

Connecting mathematics: finding factors and multiples



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TESS-India (Teacher Education through School-based Support) aims to improve the classroom practices of elementary and secondary teachers in India through the provision of Open Educational Resources (OERs) to support teachers in developing student-centred, participatory approaches. The TESS-India OERs provide teachers with a companion to the school textbook. They offer activities for teachers to try out in their classrooms with their students, together with case studies showing how other teachers have taught the topic and linked resources to support teachers in developing their lesson plans and subject knowledge.

TESS-India OERs have been collaboratively written by Indian and international authors to address Indian curriculum and contexts and are available for online and print use (<http://www.tess-india.edu.in/>). The OERs are available in several versions, appropriate for each participating Indian state and users are invited to adapt and localise the OERs further to meet local needs and contexts.

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Video resources

Some of the activities in this unit are accompanied by the following icon: . This indicates that you will find it helpful to view the TESS-India video resources for the specified pedagogic theme.

The TESS-India video resources illustrate key pedagogic techniques in a range of classroom contexts in India. We hope they will inspire you to experiment with similar practices. They are intended to complement and enhance your experience of working through the text-based units, but are not integral to them should you be unable to access them.

TESS-India video resources may be viewed online or downloaded from the TESS-India website, <http://www.tess-india.edu.in/>. Alternatively, you may have access to these videos on a CD or memory card.

What this unit is about

Finding factors and multiples is an essential part of mathematics. The use of these concepts starts from an early age, when young students are working on multiplication and sharing. In school mathematics it is built on over the years and these concepts are used in very high-level mathematics as well.

In this unit, you will think about your teaching of the concepts of factors and multiples, and use the ideas of highest common factor (HCF) and lowest common multiple (LCM). During the activities you will also think about how to extend your students' ability to think through mathematical ideas and make connections between mathematical concepts. The textbook is often a mathematics teacher's most valuable resource, but it can constrain teaching. In this unit you will think about using the textbook more creatively.

What you can learn in this unit

- How to turn textbook questions into richer and more interesting problems.
- Some suggestions to help your students focus on the process of *doing* mathematics instead of focusing on finding answers.
- How to make connections between mathematical concepts and properties.

This unit links to the teaching requirements of the NCF (2005) and NCFTE (2009) outlined in Resource 1.

1 Some issues with learning from textbooks

Making connections between mathematical concepts is a very important part of understanding mathematics as a subject. Research suggests that teachers who make connections in their teaching are more successful than those who do not (Askew et al., 1997). Making connections is also often a joyous part of mathematics. Making connections is often lost when students are using textbook questions because the focus is on completing questions as quickly as possible, and these questions are usually only about one aspect of a concept, such as listing all factors of a number.

When students use textbooks, the purpose of the learning is not always made clear to them. They can also get so bogged-down in completing the problems correctly that they lose any perspective of the learning that is supposed to take place.



Pause for thought

Think about a recent mathematics lesson in your classroom. What mathematics were your students learning? To what extent were they thinking mathematically? To what extent were they making connections between mathematical concepts and ideas? Why do you think this is?

The activities in this unit are based on problems and examples as they can be found in any textbook. Additional questions will then be asked to move the students from mechanically finding answers to really thinking about what they are doing, such as:

- How did you find that answer?
- What is the same and what is different in your answers to those questions?
- What is the same or what is different in your process of thinking?

When students are given the opportunity to think about the process of learning and making connections, they will learn to learn. However, students might initially be uncomfortable with these kinds of questions, as

they might not have been required to think in this way before. Therefore, it will be necessary to have support for students, for example through working with their peers in groups or pairs.

2 Making connections to understand mathematics

Students study and use factors and multiples from an early age. In secondary school, students are required to study the HCF (highest common factor) and LCM (lowest common multiple) of numbers. They have to be able to apply this knowledge when working with expressions. These topics and concepts are thus studied at different times, and in different years. Therefore, students can fail to see connections between the different aspects that they study, and their knowledge can become fragmented as a result. Students often rely on memorisation at each stage rather than understanding the underlying principles leading to a lack of appreciation of the power of factors and multiples.

