

Visualising, comparing and contrasting: number systems



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TESS-India (Teacher Education through School-based Support) aims to improve the classroom practices of elementary and secondary teachers in India through the provision of Open Educational Resources (OERs) to support teachers in developing student-centred, participatory approaches. The TESS-India OERs provide teachers with a companion to the school textbook. They offer activities for teachers to try out in their classrooms with their students, together with case studies showing how other teachers have taught the topic and linked resources to support teachers in developing their lesson plans and subject knowledge.

TESS-India OERs have been collaboratively written by Indian and international authors to address Indian curriculum and contexts and are available for online and print use (<http://www.tess-india.edu.in/>). The OERs are available in several versions, appropriate for each participating Indian state and users are invited to adapt and localise the OERs further to meet local needs and contexts.

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Video resources

Some of the activities in this unit are accompanied by the following icon: . This indicates that you will find it helpful to view the TESS-India video resources for the specified pedagogic theme.

The TESS-India video resources illustrate key pedagogic techniques in a range of classroom contexts in India. We hope they will inspire you to experiment with similar practices. They are intended to complement and enhance your experience of working through the text-based units, but are not integral to them should you be unable to access them.

TESS-India video resources may be viewed online or downloaded from the TESS-India website, <http://www.tess-india.edu.in/>. Alternatively, you may have access to these videos on a CD or memory card.

What this unit is about

Mathematics has several number systems: natural numbers, whole numbers, integers, rational numbers, irrational numbers and real numbers. Students often think that the distinctions between these number systems are somewhat obscure and trivial. This unit will explore the similarities and differences of these number systems, and how mathematical operations such as addition, multiplication, exponentiation, etc., function within these.

The National Curriculum Framework for Teacher Education (NCFTE, 2009) requires mathematics lessons to be interesting, student-focused and student-participatory, and to build the students' understanding of mathematics. That is not easy to accomplish. This unit aims to support teachers to achieve this by working on compare and contrast tasks and using visualisation. The unit will explain each of these approaches and provide examples of how they can be applied in the classroom. The unit will also discuss how to make small changes to existing classroom practice using a textbook.

What you can learn in this unit

- Some ideas for how to explore and analyse different ways of visualising numbers and number operations, and realise their limitations.
- To recognise the structure and effects of compare and contrast activities, and how visualisation can help your students make sense of mathematics.

This unit links to the teaching requirements of the National Curriculum Framework (NCF, 2005) and NCFTE (2009) outlined in Resource 1.

1 Comparing and contrasting activities

Comparing and contrasting are good activities to make people aware of mathematical properties and its applications. The action of comparing and contrasting forces students to think about the properties of the mathematical objects and to notice what is the same and what is different. While doing so, students make connections they might not normally consider. They are prompted into mathematical thinking processes such as generalising, conjecturing about what stays the same and what can change, and verifying these conjectures. This is an example of the national curriculum requirement of letting students use abstractions to perceive relationships, see structures, reason things out and argue the truth or falsity of statements.

Compare and contrast activities, such as in Activity 1, can help students to:

- consolidate their learning
- remind them of the different purposes of the various number systems
- become aware of the subtle similarities and differences of the systems.

Activity 1 requires your students to think about which statements are always, sometimes or never true. The aim is to become aware of differences between the number systems and to develop a mathematical argument for their reasoning. By doing so, they become more precise in their use of mathematical language and notice the more subtle similarities and differences. In the later questions there is a first step towards visualisation by referring to a number line.

Before attempting to use the activities in this unit with your students, it would be a good idea to complete all (or at least part) of the activities yourself. It would be even better if you could try them out with a colleague, as that will help you when you reflect on the experience. Trying the activities yourself will mean that you get insights

into a learner's experiences that can in turn influence your teaching and your experiences as a teacher. When you are ready, use the activities with your students. After the lesson, think about the way that the activity went and the learning that happened. This will help you to develop a more learner-focused teaching environment.

Activity 1: Always, sometimes or never true?

