

## Learning through talking: variables and constants



Teacher Education  
through School-based  
Support in India  
[www.TESS-India.edu.in](http://www.TESS-India.edu.in)



<http://creativecommons.org/licenses/>



*TESS-India (Teacher Education through School-based Support) aims to improve the classroom practices of elementary and secondary teachers in India through the provision of Open Educational Resources (OERs) to support teachers in developing student-centred, participatory approaches. The TESS-India OERs provide teachers with a companion to the school textbook. They offer activities for teachers to try out in their classrooms with their students, together with case studies showing how other teachers have taught the topic and linked resources to support teachers in developing their lesson plans and subject knowledge.*

*TESS-India OERs have been collaboratively written by Indian and international authors to address Indian curriculum and contexts and are available for online and print use (<http://www.tess-india.edu.in/>). The OERs are available in several versions, appropriate for each participating Indian state and users are invited to adapt and localise the OERs further to meet local needs and contexts.*

*TESS-India is led by The Open University UK and funded by UK aid from the UK government.*

### **Video resources**

*Some of the activities in this unit are accompanied by the following icon: . This indicates that you will find it helpful to view the TESS-India video resources for the specified pedagogic theme.*

*The TESS-India video resources illustrate key pedagogic techniques in a range of classroom contexts in India. We hope they will inspire you to experiment with similar practices. They are intended to complement and enhance your experience of working through the text-based units, but are not integral to them should you be unable to access them.*

*TESS-India video resources may be viewed online or downloaded from the TESS-India website, <http://www.tess-india.edu.in/>. Alternatively, you may have access to these videos on a CD or memory card.*

Version 2.0 EM12v1  
All India - English

*Except for third party materials and otherwise stated, this content is made available under a Creative Commons Attribution-ShareAlike licence: <http://creativecommons.org/licenses/by-sa/3.0/>*

## What this unit is about

Variables and constants are the basic concepts used in mathematical modelling and formulas. Understanding the role of variables and constants allows students to become skilled in algebraic manipulation, which is important in reasoning mathematically and in order to do well in mathematics examinations.

In this unit you will think about the roles of variables and constants in the mathematics curriculum and how understanding this helps students to give meaning to mathematical statements and algebraic expressions.

Through activities you will also learn about how talking about mathematics can help your students learn more effectively.

## What you can learn in this unit

- How to help your students understand the differences between, and roles of, variables and constants.
- Ways to help your students write mathematical statements and construct algebraic expressions.
- Some ideas to encourage your students to learn through talking, and to express themselves using mathematical vocabulary and phraseology.

This unit links to the teaching requirements of the NCF (2005) and NCFTE (2009) outlined in Resource 1.

## 1 Variables and constants in school mathematics

Understanding the role of variables and constants is essential for developing mathematical reasoning and understanding. It is required to manipulate algebraic expressions and also enables students 'to express mathematical relations in different ways, and know more about them' (Watson et al., 2013, p. 15).

Technically, there are 'dependent' variables, 'independent' variables and constants. The unknown,  $x$ , is conventionally used to denote the independent variable, and is conventionally plotted along the horizontal axis when drawing a graph.

For example, in the expression:

$$y = x + 4$$

where  $x$  and  $y$  are integers:

- $x$  is the independent variable and can stand for any value in the set for which the expression is defined. In this example this means it can be any integer.
- $y$  is described as a dependent variable. It is dependent because its value will depend on the value of  $x$ . It is a variable because, like  $x$ , it can stand for any value in the set for which the expression is defined. In this example this means it can be any integer.
- 4 is the constant, that is, a fixed quantity, whatever the values of the independent or dependent variables.

Research suggests that one of the main issues that students encounter in learning about variables and constants, and manipulating algebra in general, is not understanding the relationships between quantities and variables in algebraic expressions. This unit aims to develop this understanding by giving meaning to variables and constants by making students think and talk about connections between numbers and algebraic expressions.

## 2 Learning about variables and constants through talking

A very efficient way for students to develop understanding and give meaning to any mathematical concept is through talking:

Children need to learn how to ... use mathematical language to create, control and express their own mathematical meanings as well as to interpret the mathematical language of others.

(Pimm, 1995, p. 179)

Students who do not learn how to 'talk mathematics' lose out on many things; in particular, as Pimm says above, they do not have the resources to available to create, control and express their own mathematical ideas.