Activity 1 aims to address this fragmentation by focusing on the mathematical thinking processes involved in finding factors of numbers and expressions. Students are asked to make connections between what a factor is, between factors of numbers and factors in expressions. The activity requires students to work in pairs or small groups and exchange their ideas with other students.

Before attempting to use the activities in this unit with your students, it would be a good idea to complete all (or at least part) of the activities yourself. It would be even better if you could try them out with a colleague, as that will help you when you reflect on the experience. Trying the activities yourself will mean that you get insights into a learner's experiences that can in turn influence your teaching and your experiences as a teacher. When you are ready, use the activities with your students. After the lesson, think about the way that the activity went and the learning that happened. This will help you to develop a more learner-focused teaching environment.

Activity 1: Finding factors of numbers and expressions



Figure 1 Students working in small groups.

Tell your students:

- List the various factors of the following numbers and expressions.
 - o 60
 - o $3xy$
 - o 15
 - o $12x^2y^3$
 - o $3x^4 - 27x^4$

- o $2x^2 - 8x + 8$
- Why do you think some numbers only have two factors?
- Describe your method of finding the factors to the others in your group. Did you all use the same methods? Did all the methods work well with all questions?
- Write your own definition or description of what a factor is, with examples of where it can be found.

When your students have attempted these tasks in pairs or small groups, bring the class together and ask different students to share their definitions of a factor. Use these definitions as a starting point for exploring their answers to the other questions. Pay careful attention to who answers your questions – is it always the same students? How can you develop the confidence of other students to share their thinking?



Video: Using groupwork

<http://tinyurl.com/video-usinggroupwork>



Video: Talk for learning

<http://tinyurl.com/video-talkforlearning>

Resources 2 and 3, 'Using groupwork' and 'Talk for learning', have more information on these subjects.

Case Study 1: Mrs Kapur reflects on using Activity 1

This is the account of a teacher who tried Activity 1 with her secondary students.

I put students in groups of four so that they could offer mutual support and to give them more opportunities to come up with a rich collection of ideas. I organised the groups by putting a notice on the wall with the names of students in each one. I tried to have a mix in each group of students who were confident with learning maths and students who were not so confident. My aim was that the more confident students would support the others.

The students came up with the factors very easily in the case of pure numbers, but there were a lot of arguments about the ones involving expressions. This happened in most of the groups, so I thought it might be helpful to discuss these questions as a whole class. First I asked them to prepare a short presentation for the whole class; this is what they came up with:

- They quite accurately identified the factors of 60 as 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60 – although some groups missed out a few of them, they quickly wrote down the ones they had missed. This led to a discussion about the need to be systematic, and what approaches could help them to be systematic.
- In the second question, some groups identified only three factors: $3xy$, 3, and x and y .
- Many groups were quite good at noticing prime factors and explaining that these were special because could only be divided by themselves and by 1.

I really wanted the students to notice the similarity between factors of numbers and expressions. As a first step I asked, 'In what way are non-prime factors different from prime factors?'

They were a bit baffled by that, so I added, 'OK, look at the factors of 60 as an example. Now start your

sentence with: “Non-prime factors are different because ...”.’ There were several volunteers to have a go at answering this, but it sounded neither clear nor concise – which I did not think would encourage good discussions or learning. I decided to give them a ‘speaking frame’, like they sometimes give writing frames in language learning. So I told them, ‘Try saying it first to a partner and then I will ask you to say it to the whole class.’

They practised verbalising their thinking in that way with each other for a few minutes. I also noticed that some were writing what they had said down in their exercise books, probably so as not to forget what to say when asked. The speaking frame really seemed to have helped because when we came to sharing their sentences, their responses were clear and concise and they used mathematical language in a precise way. They had managed to both notice and express that non-prime factors were numbers that could be decomposed further into products of other numbers until only prime factors were used. I was seriously impressed! But they still had not made the connection between these kinds of factors and the factors of expressions.