Preparation

The statements can be offered as a list, or can be written on cards and shared out randomly. A fuller list of questions, and an example of a worksheet that can be cut up as cards, can be found in Resources 2 and 3. An overview of different number systems and their properties can be found in Resource 4.

Here are some suggestions for ways of working on this activity:

- You can ask students to work on this on their own, share their ideas with the class or share their ideas with their partners/classmates.
- The statements can be offered for all to be done – but this could become tedious, so ask students to complete only part of them; for example, to do odd/even/prime numbered ones.
- You could also ask students to make their own choice. For example, if they select three that they would like to tackle and two that they would not want to do, you could ask them to do all five of these with the support of their partner(s).
- Letting students make their own choice often empowers stimulating active participation and engagement in the classroom.

The activity

Ask your students which of the following statements are 'always true', 'sometimes true' or 'never true', and why?

1. The sum of two natural numbers is a natural number.
2. The sum of two integers is NOT an integer.
3. The difference of two irrational numbers is an irrational number.
4. The product of two irrational numbers is an irrational number.
5. The quotient of two whole numbers is a whole number.
6. The quotient of two real numbers is a real number.
7. There are an infinite number of pairs of whole numbers whose sum is 0.
8. There exists a pair of whole numbers whose product is 1.
9. The product of two real numbers is not a non-repeating, non-terminating decimal.
10. The exact location of a natural number cannot be determined on a number line.
11. The difference of two integers is to the left of each of the two integers on a number line.
12. There are finite natural numbers between any two real numbers.
13. The number a^2 is a natural number if a is a natural number.
14. The number ab is greater than both a and b .



Video: Talk for learning

<http://tinyurl.com/video-talkforlearning>

You may also want to have a look at the key resource 'Talk for learning' (<http://tinyurl.com/kr-talkforlearning>).

Case Study 1: Mrs Aparajeeta reflects on using Activity 1

This is the account of a teacher who tried Activity 1 with his secondary students.

I was a bit scared about trying out this activity for several reasons:

- the unusual structure of the activity
- requiring the students to develop their own reasoning
- students talking to each other when this is not usually allowed in my classroom
- my fear that they might not remember anything about the different number systems at all!

Before we started on this activity, I asked six students for a number between one and 14. These six numbers became the questions they had to do. I was very uncomfortable about allowing the students to talk to each other as I thought it would become very noisy with 79 students in the classroom; I would not be able to control what was discussed and it could become chaotic. What if the principal happened to walk past at such chaotic moment! On the other hand I do believe that talking helps learning, because when you talk, you have to organise your thinking – otherwise the listener will not understand you.

So I pushed myself to be brave and started the lesson: I asked the students to work the answers out for themselves in silence and to write down their reasoning. Then I asked them to discuss their thoughts and reasoning with their classmates, in pairs. But they had to do it quietly, so the others could not overhear what they were saying and steal their ideas! I told them that after some time I would select at random someone to explain what the thinking of the pair had been. Other students could then comment in a constructive way, just like we sometimes do in whole-class discussions. To address my fear that they would not be able to remember anything, I wrote the page numbers of their textbook where they could find information about the number systems on the blackboard – and they were free to refer to these.

To my surprise it worked really well! The classroom did not end in chaos. There was noise from the talking in pairs, but it was not loud: they seemed to like the sense of competition and secrecy. I listened into the conversations by walking around and by standing quietly against the wall, observing. What I liked very much was that the conversations were about mathematics, and that disagreements led to pointed discussions about the mathematical properties, flicking through the textbook to check their reasoning and using a number line to illustrate what they were saying. The whole class discussion was less lively than I had anticipated because there was little disagreement by then. However, I was impressed with the quality of the mathematical language used. The class discussion thus became a consolidation activity. Their responses showed me they seemed to really understand the differences in properties, and the consequences of these differences when doing operations in the different number systems.

What I found very hard in this lesson was my changed role as teacher: I was no longer standing at the front explaining and telling them what to do. Actually, I talked very little, and it was difficult not to interfere in the discussion and show them how to do it. However, it meant that this was now their learning and their thinking, which was a powerful experience.

