4 Examples in mechanism design

In this final section of Unit 1, I want to pull together the main ideas and techniques which you have studied so far, by examining some simple design problems involving displacements in mechanisms. No new principles or procedures will be presented, the objective being to give you extra insight and practice in the important location procedure which you have used already.

4.1 Designing mechanisms to produce given displacements

The two examples which follow ask for a design of a linkage mechanism which will produce a particular output motion from a given input motion. As in any design problem, many possible solutions could be suggested. Here we shall be looking for the simplest possible linkage mechanism which will perform the required task. The output specifications will be given in terms of displacements, although in actual design situations other factors such as speeds and accelerations may govern the choice of mechanism. Consideration of these other factors is deferred until later Units.

Example 8

A point on a rigid body is to be propelled back and forth (i.e. to reciprocate) along a straight path between two positions Q and R, located 0.1 m apart. The mechanism for performing this task is to be driven by a rotating shaft, located in fixed bearings at a point P as shown in Figure 84, where the distance PQ = 0.15 m.

Design the simplest linkage mechanism which will perform this task.

Solution

The simplest mechanism for converting an input rotation into an output translation is the slider-crank linkage. A crank PS is fixed to the rotating shaft (Figure 85), a coupler ST is connected to the rigid body T, and a fixed guide link forms a sliding pair with the rigid body to complete the chain.

The design problem can now be restated as follows: What are the lengths of the crank (PS) and the coupler (ST) such that for one revolution of the crank, the slider T will move back and forth between positions Q and R?

Considering first the condition for the slider to attain the extreme right-hand position R, it is evident that PS and ST must be extended in one line in order to bring T to this position (Figure 86), that is

\[ PS + ST = PQ + QR = 0.25 \text{ m} \]

Similarly for the extreme left-hand position, PS and ST will be folded over one another to bring T to position Q (Figure 87), that is

\[ ST - PS = PQ = 0.15 \text{ m} \]

We now have two equations involving PS and ST and can solve for both of these lengths

\[ ST + PS = 0.25 \]

\[ ST - PS = 0.15 \]
Adding these two equations, we get

$$2ST = 0.40$$

so

$$ST = 0.20 \text{ m}$$

Substituting this into the first equation gives

$$0.20 + PS = 0.25$$

or

$$PS = 0.05 \text{ m}$$

so that the simple linkage mechanism shown in Figure 85 with $PS = 0.05 \text{ m}$ and $ST = 0.20 \text{ m}$, will give the required displacement of the rigid body.

*Note:* You may have reasoned straight away that the crank radius $PS = \frac{1}{2} \text{QR} = 0.05 \text{ m}.$

**Example 9**

A mechanism is required which will rotate a link $DE$ of length $0.3 \text{ m}$ back and forth (oscillate) through an angle of $94^\circ$ about a fixed axis at $D$. A rotating shaft, turning in fixed bearings at a point $B$ shown in Figure 88 is to be used for driving the mechanism, where $BD = 0.55 \text{ m}$. In its extreme left-hand position the link $DE$ makes an angle of $45^\circ$ with respect to the line joining $B$ and $D$.

Design the simplest linkage mechanism which will perform this task.

![Figure 88](image)

**Solution**

The simplest linkage mechanism will be a four-bar linkage with a crank $BC$, a coupler $CE$ which transmits the motion to the follower $DE$ (Figure 89).

The design problem reduces to finding the lengths of the crank ($BC$) and coupler ($CE$) such that for one revolution of the crank, the follower will oscillate through the given angular displacement.

As in the previous example, $BC$ and $CE$ must be extended in line when $DE$ is in the extreme position to the right (Figure 90), that is

$$BC + CE = BE_R$$

Similarly for the extreme left-hand position, $BC$ and $CE$ will be folded over one another (Figure 91), that is

$$CE - BC = BE_L$$
If the line diagrams of Figure 90 and 91 are drawn to scale, as shown in Figure 92, the points $E_k$ and $E_L$ can be located.

