Secondary Mathematics

Cooperative learning and mathematical talk: triangles

Teacher Education through School-based Support in India
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TESS-India OERs have been collaboratively written by Indian and international authors to address Indian curriculum and contexts and are available for online and print use (http://www.tess-india.edu.in/). The OERs are available in several versions, appropriate for each participating Indian state and users are invited to adapt and localise the OERs further to meet local needs and contexts.

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Video resources

Some of the activities in this unit are accompanied by the following icon:  *. This indicates that you will find it helpful to view the TESS-India video resources for the specified pedagogic theme.*

The TESS-India video resources illustrate key pedagogic techniques in a range of classroom contexts in India. We hope they will inspire you to experiment with similar practices. They are intended to complement and enhance your experience of working through the text-based units, but are not integral to them should you be unable to access them.

TESS-India video resources may be viewed online or downloaded from the TESS-India website, http://www.tess-india.edu.in/). Alternatively, you may have access to these videos on a CD or memory card.

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All India - English

What this unit is about

Triangles are often thought of as the basic shape in geometry, because all other polygons can be divided into triangles. Therefore, getting your students to recognise, use and communicate with one another about triangles and their properties, using conventional terminology, is an important step in learning to 'speak like a mathematician' and understand mathematics rather than just remembering the subject.

This unit will focus on how to help your students to use talk for learning mathematics. It will also discuss and offer ideas about how students can support each other in their learning and give each other effective feedback. Such cooperative learning helps students to feel more motivated. It is also especially useful when working in large classes.

Figure 1 A group of students working with triangles.

What you can learn in this unit

- How to use cooperative learning to enable your students to support each other in their learning of mathematics.
- Some ideas to help your students learn through talking, using formal mathematical language.
- Some ideas to help your students use and benefit from effective feedback.

This unit links to the teaching requirements of the NCF (2005) and NCFTE (2009) outlined in Resource 1.

1 Talking and learning in mathematics

Encouraging students to talk about mathematics and helping them to develop the appropriate vocabulary to do this is an important part of learning. Thinking and communicating are intimately entwined (Sfard, 2010). If you want your students to think about, understand and therefore effectively learn mathematics, they will also need to learn to communicate their mathematical ideas.

If you help your students to communicate using mathematical vocabulary and phraseology, you will be able to listen to them presenting and talking during their work. By listening you will know whether they understand mathematical ideas or not. This will help you to evaluate their learning.

Students need to learn to talk about what they are thinking with one another. The act of forming thoughts in order to communicate with others will help them learn the ideas (Lee, 2006) and can sometimes enable misconceptions to be corrected.

Pause for thought

Think about your own classroom. How much communication goes on in your classroom between the students and you, or just between students? How easy do the students find it to use mathematical vocabulary? Why do you think this is?

How did you learn to communicate mathematical ideas? Who did you talk to about mathematics? Did talking help you to sort out your ideas?

It is most important to expect students to use key words and to put them into a position where they have to do so. Tasks using the new vocabulary is vital to the outcome are effective in achieving this. Activity 1 asks students to do this by creating and describing triangles.

Before attempting to use the activities in this unit with your students, it would be a good idea to complete all (or at least part) of the activities yourself. It would be even better if you could try them out with a colleague, as that will help you when you reflect on the experience. Trying the activities yourself will mean that you get insights into learners' experiences that can in turn influence your teaching and your experiences as a teacher. When you are ready, use the activities with your students. After the lesson, think about the way that the activity went and the learning that happened. This will help you to develop a more learner-focused teaching environment.

Activity 1: Talking mathematically about triangles

Preparation

For this activity, you need a stack of sticks of different lengths. You could cut suitable sticks from bamboo. The range of the length of sticks must be adequately large – say, from 2 inches to 18 inches.

Ideally there should be at least three sticks for each student. If this is not possible, use three sticks for each pair of students or even groups of three students. Each student (or pair of students) picks three sticks at random. Make sure they don't get to choose the lengths of their sticks. It might help to take your students outside where they have more room to move about.

Write the words on the board or on a large piece of paper on the wall.

The activity

Tell your students to form a triangle using the three sticks they chose.