Would I change anything next time? Yes, I think I would try to let them choose their own questions, although I am not sure how to lead the whole-class discussion then as they might have chosen all different ones. Perhaps I could ask them to prepare small presentations on their findings. I think I will also prepare the questions in 'card' format and use these when we have five or ten minutes left over at the end of a lesson. I think that could offer the repetition needed to keep all this knowledge in the mind, without it meaning doing more exercises from the book.

Reflecting on your teaching

When you do such an activity with your class, reflect afterwards about what went well and what went less well. Consider the questions that led to the students being interested and being able to progress and where you needed to clarify. Such reflection always helps with finding a 'script' that helps you engage the students to find mathematics interesting and enjoyable. If they do not understand and cannot do something, they are less likely to become involved. Use this reflective exercise every time you undertake the activities, noting, as Mrs Aparajeeta did, some quite small things that made a difference.



Pause for thought

Good questions to trigger such reflection are:

- How did it go with your class?
- What responses from students were unexpected? Why?
- Did you feel you had to intervene at any point?
- What points did you feel you had to reinforce?
- Did you modify the task in any way? If so, what was your reasoning for this?

2 Working on visualisation

Research by Dörfler (1991) and Van Hiele (1986) suggests that visualisation is a powerful tool for learning mathematics. (You can find out more about visualisation in the unit *Using visualisation: algebraic identities*.) However, most visual representations have limitations in their mathematical use. Students have to become knowledgeable about which visual representation to use for a particular purpose.

In Activity 2, students explore these limitations for the number line. The activity asks the students to identify the properties of the different number types. It also offers choice in selecting their own examples, thereby giving the participating student a sense of ownership.

Activity 2: Representing numbers on a number line

Preparation

This activity works well for students working in pairs or small groups so that ideas and opinions can be shared and challenged easily. If your students are not used to working in this way, you will need to plan how they will be organised in pairs or small groups and how they will sit so that they can easily talk to each other.

The activity

Tell your students the following:

- Plot the following numbers exactly on a number line:
 - o $2 - 4$
 - o $18/4$
 - o $\sqrt{2}$
 - o $17/3$

- Are you happy with the results? Were you able to plot all of them in exactly the right place? How do you know?
- Can you think of other numbers that will be difficult or easy to represent on the number line?

Case Study 2: Teacher Abhay reflects on using Activity 2

I tried Activity 2 first myself and could see the potential of the learning. However, it is not the kind of activity that we are used to doing at school, so I was very apprehensive. I will describe how I went about overcoming that apprehension and how my experiences influenced my planning for this activity with my students.

Because I did not feel confident in using this activity straight away with my students in class, I decided to try it out first on my colleagues. The colleagues, not all mathematics teachers, kindly marked the numbers on the number line. I then asked them how they had done this. Their methods for representing the whole and rational-terminating numbers were the same, but they had used different methods to mark the rational non-terminating and irrational numbers. For example, for some of the numbers they used the decimal representation, while for others they used constructions. A discussion followed about whether these numbers could be plotted exactly on a number line, and what the limitations are of the number line as visualisation.

Asking my colleagues to do the task first, and reflecting on this, helped me in planning to lesson with my Grade IX students. I noticed my colleagues learned from listening to each other. I had not told them how to do it. It made me think: could my students learn in the same way? What makes me think their learning would be any different?

I decided that I would like my students to learn from their classmates as well so that they can discover which methods there are and think about their effectiveness and limitations, just like my colleagues had. This meant that I had to adapt my lesson planning to make sure there would be opportunity for a knowledge exchange to happen, i.e. I had to plan for sharing time.

So, by first doing the task myself I was convinced this was a stimulating task with great learning opportunities. Asking my colleagues to do the task helped me to overcome my anxiety about trying it out in my classroom. Hearing their responses also made me question my assumptions about learning and pushed me into thinking about different teaching approaches so that shared learning could take place. I will plan some lessons to try out my 'new' ideas.



