, and those you needed to clarify. Such reflection always helps with finding a 'script' that helps you engage the students to find mathematics interesting and enjoyable. If they do not understand and cannot do something, they are less likely to become involved. Use a reflective exercise every time you undertake the activities.

## Pause for thought

Now reflect on how your class got on with Activity 1:

- How did the different groups get on with discussing the size of the objects?
- What questions did you use to probe your students' understanding?
- What responses from students were unexpected? Why?
- How did you feel about teaching a lesson in which you had to use actual mathematical terms?
- How did your students respond to this approach?


## 2 Thinking about capacity and volume and their units of measurement

'Volume' is the space that a three-dimensional object occupies or contains. Volume can be quantified in different ways depending on its physical state. For example, the volume of a cuboid solid is calculated by measuring the height, breadth and length as the students were attempting to estimate in Activity 1. In this case the volume will be quantified in $\mathrm{cm}^{3}, \mathrm{~m}^{3}$ or inches ${ }^{3}$.

Fluid displacement is another way to determine the volume of solids or gas. Fluid displacement involves immersing the object in a fluid. The volume of the object will displace the fluid. Such displacement of the fluid can be measured. In that case it will be expressed in millilitres, litres, fluid ounces, or cups.

The volume of fluids, or a quantity of small loose objects such as grains of rice, can be measured by pouring them into a measuring tool such as a measuring cup (Figure 1).


Figure 1 A domestic measuring cup or jug
Capacity, on the other hand, is the property of a container. It describes how much a container can hold. Confusion can arise from the fact that the measures used for capacity are usually the same as those used for volume.

The next activity aims to let the students understand the conceptual difference between capacity and volume of three-dimensional objects by using two 'compare and contrast' techniques. Parts 1 and 2 of the activity use the question 'Is this always, sometimes or never true?' to help students become aware of mathematical properties of volume and capacity. Part 3 poses the question 'What is the same and what is different?' to achieve the same awareness.

For the students to be able to focus on the samenesses and differences, and not to be caught up in the minutiae of measuring and calculating with precision, some of the examples used are unusual but real. Using such examples also helps to introduce a sense of playfulness into mathematics, as there are many right answers.

## Activity 2: Compare and contrast - capacity and volume

This activity works well for students working in small groups or pairs. Do not make the groups too large because then not all students will be able to contribute to the discussion. There are many right answers to these questions and not all students have to agree. As long as their reasoning is based on mathematical properties and on logic, then accept their arguments.

## Part 1: Capacity

List the objects on the blackboard. Add some more unusual ones if you want to.

| Object | Can this object <br> contain a liquid? Is <br> this always, <br> sometimes or <br> never true? |
| :--- | :--- |
| An elephant's <br> trunk |  |
| A beehive |  |
| An orange |  |
| A bucket |  |
| A water tank |  |
| A mosquito's <br> tummy |  |
| A lake |  |
| A sea |  |
| A glass |  |
| A coconut |  |

Instruct your students to:
Discuss with your classmates whether it is always, sometimes or never true that these objects [written on the blackboard] can contain a liquid? Tell them you will ask for their reasons in five minutes' time. Then ask the students to help you complete the table on the blackboard.

Discuss with the whole class the reasons for the students' decisions. Only then add the word 'capacity' to make another heading of the column in your table.

| Object | Can this object <br> contain a liquid? Is <br> this always, <br> sometimes or <br> never true? <br> (CAPACITY) | Ranking order of <br> the capacity of the <br> objects |
| :--- | :--- | :--- |
| An elephant's <br> trunk |  |  |
| A beehive |  |  |
| An orange |  |  |
| A bucket |  |  |
| A water tank |  |  |
| A mosquito's <br> tummy |  |  |
| A lake |  |  |
| A sea |  |  |
| A glass |  |  |
| A coconut |  |  |

Ask the students to number these objects on the basis of decreasing capacity: number the object with the biggest capacity 1 , the object with the second-biggest capacity 2 , and so on.
Then discuss with the whole class the reasons for the students' decisions. A good question to ask here is 'How do you know this object has the biggest capacity?' If the students do not think of the role of measurements which they explored in Activity 1, remind them of it. You may want to have a look at the key resource 'Using questioning to promote thinking' (http://tinyurl.com/kr-usingquestioning) to help you prepare for this part of the activity.