Encouraging students to talk about mathematics and helping them to develop appropriate vocabulary and phraseology to do so is an important part of learning. Thinking and communicating are so intimately entwined (Sfard, 2010) that it is not possible to know where one stops and the other starts. If you want your students to think about, understand and therefore effectively learn mathematics, they will also need to learn to communicate their mathematical ideas.

Students will also be able to talk about what they are thinking with one another. The act of forming thoughts in order to communicate with another can enable misunderstandings to be corrected. Thoughts that have to be formed into something that can be communicated, have been shown to be much more susceptible to recall by students (Lee, 2006); in other words, they are more likely to have learned those ideas.

The first activity asks students to think about identifying quantities in their experiences from their real life. It uses a picture to trigger their imagination. The activity suggests giving a short time limit for students to come up with their ideas to give a sense of urgency, competitiveness and excitement. This also means they will have little time to worry about doing algebra.

Before attempting to use the activities in this unit with your students, it would be a good idea to complete all, or at least part, of the activities yourself. It would be even better if you could try them out with a colleague as that will help you when you reflect on the experience. Trying them for yourself will mean you get insights into a learner's experiences which can, in turn, influence your teaching and your experiences as a teacher. When you are ready, use the activities with your students and, once again, reflect and make notes on how well the activity went and the learning that happened. This will help you to develop a more student-focused teaching environment.

### Activity 1: Pictures are worth a thousand words

#### Preparation

This activity is done best in pairs or small groups. Make sure the students in the group are seated so that they can hear each other well. If you feel the students need some more time at any point, give them some bonus or extra time. You may want to look at the key resource 'Using pair work' (<http://tinyurl.com/kr-usingpairwork>).

#### The activity

Ask your students the following:

How many of you have travelled in an autorickshaw? Figure 1 shows Mr Murti travelling in an autorickshaw. In your groups, think of as many (measurable) quantities as you can that are associated with an autorickshaw ride. The group who writes the greatest number of such quantities in four minutes will be the winner. Your time starts ... now.



**Figure 1** Autorickshaw driver and passenger. (Source: Muhammad Mahdi Karim)

Then, ask for their ideas to share with the rest of the class. This could be organised as follows:

- At the end of the time limit, ask them to put their pencils down.
- Give them ten seconds to count the number of things they wrote.
- Now choose the group which wrote the minimum and the group which wrote the maximum number of quantities.
- Ask two students of the group that wrote the least number of quantities and two students of the group that wrote the most to come and write the quantities they thought of on the blackboard simultaneously – this will save time. Ask the students to stay at the blackboard.
- Ask each group in the class to share any other quantities they came up with that are different to those already on the blackboard. Ask one of the students at the blackboard to write down each of the new suggestions. Having so many scribes at the blackboard at once means that this can be done quickly.
- Soon, you will have a lot of quantities related to an autorickshaw ride on the blackboard. Some of these may include:
  - o number of passengers
  - o total fare for a journey
  - o time taken for a journey
  - o number of red lights at traffic signals during the journey
  - o distance of a journey
  - o number of wheels on an autorickshaw
  - o number of bolts on each wheel of an autorickshaw
  - o registration number of the autorickshaw
  - o speed of the autorickshaw
  - o cost of the autorickshaw
  - o mileage of the autorickshaw.

If you can, leave this list on the blackboard and ask your students to copy it. They will need it for Activity 2.



## Video: Using pair work

<http://tinyurl.com/video-usingpairwork>

### Case Study 1: Mrs Bhatia reflects on using Activity 1

*This is the account of a teacher who tried Activity 1 with his elementary students.*

I myself sometimes get confused about variables and constants, and how they relate to each other, so on the one hand I thought it would be good for my own subject knowledge development to do these activities in the class, on the other hand that made me feel a bit fearful. What would happen if I got confused in the class, in front of my students?

So I did Activities 1, 2 and 3 first on my own, and then with a colleague during a lunch break. The difference between doing them on my own, and with my colleague, is that we could talk together, and help each other when something was not clear in our understanding. And also, together we came up with so many funny examples meaning we laughed a lot doing algebra! Having a go at it first made me feel very well prepared to do these activities in class.

To introduce Activity 1, I showed the students the picture of Mr Murti, and I asked them to tell the class a little bit about their own journeys in an autorickshaw. That actually helped getting some extra variables to think about, such as: how far it was to Auntie Anju's house, how far to the park, the different number of passengers on these journeys, the time it took and how this varied whether the roads were clear or a bit flooded from the monsoon rains. Only then did I ask them to do the activity.