Pause for thought

- What questions did you use to probe your students' understanding?
- What responses from students were unexpected? Why?
- Did you modify the task in any way? If so, what was your reasoning for this?



Video: Planning lessons

<http://tinyurl.com/video-planninglessons>

For more information, read Resource 5, 'Planning lessons'.

3 Creating compare and contrast activities – making small changes to existing practice

Reading about new teaching ideas might be exciting and desirable, but how does it fit in with existing practice? Like the teachers in the case studies, you could use the activities from this unit or from other sources and adapt them to fit with your teaching practice. At times, small changes can also be made with great effect on the quality and efficiency of learning.

The next activity is a planning activity that focuses on making small changes to the existing practice of textbook use. It requires a little extra lesson preparation time but it can actually save considerable teaching time because exercises from the book are used more efficiently and focus on what can be learned. A good start is to have some questions to hand that will lead students to compare and contrast, perhaps written down on a piece of paper and stuck to your desk or displayed on the wall. These questions can be used repetitively at any point in any lesson. Some simple though good questions are:

- What is the same and what is different?
- Can you make up another question like this?
- Can you make up a hard question and an easy question?
- How do you know?

Activity 3: Planning to adapt questions in textbooks

In the textbook you use, look at exercises about working in different number systems.

1. What is the same and what is different in these questions?
2. How could you make small changes to these questions so that it would allow you to compare and contrast?
3. How could you introduce these questions to your students, so that it would allow them to compare and contrast?
4. Make notes to use when you are teaching these lessons. Sharing your ideas with other teachers will build up a list of suggestions for different lessons.

You can find an example of adapted textbook questions in Resource 6.

Case Study 3: Teacher Anand reflects on using Activity 3

I teach secondary classes in a rural government school, where class sizes are about 80 students per class. I usually have a careful look at the questions given in a particular exercise of a chapter in the textbooks before I teach this to the class. I had never considered looking at the questions in the way suggested in the activity. It looked very simple. I did not expect to notice much because I think the books are good and introduce mathematical ideas gradually, step by step, to the students. However, my returning frustration is that the students find it difficult to remember what they did last lesson, last week, last month or last year, and often they do not do as well in exams as I would like them to! Why do they not notice the differences between the different number systems, for example?

So on this occasion, I looked at the chapter 'Number system' for Class IX, in particular at Exercise 9.3 of the NCERT textbook. I thought about what was the same and what was different in these questions: I

noticed that Question 1 involved conversion of fractional form to decimals; in Question 3 the opposite was asked. I decided to make a small change and combine these two questions. The students were now asked to convert $65/100$ and $13/99$ to decimal form and then convert the decimal forms back to fractional forms, instead of doing this in two separate exercises with another question in between. I thought this would help them to correlate the concept of decimal and fractions, to make connections and not think of these two concepts in isolation.

This seemed indeed to be the case. Moreover, by asking the students the questions 'What is the same, what is different?' and 'How do you know?', the students were able to compare and contrast terminating and non-terminating decimals, and explain how they are different.

On reflection, I think it is important to take a critical view of the questions in the textbooks. By asking myself simply 'What is the same and what is different?' I could see the difference in structure in the questions and the learning opportunities they offer.



Pause for thought

Identify three ideas that you have used in this unit that would work when teaching other topics. Make a note of two topics you have to teach soon where those ideas can be used with some small adjustments.

4 Summary

This unit has explored teaching what is the same and different about different number systems and how mathematical operations such as addition, multiplication, exponentiation, etc., function within these. Requirements for teaching from the NCF (2005) and NCFTE (2009) were used as ambitious goals. To work on making mathematics lessons interesting, student-focused, student-participatory and building on understanding of mathematics, the unit focused on developing pedagogical approaches of compare and contrast tasks using visualisation. The unit also discussed how to plan for making small changes to existing classroom teaching practice using a textbook.