![Figure 92 Location of $E_k$ and $E_L$](image)

From Figure 92

- $BE_k = (160 \times 5) = 800 \text{ mm} = 0.8 \text{ m}$
- $BE_L = (80 \times 5) = 400 \text{ mm} = 0.4 \text{ m}$

After substituting these values, we now have two equations involving $BC$ and $CE$ and can solve for both of these links.

1. $CE + BC = 0.8$
2. $CE - BC = 0.4$

Adding these two equations

$2CE = 1.2$

so

$CE = 0.6 \text{ m}$

Substituting this,

$0.6 + BC = 0.8$

so

$BC = 0.2 \text{ m}$

The required mechanism shown in Figure 89 with crank $BC = 0.2 \text{ m}$ and a coupler $CE = 0.6 \text{ m}$ will produce the specified angular displacement of link $DE$.

**SAQ 18**

Design a slider–crank mechanism to drive a slider $R$ back and forth along the straight line $ST$, 80 mm long (Figure 93). The mechanism is to be driven by a shaft located in a position along the line $SQ$ such that the ratio of angular displacements of the crank for forward and return motions of the slider (known as the crank time ratio) is $195^\circ/165^\circ$.
Summary of Unit 1

Let us look back at what progress we have made in this Unit towards understanding some of the basic subject matter of engineering mechanics of solids. Right from the start we focused our attention on the specific study of machines and the motion of their component parts. We began with an initial requirement of finding a way of representing the parts of a machine in a drawing such that their motion could be readily identified and the manner in which they were connected could be made clear. The rigid-body concept provided a basis for this representation. By treating machine components as rigid bodies, they could be represented simply by points and straight lines for the purpose of studying their motion.

Next we considered ways of measuring the motion in terms of the changes of position or displacements of the points and lines on the bodies. Changes of position of rigid bodies in plane motion can be measured in terms of the linear displacement of a point on the body and/or the angular displacement of a line on the body.

Having considered the motion of single isolated objects, we then looked at the motion of objects connected in assemblies. A particular class of connected objects was singled out for special attention, that is, plane-linkage mechanisms in which the objects in the assembly (or kinematic chain) were treated as rigid bodies (links) in plane motion.

Each connection between two links (a pair) allows constrained rotation or sliding movement of one link relative to the other. One method of representing such kinematic chains uses a simple pictorial convention, that is, the line diagram. The line diagram can inform us of the number and kind of links and connections in a kinematic chain. The term mechanism is restricted to a special class of kinematic chain in which there is a definite relationship between the motions of every link in the chain. The basic unit of all such mechanisms is the four-link continuous closed chain, of which the four-bar linkage and slider-crank linkage are the most common.

Finally, having (a) recognized the component parts of a machine and the manner in which their motion is constrained, and (b) drawn a line diagram representation of the mechanism, the next step is (c) to locate the mechanism at any instant during its motion. This involves the use of the geometric properties of the triangle in graphical construction or calculation by the sine rule or the cosine rule. We are now at a stage where a more detailed analysis of the motion of machines can begin.
# Glossary