## Part 2: Volume

Add another column to the table on the blackboard:

| Object | Can this object <br> contain a liquid? Is <br> this always, <br> sometimes or <br> never true? <br> (CAPACITY) | Ranking order of <br> the capacity of the <br> objects | This object, <br> without any liquid <br> in it, occupies <br> space. Is this <br> always, sometimes, <br> or never true? |
| :--- | :--- | :--- | :--- |
| An elephant's <br> trunk |  |  |  |
| A beehive |  |  |  |
| An orange |  |  |  |
| A bucket |  |  |  |
| A water tank |  |  |  |
| A mosquito's <br> tummy |  |  |  |
| A lake |  |  |  |
| A sea |  |  |  |
| A glass |  |  |  |
| A coconut |  |  |  |

Ask the students to discuss amongst themselves whether it is always, sometimes or never true that these objects [on the blackboard] occupy space when they are empty? Tell them you will be asking for their reasons in five minutes. Then complete the table on the blackboard.

Discuss with the whole class the reasons for the students' decision. Lastly, add the word 'volume' to make a final column, as shown below.

| Object | Can this object <br> contain a liquid? Is <br> this always, <br> sometimes or <br> never true? <br> (CAPACITY) | Ranking order of <br> the capacity of the <br> objects | This object, <br> without any liquid <br> in it, occupies <br> space. Is this <br> always, <br> sometimes, or <br> never true? <br> (VOLUME) | Ranking order of <br> the volume of the <br> objects |
| :--- | :--- | :--- | :--- | :--- |
| An elephant's <br> trunk |  |  |  |  |
| A beehive |  |  |  |  |
| An orange |  |  |  |  |
| A bucket |  |  |  |  |
| A water tank |  |  |  |  |
| A mosquito's <br> tummy |  |  |  |  |
| A lake |  |  |  |  |
| A sea |  |  |  |  |
| A glass |  |  |  |  |
| A coconut |  |  |  |  |

Ask your students to arrange the objects on the basis of decreasing volume. Number the object with the biggest volume 1, the object with the second-biggest volume 2 , and so on.

Discuss the reasons for the students' decisions together.

## Part 3: Comparing and contrasting capacity and volume

Ask the students to look again at the completed table. Ask what is the same and what is different for the objects? Do all objects have capacity and volume? If an object has the biggest capacity, does that mean it also has the biggest volume?

Tell them to discuss in their groups and to prepare to share their thoughts with the whole class in five minutes.

Then discuss with the whole class the reasons for the students' decisions.

## Video: Using questioning to promote thinking

http://tinyurl.com/video-usingquestioning

## Case Study 2: Mrs Meganathan reflects on using Activity 2

I thought this activity would take a whole lesson and it did. I asked the students again to work in groups of four or five.

They loved the examples! Samir, one of the students who always asks questions, asked whether it was a fully grown elephant or a baby one. So I told him he could take either and then grade them accordingly. I think all the groups ended up working it out for both!

I think the activity worked well because we had done Activity 1 in the previous lesson which had made them think about the concepts of capacity and volume. I also noticed that perhaps because the examples such as mosquito's tummy are so unusual, and actually a bit absurd, the students who normally never speak out in the mathematics lesson, and never put their hand up, were now making suggestions and they made sense as well.

When the groups were reporting on their findings, for about half of the groups I made a point of asking the 'weaker' student to report their group's ideas. I had never done this before, but was impressed with the explanations that were given. I also noticed that their reasoning had become more convincing and sophisticated as we heard more and more explanations from the different groups.

We normally do not work with mathematical activities that have many right answers - they tend to be questions that you get right or wrong. So this was new, both for me and the students. To help them be more open in that kind of thinking I told them what it said in the activity description:

There are many right answers to these questions and not all students have to agree. As long as their reasoning is based on mathematical properties and on logic, then accept their arguments.