Resources

Resource 1: NCF/NCFTE teaching requirements

The learning in this unit links to the NCF (2005) and NCFTE (2009) teaching requirements as specified below:

- View students as active participants in their own learning and not as mere recipients of knowledge; encouraging their capacity to construct knowledge; shifting learning away from rote methods.
- Engage with the curriculum, syllabuses and textbooks to critically examine them rather than taking them as 'given' and accepted without question.
- Let students see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.
- Let students use abstractions to perceive relationships, to see structures, to reason out problems, to argue the truth or falsity of statements.

Resource 2: Fuller list of statements for Activity 1

This is an extensive (but not exhaustive) list of statements that could be used for Activity 1. Select the area (operations, which number systems, etc.) that you would like your students to work on. Students are asked which of the following statements are 'always true', 'sometimes true' or 'never true', and asked to explain why.

Working on the property of closure

1. The sum/difference/product/quotient of two natural numbers is a natural number.
2. The sum/difference/product/quotient of two whole numbers is a whole number.
3. The sum/difference/product/quotient of two integers is NOT an integer.
4. The sum/difference/product/quotient of two rational numbers is a rational number.
5. The sum/difference/product/quotient of two irrational numbers is an irrational number.
6. The sum/difference/product/quotient of two real numbers is a real number.

Working on inverses

1. There are an infinite number of pairs of irrational numbers whose sum/product is 0 (or 1).
2. There are an infinite number of pairs of rational numbers whose sum/product is 0 (or 1).
3. There are an infinite number of pairs of integers whose sum/product is 0 (or 1).
4. There are an infinite number of pairs of whole numbers whose sum/product is 0 (or 1).
5. There exists a pair of whole numbers whose sum/product is 0 (or 1).

Working on decimal representations

1. A terminating decimal can be expressed as a ratio of an integer to a non-zero integer.
2. A non-terminating decimal can be expressed as a ratio of an integer to a non-zero integer.
3. A repeating decimal can be expressed as a ratio of an integer to a non-zero integer.
4. A non-repeating decimal can be expressed as a ratio of an integer to a non-zero integer.
5. The sum of a rational and an irrational number is NOT a repeating decimal.
6. The sum of two real numbers is a non-repeating, non-terminating decimal.
7. The product of a rational and an irrational number is a repeating decimal.
8. The product of two real numbers is NOT a non-repeating, non-terminating decimal.
9. The product of a rational and an irrational number is a repeating decimal.
10. The product of two real numbers is NOT a non-repeating, non-terminating decimal.

Locating numbers on a number line

1. The exact location of a natural number/integer cannot be determined on a number line.
2. The exact location of an integer cannot be determined on a number line.
3. The exact location of a rational number can be determined on a number line.
4. The exact location of an irrational number can be determined on a number line.
5. The exact location of a real number can be determined on a number line.
6. The sum of two natural numbers is to the right of each of the two numbers on a number line.
7. The difference of two integers is to the left of each of the two integers on a number line.
8. The sum of two real numbers is to the right of each of the two numbers on a number line.
9. The quotient of two integers is always to the left of each of the two integers on a number line.
10. There are infinite real numbers between any two real numbers on a number line.
11. There are finite natural numbers between any two real numbers on a number line.
12. There are infinite irrational numbers between any two rational numbers on a number line.

13. There is at least one whole number between any two whole numbers.

Exponentiation

1. The number a^2 is a natural number if a is a natural number/integer.
2. The number a^2 is positive for every real number.
3. The number ab is greater than both a and b .

Resource 3: Activity 1 in 'card' format

This resource includes the statements from Activity 1 presented in a format that can be printed on paper or card, and then cut out. These little cards can be given out to students at random to work on. They can be used as a longer activity, or as a short activity of five or ten minutes.

The sum of two natural numbers is a natural Number	The sum of two integers is NOT an integer	The difference of two irrational numbers is an irrational number
The product of two irrational numbers is an irrational number	The quotient of two whole numbers is a whole number	The quotient of two real numbers is a real number
There is an infinite number of pairs of whole numbers whose sum is 0	There exists a pair of whole numbers whose product is 1	The product of two real numbers is not a non-repeating, non-terminating decimal
The exact location of a natural number can not be determined on a number line	The difference of two integers is to the left of each of the two integers on a number line	There are finite natural numbers between any two real numbers
The number a^2 is a natural number if a is a natural number	The number ab is greater than both a and b	Make up your own statement

Figure R2.1 The statements from Activity 1 in card format.