Ask your students the following:

- Are some of you not able to form a triangle? If so, discuss why you think this is.
- If you were allowed to change only one stick, which stick would that be and why?
- Describe your triangle using as many of these words as you can (extend or contract this list as needed):

  acute, obtuse, right, perpendicular, scalene, isosceles, equilateral, angle, side, length, degree, larger, smaller, longer, shorter, area, square, opposite, adjacent.

Now give the students another stick so that they have four sticks altogether.

Ask your students to:

- make four different triangles
- tell their classmate what is the same and what is different about the four triangles they can make, again using as many of the words in the list above as possible.

Case Study 1: Mrs Chadha reflects on using Activity 1

This is the account of a teacher who tried Activity 1 with her secondary students.

I made sure that I told the class right at the start why they were doing this activity. I told them that it was really important that they could identify and talk about all the different parts of a triangle. They would then be able to understand better the questions they are asked to do in their textbook or in the exams. I also told them that at the end of the lesson, I would ask them to check that they could use the conventional terminology for every part of a triangle, so it was important that they made sure they understood and could use the names correctly.

As I have quite a large class of students, I asked them to work in groups of three on this exercise. One student from each group came out and picked up three sticks; I had to hold onto them so that they did not know if they were picking long or short sticks.

Then I asked them to make a triangle with their three sticks meeting at the ends and to hold up their hands when they had done that. Some groups very quickly held up their hands, but there was a great deal of trying this way and that in two of the groups. I told everyone to stop and asked the groups that had made a triangle to sit down.

Then I asked the two groups that were still standing what the matter was. 'We cannot make a triangle, Miss,' they said. 'Why not?' I asked. At first they started to say things like 'the sticks won't meet' or 'we cannot make a point'. I stopped them and asked the class for what the sticks and points were called when in a triangle. Danna eventually came up with 'they are sides and vertices,' so I congratulated her on using the correct terminology – reminding them of the purpose of the lesson. After much talk and help from others, the two groups were able to say that 'in order to form a triangle, the sum of the lengths of the two shorter sides must be longer than the length of the longer side'. It was real class endeavour, so I felt pretty certain that they had all contributed and understood this point. They were then allowed to swap one of their sticks with another group and we went onto the second part.

I told the students they had to introduce their triangle to the rest of the class using conventional mathematical language. I wrote the words on the blackboard that they had to use, using the textbook to check I had all that they needed. I gave them ten minutes to name their triangle and devise an introduction for it using as many of the words on the blackboard as possible.

Because I'd written down the list of words, the students could speak mathematically; also, I made a point of getting every student say something about the triangles they had made, so that everyone had to make some contribution to the discussion.

This was interesting because some of the students came up with very unusual descriptions for their triangle. Sona said that she had made 'an old triangle'. When I asked her what she meant, she said it was bending. I asked her to use a mathematical term and she couldn't think of a word for it, so I asked her to draw it on the blackboard and asked the others what they would call it. Ravi at once said that it was 'obtuse angled'. We had a discussion about these triangles and, as a result, we discussed a lot of things. As each group introduced their triangle I ticked off the words 'hypotenuse', 'opposite', 'acute', 'obtuse', etc. on the blackboard, and told them how successful they had been. No one quite used all of the words, but it came pretty close.

Reflecting on your teaching practice

When you do such an activity with your class, reflect afterwards on what went well and what went less well. Consider the questions that led to the students being interested and those where you needed to clarify. Such reflection always helps with finding a 'script' that helps you engage the students to find mathematics interesting and enjoyable. If they do not understand and cannot do something, they are less likely to become involved. Use this reflective exercise every time you undertake the activities, noting, as Mrs Chadha did, some quite small things that made a difference.

Pause for thought

Good questions to trigger such reflection are:

- How did it go with your class?
- What responses from students were unexpected? Why?
- What questions did you use to probe your students' understanding?
- What points did you feel you had to reinforce at the end of the lesson?
- How will you reinforce this learning in the next lesson?

Using a practical activity to encourage mathematical talk is a technique you can use in many different topics. Try to think of two other topics where you could use this technique. Share your ideas with fellow maths teachers in your school or local schools and keep a note of their ideas for integrating into your lesson planning.