I asked them to work in groups of four. Telling them they had only four minutes really spurred them into immediate action and there was a sense of competitiveness and eagerness. However, I also think that by giving them only four minutes I gave the students who are a bit shy an excuse to not really contribute much. Perhaps next time I will give them a bit longer and the extra instruction of making sure that everyone in the group has contributed at least two ideas.

To feedback to the class, I used the approach that is suggested in the activity. The big advantage of it was that we had loads of examples on the blackboard in record time while still valuing the contributions of the whole class. I thought that was a good idea – I have about 90 students in the class and so I often avoid 'sharing with the whole class' because it takes so much time.

## Reflecting on your teaching practice

When you do such an exercise with your class, reflect afterwards on what went well and what went less well. Consider the questions that led to the students being interested and being able to progress, and those you needed to clarify. Such reflection always helps with finding a 'script' that helps you engage the students to find mathematics interesting and enjoyable. If they do not understand and cannot do something, they are less likely to become involved. Use this reflective exercise every time you undertake the activities, noting as Mrs Bhatia did some quite small things that made a difference.



## Pause for thought

Good questions to trigger reflection are:

- How did it go with your class?
- What responses from students were unexpected? Why?
- Did you feel you had to intervene at any point?
- Did you modify the task in any way? If so, what was your reasoning for this?

## 3 Identifying variables and constants

In the previous activity the students used their experiences from real life and linked these to the mathematical concept of quantities. The second activity now moves the students on to make the distinction between variables and constants based on the quantities they identified in Activity 1.

To help students in developing their understanding of the difference between these concepts it is important that they have the opportunity to talk about it. It is very important to expect the students to use the mathematical words themselves and to create an environment where they have to do so. This will enable your students to recognise, use and communicate with one another about algebraic expressions, variables and constants. It is an important step in learning to 'speak like a mathematician' and to *understand* mathematics rather than just *remembering* it.

Asking the students 'What will be the same?' or 'What will change?' as in the next activity can help to trigger discussion and to identify variables and constants.

### Activity 2: What will stay the same, what will change?

This activity is best done in small groups or pairs. It is important that all students have a chance to say what they are thinking and to practise their mathematical vocabulary.

- Ask the students to look at the list of quantities from Activity 1 that they wrote in their books and listed on the blackboard. Then, in their groups or pairs, ask them to identify, discuss and make a note about which of these are:
  - o quantities whose values will change, called 'variables'
  - o quantities whose values do not change, that will stay the same, called 'constants'.
- As a whole class, discuss with the students their findings and reasons for categorising the quantities as variables or constants.
- Now ask your students to work in small groups, picking three of the variables and discussing what would make the variables change. Ask them to record this in their own way, but they must use the words and phrases 'variable', 'constant', 'change', 'stay the same'.
- Pick two or three of the variables that were most looked at in the previous step and invite the students to come and write down how they had recorded what would make the variables change. Discuss these records with the class, and discuss how the students think their records could be made clearer.



## Case Study 2: Mrs Mehta reflects on using Activity 2

For the first part of the activity I asked the students to work in pairs or threes so they could all see each other's writing.

I told them the question and I asked the students to draw a box around the quantities that would change. While they were doing this I walked around the classroom, but without interfering. I overheard a lot of discussions about whether a quantity would always change, or always stay the same, no matter what. At first their explanations in trying to convince each other were not always fluent, but I do think they got better at it as they tried again and again. By the time we had the whole-class discussion about the categorisation most of the students were able to express themselves pretty clearly. Those who sounded a bit muddled I asked to have another go at explaining and that helped in most cases.

We still had the list on the blackboard, and at first I also put boxes around the variables (at the students' request) but that just looked messy. So I rewrote the list in two columns – one labelled 'Variables – quantities whose values will change' and the other 'Constants – quantities whose values will stay the same'. I thought writing the mathematical terms first would help them in learning these.

They worked in groups of four to six students on the third part of the activity. I gave them a big sheet to write their records and told them we would put these on the wall at the end of the lesson (which we did). I think doing this made them think with greater precision about what they were writing. While they were working on step 3 I walked around the classroom and decided on two variables we would look at in greater detail for the last part of the activity – one was the number of wheels, and the other a rather complicated one of the number of rupees (or the price) a passenger would need to pay as their share when travelling in a group to different places.