Resource 4: Number systems and their properties

Table R3.1 gives an overview of the properties of operations on numbers in the different number systems. You could also ask your students to fill in the 'yes' and 'no'.

Table R3.1 Number systems and their properties.

Property of operations on numbers	Number system					
	Natural	Whole	Integers	Rational	Irrational	Real
Closed under addition: If a, b belong to a set, then $a + b$ also belongs to the set.	Yes	Yes	Yes	Yes	No	Yes
Additive identity: For a belonging to a set, there is a number z in the set, such that $a + z = a$.	No	Yes	Yes	Yes	No	Yes
Additive inverse: For every number a in a set, there is another number $-a$ in the set such that, $a + (-a) = z$.	No	No	Yes	Yes	No	Yes
Closed under multiplication: If a, b belong to a set, then $a \times b$ also belongs to the set.	Yes	Yes	Yes	Yes	No	Yes
Multiplicative identity: For a belonging to a set, there is a number u in the set, such that $a \times u = a$.	Yes	Yes	Yes	Yes	No	Yes
Multiplicative inverse: For every number a in a set, there is another number a^{-1} in the set such that, $a \times a^{-1} = u$.	No	No	No	Yes	No	Yes

Resource 5: Planning lessons

Why planning and preparing are important

Good lessons have to be planned. Planning helps to make your lessons clear and well-timed, meaning that students can be active and interested. Effective planning also includes some in-built flexibility so that teachers can respond to what they find out about their students' learning as they teach. Working on a plan for a series of lessons involves knowing the students and their prior learning, what it means to progress through the curriculum, and finding the best resources and activities to help students learn.

Planning is a continual process to help you prepare both individual lessons as well as series of lessons, each one building on the last. The stages of lesson planning are:

- being clear about what your students need in order to make progress

- deciding how you are going to teach in a way that students will understand and how to maintain flexibility to respond to what you find
- looking back on how well the lesson went and what your students have learnt in order to plan for the future.

Planning a series of lessons

When you are following a curriculum, the first part of planning is working out how best to break up subjects and topics in the curriculum into sections or chunks. You need to consider the time available as well as ways for students to make progress and build up skills and knowledge gradually. Your experience or discussions with colleagues may tell you that one topic will take up four lessons, but another topic will only take two. You may be aware that you will want to return to that learning in different ways and at different times in future lessons, when other topics are covered or the subject is extended.

In all lesson plans you will need to be clear about:

- what you want the students to learn
- how you will introduce that learning
- what students will have to do and why.

You will want to make learning active and interesting so that students feel comfortable and curious. Consider what the students will be asked to do across the series of lessons so that you build in variety and interest, but also flexibility. Plan how you can check your students' understanding as they progress through the series of lessons. Be prepared to be flexible if some areas take longer or are grasped quickly.

Preparing individual lessons

After you have planned the series of lessons, each individual lesson will have to be planned **based on the progress that students have made up to that point**. You know what the students should have learnt or should be able to do at the end of the series of lessons, but you may have needed to re-cap something unexpected or move on more quickly. Therefore each individual lesson must be planned so that all your students make progress and feel successful and included.

Within the lesson plan you should make sure that there is enough time for each of the activities and that any resources are ready, such as those for practical work or active groupwork. As part of planning materials for large classes you may need to plan different questions and activities for different groups.

When you are teaching new topics, you may need to make time to practise and talk through the ideas with other teachers so that you are confident.

Think of preparing your lessons in three parts. These parts are discussed below.

1 The introduction

At the start of a lesson, explain to the students what they will learn and do, so that everyone knows what is expected of them. Get the students interested in what they are about to learn by allowing them to share what they know already.