My first instinct was simply to tell them – to just pass on my knowledge to them. But they were so full of enthusiasm and engagement from being asked to come up with their ideas that I could not just tell them. I wanted them to discover the answer themselves, and experience the joy of discovering – seeing the beauty of mathematical structures and their connectivity. But what question could I pose so that they could become aware of this connection between factors of numbers and factors of expressions? So many questions and approaches came up in my mind, but they were too complicated, or simply disguised ‘telling them what it is’ questions! What if I just told them what I really wanted them to do? So I said:

I really want you to discover the connections and similarities and differences between finding factors of numbers, and finding factors of expressions. For example, between the factors of 60 and the factors of $3xy$. And I don’t just want to tell you, I want you to think about it and discover it for yourself. So think for a moment about all these discussions we have had during this lesson, about the difference between prime and non-prime factors, and look at the factors you have found for 60 and $3xy$. What is the same, what is different? Have you got all the factors for all? Have a go at it in your groups.

And they did find the missing factors for $3xy$. They found the factors for $3x^4 - 27x^2$: from re-writing it as $3x^2(x^2 - 9)$ and then going onto identifying the factors as 3, x , x^2 , $x^2 - 9$, $(x^2 - 9)$, $x^2(x^2 - 9)$ and $3x^2(x^2 - 9)$.

Only then did we move on to think about methods used. We had to do that in the next lesson, because we ran out of time. This was not a problem, because in the next lesson I was able to ask them to think back to the previous lesson. I asked them to look in their exercise books at what they had done and what their thinking had been, asking them to re-enter that thinking. Coming up with descriptions of methods went rather smoothly, probably because we had done so much thinking about it already. We shared the different methods with the whole class, and they corrected each other if the method was lacking or overcomplicated. We actually ended up with a ‘Suggested method of Mrs T’s Class’, and wrote that on a large piece of paper that was then displayed on the wall.

Reflecting on your teaching practice

When you do such an activity with your class, reflect afterwards on what went well and what went less well. Such reflection always helps with finding a ‘script’ that helps you engage the students to find mathematics interesting and enjoyable. If they do not understand and cannot do something, they are less likely to

become involved. Use this reflective exercise every time you undertake the activities, noting, as Mrs Kapur did, some quite small things that made a difference.



Pause for thought

After the lesson, try to find time to talk about these questions with a fellow teacher:

- How did it go with your class?
- What responses from students were unexpected? Why?
- What questions did you use to probe your students' understanding?
- Did you feel you had to intervene at any point?
- What points did you feel you had to reinforce?

3 Practising techniques and noticing differences between LCM and HCF

The next activity develops the practice of asking students to think about the methods they have used. This activity is, again, very similar to an activity that can be found in textbooks. The difference is that it gives students a mixture of problems in terms of having to find common multiples and factors in one activity, and mixing up numbers and expressions. The other difference is the request to make notes on the methods they have used. The aims of these modifications to the textbook activity are to make students aware of connections between topics, noticing the differences and sameness, and for the mathematical thinking processes involved to become explicit. Again, to help students engage with this new way of working, it might help to let them work in pairs or small groups when you facilitate this activity in your classroom.

Activity 2: Practising techniques and noticing differences between LCM and HCF

Tell your students:

- Find common factors and multiples of the following: [Write these problems on the board.]
 - o 48 and 72
 - o x^2 and $3xy$
 - o $\sqrt{18}$ and $\sqrt{32}$
 - o $(a - b)^2$ and $(a - b)^3$
 - o $(a^2 - b^2)$ and $(a^3 - b^3)$
- Write down the methods you used to work these out.
- Now convince your partner that these methods are mathematically correct. If you are working with a partner, try to convince another pair of students.