2 Effective feedback

Feedback has been shown to make an enormous difference to learning (Hattie and Timperley, 2007) when it clearly provides answers to these three questions:

- Where am I going?
- How am I doing?
- Where to next?

First the student needs to be very clear about the purpose or objective of the learning ('Where am I going?'). Feedback on 'How am I doing?' has been shown to lead to students displaying enhanced engagement and motivation to close the gap between where they are going and where they currently are in their learning. However, they will only be able to employ that motivation if they can close the gap further by knowing the answer to the third question, 'Where to next?'

Giving and receiving feedback requires much skill from both teacher and student alike. It requires a classroom ethos that allows students to expose their thoughts without fear of ridicule from anyone and where the focus is entirely on everyone learning and improving together. Within such an ethos it is possible for teachers or other students to give feedback focused on 'How am I doing?' and 'Where next?', and for students to listen to and act on that feedback.

If students try out their ideas together, they can receive frequent feedback from one another. The feedback will not be of the same quality as the teacher's, because students will not have the depth of subject knowledge nor the ability to make connections that the teacher has; however, the opportunity to gain feedback quickly often outweighs the issue of quality.

The purpose of the next activity is to learn more about triangles, but it also gives the class the opportunity to try out using the three questions for effective feedback ('Where am I going?', 'How am I doing?' and 'Where to next?'). You can also apply the ideas from this activity to other mathematical topics.

Activity 2: What happens if ...?

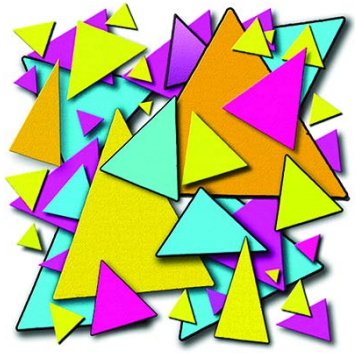


Figure 2 How many different triangles can the students make?

Every student, or group of students, should have in front of them a triangle made from sticks.

Ask your students to discuss the following questions in groups:

- Using the same three sticks, how many different triangles can you make? Why? What happens if you interchange the position of two sticks?
- What happens to the triangle if you replace a stick with a shorter or longer one?
- What happens if you increase or decrease the angle between two of the sticks?
- Measure the three sides and the three angles. Arrange these measurements in decreasing order. What do you observe? Do other students observe the same thing? State your observation as a result for the triangles.
- Formulate a result about two triangles, the lengths of whose corresponding sides are the same. Do you think this statement is true for all triangles? Can you extend this statement to other polygons?

Also ask the students to reflect during and after their discussions on the questions:

- Where am I going?
- How am I doing?
- Where to next?

Then ask them to share their findings with the class. Tell the groups to include their thoughts on 'Where am I going?', 'How am I doing?' and 'Where to next?' Tell the students listening to other groups and to prepare and give feedback on these questions.

Then ask different students to present their thinking to the class. Other students should be encouraged to comment. Try to ensure that students from different groups all get an opportunity to speak.

Case Study 2: Mrs Chadha reflects on using Activity 2

I used Activity 2 straight after Activity 1. The students spent quite a lot of time trying to make different triangles and I had to keep reminding them about the sides meeting at the ends of the sticks. Eventually they all felt that they had answers to the first four questions. This time I asked them to present their results to another group and to try and convince them that they were correct. I introduced ideas about feedback to the class, asking them to think before they started to listen to one another about the purpose of the lesson or 'Where am I going?', which I had talked about several times by this time. I then told the listening group to provide feedback on 'How am I doing?' and 'Where to next?' to the group presenting their ideas. After five minutes I reminded the class to change over from listening to presenting.

After that, we had a class discussion. I asked first, 'What was easy?' The class said that they found it very easy to talk about their triangles and the sides, but it wasn't until someone remembered about reflection and rotation that they could say why the triangles were the same – but different! They also came up with the word 'similar' to describe the triangles, which I was very pleased to hear. We discussed the mathematical meaning of 'similar' and I asked each group to write down their own mathematical definition of what 'similar' means in mathematical triangles and shapes.

