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TI-AIE

Visualising, comparing and contrasting: number systems

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What this unit is about

Mathematics has several number systems: natural numbers, whole numbers, integers, rational numbers, irrational numbers and real numbers. Students often think that the distinctions between these number systems are somewhat obscure and trivial. This unit will explore the similarities and differences of these number systems, and how mathematical operations such as addition, multiplication, exponentiation, etc., function within these.

The National Curriculum Framework for Teacher Education (NCFTE, 2009) requires mathematics lessons to be interesting, student-focused and student-participatory, and to build the students' understanding of mathematics. That is not easy to accomplish. This unit aims to support teachers to achieve this by working on compare and contrast tasks and using visualisation. The unit will explain each of these approaches and provide examples of how they can be applied in the classroom. The unit will also discuss how to make small changes to existing classroom practice using a textbook.

What you can learn in this unit

- Some ideas for how to explore and analyse different ways of visualising numbers and number operations, and realise their limitations.
- To recognise the structure and effects of compare and contrast activities, and how visualisation can help your students make sense of mathematics.

This unit links to the teaching requirements of the National Curriculum Framework (NCF, 2005) and NCFTE (2009) outlined in Resource 1.

1 Comparing and contrasting activities

Comparing and contrasting are good activities to make people aware of mathematical properties and its applications. The action of comparing and contrasting forces students to think about the properties of the mathematical objects and to notice what is the same and what is different. While doing so, students make connections they might not normally consider. They are prompted into mathematical thinking processes such as generalising, conjecturing about what stays the same and what can change, and verifying these conjectures. This is an example of the national curriculum requirement of letting students use abstractions to perceive relationships, see structures, reason things out and argue the truth or falsity of statements.

Compare and contrast activities, such as in Activity 1, can help students to:

- consolidate their learning
- remind them of the different purposes of the various number systems
- become aware of the subtle similarities and differences of the systems.

Activity 1 requires your students to think about which statements are always, sometimes or never true. The aim is to become aware of differences between the number systems and to develop a mathematical argument for their reasoning. By doing so, they become more precise in their use of mathematical language and notice the more subtle similarities and differences. In the later questions there is a first step towards visualisation by referring to a number line.

Before attempting to use the activities in this unit with your students, it would be a good idea to complete all (or at least part) of the activities yourself. It would be even better if you could try them out with a colleague, as that will help you when you reflect on the experience. Trying the activities yourself will mean that you get insights into a learner's experiences that can in turn influence your teaching and your experiences as a teacher. When you are ready, use the activities with your students. After the lesson, think about the way that the activity went and the learning that happened. This will help you to develop a more learner-focused teaching environment.

Activity 1: Always, sometimes or never true?

Preparation

The statements can be offered as a list, or can be written on cards and shared out randomly. A fuller list of questions, and an example of a worksheet that can be cut up as cards, can be found in Resources 2 and 3. An overview of different number systems and their properties can be found in Resource 4.

Here are some suggestions for ways of working on this activity:

- You can ask students to work on this on their own, share their ideas with the class or share their ideas with their partners/classmates.
- The statements can be offered for all to be done – but this could become tedious, so ask students to complete only part of them; for example, to do odd/even/prime numbered ones.
- You could also ask students to make their own choice. For example, if they select three that they would like to tackle and two that they would not want to do, you could ask them to do all five of these with the support of their partner(s).
- Letting students make their own choice often empowers stimulating active participation and engagement in the classroom.

The activity

Ask your students which of the following statements are 'always true', 'sometimes true' or 'never true', and why?

1 The sum of two natural numbers is a natural number.
2 The sum of two integers is NOT an integer.
3 The difference of two irrational numbers is an irrational number.
4 The product of two irrational numbers is an irrational number.
5 The quotient of two whole numbers is a whole number.
6 The quotient of two real numbers is a real number.
7 There are an infinite number of pairs of whole numbers whose sum is 0.
8 There exists a pair of whole numbers whose product is 1.
9 The product of two real numbers is not a non-repeating, non-terminating decimal.

10 The exact location of a natural number cannot be determined on a number line.
11 The difference of two integers is to the left of each of the two integers on a number line.
12 There are finite natural numbers between any two real numbers.
13 The number a2 is a natural number if a is a natural number.
14 The number ab is greater than both a and b.

Video: Talk for learning

You may also want to have a look at the key resource 'Talk for learning'.

Case Study 1: Mrs Aparajeeta reflects on using Activity 1

This is the account of a teacher who tried Activity 1 with his secondary students.

I was a bit scared about trying out this activity for several reasons:

- the unusual structure of the activity
- requiring the students to develop their own reasoning
- students talking to each other when this is not usually allowed in my classroom
- my fear that they might not remember anything about the different number systems at all!

Before we started on this activity, I asked six students for a number between one and 14. These six numbers became the questions they had to do. I was very uncomfortable about allowing the students to talk to each other as I thought it would become very noisy with 79 students in the classroom; I would not be able to control what was discussed and it could become chaotic. What if the principal happened to walk past at such chaotic moment! On the other hand I do believe that talking helps learning, because when you talk, you have to organise your thinking – otherwise the listener will not understand you.

So I pushed myself to be brave and started the lesson: I asked the students to work the answers out for themselves in silence and to write down their reasoning. Then I asked them to discuss their thoughts and reasoning with their classmates, in pairs. But they had to do it quietly, so the others could not overhear what they were saying and steal their ideas! I told them that after some time I would select at random someone to explain what the thinking of the pair had been. Other students could then comment in a constructive way, just like we sometimes do in whole-class discussions. To address my fear that they would not be able to remember anything, I wrote the page numbers of their textbook where they could find information about the number systems on the blackboard – and they were free to refer to these.

To my surprise it worked really well! The classroom did not end in chaos. There was noise from the talking in pairs, but it was not loud: they seemed to like the sense of competition and secrecy. I listened into the conversations by walking around and by standing quietly against the wall, observing. What I liked very much was that the conversations were about mathematics, and that disagreements led to pointed discussions about the mathematical properties, flicking through the textbook to check their reasoning and using a number line to illustrate what they were saying. The whole class discussion was less lively than I had anticipated because there was little disagreement by then. However, I was impressed with the quality of the mathematical language used. The class discussion thus became a consolidation activity. Their responses showed me they seemed to really understand the differences in properties, and the consequences of these differences when doing operations in the different number systems.

What I found very hard in this lesson was my changed role as teacher: I was no longer standing at the front explaining and telling them what to do. Actually, I talked very little, and it was difficult not to interfere in the discussion and show them how to do it. However, it meant that this was now their learning and their thinking, which was a powerful experience.

Would I change anything next time? Yes, I think I would try to let them choose their own questions, although I am not sure how to lead the whole-class discussion then as they might have chosen all different ones. Perhaps I could ask them to prepare small presentations on their findings. I think I will also prepare the questions in ‘card’ format and use these when we have five or ten minutes left over at the end of a lesson. I think that could offer the repetition needed to keep all this knowledge in the mind, without it meaning doing more exercises from the book.

Reflecting on your teaching

When you do such an activity with your class, reflect afterwards about what went well and what went less well. Consider the questions that led to the students being interested and being able to progress and where you needed to clarify. Such reflection always helps with finding a ‘script’ that helps you engage the students to find mathematics interesting and enjoyable. If they do not understand and cannot do something, they are less likely to become involved. Use this reflective exercise every time you undertake the activities, noting, as Mrs Aparajeeta did, some quite small things that made a difference.