Case Study 2: Teacher Faraz reflects on using Activity 2

The students did the first question with great confidence. The second one provoked some discussion but the third one was left by most. For this third question I gave a hint of getting factors within the root sign, and then some of them got the answer almost at once. The fourth question resulted in a bit of discussion, but they came up with an answer. However, for the last question, some pairs came up with $a^2 - b^2$ as the

common factor and $a^3 - b^3$ as the multiple.

They described their methods in terms of an algorithm. They kept repeating that they had given the rule and they had learned that this was the rule and that I had told them so! I tell you, this did cause some soul-searching on my part! But I insisted and kept asking how they knew they were allowed to do each step and why they were doing each step. I asked them to imagine their little sister keeping asking that question, 'Why?', and that she would not be happy with 'Because I tell you' as an answer.



Pause for thought

- What responses from students were unexpected? Why?
- What questions did you use to probe your students' understanding?
- Did you modify the task in any way? If so, what was your reasoning for this?

4 Learning from the work of fictitious students

You have now engaged with a number of activities that help your students to think about the mathematical processes involved in finding common factors and multiples, in order to analyse and describe their own methods.

The next activity shows another way to encourage students to think critically about what they are doing and what they are leaning. This activity asks students to critically evaluate the working methods of some fictitious students and come up with reasons why the students' way of working is (or is not) mathematically valid. Using work from fictitious students often works well in exposing possible misconceptions, because it avoids emotional reactions and feelings of embarrassment. Since it is not work from the students themselves or any of their classmates, they can treat it without concern for anyone who may be embarrassed or upset about what they are saying. The task is further enriched because one of the methods offered is not incorrect, as such, but could be – that is, it has limitations.

Activity 3: Using fictitious students' work

Tell your students that students Meena, Deepak, Aditya and Aneesh were asked to solve the following:

$$\frac{5}{12} - \frac{3}{10}$$

a. Meena did it thus:

$$\begin{aligned} &= \frac{5}{12 \times 5} - \frac{3}{10 \times 6} \\ &= \frac{5-3}{60} \end{aligned}$$

b. Deepak thought this was what should be done:

$$\begin{aligned} &= \frac{5 \times 10 - 3 \times 12}{120} \\ &= \frac{50 - 36}{120} \\ &= \frac{14}{120} \end{aligned}$$

c. Aditya was sure this was the method:

$$\begin{aligned}
 &= \frac{5 \times 5}{12 \times 5} - \frac{3 \times 6}{10 \times 6} \\
 &= \frac{25}{60} - \frac{18}{60} \\
 &= \frac{25 - 18}{60 + 60} \\
 &= \frac{7}{120}
 \end{aligned}$$

d. Aneesh solved it in this way:

$$\begin{aligned}
 &= \frac{5 \times 5}{12 \times 5} - \frac{3 \times 6}{10 \times 6} \\
 &= \frac{25 - 18}{60} \\
 &= \frac{7}{60}
 \end{aligned}$$

Discuss each of these methods with your partner.

- Are they mathematically correct, incorrect or partly correct?
- Why?
- How do you know?

Then bring the class together and ask different pairs of students to present their responses. For each response, ask if other students agree and encourage them to share their answers. In this type of class discussion your role as a teacher is to facilitate, not to give your own opinions. So you could use phrases like 'Does everyone agree?' and 'Who would like to give a different view?' You should encourage the students to listen to each other and build on each other's understanding.

Case Study 3: Teacher Faraz reflects on using Activity 3

The students were grouped in pairs so that there could be input and ideas from both students. I decided not to go for bigger groups this time because I thought the work needed quite detailed reading and discussion. Pair groupings offer a closer working environment where the work can be more thoroughly examined by each student.

I could hear a lot of arguing, with students questioning each other and asking for reasons when they disagreed. After they had had sufficient time to get to grips with each item, we discussed the methods of the fictitious students one by one at the blackboard. For each one I asked a student to come to the blackboard to be the teacher and guide the discussions. The first attempt was slightly chaotic, with some students shouting out, but after I told them there was to be no shouting out at all and hands-up only, the discussion really improved and seemed valuable. I stood at the side, sometimes offering another thought, but with actually very little intervention on my part.