<table>
<thead>
<tr>
<th>Term</th>
<th>Text reference</th>
<th>Explanation</th>
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</thead>
<tbody>
<tr>
<td>Angular displacement</td>
<td>1.3</td>
<td>The angle between two positions of a rotating line</td>
</tr>
<tr>
<td>Assembly</td>
<td>2</td>
<td>Group of connected rigid bodies</td>
</tr>
<tr>
<td>Baseline</td>
<td>3.2</td>
<td>Line from which measurements are made in the location procedure</td>
</tr>
<tr>
<td>Binary link</td>
<td>2.2</td>
<td>Forms pairs with two other links</td>
</tr>
<tr>
<td>Combined motion</td>
<td>1.2</td>
<td>Form of plane motion considered as simultaneous translation and rotation</td>
</tr>
<tr>
<td>Compound link</td>
<td>2.2</td>
<td>Forms pairs with three or more links</td>
</tr>
<tr>
<td>Coupler</td>
<td>2.3</td>
<td>Intermediate link in four-bar linkage and slider–crank linkage</td>
</tr>
<tr>
<td>Crank</td>
<td>2.3</td>
<td>Link which provides rotation about a fixed axis, often a full turn</td>
</tr>
<tr>
<td>Curvilinear motion</td>
<td>1.2</td>
<td>When a body moves on a curved path without rotation</td>
</tr>
<tr>
<td>Degrees of freedom</td>
<td>2.2, 3.4</td>
<td>Number of independent quantities which are necessary to fix the location of an object</td>
</tr>
<tr>
<td>Displacement</td>
<td>1.3</td>
<td>Change of position of an object</td>
</tr>
<tr>
<td>Fixed-axis rotation</td>
<td>1.2</td>
<td>When paths of all points on a rigid body are circular arcs about a fixed axis</td>
</tr>
<tr>
<td>Fixed link</td>
<td>2.1</td>
<td>Link which is considered to be held stationary, e.g. engine casing, machine body, ground</td>
</tr>
<tr>
<td>Four-bar linkage</td>
<td>2.3</td>
<td>Consists of four binary links all forming rotating pairs</td>
</tr>
<tr>
<td>Frame</td>
<td>2.3</td>
<td>The fixed link of four-bar linkage and slider–crank linkage</td>
</tr>
<tr>
<td>Geometric similarity</td>
<td>3.5</td>
<td>Process of scaling length dimensions between object and drawing to ensure similarity of proportions.</td>
</tr>
<tr>
<td>Inclination</td>
<td>3.4</td>
<td>Angle of a line measured anticlockwise from the baseline</td>
</tr>
<tr>
<td>Inversion of mechanisms</td>
<td>2.4</td>
<td>Process of substituting a ‘moving’ link for the fixed link of a mechanism</td>
</tr>
<tr>
<td>Kinematic chain</td>
<td>2.1</td>
<td>Series of links connected allowing motion of links</td>
</tr>
<tr>
<td>Line diagram</td>
<td>2.2</td>
<td>Representation of the links and connections in a kinematic chain</td>
</tr>
<tr>
<td>Linear displacement</td>
<td>1.3</td>
<td>Straight-line distance between any two positions of a point</td>
</tr>
<tr>
<td>Link</td>
<td>2.1</td>
<td>A rigid body which is connected to other rigid bodies to transmit motion or to guide motion</td>
</tr>
<tr>
<td>Location procedure</td>
<td>3.2, 3.3</td>
<td>Procedure for locating a point given a baseline and two further quantities (side lengths and/or angles of a triangle)</td>
</tr>
<tr>
<td>Mechanism</td>
<td>2.3</td>
<td>A kinematic chain by which an input motion can be transmitted to produce a specific output motion</td>
</tr>
<tr>
<td>Oscillating cylinder</td>
<td>2.4</td>
<td>Inversion of slider–crank linkage</td>
</tr>
<tr>
<td>Oscillating motion</td>
<td>1.2</td>
<td>Fixed-axis rotation back and forth through a given angular displacement</td>
</tr>
<tr>
<td>Pair</td>
<td>2.1</td>
<td>Two links connected so that there is relative motion between them</td>
</tr>
<tr>
<td>Path</td>
<td>1.2</td>
<td>Line traced out by a moving point</td>
</tr>
<tr>
<td>Path length</td>
<td>1.3</td>
<td>Distance measured along the path between two positions of a point</td>
</tr>
<tr>
<td>Plane-linkage mechanism</td>
<td>2.2</td>
<td>Mechanism in which all links are in plane motion (or stationary) and all pairs are either rotating or sliding pairs</td>
</tr>
<tr>
<td>Plane motion</td>
<td>1.2</td>
<td>When the paths of all moving points on a rigid body lie in the same or parallel planes</td>
</tr>
<tr>
<td>Quick-return mechanism</td>
<td>2.4</td>
<td>Inversion of slider–crank linkage</td>
</tr>
<tr>
<td>Term</td>
<td>Text reference</td>
<td>Explanation</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>----------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Reciprocating motion</td>
<td>1.2</td>
<td>Rectilinear translation back and forth through a given linear displacement</td>
</tr>
<tr>
<td>Rectilinear translation</td>
<td>1.2</td>
<td>When a body moves in a straight-line path without rotation</td>
</tr>
<tr>
<td>Rigid body</td>
<td>1.1</td>
<td>Hypothetical solid object which does not change its size or shape during motion or when resisting loads</td>
</tr>
<tr>
<td>Rocker</td>
<td>2.3</td>
<td>Rotating link in four-bar linkage, usually the output</td>
</tr>
<tr>
<td>Rotating pair</td>
<td>2.2</td>
<td>Pairs of links in which the relative motion is constrained to rotation</td>
</tr>
<tr>
<td>Rotation</td>
<td>1.2</td>
<td>General term for a body which turns during its motion, either fixed-axis rotation or combined motion</td>
</tr>
<tr>
<td>Slider</td>
<td>2.3</td>
<td>Sliding link in slider–crank linkage</td>
</tr>
<tr>
<td>Slider–crank linkage</td>
<td>2.3</td>
<td>Consists of four binary links forming three rotating pairs and one sliding pair</td>
</tr>
<tr>
<td>Sliding pair</td>
<td>2.2</td>
<td>Pair of links in which one link slides over the ends of its travel</td>
</tr>
<tr>
<td>Time ratio</td>
<td>4.1</td>
<td>Ratio of angular displacements of input crank for forward and return motions of output rocker or slider</td>
</tr>
<tr>
<td>Total angle turned through</td>
<td>1.3</td>
<td>Total angle traversed by a rotating line</td>
</tr>
<tr>
<td>Total distance</td>
<td>1.3</td>
<td>Sum of the path lengths covered during the travel of a moving point</td>
</tr>
<tr>
<td>Translation</td>
<td>1.2</td>
<td>General term for the motion of a body which does not turn (see rectilinear and curvilinear translation)</td>
</tr>
</tbody>
</table>
Answers to Self-Assessment Questions