I then asked the students what they found difficult. They said that giving proper feedback was hard, but that it really made them think. Trying to give a useful reply to 'How am I doing?' and 'Where to next?' was not easy, but meant that you had to think about what was said. I replied, 'I know!' This too turned out to be an interesting session and I think that the debate must have helped a lot of students, because they looked quite happy after the class.

Video: Involving all

http://tinyurl.com/video-involvingall

You may also want to have a look at the key resource on 'Involving all' (http://tinyurl.com/kr-involvingall).

Pause for thought

- What responses from students were unexpected? Why?
- What questions did you use to probe your students' understanding?
- Did you modify the task in any way? If so, what was your reasoning for this?

If you have access to the internet you could also use dynamic geometry software (such as the free-to-download programme GeoGebra) to create similar tasks.

3 Cooperative learning

In order to maximise the opportunities to learn in a crowded classroom, students should help one another as they did in Mrs Chadha's class in Case Study 2. However, cooperative learning is rarely deployed in secondary mathematics classrooms and where it is, it is often by default rather than purposeful. Slavin et al. (2003) reviewed a great deal of evidence and concluded that 'cooperative learning is one of the greatest success stories in the history of modern research' (p. 177). The four main reasons (Wiliam, 2011) for this success seem to be:

1. **Motivation**: Students help one another learn because it is in their own interests to do so. This has the effect of increasing all-round effort, leading to more success in learning and therefore more motivation to work on challenging ideas.
2. **Social cohesion:** Students help their peers because they are part of the same group and it matters to them that the group succeeds.
3. **Personalisation:** If a particular student is having difficulties, it is likely that there will be another in the group who can help out. Where groups are well-structured it is not always the same people helping or receiving help.
4. **Cognitive elaboration:** Those who contribute to discussions are forced to think through the ideas and clarify them for themselves and others.

If students are to get the help they need in a large class, they must be available to help one another. Students teaching one another can be surprisingly effective: in one study students learned almost as much when peer-tutored as they did from one-to one instruction from their teacher, possibly because they feel less intimidated asking questions of a peer (Schacter, 2000).

The purpose of the next activity is to ask the students to work together cooperatively in order to make connections with other mathematical ideas to solve a problem.

Activity 3: Making connections

For this activity you need at least three sticks for each student. If this is not possible, use three sticks for each pair of students or even groups of three students. Each student (or pair of students) picks three sticks at random. Make sure they don't get to choose the lengths of their sticks.

Tell your students:

- You can now slide your sticks in order to 'cut' a portion of the longest side of the triangle you have created so that the three lengths form a right-angled triangle.
- Discuss in your groups how you will find the exact length that needs to be cut. What fact(s) do you use to find this length?
- Measure the three sticks you used to form the right-angled triangle as accurately as possible.
- Find the ratios of the lengths of each of the three sticks to each other.
- Find another student for whom any of these ratios coincide.
- Investigate with the other student what is common between your triangles that resulted in the ratios coinciding.
- Formulate and present your findings in class.

Case Study 3: Mrs Chadha reflects on using Activity 3

I had taught Pythagoras' theorem to the class a while ago and wanted to move onto trigonometry. Now that the class was much happier talking about triangles together and using the correct terminology, I thought I would also be able to use this exercise to judge how well they remembered Pythagoras and whether or not I needed to teach it again.

I told them that the purpose of this session was to work with right-angled triangles, to make connections to other mathematical ideas they had learned and to extend what they knew. That meant that they should note for themselves when they made connections so that they could share them with the class and that I would ask

them at the end about anything new that they had learned.

They naturally formed into the groups that they had used yesterday when doing Activities 1 and 2 and got hold of their sticks. They started talking about how they could be sure that they had the correct length to 'cut' – I didn't let them actually cut the stick, just mark them with a biro. They first started talking about measuring, saying if we measure 90° exactly then that will be correct, so they got their protractors out and started to measure. I wanted to see what would happen as they talked together, so I just listened in to the various groups. I heard some people saying 'Check it', 'Oh no, it's moved' and 'It's difficult to do this'. I asked the whole class to think about what connections they could be making at that point with other mathematics that they knew, as they seemed engrossed with measuring. I told them all to be quiet and think for 30 seconds, then to go back to work.