I think this helped both me and the students to indeed focus on what were the mathematical properties, and on flaws in logical thinking. I think it is the first time I actually felt confident in understanding the difference between capacity and volume.

## Pause for thought

Mrs Meganathan explained how she had chosen some of the 'weaker' students to report their group's ideas. What do you think are the benefits of doing this, and what strategies might you use to ensure that it is a positive experience for the student giving the report?

Now think about how your class got on with the activity and reflect on the following questions:

- What questions did you use to probe your students' understanding?
- Did you modify the task in any way like Mrs Meganathan did?
- If so, what was your reasoning for this?


## 3 Consolidating knowledge

To help students really get to grips with mathematical concepts it is good practice to think carefully about including consolidation activities in your lessons. Such activities give the students more opportunity for practising their thinking. Good consolidation activities can also ask students to use their newly acquired knowledge from a different perspective. The next activity aims to do this by making the students think about subtle changes, and then by asking them to construct their own questions.

## Activity 3: Consolidation activity - sometimes, always, never true

In preparation for this activity ask each student to bring one bottle or container from home. In class, ask them to randomly exchange their containers with some other student. Alternatively, bring in a variety of bottles yourself and put them where all the students can see them.

## Part 1: Evaluating statements

Ask your students which of the following statements are sometimes true, always true or never true? Why?
a) The volume of the shampoo bottle is 150 ml .
b) The capacity of the shampoo bottle is 150 ml .
c) The volume of shampoo in the bottle is 150 ml .
d) The capacity of shampoo in the bottle is 150 ml .
e) The volume of shampoo that a bottle can hold is 150 ml .

## Part 2: Constructing questions of their own

1. Looking at the container the students have been given (or they can see displayed), ask the students to write in a random order:

- two correct statements - one using the term 'volume' and the other using the term 'capacity'
- two incorrect statements - one using the term 'volume' and the other using the term 'capacity'.

2. Ask the students to exchange their statements with the question:

Which of the following statements are sometimes true, always true or never true? Why?

## Pause for thought

- How effective do you think the activity was in consolidating your students' understanding?
- Did the activity uncover any misconceptions? If so, how might you address these in future lessons?
- Did you modify the task in any way? If so, what was your reasoning for this?


## 4 Summary

In studying this unit you have explored what is the same and what is different about capacity and volume. You have considered how the activity of 'compare and contrast' allows students to discover and understand mathematical properties and subtle differences.

Requirements for teaching from NCF (2005) and NCFTE (2009) were used as ambitious goals.


## Pause for thought

Identify three ideas that you have used in this unit that would work when teaching other topics. Make a note of two topics that you have to teach soon where those ideas can be used with some small adjustments.

## Resources

## Resource 1: NCF/NCFTE teaching requirements

- View students as active participants in their own learning and not as mere recipients of knowledge; how to encourage their capacity to construct knowledge; how to shift learning away from rote methods.
- Let students see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.


## Resource 2: Managing groupwork

You can set up routines and rules to manage good groupwork. When you use groupwork regularly, students will know what you expect and find it enjoyable. Initially it is a good idea to work with your class to identify the benefits of working together in teams and groups. You should discuss what makes good groupwork behaviour and possibly generate a list of 'rules' that might be displayed; for example, 'Respect for each other', 'Listening','Helping each other', 'Trying more than one idea', etc.

It is important to give clear verbal instructions about the groupwork that can also be written on the blackboard for reference. You need to:

- direct your students to the groups they will work in according to your plan, perhaps designating areas in the classroom where they will work or giving instructions about moving any furniture or school bags
- be very clear about the task and write it on the board in short instructions or pictures. Allow your students to ask questions before you start.

During the lesson, move around to observe and check how the groups are doing. Offer advice where needed if they are deviating from the task or getting stuck.