**Important note:** When checking answers to SAQs that involve drawing, do not expect your answers to be exactly the same as mine. Allow some latitude for slight differences in eye alignment. In some cases measurements given in the text may vary slightly from the drawings as reproduced.

**SAQ 1**

From Figure 4 in the text, the distance PR = 60 mm. This represents an actual distance between the centre of the piston and the centre of the crankshaft of

\[ 5 \times 60 = 300 \text{ mm} = 0.30 \text{ m} \]

according to the scale factor for this particular crank position.

**SAQ 2**

(a) Relative to the ground:
Bicycle back wheel – combined motion
Pedal when horizontal – curvilinear translation
Links on chain – combined motion

(b) Relative to the fixed support:
Pulley A – fixed-axis rotation
Pulley B – combined motion
Block C – rectilinear translation

(c) Relative to the ground:
Component A – fixed-axis rotation
Component B – curvilinear translation (it slides in and out of the collar and also rises and falls in lifting and lowering the load)
Component C – combined motion (it rotates about moving axes)

**SAQ 3**

Answers (ii) and (iii) are incorrect.

**SAQ 4**

Figure 94 shows one circuit for the bus.

![Figure 94](image)

Distance travelled = OSTVO

\[ = OS + STV + VO \]

\[ = 0.75 + \left( 0.75 \times \frac{3\pi}{2} \right) + 0.75 \]

\[ = 5.03 \text{ km} \]

Linear displacement = 0, since the end position is at the same point as the starting position.

After 6\(\frac{1}{2}\) circuits, the bus breaks down at point T:

(a) total distance travelled

\[ = 6\frac{1}{2} \times 5.03 = 32.7 \text{ km} \]

(b) path length = OST

\[ = OS + ST \]

\[ = 0.75 + \left( 0.75 \times \frac{3\pi}{4} \right) \]

\[ = 2.52 \text{ km} \]

(c) linear displacement = OT = 0.75 km.

The repair man, interested in finding as direct a route as possible to the breakdown, would be looking at the linear displacement as a guide. The beleaguered passenger would probably be contemplating catching another bus at the depot and would be interested in the path length back to his destination. The clerk would be filing his breakdown statistics on the basis of total distance travelled.