The quiet thinking time did the trick and someone started saying, 'We must find the hypotenuse, so what if we use, umm p, p …' Someone else came in with 'Pythagoras – now what was that!' I saw several people look Pythagoras' theorem up in their textbook. It seemed they were just double-checking as within a very few minutes, they were squaring and trying to find the square roots. I was so pleased that all I had had to do was to give them time to think and then the whole class supported one another to use Pythagoras' theorem easily and naturally to find an exact answer.

I felt that most of the class had a good understanding of Pythagoras but I saw that Pavendeep and a couple of others were confused. I asked them to talk to me once the rest of the class had started Part 2 and it turned out they had missed the lessons on Pythagoras' theorem. Together we made a plan about how they would make up that knowledge, including using the textbook, using the internet and coming back to tell me what they knew next week, before their next lesson.

I asked for volunteers who were ready to come out and say how they had achieved the task. I also asked them if there were some different ways that they had done the activity. After that they were grouped again in fours and the second part of the activity was done, along with a discussion following that as to what was happening.

They set about Part 2 with a will. They knew there was something to find out and they wanted to find it. They did the arithmetic, double-checking that they had the correct answer. (When do they check normally? Normally they are just trying to finish the exercise!) They could not compare the ratios when they kept them as fractions, so they realised that they would have to do something else. Some said we could make equivalent fractions; some found it easier to represent in decimal forms. Luckily there were several groups with isosceles triangles of different sizes and some 30-, 60- and 90-degree triangles as well. So groups of students formed naturally with the same answers and they could see that if the angles were the same, the ratios for all the sides were the same – some even saw that the sides were doubled in one case. This was the perfect start for learning about the trigonometry ratios but that had to wait for the next lesson. First we had some questions to answer so that they could see that they had successfully fulfilled the purpose of the lesson.

Pause for thought

How often do you use problem solving to evaluate students' learning? Do you think Mrs Chadha found out what each of her class could do? Many of them would have received some help in remembering how to use Pythagoras' theorem, does that mean that they did not know about it?

Observing your class solving problems can help you know who can understand how to use a mathematical concept, who can only follow an algorithm and is therefore lost out of context, and who has no idea at all.

Perhaps the most important thing here is that the students find out what they can and cannot do. You as the teacher, can then give really good feedback on the 'Where to next?' question, just as Mrs Chadha did.

Video: Assessing progress and performance

http://tinyurl.com/video-assessingprogress

For more detail, read Resource 2, 'Assessing progress and performance'.

4 Summary

This unit has used ideas about teaching triangles to focus on three important issues:

- The need to help students to be able to **talk** about their mathematical ideas. Students need to be able to express their ideas if they are to fully understand them and use them beyond the immediate context where they are learned. Because thinking requires language, students that are helped to use specific mathematical vocabulary and phraseology themselves are at an advantage when asked to think.
- The role of **feedback** to help students engage more with their learning and feel more motivated.
- **Cooperative learning**, where students support each other effectively in their learning. This is especially useful when teaching large classes.

Pause for thought

Identify three ideas that you have used in this unit that would work when teaching other topics. Make a note now of two topics you have to teach soon where those ideas can be used with some small adjustments.

Resources

Resource 1: NCF/NCFTE teaching requirements

The learning in this unit links to the NCF (2005) and NCFTE (2009) teaching requirements as specified below:

- View learners as active participants in their own learning and not as mere recipients of knowledge; to encourage their capacity to construct knowledge; to ensure that learning shifts away from rote methods.
- View learning as a search for meaning out of personal experiences and knowledge generation as a continuously evolving process of reflective learning.
- Support students to learn to enjoy mathematics rather than fear it.

Resource 2: Assessing progress and performance

Assessing students' learning has two purposes:

- **Summative assessment** looks back and makes a judgement on what has already been learnt. It is often conducted in the form of tests that are graded, telling students their attainment on the questions in that test. This also helps in reporting outcomes.
- **Formative assessment** (or assessment for learning) is quite different, being more informal and diagnostic in nature. Teachers use it as part of the learning process, for example questioning to

check whether students have understood something. The outcomes of this assessment are then used to change the next learning experience. Monitoring and feedback are part of formative assessment.