2 The main part of the lesson

Outline the content based on what students already know. You may decide to use local resources, new information or active methods including groupwork or problem solving. Identify the resources to use and the way that you will make use of your classroom space. Using a variety of activities, resources, and timings is an important part of lesson planning. If you use various methods and activities, you will reach more students, because they will learn in different ways.

3 The end of the lesson to check on learning

Always allow time (either during or at the end of the lesson) to find out how much progress has been made. Checking does not always mean a test. Usually it will be quick and on the spot – such as planned questions or observing students presenting what they have learnt – but you must plan to be flexible and to make changes according to what you find out from the students' responses.

A good way to end the lesson can be to return to the goals at the start and allowing time for the students to tell each other and you about their progress with that learning. Listening to the students will make sure you know what to plan for the next lesson.

Reviewing lessons

Look back over each lesson and keep a record of what you did, what your students learnt, what resources were used and how well it went so that you can make improvements or adjustments to your plans for subsequent lessons. For example, you may decide to:

- change or vary the activities
- prepare a range of open and closed questions
- have a follow-up session with students who need extra support.

Think about what you could have planned or done even better to help students learn.

Your lesson plans will inevitably change as you go through each lesson, because you cannot predict everything that will happen. Good planning will mean that you know what learning you want to happen and therefore you will be ready to respond flexibly to what you find out about your students' actual learning.

Resource 6: Examples of Activity 3

Adapting Exercise 1.6 (p. 26 in NCERT textbook Class IX, Chapter 1)

Exercise 1.6

- Find: (i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$
- Find: (i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$ (iv) $125^{\frac{-1}{3}}$
- Simplify: (i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ (ii) $(\frac{1}{3})^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ (iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

Adapt by asking the students these additional questions:

- What is the same and what is different in the methods for solving these questions?
- Make up two additional parts for Questions 1, 2 and 3 above. One of these questions has to be easy and one has to be hard. Make sure you can work out the solution yourself!

Adapting Exercise 1.2, Question 1 (p. 8 in NCERT textbook Class IX, Chapter 1)

Exercise 1.2

- State whether the following statements are true or false. Justify your answers.
 - Every irrational number is a real number.
 - Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Adapt this exercise by asking the students these additional questions:

- How do you know you are right?
- Now try and convince your partner of the opposite of what you think are the correct answers for Question 1.

Additional resources

- A newly developed maths portal by the Karnataka government: <http://karnatakaeducation.org.in/KOER/en/index.php/Portal:Mathematics>
- Class X maths study material: http://www.zietmysore.org/stud_mats/X/maths.pdf
- National Centre for Excellence in the Teaching of Mathematics: <https://www.ncetm.org.uk/>
- National STEM Centre: <http://www.nationalstemcentre.org.uk/>
- OpenLearn: <http://www.open.edu/openlearn/>
- BBC Bitesize: <http://www.bbc.co.uk/bitesize/>
- Khan Academy's math section: <https://www.khanacademy.org/math>
- NRIC: <http://nrich.maths.org/frontpage>
- Mathcelebration: <http://www.mathcelebration.com/>
- Art of Problem Solving's resources page: <http://www.artofproblemsolving.com/Resources/index.php>
- Teachnology: <http://www.teach-nology.com/worksheets/math/>
- Maths is Fun: <http://www.mathsisfun.com/>
- National Council of Educational Research and Training's textbooks for teaching mathematics and for teacher training of mathematics: <http://www.ncert.nic.in/ncerts/textbook/textbook.htm>
- LMT-01 *Learning Mathematics*, Block 1 ('Approaches to Learning') Block 2 ('Encouraging Learning in the Classroom'), Block 6 ('Thinking Mathematically'): <http://www.ignou4ublog.com/2013/06/ignou-lmt-01-study-materialbooks.html>
- *Learning Curve* and *At Right Angles*, periodicals about mathematics and its teaching: http://azimpremijifoundation.org/Foundation_Publications
- Central Board of Secondary Education's books and support material (also including the *Teachers Manual for Formative Assessment – Mathematics (Class IX)*) – select 'CBSE publications', then 'Books and support material': <http://cbse.nic.in/welcome.htm>

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