**SAQ 5**

The path length of any point on the rim of the wheel in moving between the two positions is 0.8 m. Then the angular displacement of the wheel is given by

\[ \theta = \frac{p}{r} = \frac{0.8}{0.75} = 1.07 \text{ radians} = 61^\circ \]

Figure 95 shows the scaled drawing of the positions of line OV, from which the linear displacement of V is

\[ s = VV' = 41 \text{ mm} \]

or 0.41 m scaled up.
SAQ 6

The penknife would be represented by the diagram shown in Figure 96. Notice that the number of individual links is 8, including the handle. The number of rotating pairs is 4 at one end and 3 at the other end. Also notice that the individual shapes of the components are not shown or needed to indicate the links and pair connections.

Figure 96 Penknife

SAQ 7

The 'exploded' diagrams for the kinematic chains of SAQ 7 are shown in Figure 97. The table can then be completed as follows:

<table>
<thead>
<tr>
<th>Chain</th>
<th>Chain</th>
<th>Chain</th>
<th>Chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>Number of links</td>
<td>4</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Number of pairs</td>
<td>4</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Number of binary links with two rotating pair connections</td>
<td>2</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Number of binary links with one rotating and one sliding pair connection</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Number of compound links</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 97 SAQ 7, referring to Figure 43
SAQ 8
Figure 98(a) shows the line diagram of the quick-return mechanism and Figure 98(b) the exploded line diagram of the links. There are 6 links and 7 pairs.

SAQ 9
Figure 99 shows a translation–translation mechanism. The centre point C of the rotating link traces a circle with all other points on the link (except the ends) tracing ellipses.

SAQ 10
With link 4 fixed, the line diagram for this inversion looks like Figure 100. This mechanism finds use as a hand pump, in which link 1 is vertical and link 2 is extended to form the pump handle.
**SAQ 11**
The graphical solution is shown in Figure 101. Use the Case 3 procedure – you are given two sides and one angle (not included). There is only one solution shown as \( A_1 \), since the other intersection at \( A_2 \) does not give an angle of 60° for \( \beta \).

Angle \( A_1 \text{BC} = 74° \)

![Figure 101](image)

**SAQ 12**
The graphical solution is shown in Figure 102.

From the baseline SP, point R is located by Case 4 procedure – three sides given. There are two solutions, but one of the solutions would involve the bottom of the door initially moving inwards, which would not be physically possible. (The door would have to pass through the pivot arm!) Once point R is located, the distance TQ can be measured; that is, the door projects a distance \( TQ = 4.5 \text{ mm} \), which scales to 0.9 m.

![Figure 102](image)

**SAQ 13**
(a) \( \sin 108° = 0.951; \sin 72° = 0.951 \)
\( \cos 22° = 0.927; \cos 158° = -0.927 \)
\( \tan 34° = 0.675; \tan 89° = 57.3 \)
\( \tan 130° = -1.19 \)

(b) \( \sin^{-1} (0.85) = 58° \) and 122°

**SAQ 14**
Referring back to SAQ 11, the information given is

\( \text{BC} = a = 50 \text{ mm} \)
\( \text{AB} = c = 60 \text{ mm} \)
\( \text{Angle ACB} = \gamma = 60° \) (see Figure 103).

Hence using the Case 3 procedure:

(i) Use sine rule to find \( \alpha \):
\[
\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}
\]
\[
\sin \alpha = \frac{a \cdot \sin \gamma}{c} = \frac{50 \times \sin 60°}{60} = 0.72
\]

Hence \( \alpha = \sin^{-1} (0.72) = 46° \) or 134°

However, if \( \alpha = 134° \), \( \alpha + \gamma \) would be greater than 180°, which is invalid. Therefore \( \alpha = 46° \).

(ii) Calculate \( \beta \):
\[
\beta = 180° - (\alpha + \gamma) = 180° - (46° + 60°) = 74°
\]

which checks with the graphical solution of SAQ 11.

![Figure 103](image)
**SAQ 15**
The information given is
(a) PT = 330 mm (110 mm scaled)
(b) Angle QPT = 100°
(c) PQ = 60 mm (20 mm scaled)
(d) QS = 360 mm (12 mm scaled)
(e) ST = 30 mm (10 mm scaled)

(a), (b) and (c) are sufficient to establish a baseline QT shown in Figure 104.