Pause for thought

- How did you organise your students in pairs? Did you let them choose their partner or did you tell them who to work with?
- What questions did you use to probe your students' understanding?
- Did you feel you had to intervene at any point?

- What points did you feel you had to reinforce?

You can adapt this activity in many different areas of the maths curriculum. Think about the topics you will be teaching over the next month. Which topics do students usually find very difficult? Do you have examples of students' work from previous years? Plan an activity like Activity 3 to use with your class. Share your ideas with other teachers in your school or cluster. In this way you can develop a folder of interesting activities for your teaching based on the textbook.

5 Adapting questions from textbooks

The last activity in this unit shows how you can help students to tackle some harder questions and to identify the different places in their mathematics course where they have to use HCF and LCM, and, hence, get equivalent numbers before they can add or subtract. They will also be asked to notice what is the same and what is different in these questions, so they can become aware of which methods and processes to use in which situation. Asking students to engage in this way with textbook questions encourages them to think about the mathematical thinking processes involved.

Activity 5: Adapting questions from textbooks

Tell your students to look at the following problems:

$$\frac{-3}{10} + \frac{5}{6}$$

$$\frac{\cos A}{1 - \sin A} + \frac{1 + \sin A}{\cos A}$$

$$\frac{24}{18 - x} - \frac{24}{18 + x} = 1$$

$$\frac{5}{(x+1)(x-2)} + \frac{2}{(x-2)(x+3)}$$

- Are there any similarities in the above questions? What 'mathematical differences' can you see?
- How would you solve these problems?
- Is there any one correct or incorrect way of doing this?
- Look at exercises in the NCERT textbook for different topics and see if you can identify the exercises where you have encountered such questions.
- What is same and what is different in these questions?

When most of your students have found some more exercises, stop their discussions and ask them to share their ideas with the whole class. Again, your role is as a facilitator, challenging their thinking and encouraging them to ask each other for clarification.



Pause for thought

- What responses from students were unexpected? Why?
- How did you evaluate your students understanding? Did this exercise help to improve their learning?
- Did you modify the task in any way? If so, what was your reasoning for this?

Again, you can use this exercise for different topics to help your students to build mathematical connections. This will develop their confidence with mathematics and in particular when they are asked to solve an unfamiliar problem.

6 Summary

This unit has explored ideas on teaching multiples and factors in order to illustrate how one idea is threaded through many parts of the mathematical curriculum. The requirements for teaching from the NCF (2005) and NCFTE (2009) are often seen as ambitious goals, but the ideas in this unit show that using textbook questions creatively will allow students to perceive relationships and to see structures. If the students use reasoning, argue the truth or falsity of statements, and look for similarities and differences, they will begin to understand and see the web of ideas in mathematics. Understanding that fluency with one idea met in the early years of schooling is necessary in order to work with much more complex ideas later helps students understand why they are being asked to engage with new ideas that seem to have no immediate application.

Resources

Resource 1: NCF/NCFTE teaching requirements

This unit links to the following teaching requirements of the NCF (2005) and NCFTE (2009), and will help you to meet those requirements:

- View students as active participants in their own learning and not as mere recipients of knowledge; how to encourage their capacity to construct knowledge; how to shift learning away from rote methods
- Let students see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on
- Let students use abstractions to perceive relationships, to see structures, to reason out things, to argue the truth or falsity of statements
- Engage with the curriculum, syllabuses and textbooks critically by examining them rather than taking them as 'given' and accepted without question.

Resource 2: Using groupwork

Groupwork is a systematic, active, pedagogical strategy that encourages small groups of students to work together for the achievement of a common goal. These small groups promote more active and more effective learning through structured activities.

The benefits of groupwork

Groupwork can be a very effective way of motivating your students to learn by encouraging them to think, communicate, exchange ideas and thoughts, and make decisions. Your students can both teach and learn from others: a powerful and active form of learning.