Formative assessment enhances learning because in order to learn, most students must:

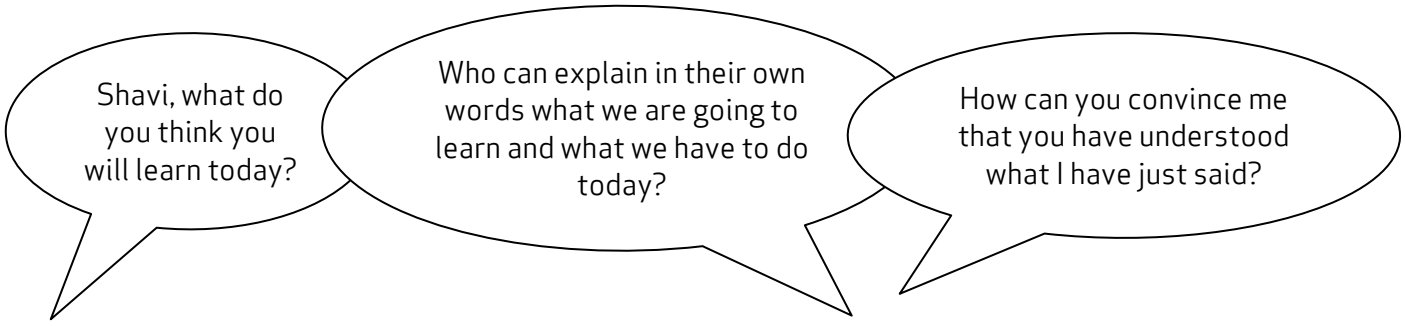
- understand what they are expected to learn
- know where they are now with that learning
- understand how they can make progress (that is, what to study and how to study)
- know when they have reached the goals and expected outcomes.

As a teacher, you will get the best out of your students if you attend to the four points above in every lesson. Thus assessment can be undertaken before, during and after instruction:

- **Before:** Assessing before the teaching begins can help you identify what the students know and can do prior to instruction. It determines the baseline and gives you a starting point for planning your teaching. Enhancing your understanding of what your students know reduces the chance of re-teaching the students something they have already mastered or omitting something they possibly should (but do not yet) know or understand.
- **During:** Assessing during classroom teaching involves checking if students are learning and improving. This will help you make adjustments in your teaching methodology, resources and activities. It will help you understand how the student is progressing towards the desired objective and how successful your teaching is.
- **After:** Assessment that occurs after teaching confirms what students have learnt and shows you who has learnt and who still needs support. This will allow you to assess the effectiveness of your teaching goal.

Before: being clear about what your students will learn

When you decide what the students must learn in a lesson or series of lessons, you need to share this with them. Carefully distinguish what the students are expected to learn from what you are asking them to do. Ask an open question that gives you the chance to assess whether they have really understood. For example:



Give the students a few seconds to think before they answer, or perhaps ask the students to first discuss their answers in pairs or small groups. When they tell you their answer, you will know whether they understand what it is they have to learn.

Before: knowing where students are in their learning

In order to help your students improve, both you and they need to know the current state of their knowledge and understanding. Once you have shared the intended learning outcomes or goals, you could do the following:

- Ask the students to work in pairs to make a mind map or list of what they already know about that topic, giving them enough time to complete it but not too long for those with few ideas. You should then review the mind maps or lists.
- Write the important vocabulary on the board and ask for volunteers to say what they know about each word. Then ask the rest of the class to put their thumbs up if they understand the word, thumbs down if they know very little or nothing, and thumbs horizontal if they know something.

Knowing where to start will mean that you can plan lessons that are relevant and constructive for your students. It is also important that your students are able to assess how well they are learning so that both you and they know what they need to learn next. Providing opportunities for your students to take charge of their own learning will help to make them life-long learners.

During: ensuring students' progress in learning

When you talk to students about their current progress, make sure that they find your feedback both useful and constructive. Do this by:

- helping students know their strengths and how they might further improve
- being clear about what needs further development
- being positive about how they might develop their learning, checking that they understand and feel able to use the advice.