By asking the students to come and copy their record of this on the blackboard we ended up with spider diagrams, whole sentences, algebraic expressions, and a mixture of these, all on the blackboard. What I liked about it was how it showed all these connections and different representations, and that we were able to discuss the similarities and differences in these representations.



### Pause for thought

Mrs Mehta's lesson involved quite a lot of writing and recording on the blackboard, by the students themselves as well as the teacher. What do you think are the advantages and potential disadvantages of this approach?

Now think about how the activity went with your students, and reflect on the following questions:

- What responses from students were unexpected? Why?
- What questions did you use to probe your students' understanding?
- Did you modify the task in any way like Mrs Mehta did? If so, what was your reasoning for this?



## 4 Moving on to write formal algebraic statements and expressions

Professional mathematicians develop models to predict and describe dynamics and changes in what is happening. In doing this they make it possible to foresee what might be needed when changes happen, which is very important in all planning. This mathematical modelling relies on deciding what the variables and constants are, which ones are connected and how they are connected. This has been considered in Activities 1 and 2. The next step is to decide how these variables and constants influence and relate to each other and to record this 'model' in a mathematical way by making mathematically expressed statements.

The next activity will develop your thinking about how to make simple versions of such mathematical models, and build on the learning from Activities 1 and 2. These tasks work particularly well for students working in pairs or small groups because this allows more ideas to be generated and students can offer mutual support when they are stuck.

### Activity 3: Variables and constants in algebraic expressions

[Note for the teacher: this task can be simplified by using whole numbers only.]

#### Part 1: Mr Murti calculates his autorickshaw fare

- Remind the students of Mr Murti and his travels around the city in an autorickshaw. To make sure he does not get overcharged, Mr Murti likes to calculate how much he has to pay himself.
- The autorickshaw driver charges Rs. 25 for a journey that is up to 2 km. After that the fare is Rs. 0.80 for every extra 0.1 km.

Ask the students to do the following:

- Calculate the fare that Mr Murti pays for travelling 3.6 km, 6.7 km, 12.3 km, 25.9 km, 31 km, 1,000 km, 1 crore km, and finally 'x' km.
- Write down the way that you worked out each answer. Did you need to change your method to find the cost for x km? Check your answers with your classmates.
- Now complete this statement in algebra or in words:

If Mr. Murti travels x km, he will have to pay a fare of \_\_\_\_\_.

Note that the whole sentence above, 'If Mr Murti ...', is called a statement. What the students have to fill in at the end, in this case something similar to  $25 + 8(x - 2)$ , where  $x > 2$ , is called 'the algebraic expression'. If students cannot write it yet in algebraic notation they should write it in their own words in a sentence.

#### Part 2: Make up your own statements

Ask the students to use the list of variables and constants they made in Activity 2 to construct their own statements in words or with algebraic expressions using variables and constants for concepts like cost or time.

Your students will probably not all be at the same stage in their understanding of how to construct their own statements using variables and constants. This activity should provide you with an excellent opportunity to monitor their performance and provide them with constructive feedback. You may wish to have a look at the key resource 'Monitoring and giving feedback' (<http://tinyurl.com/kr-monitoringandfeedback>) to help you prepare for this aspect of the activity.



## Video: Monitoring and giving feedback

<http://tinyurl.com/video-monitoringandfeedback>

### Case Study 3: Mrs Aparajeeta reflects on using Activity 3

I asked the students to work on part 1 of Activity 3 in pairs or in groups of three because I thought that might help them to get more ideas and to get un-stuck if they did not know how to do it.

While they were working on this, I walked around observing how they went about working out the fare Mr Murti would have to pay. I noticed that they used different methods. I thought it would be nice to share those with the whole class so the students could see that there were several ways of solving a question. So after about five minutes I stopped the class and asked two students that I knew had used different approaches to come to the blackboard and write them down. I then asked who had done it differently and asked them to explain how they had done their calculations.

I noticed that not all students were listening, so I then asked all the students, in their pairs or groups, to find justification for the methods others had used, and then we shared this again with the whole class. This led them to discover and discuss misunderstandings. For example, Seema and her partner had found multiples of 0.8 km and the costs for each of those multiples. They had then used the one closest to what was asked in the question. She had, however, forgotten that the first two kilometres was a fixed price. Jai's group examined her method and pointed out that they had forgotten this fixed price. So what then happened was, not only could they see the different ways of getting to a solution, but they could also see what they had missed.