Groupwork is more than students sitting in groups; it involves working on and contributing to a shared learning task with a clear objective. You need to be clear about why you are using groupwork for learning and know why this is preferable to lecturing, pair work or to students working on their own. Thus groupwork has to be well-planned and purposeful.

Planning groupwork

When and how you use groupwork will depend on what learning you want to achieve by the end of the lesson. You can include groupwork at the start, the end or midway through the lesson, but you will need to allow enough time. You will need to think about the task that you want your students to complete and the best way to organise the groups.

As a teacher, you can ensure that groupwork is successful if you plan in advance:

- the goals and expected outcomes of the group activity
- the time allocated to the activity, including any feedback or summary task
- how to split the groups (how many groups, how many students in each group, criteria for groups)
- how to organise the groups (role of different group members, time required, materials, recording and reporting)
- how any assessment will be undertaken and recorded (take care to distinguish individual assessments from group assessments)
- how you will monitor the groups' activities.

Groupwork tasks

The task that you ask your students to complete depends on what you want them to learn. By taking part in groupwork, they will learn skills such as listening to each other, explaining their ideas and working cooperatively. However, the main aim is for them to learn something about the subject that you are teaching. Some examples of tasks could include the following:

- **Presentations:** Students work in groups to prepare a presentation for the rest of the class. This works best if each group has a different aspect of the topic, so they are motivated to listen to each other rather than listening to the same topic several times. Be very strict about the time that each group has to present and decide on a set of criteria for a good presentation. Write these on the board before the lesson. Students can then use the criteria to plan their presentation and assess each other's work. The criteria could include:
 - o Was the presentation clear?
 - o Was the presentation well-structured?
 - o Did I learn something from the presentation?
 - o Did the presentation make me think?
- **Problem solving:** Students work in groups to solve a problem or a series of problems. This could include conducting an experiment in science, solving problems in mathematics, analysing a story or poem in English, or analysing evidence in history.
- **Creating an artefact or product:** Students work in groups to develop a story, a piece of drama, a piece of music, a model to explain a concept, a news report on an issue or a poster to summarise information or explain a concept. Giving groups five minutes at the start of a new topic to create a brainstorm or mind map will tell you a great deal about what they already know, and will help you pitch the lesson at an appropriate level.
- **Differentiated tasks:** Groupwork is an opportunity to allow students of different ages or attainment levels to work together on an appropriate task. Higher attainers can benefit from the opportunity to

explain the work, whereas lower attainers may find it easier to ask questions in a group than in a class, and will learn from their classmates.

- **Discussion:** Students consider an issue and come to a conclusion. This may require quite a bit of preparation on your part in order to make sure that the students have enough knowledge to consider different options, but organising a discussion or debate can be very rewarding for both you and them.

Organising groups

Groups of four to eight are ideal but this will depend on the size of your class, the physical environment and furniture, and the attainment and age range of your class. Ideally everyone in a group needs to see each other, talk without shouting and contribute to the group's outcome.

- Decide how and why you will divide students into groups; for example, you may divide groups by friendship, interest or by similar or mixed attainment. Experiment with different ways and review what works best with each class.
- Plan any roles you will give to group members (for example, note taker, spokesperson, time keeper or collector of equipment), and how you will make this clear.

Managing groupwork

You can set up routines and rules to manage good groupwork. When you use groupwork regularly, students will know what you expect and find it enjoyable. Initially it is a good idea to work with your class to identify the benefits of working together in teams and groups. You should discuss what makes good groupwork behaviour and possibly generate a list of 'rules' that might be displayed; for example, 'Respect for each other', 'Listening', 'Helping each other', 'Trying more than one idea', etc.

It is important to give clear verbal instructions about the groupwork that can also be written on the blackboard for reference. You need to:

- direct your students to the groups they will work in according to your plan, perhaps designating areas in the classroom where they will work or giving instructions about moving any furniture or school bags
- be very clear about the task and write it on the board in short instructions or pictures. Allow your students to ask questions before you start.