You might want to change the groups during the task. Here are two techniques to try when you are feeling confident about groupwork - they are particularly helpful when managing a large class:

- 'Expert groups’: Give each group a different task, such as researching one way of generating electricity or developing a character for a drama. After a suitable time, re-organise the groups so that each new group is made up of one 'expert' from all the original groups. Then give them a task that involves collating knowledge from all the experts, such as deciding on what sort of power station to build or preparing a piece of drama.
- 'Envoys': If the task involves creating something or solving a problem, after a while, ask each group to send an envoy to another group. They could compare ideas or solutions to the problem and then report back to their own group. In this way, groups can learn from each other.

At the end of the task, summarise what has been learnt and correct any misunderstandings that you have seen. You may want to hear feedback from each group, or ask just one or two groups who you think have some good ideas. Keep students' reporting brief and encourage them to offer feedback on work from other groups by identifying what has been done well, what was interesting and what might be developed further.

Even if you want to adopt groupwork in your classroom, you may at times find it difficult to organise because some students:

- are resistant to active learning and do not engage
- are dominant
- do not participate due to poor interpersonal skills or lack of confidence.

To become effective at managing groupwork it is important to reflect on all the above points, in addition to considering how far the learning outcomes were met and how well your students responded (did they all benefit?). Consider and carefully plan any adjustments you might make to the group task, resources, timings or composition of the groups.

Research suggests that learning in groups need not be used all the time to have positive effects on student achievement, so you should not feel obliged to use it in every lesson. You might want to consider using groupwork as a supplemental technique, for example as a break between a topic change or a jump-start for class discussion. It can also be used as an ice-breaker or to introduce experiential learning activities and problem solving exercises into the classroom, or to review topics.

## Additional resources

- A newly developed maths portal by the Karnataka government: http://karnatakaeducation.org.in/KOER/en/index.php/Portal:Mathematics
- National Centre for Excellence in the Teaching of Mathematics: https://www.ncetm.org.uk/
- National STEM Centre: http://www.nationalstemcentre.org.uk/
- National Numeracy:http://www.nationalnumeracy.org.uk/home/index.html
- BBC Bitesize: http://www.bbc.co.uk/bitesize/
- Khan Academy's math section:https://www.khanacademy.org/math
- NRICH: http://nrich.maths.org/frontpage
- Art of Problem Solving's resources page: http://www.artofproblemsolving.com/Resources/index.php
- Teachnology:http://www.teach-nology.com/worksheets/math/
- Math Playground's logic games: http://www.mathplayground.com/logicgames.html
- Maths is Fun: http://www.mathsisfun.com/
- Coolmath4kids.com:http://www.coolmath4kids.com/
- National Council of Educational Research and Training's textbooks for teaching mathematics and for teacher training of mathematics: http://www.ncert.nic.in/ncerts/textbook/textbook.htm
- AMT-01 Aspects of Teaching Primary School Mathematics, Block 1 ('Aspects of Teaching Mathematics'), Block 2 ('Numbers (I)'), Block 3 ('Numbers (II)'), Block 5 ('Measurement'): http://www.ignou4ublog.com/2013/06/ignou-amt-01-study-materialbooks.html
- LMT-01 Learning Mathematics, Block 1 ('Approaches to Learning') Block 2 ('Encouraging Learning in the Classroom'), Block 4 ('On Spatial Learning'), Block 6 ('Thinking Mathematically'): http://www.ignou4ublog.com/2013/06/ignou-Imt-01-study-materialbooks.html
- Manual of Mathematics Teaching Aids for Primary Schools, published by NCERT: http://www.arvindguptatoys.com/arvindgupta/pks-primarymanual.pdf
- Learning Curve and At Right Angles, periodicals about mathematics and its teaching: http://azimpremjifoundation.org/Foundation Publications
- Textbooks developed by the Eklavya Foundation with activity-based teaching mathematics at the primary level: http://www.eklavya.in/pdfs/Catalouge/Eklavya Catalogue 2012.pdf
- Central Board of Secondary Education's books and support material (also including List of Hands-on Activities in Mathematics for Classes /// to VIII) - select 'CBSE publications', then 'Books and support material': http://cbse.nic.in/welcome.htm


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