Most of the students were able to complete the statement in words, and about a third of the class attempted to then complete the statement with an algebraic expression. The algebraic expression was seldom correct however. I asked a couple of students to come and write their algebraic expression, together with their statements, in words on the blackboard. We then discussed how they were linked, and whether we could improve on the mathematical notation.

Part 2 of the activity helped them in experimenting with this more, which was useful. I think for the student to be able to talk about algebraic notation and how it relates to a statement in words was really helpful and I realised I had actually never given them a chance to do that before.



### Pause for thought

- Did you feel you had to intervene at any point?
- Were all your students engaged in the activity?
- What points did you feel you had to reinforce?
- Did you modify the task in any way like Mrs Aparajeeta did? If so, what was your reasoning for this?

## 5 Putting algebra into cricket

The National Curriculum Framework (NCF, 2005) lists one of its principles regarding the approach to knowledge in the curriculum as:

Connecting with the local and the contextualised in order to 'situate' knowledge and realising its 'relevance' and 'meaningfulness'; to reaffirm one's experiences outside school; to draw one's learning from observing, interacting with, classifying, categorising, questioning, reasoning and arguing in relation to these experiences.

(NCF, 2005, p. 33)

Activity 4 aims to address this. The activity can be used as a consolidation exercise, where students can use their learning from the earlier activities in a different context, one they are likely to be familiar with – cricket.



Video: Using local resources

<http://tinyurl.com/video-usinglocalresources>

### Activity 4: Runs per over

This activity allows students to become aware of variable quantities in a cricket game.



Figure 2 Students playing cricket.

### Preparation

For this activity the students could go outside and play the game for real. Alternatively, it could be played inside the classroom by throwing a dice for the number of runs that each player gets for each ball. (Let the number five on the dice be zero runs, since you only rarely score five runs from one ball in cricket!) If the dice falls off the desk, they are out!

## The activity

Say to the students:

- Let's play a 5–5 cricket match. For this, we are going to create groups of two teams of five students. Each team is to include both girls and boys. For each team, one student will be designated the score keeper (not always the girls!). Each team gets to bowl five overs.
- After six balls have been bowled, add up the score for that over.
- The two scorers jointly fill in Table 1 by recording the number of runs scored in each over:

*Table 1 Scoring card.*

| Over   | Team 1 | Team 2 |
|--------|--------|--------|
| 1      |        |        |
| 2      |        |        |
| 3      |        |        |
| 4      |        |        |
| 5      |        |        |
| Totals |        |        |

After the match, ask the class to discuss the following questions. In larger classes it works well to ask the students to first discuss these questions in small groups and then share with the whole class.

- Did each team score the same number of runs in each over? Why?
- What is the maximum number of runs that could be scored per over? Why? (Note for the teacher: You can score one, two or three runs if you really run between the wickets, four for reaching the boundary having hit the field first, and six for hitting the ball over the boundary without hitting the field first – so the maximum number of runs is six sixes. This is a case of a variable having limited values that it can take.)
- For each team, is there a visible trend in the number of runs scored in each over? Is the trend the same for both teams? If not, why do you think the trend is different?
- If this was a six-over match, what could have been the runs scored by each team? Would the result of the game be different or the same if each team got six overs?
- Which of the following quantities are variables? In other words, what may have varied during the match?
  - number of wickets taken by each bowler
  - number of overs bowled by each team
  - number of boundaries scored by different batsman
  - weight of the ball used in the match.
- What other quantities may have varied during the match? What quantities are constants (which may have remained unchanged during the match)?

Similar to Part 2 in Activity 3, ask the students to use their list of variables and constants to construct their own statements with algebraic expressions.



## Video: Involving all

<http://tinyurl.com/video-involvingall>

### Case Study 4: Mr Kapur reflects on using Activity 4

Cricket being a game very close to all their hearts, the students could really get into the discussion and contribute a lot to the activity. The day before, I asked the students to bring their cricket bats and balls to the school the next day if that was possible.

I have both girls and boys in the class and I thought the girls might feel a bit left out, or the boys might think this would be all about them! So I introduced the activity by saying I had read in the newspaper about the National Indian women's cricket team and how well they were doing. I mentioned some names such as Mithali Raj, team captain from Rajasthan, and Jhulan 'Babul' Goswami, from Bengal, who, like Mithali Raj also has won the government's Arjuna Award for excellence in the game and had also been team captain in the past.