During the lesson, move around to observe and check how the groups are doing. Offer advice where needed if they are deviating from the task or getting stuck.

You might want to change the groups during the task. Here are two techniques to try when you are feeling confident about groupwork – they are particularly helpful when managing a large class:

- **'Expert groups':** Give each group a different task, such as researching one way of generating electricity or developing a character for a drama. After a suitable time, re-organise the groups so that each new group is made up of one 'expert' from all the original groups. Then give them a task that involves collating knowledge from all the experts, such as deciding on what sort of power station to build or preparing a piece of drama.
- **'Envoys':** If the task involves creating something or solving a problem, after a while, ask each group to send an envoy to another group. They could compare ideas or solutions to the problem and then report back to their own group. In this way, groups can learn from each other.

At the end of the task, summarise what has been learnt and correct any misunderstandings that you have seen. You may want to hear feedback from each group, or ask just one or two groups who you think have

some good ideas. Keep students' reporting brief and encourage them to offer feedback on work from other groups by identifying what has been done well, what was interesting and what might be developed further.

Even if you want to adopt groupwork in your classroom, you may at times find it difficult to organise because some students:

- are resistant to active learning and do not engage
- are dominant
- do not participate due to poor interpersonal skills or lack of confidence.

To become effective at managing groupwork it is important to reflect on all the above points, in addition to considering how far the learning outcomes were met and how well your students responded (did they all benefit?). Consider and carefully plan any adjustments you might make to the group task, resources, timings or composition of the groups.

Research suggests that learning in groups need not be used all the time to have positive effects on student achievement, so you should not feel obliged to use it in every lesson. You might want to consider using groupwork as a supplemental technique, for example as a break between a topic change or a jump-start for class discussion. It can also be used as an ice-breaker or to introduce experiential learning activities and problem solving exercises into the classroom, or to review topics.

Resource 3: Talk for learning

Why talk for learning is important

Talk is a part of human development that helps us to think, learn and make sense of the world. People use language as a tool for developing reasoning, knowledge and understanding. Therefore, encouraging students to talk as part of their learning experiences will mean that their educational progress is enhanced.

Talking about the ideas being learnt means that:

- those ideas are explored
- reasoning is developed and organised
- as such, students learn more.

In a classroom there are different ways to use student talk, ranging from rote repetition to higher-order discussions.

Traditionally, teacher talk was dominant and was more valued than students' talk or knowledge. However, using talk for learning involves planning lessons so that students can talk more and learn more in a way that makes connections with their prior experience. It is much more than a question and answer session between the teacher and their students, in that the students' own language, ideas, reasoning and interests are given more time. Most of us want to talk to someone about a difficult issue or in order to find out something, and teachers can build on this instinct with well-planned activities.

Planning talk for learning activities in the classroom

Planning talking activities is not just for literacy and vocabulary lessons; it is also part of planning mathematics and science work and other topics. It can be planned into whole class, pair or groupwork, outdoor activities, role play-based activities, writing, reading, practical investigations, and creative work.

Even young students with limited literacy and numeracy skills can demonstrate higher-order thinking skills if the task is designed to build on their prior experience and is enjoyable. For example, students can make

predictions about a story, an animal or a shape from photos, drawings or real objects. Students can list suggestions and possible solutions about problems to a puppet or character in a role play.

Plan the lesson around what you want the students to learn and think about, as well as what type of talk you want students to develop. Some types of talk are exploratory, for example: 'What could happen next?', 'Have we seen this before?', 'What could this be?' or 'Why do you think that is?' Other types of talk are more analytical, for example weighing up ideas, evidence or suggestions.

Try to make it interesting, enjoyable and possible for all students to participate in dialogue. Students need to be comfortable and feel safe in expressing views and exploring ideas without fear of ridicule or being made to feel they are getting it wrong.

Building on students' talk

Talk for learning gives teachers opportunities to:

- listen to what students say
- appreciate and build on students' ideas
- encourage the students to take it further.