You will also need to provide opportunities for students to improve their learning. This means that you may have to modify your lesson plans to close the gap between where your students are now in their learning and where you wish them to be. In order to do this you might have to:

- go back over some work that you thought they knew already
- group students according to needs, giving them differentiated tasks
- encourage students to decide for themselves which of several resources they need to study so that they can 'fill their own gap'
- use 'low entry, high ceiling' tasks so that all students can make progress – these are designed so that all students can start the task but the more able ones are not restricted and can progress to extend their learning.

By slowing the pace of lessons down, very often you can actually speed up learning because you give students the time and confidence to think and understand what they need to do to improve. By letting students talk about their work among themselves, and reflect on where the gaps are and how they might close them, you are providing them with ways to assess themselves.

After: collecting and interpreting evidence, and planning ahead

While teaching–learning is taking place and after setting a classwork or homework task, it is important to:

- find out how well your students are doing
- use this to inform your planning for the next lesson
- feed it back to students.

The four key states of assessment are discussed below.

Collecting information or evidence

Every student learns differently, at their own pace and style, both inside and outside the school. Therefore, you need to do two things while assessing students:

- Collect information from a variety of sources – from your own experience, the student, other students, other teachers, parents and community members.
- Assess students individually, in pairs and in groups, and promote self-assessment. Using different methods is important, as no single method can provide all the information you need. Different ways of collecting information about the students' learning and progress include observing, listening, discussing topics and themes, and reviewing written class and homework.

Recording

In all schools across India the most common form of recording is through the use of report card, but this may not allow you to record all aspects of a student's learning or behaviours. There are some simple ways of doing this that you may like to consider, such as:

- noting down what you observe while teaching–learning is going on in a diary/notebook/register
- keeping samples of students' work (written, art, craft, projects, poems, etc.) in a portfolio
- preparing every student's profile
- noting down any unusual incidents, changes, problems, strengths and learning evidences of students.

Interpreting the evidence

Once information and evidence have been collected and recorded, it is important to interpret it in order to form an understanding of how each student is learning and progressing. This requires careful reflection and analysis. You then need to act on your findings to improve learning, maybe through feedback to students or finding new resources, rearranging the groups, or repeating a learning point.

Planning for improvement

Assessment can help you to provide meaningful learning opportunities to every student by establishing specific and differentiated learning activities, giving attention to the students who need more help and challenging the students who are more advanced.

Additional resources

- A newly developed maths portal by the Karnataka government: http://karnatakaeducation.org.in/KOER/en/index.php/Portal:Mathematics
- Class X maths study material: http://www.zietmysore.org/stud_mats/X/maths.pdf
- National Centre for Excellence in the Teaching of Mathematics: https://www.ncetm.org.uk/
- National STEM Centre: http://www.nationalstemcentre.org.uk/
- OpenLearn: http://www.open.edu/openlearn/
- BBC Bitesize: http://www.bbc.co.uk/bitesize/
- Khan Academy's math section: https://www.khanacademy.org/math
- NRICH: http://nrich.maths.org/frontpage
- Mathcelebration: http://www.mathcelebration.com/
- Art of Problem Solving's resources page: http://www.artofproblemsolving.com/Resources/index.php
- Teachnology: http://www.teach-nology.com/worksheets/math/

- Maths is Fun: http://www.mathsisfun.com/
- National Council of Educational Research and Training's textbooks for teaching mathematics and for teacher training of mathematics: http://www.ncert.nic.in/ncerts/textbook/textbook.htm
- LMT-01 *Learning Mathematics*, Block 1 ('Approaches to Learning') Block 2 ('Encouraging Learning in the Classroom'), Block 6 ('Thinking Mathematically'): http://www.ignou4ublog.com/2013/06/ignou-lmt-01-study-materialbooks.html
- *Learning Curve* and *At Right Angles*, periodicals about mathematics and its teaching: http://azimpremjifoundation.org/Foundation_Publications
- Central Board of Secondary Education's books and support material (also including the *Teachers Manual for Formative Assessment – Mathematics (Class IX)*) – select 'CBSE publications', then 'Books and support material': http://cbse.nic.in/welcome.htm

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