I also made sure we discussed some rules of the games that they needed to know to complete the activity, so there would be no issues about that. We then went outside and played the games. We had mixed gender teams, we had girls against boys, we had boys against boys and girls against girls – a real mix.

In preparation for the whole-class discussion I decided to make some changes and work on Questions 1 to 3 first, and only then move on the other questions. I thought otherwise the discussion about variables and constants might get lost and it was important that did not happen as that is about the mathematics I wanted them to learn. So I wrote the Questions 1 to 3 on the blackboard and asked the students to discuss these in their teams of five. We then discussed this as a whole class.

I then wrote the Questions 4 and 5 on the blackboard, asked them to discuss these in their teams again and to make a list in their books with 'Variables and constants in the cricket game' as the title. I gave them five minutes to do this. We then had a whole-class discussion. I asked them to use the words 'variable' and 'constant' all the time so they could get used to the vocabulary.

By the end of the lesson I think I can safely say that most of them were confident of what varying quantities were and what was meant by constants. They also could see that even when denoting the varying quantity by a letter, it was a number. Some of the students managed to write their statements in algebraic notation, others could describe the statements in words. I was happy with that differentiation – they had all learned, and moved on their learning from where they were.

At the end of the lesson I told them to now go and discuss 'variables and constants in the cricket game' at home and with their friends. I do not know whether they did this, but they seemed to like the idea!



### Pause for thought

- What questions did you use to probe your students' understanding?
- Did you feel you had to intervene at any point? What points did you feel you had to reinforce?
- Which students might need further reinforcement?
- Did you modify the task in any way like Mr Kapur did? If so, what was your reasoning for this?

## 6 Summary

This unit has used ideas about teaching variables and constants to focus on two important issues.

The first was the need to help students to be able to talk about their mathematical ideas and algebraic notation. The students need to be able to express their ideas if they are to fully understand them and use them beyond the immediate context of the classroom. Thinking requires language, and students that are helped to use specific mathematical vocabulary and phraseology themselves are at an advantage when asked to think.

The second big idea in this unit was the importance of students understanding the role of constants and variables to allow them to give meaning to mathematical statements and algebraic expressions.



### Pause for thought

Identify three ideas that you have used in this unit that would work when teaching other topics. Make a note of two topics you have to teach soon where those ideas can be used with some small adjustments.

## Resources

### Resource 1: NCF/NCFTE teaching requirements

This unit links to the following teaching requirements of the NCF (2005) and NCFTE (2009) and will help you to meet those requirements:

- View students as active participants in their own learning and not as mere recipients of knowledge; how to encourage their capacity to construct knowledge; how to shift learning away from rote methods.
- Let students learn important mathematics and see mathematics is more than formulas and mechanical procedures.
- Reaffirm one's experiences outside school: to draw one's learning from observing, interacting with, classifying, categorising, questioning, reasoning and arguing in relation to these experiences.

## Additional resources

- *Many Right Answers: Learning in Mathematics through Speaking and Listening* by Els De Geest: <http://shop.niace.org.uk/media/catalog/product/m/a/manyrightanswers.pdf>
- A newly developed maths portal by the Karnataka government: <http://karnatakaeducation.org.in/KOER/en/index.php/Portal:Mathematics>
- National Centre for Excellence in the Teaching of Mathematics: <https://www.ncetm.org.uk/>
- National STEM Centre: <http://www.nationalstemcentre.org.uk/>
- National Numeracy: <http://www.nationalnumeracy.org.uk/home/index.html>
- BBC Bitesize: <http://www.bbc.co.uk/bitesize/>
- Khan Academy's math section: <https://www.khanacademy.org/math>
- NRICH: <http://nrich.maths.org/frontpage>
- Art of Problem Solving's resources page: <http://www.artofproblemsolving.com/Resources/index.php>