Not all responses have to be written or formally assessed, because developing ideas through talk is a valuable part of learning. You should use their experiences and ideas as much as possible to make their learning feel relevant. The best student talk is exploratory, which means that the students explore and challenge one another's ideas so that they can become confident about their responses. Groups talking together should be encouraged not to just accept an answer, whoever gives it. You can model challenging thinking in a whole class setting through your use of probing questions like 'Why?', 'How did you decide that?' or 'Can you see any problems with that solution?' You can walk around the classroom listening to groups of students and extending their thinking by asking such questions.

Your students will be encouraged if their talk, ideas and experiences are valued and appreciated. Praise your students for their behaviour when talking, listening carefully, questioning one another, and learning not to interrupt. Be aware of members of the class who are marginalised and think about how you can ensure that they are included. It may take some time to establish ways of working that allow all students to participate fully.

Encourage students to ask questions themselves

Develop a climate in your classroom where good challenging questions are asked and where students' ideas are respected and praised. Students will not ask questions if they are afraid of how they will be received or if they think their ideas are not valued. Inviting students to ask the questions encourages them to show curiosity, asks them to think in a different way about their learning and helps you to understand their point of view.

You could plan some regular group or pair work, or perhaps a 'student question time' so that students can raise queries or ask for clarification. You could:

- entitle a section of your lesson 'Hands up if you have a question'
- put a student in the hot-seat and encourage the other students to question that student as if they were a character, e.g. Pythagoras or Mirabai
- play a 'Tell Me More' game in pairs or small groups
- give students a question grid with who/what/where/when/why questions to practise basic enquiry

- give the students some data (such as the data available from the World Data Bank, e.g. the percentage of children in full-time education or exclusive breastfeeding rates for different countries), and ask them to think of questions you could ask about this data
- design a question wall listing the students' questions of the week.

You may be pleasantly surprised at the level of interest and thinking that you see when students are freer to ask and answer questions that come from them. As students learn how to communicate more clearly and accurately, they not only increase their oral and written vocabulary, but they also develop new knowledge and skills.

Additional resources

- A newly developed maths portal by the Karnataka government: <http://karnatakaeducation.org.in/KOER/en/index.php/Portal:Mathematics>
- Class X maths study material: http://www.zietmysore.org/stud_mats/X/maths.pdf
- National Centre for Excellence in the Teaching of Mathematics: <https://www.ncetm.org.uk/>
- National STEM Centre: <http://www.nationalstemcentre.org.uk/>
- OpenLearn: <http://www.open.edu/openlearn/>
- BBC Bitesize: <http://www.bbc.co.uk/bitesize/>
- Khan Academy's math section: <https://www.khanacademy.org/math>
- NRIC: <http://nrich.maths.org/frontpage>
- Mathcelebration: <http://www.mathcelebration.com/>
- Art of Problem Solving's resources page: <http://www.artofproblemsolving.com/Resources/index.php>
- Teachnology: <http://www.teach-nology.com/worksheets/math/>
- Maths is Fun: <http://www.mathsisfun.com/>
- National Council of Educational Research and Training's textbooks for teaching mathematics and for teacher training of mathematics: <http://www.ncert.nic.in/ncerts/textbook/textbook.htm>
- LMT-01 *Learning Mathematics*, Block 1 ('Approaches to Learning') Block 2 ('Encouraging Learning in the Classroom'), Block 6 ('Thinking Mathematically'): <http://www.ignou4ublog.com/2013/06/ignou-lmt-01-study-materialbooks.html>
- *Learning Curve* and *At Right Angles*, periodicals about mathematics and its teaching: http://azimpremjiifoundation.org/Foundation_Publications
- Central Board of Secondary Education's books and support material (also including the *Teachers Manual for Formative Assessment – Mathematics (Class IX)*) – select 'CBSE publications', then 'Books and support material': <http://cbse.nic.in/welcome.htm>

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