Then point S can be located by the Case 4 procedure, with three sides given. There are two solutions, but only one is physically feasible.

(i) The inclination of the coupler QS = 174°.

(ii) Angle QST = 58°

To solve the problem by trigonometry involves two stages: first to establish the baseline QT, second to locate point S.

The baseline QT can be calculated from the cosine rule. Considering triangle PQT in Figure 104,

\[ QT^2 = PQ^2 + PT^2 - 2 \times PQ \times PT \times \cos QPT \]

\[ = 0.06^2 + 0.33^2 - 2(0.06)(0.33) \cos 100° \]

\[ = 0.1194 \]

or

\[ QT = 0.346 \text{ m} \]

We can now locate point S by the Case 4 procedure.

Considering triangle QST, the law of cosines can be used to find any angle. In this case, we wish to find angle QST. Hence

\[ \cos QST = \frac{(QS^2 + ST^2) - QT^2}{2 \times QS \times ST} \]

\[ = \frac{(0.36^2 + 0.03^2) - 0.1194}{2 \times 0.36 \times 0.03} \]

\[ = 0.514 \]

Hence angle QST = \cos^{-1} 0.514 = 59°, which checks with the approximate graphical solution.

**SAQ 16**
The pantograph linkage is shown in Figure 105. Since CDEF is a parallelogram, CD = 75 mm and DF = 60 mm. Also the triangles BCE and BDG are geometrically similar since they have equal angles. Then the scaling ratio

\[ \frac{BE}{BG} = \frac{CE}{DG} = \frac{BC}{BD} = \frac{125 - 75}{125} = \frac{50}{125} = \frac{2}{5} \]

\[ \text{Figure 105} \]

Hence

\[ DG = \frac{5}{2} \times CE = \frac{5}{2} \times 60 = 150 \text{ mm} \]

**SAQ 17**
Figure 106 shows the block after the wires have stretched, by amounts CC' and GG'.

\[ \text{Figure 106} \]

Using the small-angle approximation

\[ CC' = 1.0 \times \theta \text{ rad} \]

\[ GG' = 0.5 \times \theta \text{ rad} \]

Knowing CC' = 0.005 m, these two equations can be solved for \( \theta \) and GG', that is

\[ \theta = \frac{0.005}{1.0} = 0.005 \text{ radians} = 0.29° \]

and

\[ GG' = 0.5 \times 0.005 = 0.0025 \text{ m} \]

(a) The angle through which the block has deflected is 0.29°.

(b) The wire FG has stretched by 2.5 mm, i.e. half as much as BC.

\[ \text{Figure 106} \]

scale 1 mm : 3 mm
SAQ 18

Figure 107 shows the slider–crank mechanism which is to be designed. The problem can be restated as follows: What are the lengths of the crank OP and coupler PR such that for one revolution of the crank the slider will move back and forth between S and T?

The slider will be in its extreme right-hand position at T when OP and PR are extended in line, as shown in Figure 108.

OP + PR = OT

The slider will be in its extreme left-hand position at S when OP and PR are folded over one another as shown in Figure 109.

PR - OP = OS

In order to achieve the required time ratio, as the crank moves between the positions shown in Figures 108 and 109, its angular displacements are 195° and 165° (Figure 110).

Then

angle SOT = 15°

and

angle OTS = 180° - (143° + 15°) = 22°

Figures 108 and 109 can now be drawn to scale in Figure 111 which locates the crankshaft axis O.

From Figure 111

OT = 194 mm and OS = 125 mm

Substituting these values, we can solve for the link sizes OP and PR

PR + OP = 194
PR - OP = 125

Adding these equations gives

2PR = 319

so

PR = 159.5 mm

Substituting into the first of the equations,

159.5 + OP = 194

or

OP = 34.5 mm.

The mechanism shown in Figure 107 with OP = 34.5 mm and PR = 159.5 mm will produce the required linear displacement of the slider R with a crank time ratio of 195°/165°.
Figure III

O

S

125 mm

194 mm

80 mm

143°

15°

22°