- Teachnology: <http://www.teach-nology.com/worksheets/math/>
- Math Playground's logic games: <http://www.mathplayground.com/logicgames.html>
- Maths is Fun: <http://www.mathsisfun.com/>
- Coolmath4kids.com: <http://www.coolmath4kids.com/>
- National Council of Educational Research and Training's textbooks for teaching mathematics and for teacher training of mathematics: <http://www.ncert.nic.in/ncerts/textbook/textbook.htm>
- AMT-01 *Aspects of Teaching Primary School Mathematics*, Block 1 ('Aspects of Teaching Mathematics'), Block 2 ('Numbers (I)'), Block 3 ('Numbers (II)'), Block 4 ('Fractions'): <http://www.ignou4ublog.com/2013/06/ignou-amt-01-study-materialbooks.html>
- LMT-01 *Learning Mathematics*, Block 1 ('Approaches to Learning') Block 2 ('Encouraging Learning in the Classroom'), Block 4 ('On Spatial Learning'), Block 6 ('Thinking Mathematically'): <http://www.ignou4ublog.com/2013/06/ignou-lmt-01-study-materialbooks.html>
- *Manual of Mathematics Teaching Aids for Primary Schools*, published by NCERT: <http://www.arvindguptatoys.com/arvindgupta/pks-primarymanual.pdf>
- *Learning Curve* and *At Right Angles*, periodicals about mathematics and its teaching: [http://azimpremjifoundation.org/Foundation\\_Publications](http://azimpremjifoundation.org/Foundation_Publications)
- Textbooks developed by the Eklavya Foundation with activity-based teaching mathematics at the primary level: [http://www.eklavya.in/pdfs/Catalogue/Eklavya\\_Catalogue\\_2012.pdf](http://www.eklavya.in/pdfs/Catalogue/Eklavya_Catalogue_2012.pdf)
- Central Board of Secondary Education's books and support material (also including *List of Hands-on Activities in Mathematics for Classes III to VIII*) – select 'CBSE publications', then 'Books and support material': <http://cbse.nic.in/welcome.htm>

## References/bibliography

- Bruner, J. (1986) *Actual Minds, Possible Worlds*. Cambridge, MA: Harvard University Press.
- Lee, C. (2006) *Language for Learning Mathematics: Assessment for Learning in Practice*. Buckingham: Open University Press.
- National Council for Teacher Education (2009) *National Curriculum Framework for Teacher Education* (online). New Delhi: NCTE. Available from: [http://www.ncte-india.org/publicnotice/NCFTE\\_2010.pdf](http://www.ncte-india.org/publicnotice/NCFTE_2010.pdf) (accessed 12 April 2014).
- National Council of Educational Research and Training (2005) *National Curriculum Framework (NCF)*. New Delhi: NCERT.
- Pimm, D. (1995) *Symbols and Meanings in School Mathematics*. London: Routledge.
- Sfard, A. (2010) *Thinking as Communicating*. Cambridge, UK: Cambridge University Press.
- Watson, A., Jones, K. and Pratt, D. (2013) *Key Ideas in Teaching Mathematics*. Oxford: Oxford University Press.
- Zack, V. and Graves, B. (2001). 'Making mathematical meaning through dialogue: "Once you think of it, the Z minus three seems pretty weird"', *Educational Studies in Mathematics*, vol. 46, pp. 229–71.

## Acknowledgements

Except for third party materials and otherwise stated below, this content is made available under a Creative Commons Attribution-ShareAlike licence (<http://creativecommons.org/licenses/by-sa/3.0/>). The material acknowledged below is Proprietary and used under licence for this project, and not subject to the



Creative Commons Licence. This means that this material may only be used unadapted within the TESS-India project and not in any subsequent OER versions. This includes the use of the TESS-India, OU and UKAID logos.

Grateful acknowledgement is made to the following sources for permission to reproduce the material in this unit:

Figure 1: © Muhammad Mahdi Karim, [http://commons.wikimedia.org/wiki/File:Authorickshaw\\_Bangalore.jpg](http://commons.wikimedia.org/wiki/File:Authorickshaw_Bangalore.jpg).

Figure 2: © rajkumar 1220, [http://commons.wikimedia.org/wiki/File:Tribal\\_Street\\_Cricket\\_Orissa\\_India\\_1.jpg](http://commons.wikimedia.org/wiki/File:Tribal_Street_Cricket_Orissa_India_1.jpg) made available under <http://creativecommons.org/licenses/by/2.0/deed.en>.

Every effort has been made to contact copyright owners. If any have been inadvertently overlooked the publishers will be pleased to make the necessary arrangements at the first opportunity.

Video (including video stills): thanks are extended to the teacher educators, headteachers, teachers and students across India who worked with The Open University in the productions.